String Phenomenology 2024

Decay of Kaluza-Klein Vacuum via Singular Instanton

Based on the study with Yutaka Ookouchi and Ryota Sato (arXiv: 2404.13917[hep-th]).

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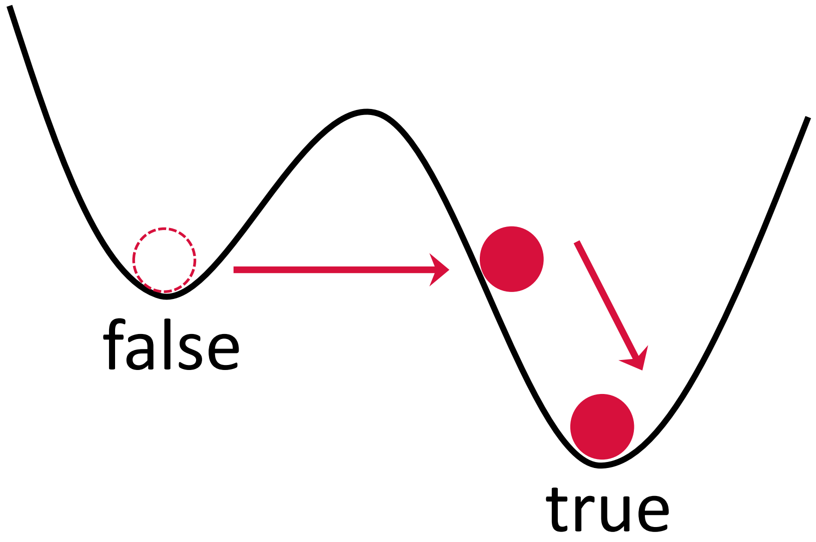
- Absence of guiding principle for compactification implies a huge number of (meta)stable states.

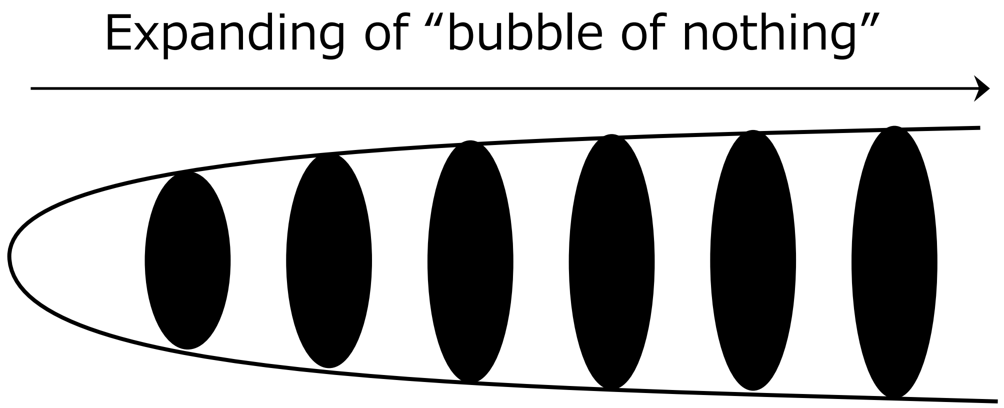
e.g. 1 : metastable (anti) de Sitter



✓ Potential barriar.

✓ Stringy vacua are often metastable (can transit to a true vacuum with a lower vacuum energy).

e.g. 2 : bubble of nothing



[E. Witten '81]

✓ The unique decay mode to compactified spacetime.

✓ Expanding bubble with no degree of freedom.

- We considered a decay of Kaluza-Klein vacuum mediated by a singular instanton.
- We have evaluated the on-shell contribution of the singularity and find that it reduces a total bounce action.

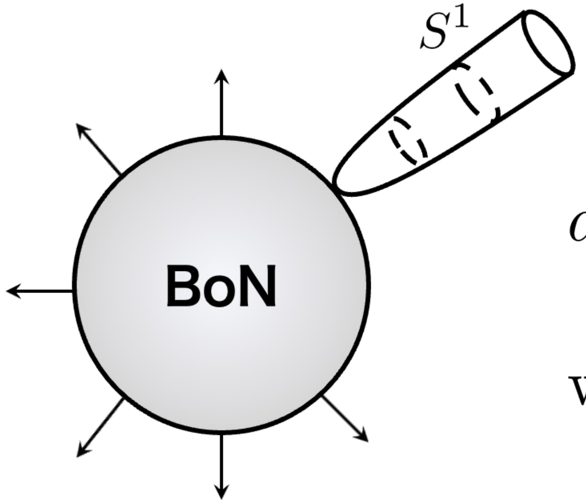
Take-home message

Decay via singular instanton may be more dominant channel in the context of BoN.

Contents

[E. Witten, Nucl.Phys.B 195 (1982) 481-492]

- "Bubble of Nothing" (often abbreviated as BoN) is a catastrophic decay phenomenon particular to compactified spacetime.
- Kaluza-Klein vacuum ($M_4 \times S^1$) can decay as the BoN expands at the speed of light.



$$ds_5^2 = \left(1 - \left(\frac{R}{r}\right)^2\right) d\phi^2 + \left(1 - \left(\frac{R}{r}\right)^2\right)^{-1} dr^2 + r^2 ds_{dS_3}^2,$$

where $ds_{dS_3}^2 = -d\tau^2 + \cosh^2 \tau ds_{S^2}^2$

[E. Witten, Nucl.Phys.B 195 (1982) 481-492]

- BoN instantons take the form of Euclidean black hole solutions.

$$ds_E^2 = \left(1 - \left(\frac{\sqrt{\alpha}}{r}\right)^2\right) d\phi^2 + \left(1 - \left(\frac{\sqrt{\alpha}}{r}\right)^2\right)^{-1} dr^2 + r^2 ds_{S^3}^2$$

- Euclidean black hole solutions have conical singularity at the position of event horizon.

  - Fix the periodicity of the imaginary time to appropriate value.

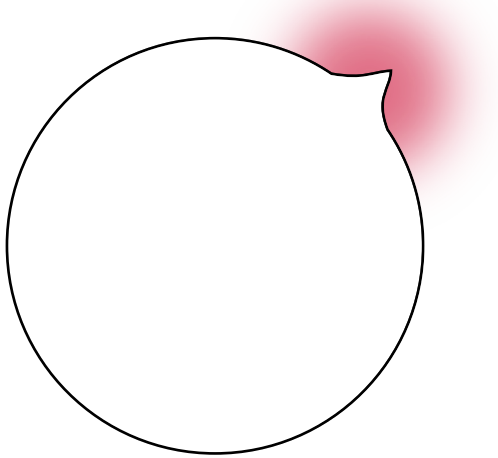
$$\alpha = R_{KK}^2$$ (Smoothness condition)

- if the contribution to the on-shell action from the singularity is finite, the condition may be relaxed.

Contents

- The study of singular instantons was initiated by Hawking and Turok and then explored mainly in the context of open universe.

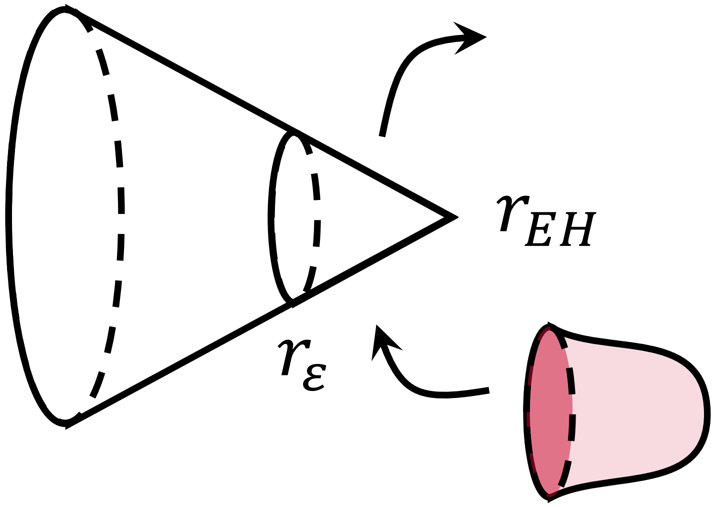
[S.W. Hawking and N. Turok, Phys. Lett. B 425 (1998) 25 [hep-th/9802030]]
[N. Turok and S.W. Hawking, Phys. Lett. B 432 (1998) 271 [hep-th/9803156]]



dS instanton + singularity

- Gregory, Moss and Withers have refined the geometrical technology for more precise treatment of singularities.

[D. V. Fursaev, A. N. Solodukhin, Phys.Rev.D 52 (1995) 2133-2143 [arXiv:9501127[hep-th]]]
[R. Gregory, I. G. Moss and B. Withers, JHEP 03 (2014) 081 [arXiv:1401.0017[hep-th]]]



$$I_{\mathcal{B}} = -\frac{1}{16\pi G_n}\int_{\mathcal{B}} d^n x\sqrt{g}R - \frac{1}{8\pi G_n}\int_{\partial\mathcal{B}} d^{n-1}y\sqrt{h}\,(K - K_0)$$

$$= -\frac{\mathcal{A}_{EH}}{4G_n}$$

Entropy-like formula

Smooth cap

Conical deficit

Introducing $\rho \equiv r\sqrt{f(r)}$, we can rewrite the instanton solution as

$$ds^2 = F(\rho)^2 d\chi^2 + d\rho^2 + r(\rho)^2 d\Omega_3^2, \quad F(\rho)^2 \equiv \left(1 - \left(\frac{\sqrt{\alpha}}{r(\rho)}\right)^2\right) R^2$$

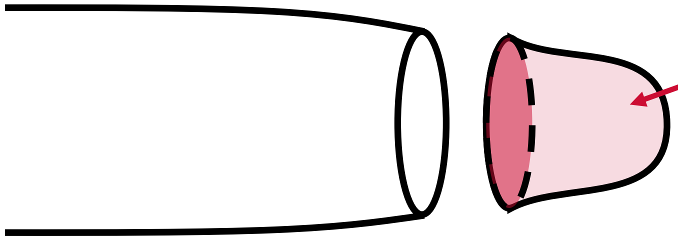
2π periodic

$$ds^2 \simeq d\rho^2 + \rho^2 d(F'(0)\chi)^2 + r(0)^2 d\Omega_3^2$$ (near the singularity)

Since $F'(0) \neq 1$ in general, there would be a deficit angle defined as

$$2\pi\delta = 2\pi\left(1 - F'(0)\right) = 2\pi\left(1 - \frac{R}{\sqrt{\alpha}}\right)$$

[D. V. Fursaev, A. N. Solodukhin, Phys.Rev.D 52 (1995) 2133-2143 [arXiv:9501127[hep-th]]]
[R. Gregory, I. G. Moss and B. Withers, JHEP 03 (2014) 081 [arXiv:1401.0017[hep-th]]]



$$ds^2 = \widetilde{F}(\rho)^2 d\chi^2 + d\rho^2 + r(\rho)^2 d\Omega_3^2$$

$\widetilde{F}'(0) = 1$, but $\delta \simeq 1 - \widetilde{F}'(\varepsilon)$

smooth

$$-\frac{1}{16\pi G_n}\int_{\mathcal{B}} \mathcal{R} = -\frac{\mathcal{A}}{4G_n}\delta \ , \quad -\frac{1}{8\pi G_n}\oint_{\partial\mathcal{B}} (\mathcal{K} - \mathcal{K}_0) = -\frac{\mathcal{A}}{4G_n}(1-\delta)$$

$$I_{\mathcal{B}} = -\frac{\mathcal{A}}{4G_n}\delta - \frac{\mathcal{A}}{4G_n}(1-\delta) = -\frac{\mathcal{A}}{4G_n}$$

cancel each other out

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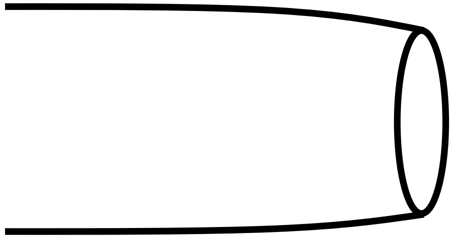
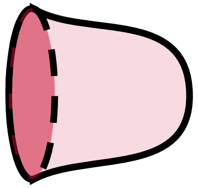
Bounce action

Split the manifold into two and calculate the Euclidean action for each.



$$I_{\mathcal{M}-\mathcal{B}} = -\frac{1}{8\pi G_n}\int d^{n-1}y\sqrt{h}\,(K-K_0)\Big|_{r_\varepsilon, r_\infty} = \frac{3\pi\alpha}{8G_4}$$

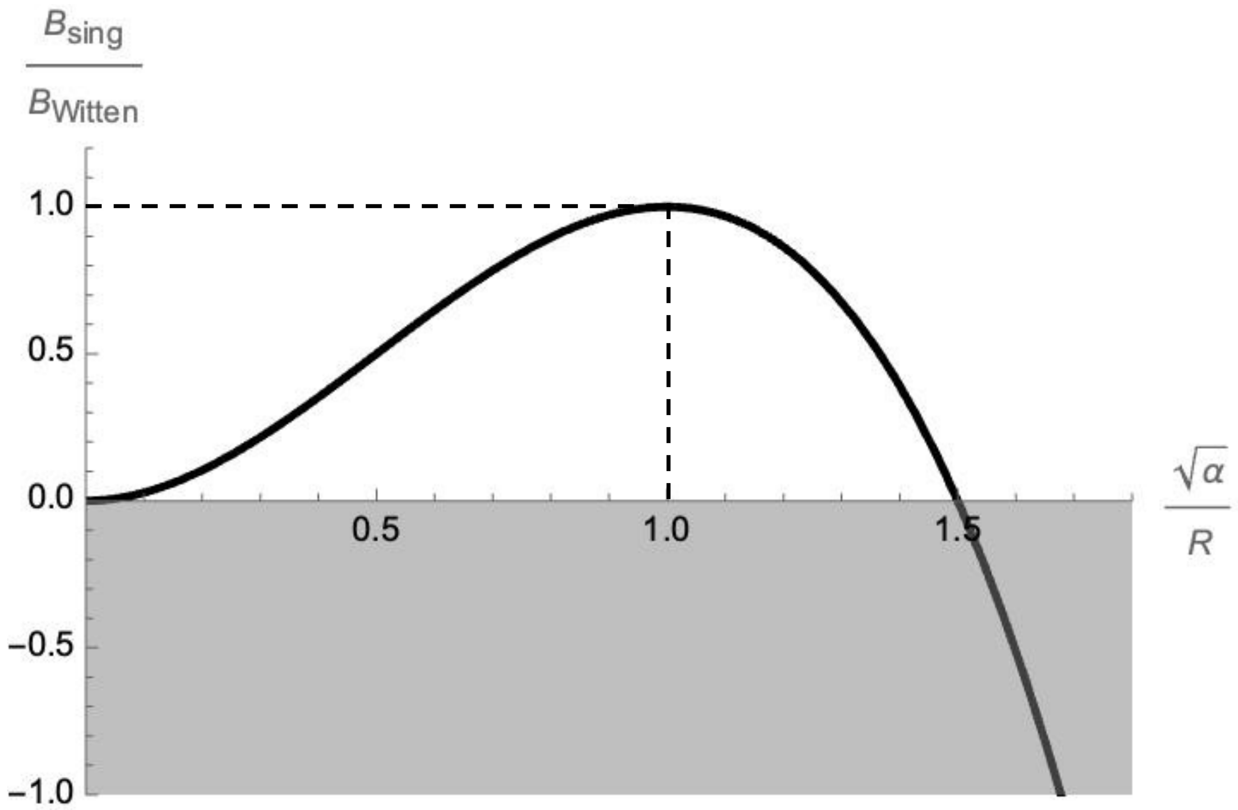


$$I_{\mathcal{B}} = -\frac{2\pi^2\alpha^{3/2}}{4G_5} = -\frac{\pi\alpha^{3/2}}{4G_4 R}$$

$$B = I_{\mathcal{M}-\mathcal{B}} + I_{\mathcal{B}} = \frac{\pi R^2}{8G_4}\left(\frac{3\alpha}{R^2} - \frac{2\alpha^{3/2}}{R^3}\right)$$ (Bounce action)

✓ Conical singularities in Euclidean solutions play an important role as a catalyst which reduce bounce actions.

✓ Our semiclassical analysis is unreliable in the shaded region.



Perhaps we should pay more attention to “singular” BoN.

Contents

Thermodynamical interpretation

We can reproduce the bounce action with thermodynamic functions.

ADM energy
$$E = -\frac{1}{8\pi G_5}\oint_{S_{\phi r}} (k - k_0)\sqrt{\sigma} d^3\theta = \frac{3\pi\alpha}{8G_5}$$

Entropy
$$S = \frac{2\pi^2\alpha^{3/2}}{4G_5}$$

$$B = \frac{W}{T} = \frac{E - TS}{T} = \frac{3\pi\alpha}{8G_4} - \frac{\pi\alpha^{3/2}}{4RG_4}$$

Let us consider shifting α slightly from R^2 up to the second order.

$$\Delta E = \frac{3}{16 G_4 R} \Delta\alpha \ , \qquad \Delta S = \frac{3\pi}{8 G_4} \Delta\alpha + \frac{3\pi}{32 G_4 R^2} \Delta\alpha^2$$

$$B \simeq B_{\text{witten}} + \frac{3\pi}{8 G_4} \Delta\alpha - \frac{3\pi}{8 G_4} \Delta\alpha - \frac{3\pi}{32 G_4 R^2} \Delta\alpha^2$$

$$= B_{\text{witten}} - \frac{3\pi}{32 G_4 R^2} \Delta\alpha^2$$

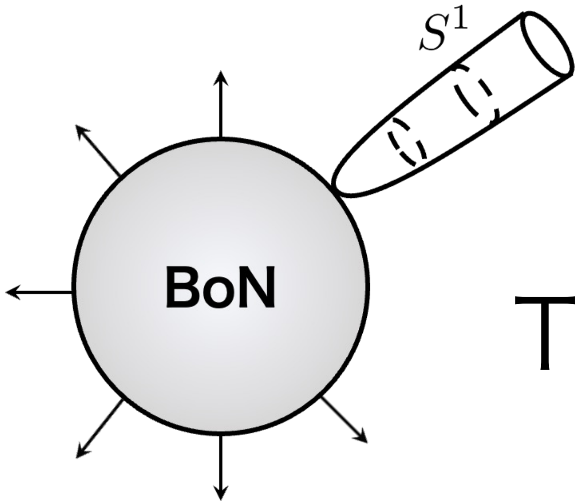
Negative contribution

Contents

Summary and future work

- Bubble of nothing is a catastrophic decay phenomenon which "nothing" overwhelms the spacetime.
- Singular instanton may play an important role in decays of higher-dimensional spacetime.
  - Conical singularity works as reducing the value of bounce action.
  - Our calculation is consistent with Witten's original argument.
  - We can reproduce the bounce action with thermodynamic functions and give an interpretation.

✓ Validity of the regularization method.
✓ Uniform flux?
✓ Embedding into stringy model?



Thank you!

BACKUP

Background of conical deficit regularizaiton

Conical deficit regularization has been gradually established in studies since the 1980s.

1985 Sprouting of the idea "replacing the conical singularity by a smooth cap".

[W. A. Hiscock, Phys. Rev. D 31, 3288 (1985); J. R. Gott, As- trophys. J. 288, 422 (1985)]

1990 Some discussions on such a regularization.

e.g. [G. Hayward, J. Louko, Phys.Rev.D 42 (1990) 4032-4041; B. Allen, A. C. Ottewill, Phys.Rev.D 42 (1990) 2669-2677]

1995 Fursaev and Solodukhin developed a natural recipe for treating singular spaces.

[D. V. Fursaev, A. N. Solodukhin, Phys.Rev.D 52 (1995) 2133-2143 [arXiv:9501127[hep-th]]]

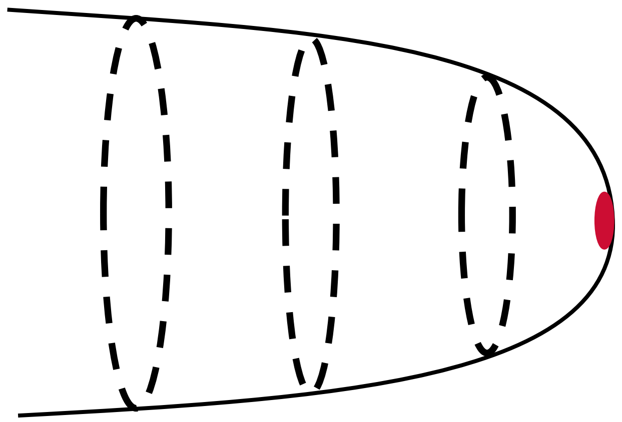
2014 Refined discussion including GHY term. Contribution from conical singularities does not depend on deficit angle. [R. Gregory, I. G. Moss and B. Withers, JHEP 03 (2014) 081 [arXiv:1401.0017[hep-th]]]

Many applications mainly in the context of phase transitions in early universe.

e.g. [P. Burda, R. Gregory and I. Moss, JHEP 08 (2015) 114 [arXiv:1503.07331 [hep-th]]]

Cobordism conjecture predicts that all string compactifications have defects (end-of-the-world branes) at the tip of cigar geometry.

[4D effective theory description]



$$S_{4D} = \int_{\mathcal{M}} \sqrt{g} \left(-\frac{1}{2}\mathcal{R}_4 + \frac{1}{2}(\partial\phi)^2 + V(\phi) \right) - \int_{\partial\mathcal{M}} \sqrt{h} \, (\mathcal{K}_4 - T_4)$$

[A bordism with defect, from Friedrich,Hebecker&Walcher'23]

[T. Higaki, K. Kamada, K. Nishimura, Phys.Rev.D 106 (2022) 9, 096022 [arXiv: 2207.00212[hep-th]]]
[M. Eto, M. Nitta, JHEP 09 (2022) 077 [arXiv: 2207.00211[hep-th]]]

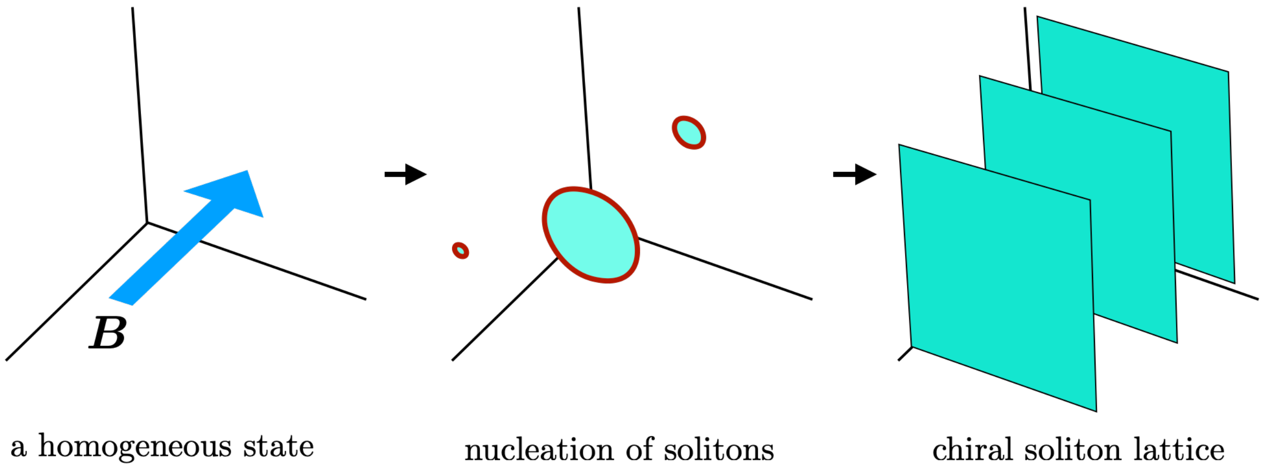
Example: Chiral soliton lattice

They considered the nucleation of domain walls with a constant background magnetic field. Their defects have negative energy due to the topological interaction.

$$\mathcal{L}_{\rm UV} = |\partial_\mu \phi|^2 - \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2 + v m^2 \left(\phi + \phi^*\right) + c\boldsymbol{j}\cdot\boldsymbol{B}$$

$$j^\mu = -\frac{i}{2}\left(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*\right) = |\phi|^2 \partial^\mu \theta, \quad \phi = |\phi| e^{i\theta}.$$

$$\mathcal{H}_{\rm UV} = |\dot{\phi}|^2 + |\nabla \phi|^2 + \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2 - v m^2 \left(\phi + \phi^*\right) - c\boldsymbol{j}\cdot\boldsymbol{B}$$



[Picture from Eto&Nitta'22]

- Bubble of nothing (BoN) is attracting more and more attention because it would be the best candidate for a universal decay channel of non-supersymmetric string vacua.

  [I. Garcia Etxebarria, M. Montero, K. Sousa and I. Valenzuela, JHEP 12 (2020) 032 [arXiv: 2005.06494[hep-th]]]

  [G. Dibietto, N. Petri, and M. Schillo, JHEP 08 (2020) 040 [arXiv: 2002.01764[hep-th]]]

- While in conventional discussion, BoN is forbidden even for SUSY broken vacua if fermions with SUSY preserving boundary conditions exist, some counterexamples have been proposed in recent years.

  [J.J. Blanco-Pillado, B. Shlaer, K. Sousa and J. Urrestilla, JCAP 10 (2016) 002 [arXiv: 1606.03095[hep-th]]]

  [P. Draper, B. Lillard and C. Skye, JHEP 10 (2023) 049 [arXiv: 2305.17838[hep-th]]]

"Bubble of nothing" is more universal than you think.