Instituto de Física Teórica
UAM-CSIC

CSIC
Consejo Superior de Investigaciones Científicas

Flat manifolds, fluxes & Casimir energies

A recipe for de Sitter?

Work in progress w/ Miguel Montero

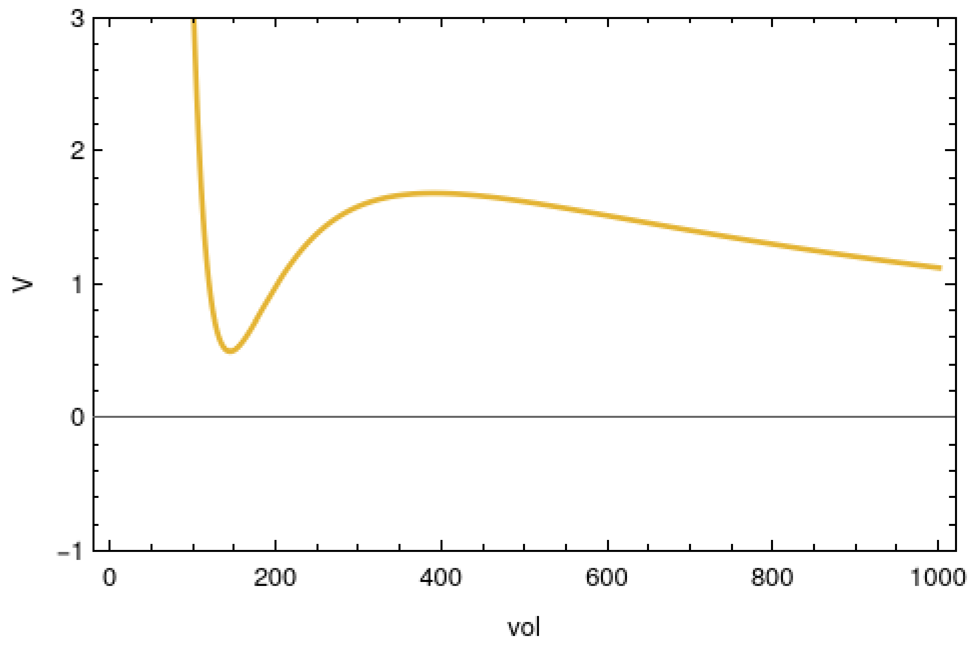
String Pheno 2024, Padova

Bruno Valeixo Bento

The de Sitter conundrum

Does String Theory have dS vacua?

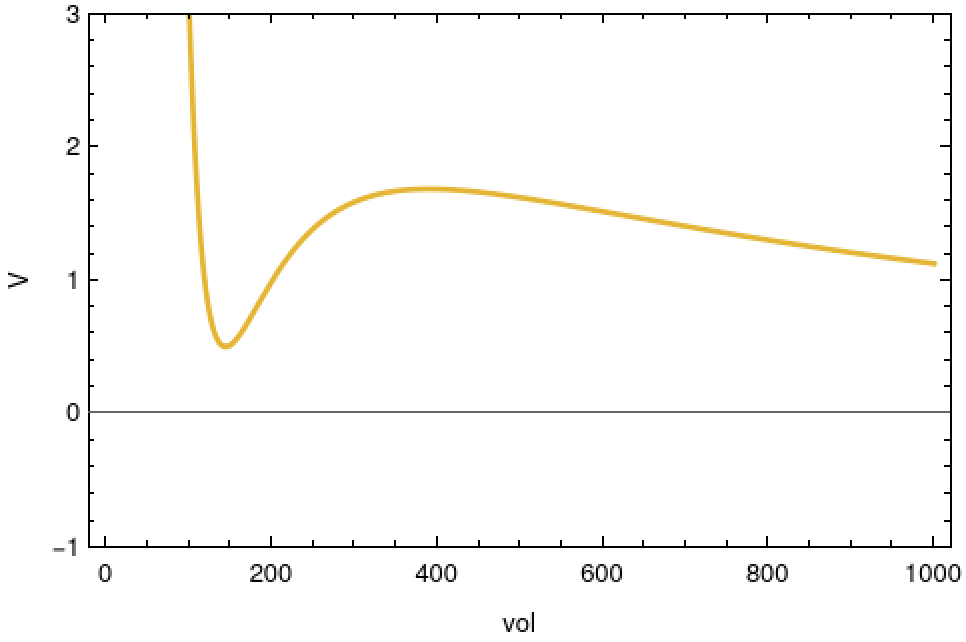
[a lot of work on this]



The de Sitter conundrum

~~Does String Theory have dS vacua?~~

Can we get dS vacua in String Theory?



Our ability to compute (and control) is part of the problem!

[D. Junghans, X. Gao, A. Hebecker, S. Schreyer, G. Venken, I. Bena, E. Dudas, M. Graña, S. Lust, F. Carta, J. Moritz, L. McAllister, R. Nally, A. Schachner, A. Westphal, D. Chakraborty, S. Parameswaran, I. Zavala, ...]

Basic setup

D-dimensional supergravity

↓ n-dimensional manifold

d-dimensional EFT

de Sitter??

What's the simplest thing we can try?

Basic setup

D-dimensional supergravity

↓ n-dimensional manifold

d-dimensional EFT

de Sitter??

~~Warped throats~~

~~D-branes~~

~~Calabi-Yau~~

~~Non-perturbative corrections~~

What's the simplest thing we can try?

Basic setup

What's the simplest thing we can try?

1) Flat manifolds ~~(curvature)~~

We consider internal manifolds of the form T^n/Γ

Isometries

$$V_{\text{curv}} = 0$$

E.g. Klein bottle

$$\text{KB} \sim T^2/\mathbb{Z}_2$$

Parametrise as $(R_1, R_2, \ldots, R_2) \to R^n = R_1 (R_2)^{n-1}, \;\; x = R_2/R_1$.

cf. G. B. De Luca, E. Silverstein, G. Torroba '21

Basic setup

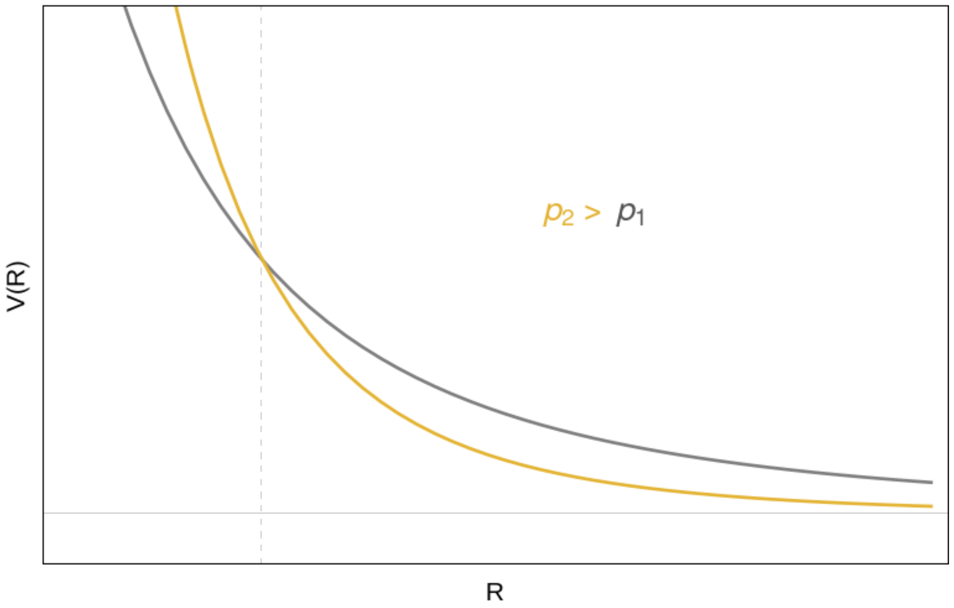
What's the simplest thing we can try?

2) Fluxes

Fluxes always contribute positively.

$$V_p^{(\alpha)} \propto \frac{1}{R^{\frac{d\cdot n}{d-2}}} \cdot \frac{n_{p,\alpha}^2}{R^{2p-n}} \cdot x^{2\alpha - \frac{2p}{5}}$$

Einstein frame



Basic setup

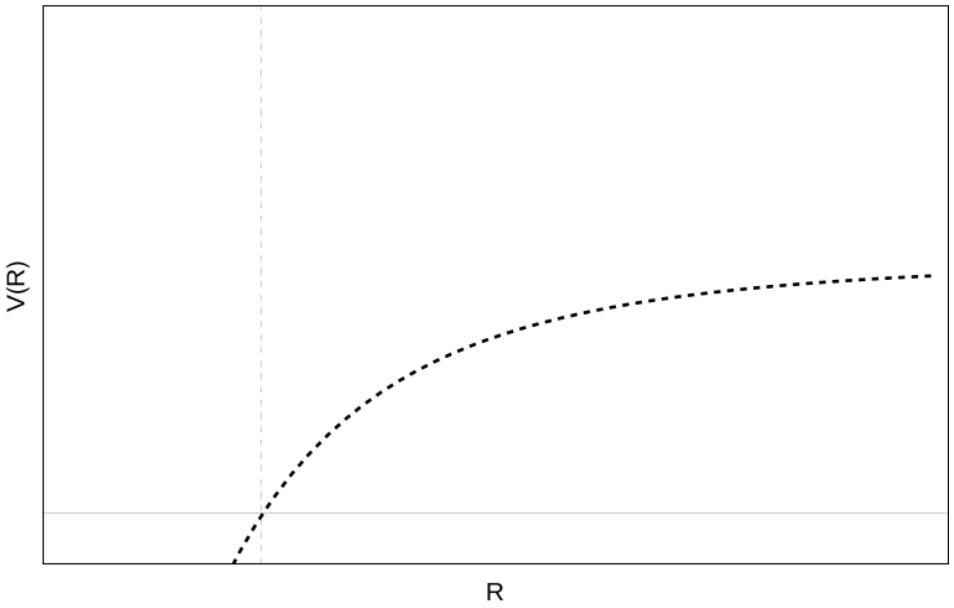
What's the simplest thing we can try?

3) Casimir energy

Quantum leftover of compact space
→ can contribute negatively

$$V_{\text{cas}} \propto -\frac{1}{R^{\frac{d \cdot n}{d-2}}} \cdot \frac{\mathcal{C}}{R^d} \cdot x^{\frac{d(n-1)}{n}} f(x)$$

Einstein frame



cf. G. B. De Luca, E. Silverstein, G. Torroba '21

Basic setup

What's the simplest thing we can try?

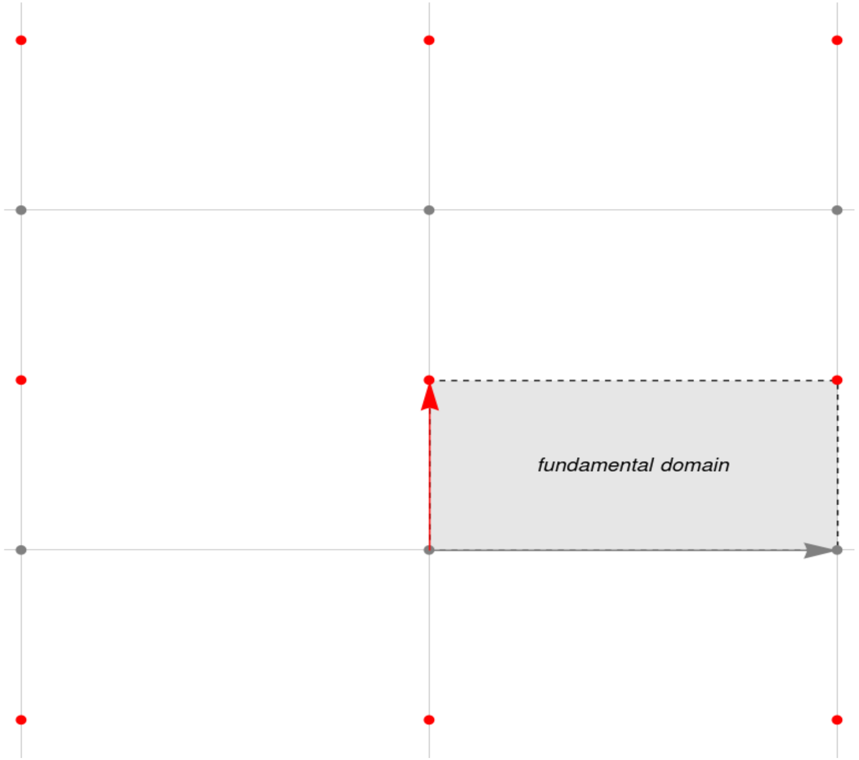
3) Casimir energy

Can we enhance it with
a good choice of Γ ?

$$V_{\mathrm{cas}} \propto -\frac{x^{\frac{d(n-1)}{n}}}{R^d} \sum_{m,\vec{n}} \frac{1}{[m^2 + x^2|\vec{n}|^2]^{\frac{d}{2}}}$$

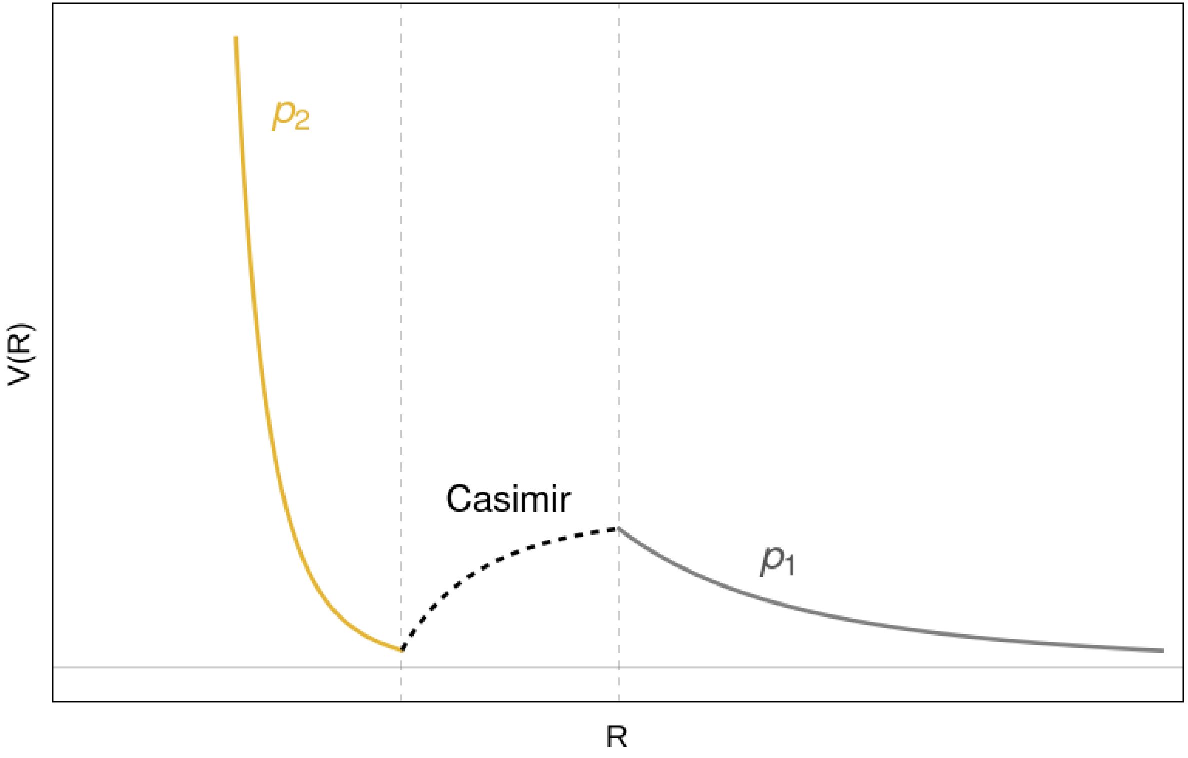
$$\frac{\Gamma(\frac{D}{2})}{2\pi^{D/2}} \sim 10^{-2}$$

It's a propagator



Fluxes vs Casimir

How can a dS minimum arise from fluxes and Casimir? (3-term potential)



Fluxes vs Casimir

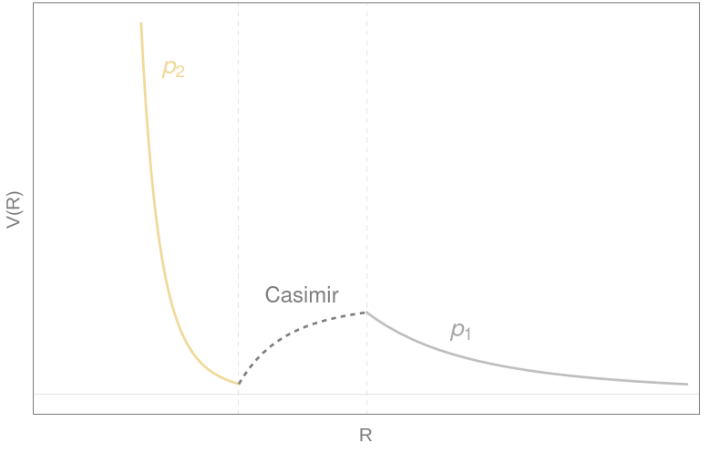
For a dS minimum to arise from fluxes and Casimir (in 3-term potential)

$$V_{p_2} \propto \frac{1}{R^{2p_2-n}} \qquad V_{\text{cas}} \propto \frac{1}{R^d} \qquad V_{p_1} \propto \frac{1}{R^{2p_1-n}}$$

we need $D > 2d$ and some $n \geq p_2 > D/2$.

d	Possible D
5	~~11~~
4	$9, 10, 11$
3	$7, 8, 9, 10, 11$

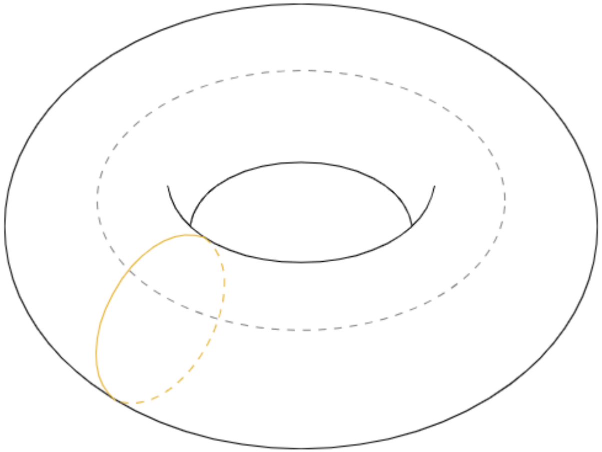
→ 16 or 32 supercharges



What about D-dim moduli?

We can stabilise D-dimensional moduli in two ways:

1) Flux potential (needs appropriate structure)
2) Duality freeze-out ("Scherk-Schwarz mechanism")



If the D-dimensional theory has a symmetry,
we can use it when imposing boundary conditions

$$\Phi(2\pi R_1) = \mathbf{g} \cdot \Phi(0)$$

e.g. $\mathbb{Z}_2 : r \to \frac{\alpha'}{r}\,, \quad r = \frac{\alpha'}{r} \Rightarrow r = \sqrt{\alpha'}$

Bigger symmetry (duality) groups will provide more options for the "freeze-out".

(e.g. IIB D=10 → SL(2, $\mathbb{Z}$) , max.susy D=7 → SL(5, $\mathbb{Z}$) , ...)

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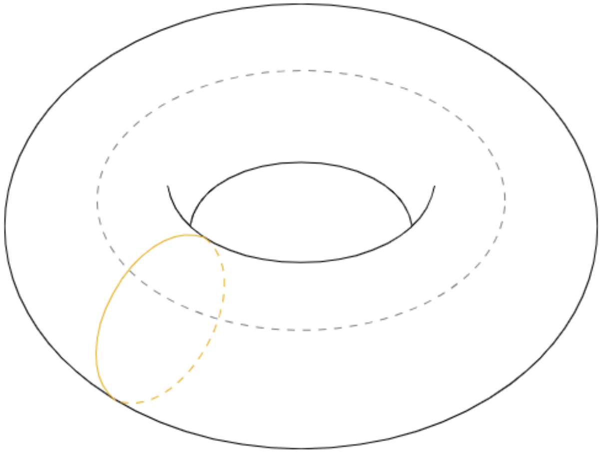


Warning: a duality freeze-out might kill some fluxes!

E.g. Type IIB → SL(2,$\mathbb{Z}$)

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$$

$$\mathbf{g} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \tau = i, \quad B_2 = C_2 = 0$$

$F_1 = F_3 = H_3 = F_7 = H_7 = 0$

32 supercharges

d = 4

$$\int G_4 \wedge G_4 \wedge C_3$$

(n = 7)

D = 11* → M-theory with G_4/G_7 → Axion monodromy runaway ❌

D = 10 → No F_6 in IIB / dilaton unstabilised in IIA ❌

D = 9 → F_2 from KK vectors (cf. M-theory on T^2)

Equivalent to replacing G_4 with nilmanifold curvature

*cf. G. B. De Luca, E. Silverstein, G. Torroba '21

32 supercharges

d = 3

$$\int G_4 \wedge G_4 \wedge C_3$$

(n = 8)

D = 11 → M-theory with G_4/G_7 → ~~Axion monodromy runaway~~

D = 10 → H_7/F_7 in IIB* / H_7 in IIA

D = 9 → Duality freeze-out & fluxes

D = 8 → 1 unstabilised scalar (duality freeze-out vs F_5)

D = 7 → 1 unstabilised scalar (duality freeze-out vs F_4)

*we cannot freeze-out axio-dilaton, because (H_7,F_7) would also vanish.

16 supercharges

We could also start with a theory with 16 supercharges.

1) $D \leq 10$

2) Only 2-forms/3-forms → $p_2 \geq D - 3$ → $d \leq 3$

   ⇒ dS_3 with H_{D-3}

3) Non-trivial elements in duality group for $D \leq 8$

We need to check each case to decide if they have dS_3.

Semidefinite optimisation

To check if the surviving cases actually work, we can turn this into an optimisation problem.

$(\mathcal{C}', \mathcal{C}'')$ → **Optimisation** max R → (n_p)

But we cannot always turn on all fluxes...

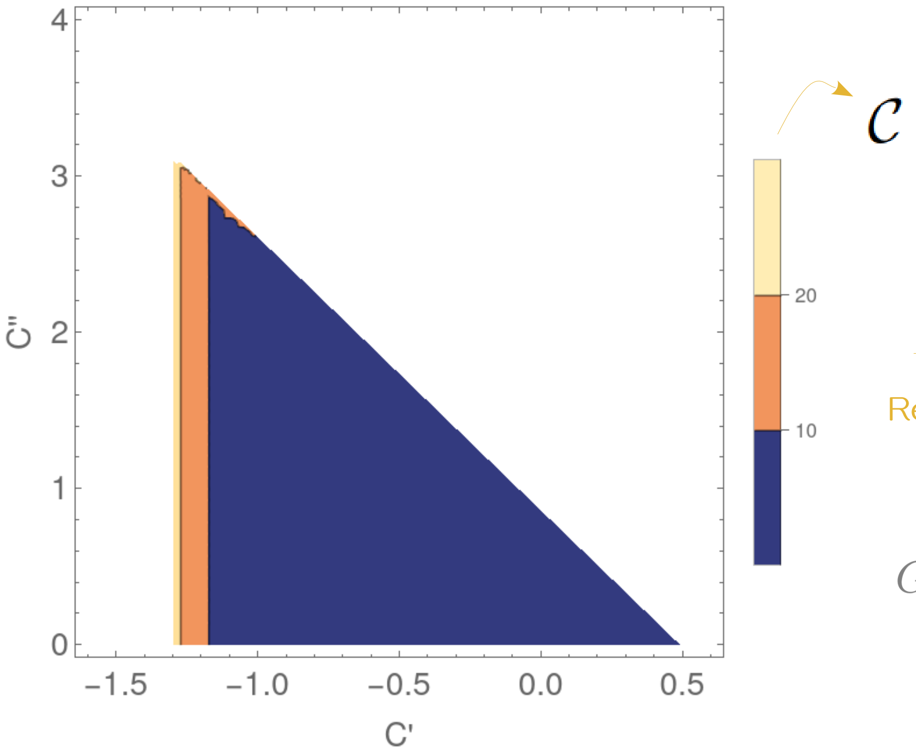
$$\partial_x V = \partial_R V = 0$$

$$\partial^2 V \succeq 0$$

*we need to check several constraints (e.g. $R \gg 1$, $n_p \gg 1$, ...)

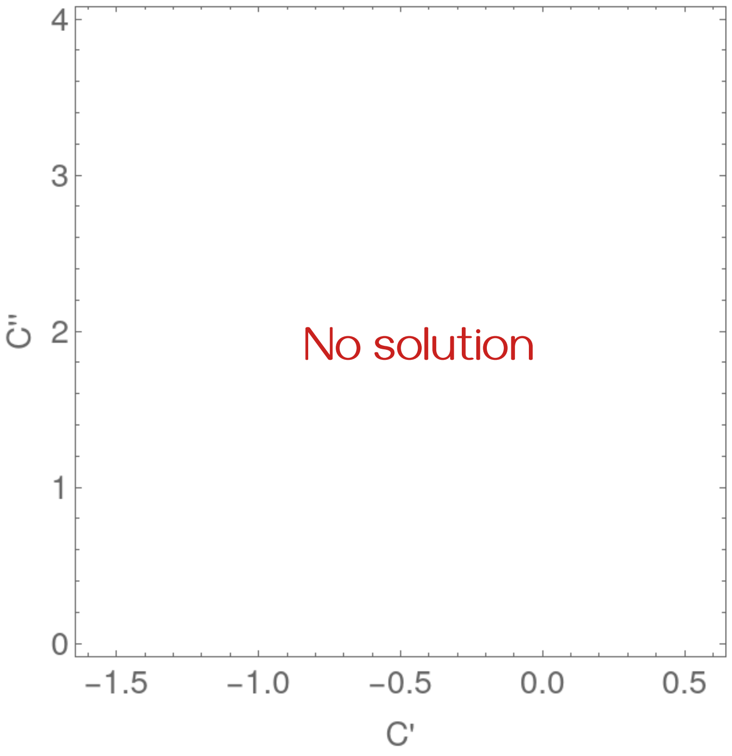
Numerical optimisation

Example: D = 11 , d = 3 → M-theory with G_4/G_7 ✗



Restrict fluxes to avoid axion monodromy

$$G_4 \wedge G_4 = 0$$



Take away

Our analysis: flat manifold + fluxes + Casimir

1) No dS_d for $d > 4$

2) dS_4 only for $D = 9$ with F_2 fluxes (maybe)

3) dS_3 possible in many cases (but duality freeze-out not enough)

This analysis focused on a dS minimum – we are also looking at saddle points and the possibility of having quintessence (less constrained).

→ Work in progress – stay tuned!