Flux Landscape with Enhanced Symmetry Not on $SL(2,\mathbb{Z})$ Elliptic Points

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Introduction

Type IIB flux compactification on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold

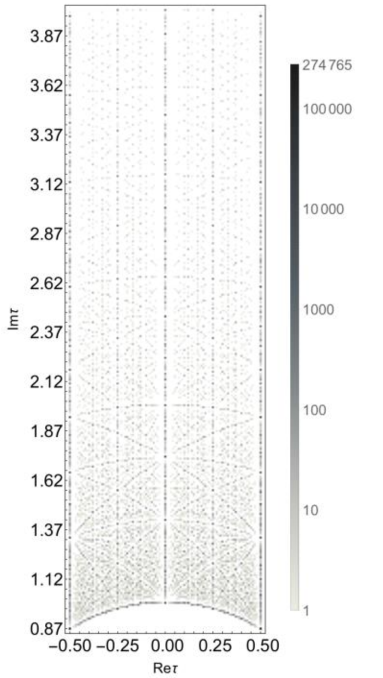
Background 3–form flux: $G_3 \to \{a^0, a^i, b_i, b_0\}$

$\Rightarrow$ Various VEVs of complex structure moduli

The distribution of VEVs regarding complex structure moduli is known to have a peak at the fixed points of $SL(2,\mathbb{Z})$.

VEVs with enhanced symmetry are favored in the Flux Landscape.

$\mathbb{Z}_3$: 40.3%



The distribution of VEVs (CS moduli) on the fundamental region

Ishiguro, Kobayashi, Otsuka 2011.09154

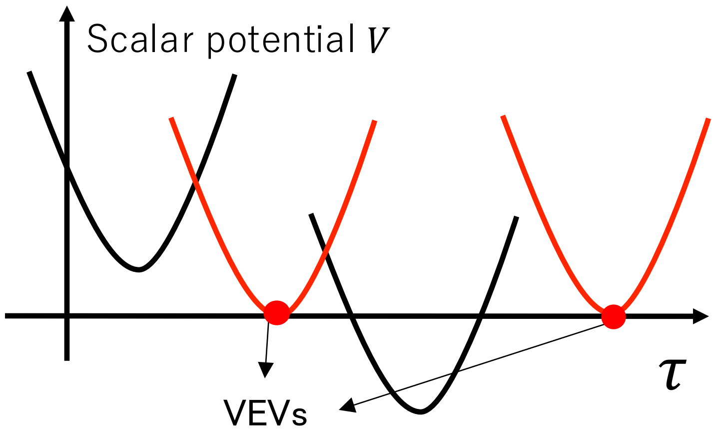
DeWolfe, Giryavets, Kachru, Taylor hep-th/0411061

Research objective

Considering Type IIB flux compactification on $T^6/\mathbb{Z}_{6-\mathrm{II}}$ and $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_4)$ orientifolds, we analyze the structure of vacua in the Flux Landscape of complex structure moduli.

Outline

Moduli stabilization



Scalar potential

$$V = e^K\left(K^{i\bar{j}}(D_i W)(D_{\bar{j}}\bar{W}) - 3|W|^2\right)$$

$i, \bar{j}$ = axio-dilaton S, complex structure moduli τ^α, $\quad K_{i\bar{j}} \equiv \partial_i \partial_{\bar{j}} K$, $\quad D_i \equiv \partial_i W + W \partial_i K$, $\quad M_{\mathrm{Pl}} = 1$

Kähler potential

$$K = -2\log \mathcal{V}_W - \log\left(-i(S-\bar{S})\right) - \log\left(i\int \Omega \wedge \bar{\Omega}\right),$$

$\mathcal{V}_W$: volume modulus

Superpotential

$$W = \int G_3 \wedge \Omega,$$

Holomorphic 3-form : $\Omega = X^I \alpha_I - F_I \beta^I$, $\quad \begin{pmatrix} X^\alpha / X^0 = \tau^\alpha, & \alpha = 1, \dots h^{2,1} \\ F_I \equiv \partial_I F, & \text{prepotential}: F \end{pmatrix}$

3-form flux : $G_3 = F_3 - S H_3$, $\quad (F_3 = a^I \alpha_I + b_I \beta^I, \quad H_3 = c^I \alpha_I + d_I \beta^I,)$

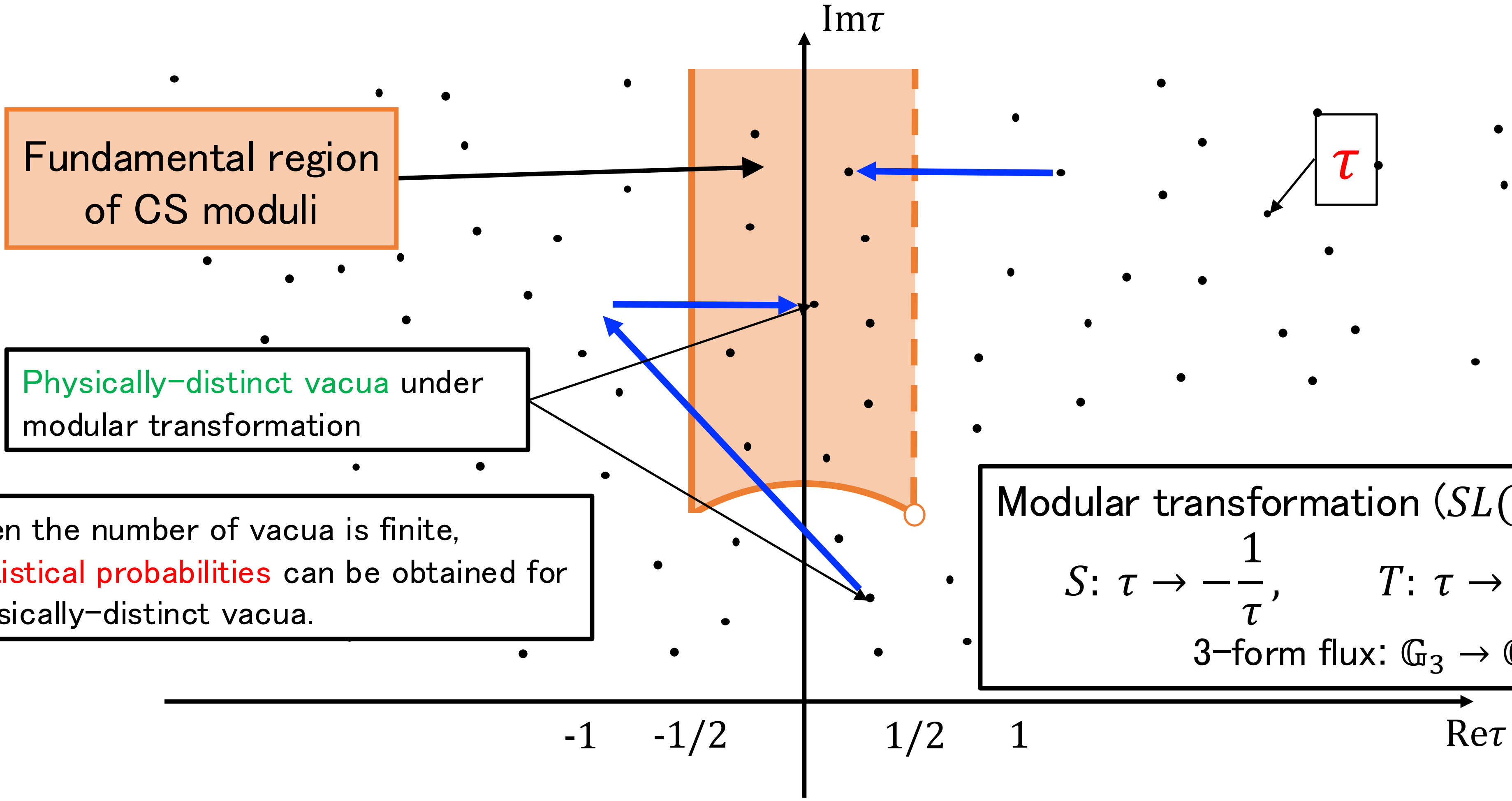
Cohomology basis : $\int \alpha_I \wedge \beta^J = \delta_I^J$, $\quad (I, J = 0, \dots, h^{2,1})$

SUSY Minkowski solutions

$$\partial_{\tau^\alpha} W = 0, \qquad \partial_S W = 0, \qquad W = 0$$

By taking arbitrary integer values for the 3-form flux, the moduli fields acquire a wide variety of VEVs.

The distribution of VEVs



When the number of vacua is finite, statistical probabilities can be obtained for physically−distinct vacua.

Modular transformation $(SL(2,\mathbb{Z}))$(→)

$$S:\ \tau \to -\frac{1}{\tau},\qquad T:\ \tau \to \tau + 1$$

3−form flux: $\mathbb{G}_3 \to \mathbb{G}'_3$

Outline

1. Introduction
2. The distribution of VEVs and modular symmetry
   - ✓ Moduli stabilization
   - ✓ The distribution of VEVs
3. Geometric structure of $T^6/\mathbb{Z}_{6-\mathrm{II}}$ and $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_4)$
   - ☐ Construction of complex bases and orbits on orbifolds
   - ☐ $SL(2,\mathbb{Z})$ symmetry breaking due to $Sp(4,\mathbb{Z})$ basis transformation
   - ☐ Symmetry in the effective theory on $T^6/\mathbb{Z}_{6-\mathrm{II}}$
   - ☐ Analysis of $T^6/\mathbb{Z}_{6-\mathrm{II}}$ and $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_4)$
4. Summary

Construction of complex bases on the orbifold

<u>Coxeter element on the $SU(6) \times SU(2)$ root lattice on $\mathbb{Z}_{6-\mathrm{II}}$ orbifold</u>

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \qquad Q^6 = 1$$

Lüst, Reffert, Schulgin, Steinberger hep-th/0506090

Bases dz^i which satisfy with $Q^t dz^i = e^{2\pi i v^i} dz^i,\ (v^1, v^2, v^3) = \left(-\frac{1}{6}, -\frac{2}{6}, \frac{3}{6}\right)$

$$dz^1 = dx^1 + e^{-\frac{2\pi i}{6}} dx^2 + e^{-\frac{2\pi i}{3}} dx^3 - dx^4 + e^{\frac{2\pi i}{3}} dx^5,$$
$$dz^2 = dx^1 + e^{-\frac{2\pi i}{3}} dx^2 + e^{\frac{2\pi i}{3}} dx^3 + dx^4 + e^{-\frac{2\pi i}{3}} dx^5,$$
$$dz^3 = \frac{1}{2\sqrt{3}} \left[\frac{1}{3} (dx^1 - dx^2 + dx^3 - dx^4 + dx^5) + \tau\, dx^6 \right]$$

Holomorphic 3-form: $\Omega \equiv dz^1 \wedge dz^2 \wedge dz^3,$

Construction of orbits on the orbifold

Four independent orbits on $T^6/\mathbb{Z}_{6-\mathrm{II}}$ orbifold $\quad (h^{2,1}_{\mathrm{untw.}} = 1)$

$$\mathbf{1}_1 \equiv \sum \Gamma(\alpha_0)\,, \qquad \mathbf{1}_2 \equiv \sum \Gamma(\alpha_1)\,, \qquad \mathbf{1}_3 \equiv -\left(\sum \Gamma(\beta^1) + \sum \Gamma(\beta^0)\right), \qquad \mathbf{1}_1 \equiv \sum \Gamma(\beta^0)\,,$$

e.g. $\alpha_0 = dx^1 \wedge dx^3 \wedge dx^5$

Γ : orbifold twist on the cohomology basis induced by Coxeter element (Q)

$$\sum \Gamma(\alpha_0) = \alpha_0 + Q\alpha_0 + Q^2\alpha_0 + Q^3\alpha_0 + Q^4\alpha_0 + Q^5\alpha_0, \qquad (dx^j = Q^j{}_i dx^i, \qquad Q^6 = 1)$$

e.g. $Q\alpha_0 = 3\alpha_0 - \alpha_1 - \alpha_2 + \beta_3$
$-\delta_5 + \delta_6 + \gamma_1 - \gamma_2 - \gamma_3 + \gamma_4$

All of these are real three-forms

Intersection number of dual cycles

$$\int_{T^6} \mathbf{1}_1 \wedge \mathbf{1}_4 = 6, \qquad \int_{T^6} \mathbf{1}_2 \wedge \mathbf{1}_3 = -6$$

$Sp(4,\mathbb{Z})$ transformation on $T^6/\mathbb{Z}_{6-\mathrm{II}}$ orbifold

<u>Period vector</u>

$$\Pi = \begin{pmatrix} \int_{A^0}\Omega \\ \int_{A^1}\Omega \\ \int_{B^0}\Omega \\ \int_{B^1}\Omega \end{pmatrix} = \begin{pmatrix} \int_{T^6/\mathbb{Z}_{6-\mathrm{II}}}\Omega\wedge\mathbf{1}_4 \\ \int_{T^6/\mathbb{Z}_{6-\mathrm{II}}}\Omega\wedge(-\mathbf{1}_3) \\ \int_{T^6/\mathbb{Z}_{6-\mathrm{II}}}\Omega\wedge\mathbf{1}_1 \\ \int_{T^6/\mathbb{Z}_{6-\mathrm{II}}}\Omega\wedge\mathbf{1}_2 \end{pmatrix} = -\frac{1}{6}\begin{pmatrix} i \\ \sqrt{3} \\ 3i\tau \\ \sqrt{3}\tau \end{pmatrix}.$$

holomorphic 3-form : $\Omega = X^I\alpha_I - F_I\beta^I$

By Kähler transformation and $Sp(4,\mathbb{Z})$ transformation, $\Pi \to \Pi' = \left(1,\ -\sqrt{3}i\tau,\ 3\tau,\ \sqrt{3}i\right)^T$

<u>$SL(2,\mathbb{Z})$ transformation for period vector</u>

$c \equiv 0 \mod 3$

$$\Pi' \to (c\tau+d)^{-1}\begin{pmatrix} d & 0 & \frac{c}{3} & 0 \\ 0 & a & 0 & -b \\ 3b & 0 & a & 0 \\ 0 & -c & 0 & d \end{pmatrix}\Pi' \equiv (c\tau+d)^{-1}M\Pi' \qquad \left(\tau \to \frac{a\tau+b}{c\tau+d}\right)$$

Since we consider a basis transformation of $H_3(T^6/\mathbb{Z}_{6-\mathrm{II}},\mathbb{Z})$,
the modular transformation must be $\Gamma_0(3)$ (Hecke congruence subgroup).

"Scaling transformation" on $T^6/\mathbb{Z}_{6-\mathrm{II}}$ orbifold

Another symmetry for period vector

"Scaling transformation": $S_{(3)} \equiv \tau \to -\dfrac{1}{3\tau}$

Taking account of $\Pi' = \left(1,\ -\sqrt{3}i\tau,\ 3\tau,\ \sqrt{3}i\right)^T$, the scaling transformation is

$$\Pi' \to \tau^{-1}\frac{i}{\sqrt{3}}\underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\notin \Gamma_0(3)}\Pi' \equiv \tau^{-1}\frac{i}{\sqrt{3}}M\Pi'.$$

Congruence subgroup : $\Gamma_0(3)$

$$\boxed{\Pi' \to (c\tau + d)^{-1}\begin{pmatrix} d & 0 & \frac{c}{3} & 0 \\ 0 & a & 0 & -b \\ 3b & 0 & a & 0 \\ 0 & -c & 0 & d \end{pmatrix}\Pi', \qquad \left(\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})\right)}$$

The outer semidirect product group

$$\boxed{\Gamma_0(3) \rtimes_{\varphi(S_{(3)})} \mathbb{Z}_2}$$

Symmetry in the effective theory on $T^6/\mathbb{Z}_{6-\mathrm{II}}$

Difference between $T^6/\mathbb{Z}_{6-\mathrm{II}}$ and $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$

$\longrightarrow$ Modular symmetry in effective theory is not $SL(2,\mathbb{Z})$ symmetry

Hecke congruence subgroup

$$\Gamma_0(3) = \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}) \middle| c \equiv 0 \mod 3 \right\}$$

Mayr, Steiberger, hep-th/9303017

" Scaling group" not included in $SL(2,\mathbb{Z})$

$$S_{(3)} \equiv \tau \to -\frac{1}{3\tau} \quad \in SL(2,\mathbb{R}) \quad \notin SL(2,\mathbb{Z})$$

Variation of the size of the fundamental region and the position of the fixed point

The distribution on VEVs on $T^6/\mathbb{Z}_{6-\mathrm{II}}$

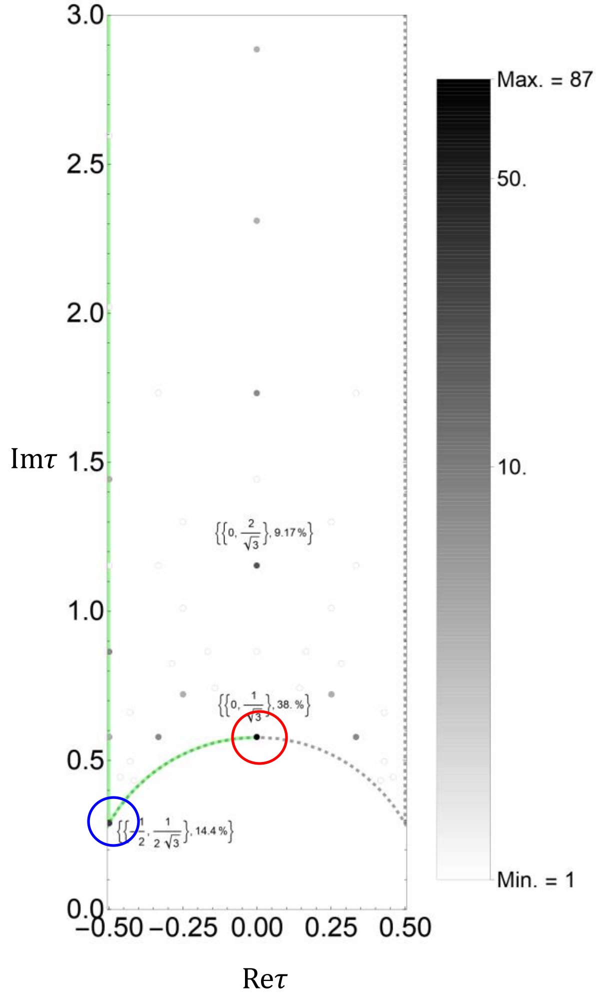
Ishiguro, T. K., Kobayashi, Otsuka, 2311.12425

- The largest number of VEVs are clustered at the fixed point (elliptic point) associated with Scaling duality

- The fixed point related to $\Gamma_0(3)$ has the second highest concentration of VEVs

In the fundamental region, the fixed points $(\mathbb{Z}_2, \mathbb{Z}_3)$ are very strong candidates for Landscape.

Ratio	38.0%	14.4%	9.17%	3.49%	1.75%
τ	$\boxed{\frac{1}{\sqrt{3}}i}$	$\boxed{-\frac{1}{2}+\frac{1}{2\sqrt{3}}i}$	$\frac{2}{\sqrt{3}}i$	$\frac{1}{3}+\frac{1}{\sqrt{3}}i$ $\sqrt{3}i$ $-\frac{1}{3}+\frac{1}{\sqrt{3}}i$ $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$	$-\frac{1}{2}+\frac{1}{\sqrt{3}}i$ $-\frac{1}{4}+\frac{5}{4\sqrt{3}}i$ $\frac{5}{\sqrt{3}}i$ $\frac{4}{\sqrt{3}}i$ $\frac{1}{4}+\frac{5}{4\sqrt{3}}i$ $-\frac{1}{2}+\frac{5}{2\sqrt{3}}i$



The distribution on VEVs on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_4)$

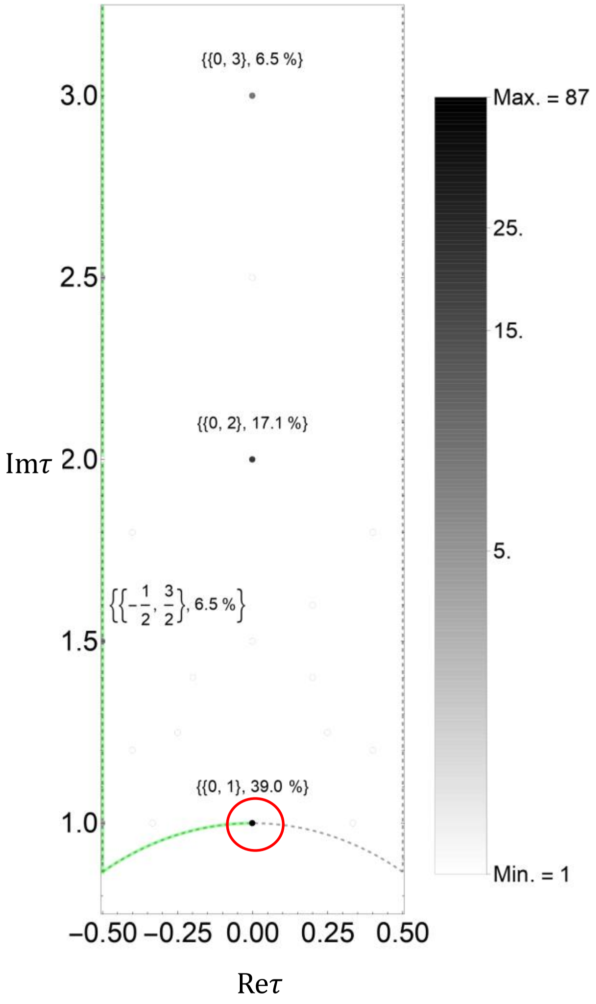
Ishiguro, T. K., Kobayashi, Otsuka, 2311.12425

- The largest number of VEVs are clustered at the fixed point $(\mathbb{Z}_2)$ associated with $SL(2,\mathbb{Z})$.

- No vacua are realized at the $\mathbb{Z}_3$ fixed point $(\tau = \omega)$.

In the case of $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_4)$, the fixed points $(\mathbb{Z}_2)$ is also very strong candidate for Landscape.

Ratio	39.0%	17.1%	6.50%	3.25%
τ	$\boxed{i}$	$2i$	$3i$	$4i$
			$-\frac{1}{2}+\frac{3}{2}i$	$-\frac{1}{2}+i$
				$5i$
				$-\frac{1}{2}+\frac{5}{2}i$



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Summary

Motivation and Previous Research

- When the number of vacua is finite, statistical probabilities can be obtained for physically distinct vacua.
- We investigated the structure of VEVs on $T^6/\mathbb{Z}_{6-\mathrm{II}}$ and $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_4)$ orbifold

Conclusion

- In the analysis of $T^6/\mathbb{Z}_{6-\mathrm{II}}$ orientifold, the modular symmetry in the effective theory changed
- There was a tendency for VEVs to cluster at the fixed points of the obtained modular symmetry

Outlook

- Analysis of modular symmetry in effective theories including twisted moduli

(Our work regarding a twisted modulas : Ishiguro, T. K., Otsuka, 2406.08970)

- Identification of the moduli field landscape by the symmetry of the vacuum structure

Back up

Cohomology basis on T^6

Real 3–form on $H^3(T^6,\mathbb{Z})$

$$
\begin{aligned}
\alpha_0 &= dx^1 \wedge dx^3 \wedge dx^5, & \beta^0 &= dx^2 \wedge dx^4 \wedge dx^6,\\
\alpha_1 &= dx^2 \wedge dx^3 \wedge dx^5, & \beta^1 &= -dx^1 \wedge dx^4 \wedge dx^6,\\
\alpha_2 &= dx^1 \wedge dx^4 \wedge dx^5, & \beta^2 &= -dx^2 \wedge dx^3 \wedge dx^6,\\
\alpha_3 &= dx^1 \wedge dx^3 \wedge dx^6, & \beta^3 &= -dx^2 \wedge dx^4 \wedge dx^5,\\
\gamma_1 &= dx^1 \wedge dx^2 \wedge dx^3, & \delta^1 &= -dx^4 \wedge dx^5 \wedge dx^6,\\
\gamma_2 &= dx^1 \wedge dx^2 \wedge dx^5, & \delta^2 &= -dx^3 \wedge dx^4 \wedge dx^6,\\
\gamma_3 &= dx^1 \wedge dx^3 \wedge dx^4, & \delta^3 &= -dx^2 \wedge dx^5 \wedge dx^6,\\
\gamma_4 &= dx^3 \wedge dx^4 \wedge dx^5, & \delta^4 &= -dx^1 \wedge dx^2 \wedge dx^6,\\
\gamma_5 &= dx^1 \wedge dx^5 \wedge dx^6, & \delta^5 &= -dx^2 \wedge dx^3 \wedge dx^4,\\
\gamma_6 &= dx^3 \wedge dx^5 \wedge dx^6, & \delta^6 &= -dx^1 \wedge dx^2 \wedge dx^4,
\end{aligned}
$$

$$
\int_{T^6} \alpha_I \wedge \beta^J = \delta_I^J, \qquad \int_{T^6} \gamma_I \wedge \delta^J = \delta_I^J
$$

Moduli stabilization on $T^6/\mathbb{Z}_{6-\mathrm{II}}$ and $T^6/(\mathbb{Z}_2\times\mathbb{Z}_4)$

SUSY Minkowski solutions

$$\begin{cases} D_\tau W = \partial_\tau W + K_\tau W = 0 \\ D_S W = \partial_S W + K_S W = 0 \\ W = 0 \end{cases} \Rightarrow \begin{cases} C + DS = 0 \\ B + D\tau = 0\,, \\ A + C\tau = 0 \end{cases}$$

Superpotential: $W = A + BS + \tau[C + DS]$

The set of (fluxes, moduli VEVs)

$$\langle S\rangle = -\frac{C}{D}, \qquad \langle\tau\rangle = -\frac{B}{D}, \qquad AD - BC = 0$$

$T^6/\mathbb{Z}_{6-\mathrm{II}}$ orientifold

$$A = -ib_0 - \frac{\sqrt{3}}{2}b_1, \qquad C = -ia^0 - \left(\sqrt{3}a^1 + \frac{1}{\sqrt{3}}a^0\right),$$

$$B = -id_0 - \frac{\sqrt{3}}{2}d_1, \qquad D = -ic^0 - \left(\sqrt{3}c^1 + \frac{1}{\sqrt{3}}c^0\right),$$

$T^6/(\mathbb{Z}_2\times\mathbb{Z}_4)$ orientifold

$$A = \frac{-1+i}{2}[a^1 + (-1+i)b_2], \qquad C = \frac{-1+i}{2}[(-1+i)a^2 + (-2i)b_1],$$

$$B = -\frac{-1+i}{2}[c^1 + (-1+i)d_2], \qquad D = -\frac{-1+i}{2}[(-1+i)c^2 + (-2i)d_1],$$