Stabilizing massless fields in Landau Ginzburg models

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String Pheno '24, Padova

Based on 2406.03435 Becker, MR, Sengupta, Walcher, Wrase

What?

What?

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How?

Use higher-than-quadratic order terms to lift massless deformations.

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Why?

Is there a fully stabilized $\mathcal{N} = 1$ SUSY Minkowski vacuum?

How?

Use higher-than-quadratic order terms to lift massless deformations.

How?

- Consider the simple example of $W = \frac{1}{2}(\phi - \psi^2)^2$.

- This function clearly has one flat direction along $\phi = \psi^2$.

- Let us apply our algorithm for stabilizing moduli order by order to this function,

- At quadratic order in the fields, $W_2 = \frac{1}{2}\phi^2$. Solving the critical point equations gives us one non-trivial constraint ,

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$$\phi_{(1)} + \phi_{(2)} + \dots$$

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Why?

- Moduli Stabilization remains a major obstacle to string model building.

[Graña 05, McAllister, Quevedo '23]

- Swampland criteria provide concrete characterizations of the obstacles.
- In this work, we will not build models viable for phenomenology.
- Expanding the String Landscape is an interesting problem in its own right.

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Swamplandr Conjectures

Explicit String Constructions

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Tadpole Conjecture (Type IIB) - The number of moduli stabilized by fluxes is constrained by,

$$N_{flux} > \frac{1}{3} n_{stab}$$

Bena, Blåbäck, Graña, Lüst '20]

Becker, Bena, Blåbäck, Brodie, Coudarchet, Gonzalo, Graña, Grimm, van de Heisteeg, Herraez, Lüst, Marchesano, Monnee, Plauschinn, Prieto, Tsagkaris, Walcher, Wiesner, Wrase …

Why?

- We need to clarify what we mean by n_{stab} -
    - $n_{stab} := \text{rank}(\partial_i \partial_j W_{flux})$
    - $n_{stab} := \text{codim}\{\partial_i W_{flux} = 0\}$

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Why?

- Conjecture has been studied extensively in the asymptotic limits of moduli space. *[Grimm, Plauschinn, van de Heisteeg '21, Graña, Grimm, van de Heisteeg, Herraez, Plauschinn '22]*

- Does it continue to hold in the interior? *[Becker, Gonzalo, Walcher, Wrase '22, Lüst, Wiesner '22]*

- Even if it continues to hold, are there models where all moduli can be stabilized?

- Fully Stabilized $\mathcal{N} = 1$ SUSY Minkowski vacua?

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What?

- DGKT showed that it is possible to stabilize all moduli in type IIA compactified on a rigid Calabi-Yau ($h^{2,1} = 0$). *[De Wolfe, Giryvayets, Kachru, Taylor '05]*

- Motivated by these results in type IIA, BBVW constructed the mirror dual in type IIB. *[Becker, Becker, Vafa, Walcher '06]*

- The mirror manifold admits no geometric interpretation, but there exists a LG description. *[Vafa '89, Witten '93, Hori, Iqbal, Vafa '00]*

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What?

- Perturbative consistency of the superstring requires that $c = 15$.
- This can be achieved via - $\mathbb{R}^{(1,3)} \times (\mathcal{N} = 2, c = 9 \text{ SCFT})$.
- The $\mathcal{N} = 2, c = 9$ SCFT does not always have to describe a geometric manifold.

$$S = \int d^2z d^4\theta \, K\left(\{x_i, \bar{x}_i\}\right) + \left(\int d^2z d^2\theta \, \mathcal{W}\left(\{x_i\}\right) + \text{complex conj.}\right)$$

where x_i are chiral super fields and the superpotential is quasi-homogeneous.

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What?

For the 1^9 model we have 9 chiral fields with the following world sheet superpotential,

$$\mathscr{W}(\{x_i\}) = \sum_{i=1}^{9} x_i^3$$

$$g : x_i \mapsto \omega x_i \ , \ \ \omega = e^{\frac{2\pi \mathrm{i}}{3}}$$

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What?

Where is the 4d physics?

$$W_{GVW} = \int_M G_3 \wedge \Omega$$

What?

- Consider the single variable building block of the 1^9 model,

$$\mathscr{W} = x^3, \qquad g : x \to e^{\frac{2\pi i}{3}} x$$

- The A-branes of this model are the contours in the complex-x plane given by $Im(\mathscr{W}) = 0$

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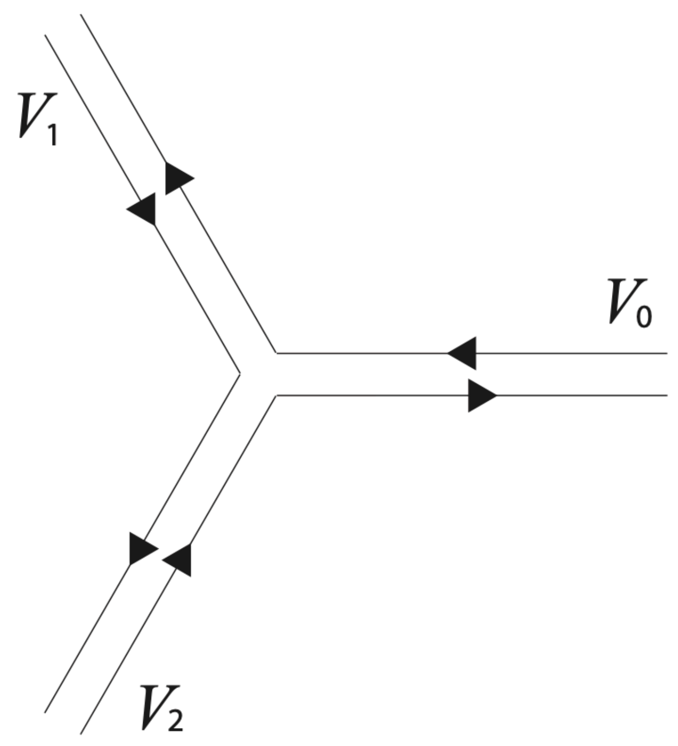

[Hori, Iqbal, Vafa '00]

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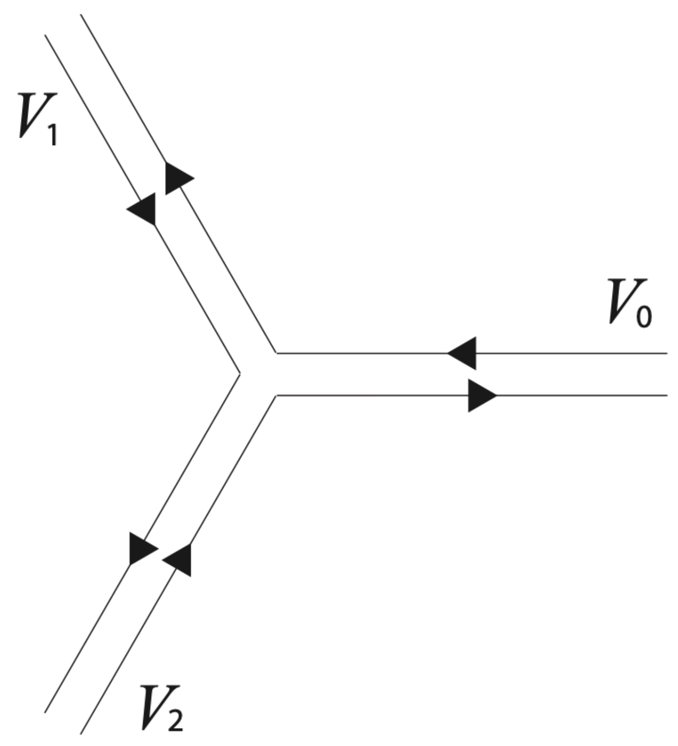

$$V_0 + V_1 + V_2 = 0$$

[Hori, Iqbal, Vafa '00]

What?

$$V_{\mathbf{n}} = V_1 \times V_2 \times \ldots \times V_9$$

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$$G_3 = \sum_{\mathbf{n}} \left(N^{\mathbf{n}} - \tau M^{\mathbf{n}}\right)\gamma_{\mathbf{n}}$$

What?

- The RR ground states of the minimal model,

$$|\, l = 1{,}2\rangle$$

What?

- The RR ground states of the full model are labelled by $\Omega_{\mathbf{l}}$ where $\mathbf{l} = (l_1, l_2 \ldots, l_9)$ with $l_i = 1,2$ -

$\sum_i l_i$	9	12	15	18
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What?

- The $1^9/\mathbb{Z}_3$ model has $h^{(2,1)} = 84$ and $h^{(1,1)} = 0$.

- We would like to study orientifolds of these models. In particular, we will restrict to,

$$\sigma : (x_1, x_2 \ldots, x_9) \to -(x_2, x_1 \ldots, x_9)$$

[Becker, Becker, Vafa, Walcher '06]

which has an orientifold charge of 12 that has to be cancelled by fluxes.

$$h^{(2,1)} = 63 \qquad h^{(1,1)} = 0$$

What?

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What?

There are 63 complex structure moduli arising from the (c, c) ring

There are 0 Kähler moduli arising from the (a, c) ring

What?

- The overlap integral between the cycles and RR ground states is then calculable,

$$\langle V_n | l \rangle = \int_{V_n} x^{l-1} e^{-x^3} dx = \frac{1}{3}\Gamma\left(\frac{l}{3}\right)(1-\omega^l)\omega^{ln}$$

with $l = 1,2$, $n = 0,1,2$ and $\omega = e^{\frac{2\pi \mathrm{i}}{3}}$

What?

- When the worldsheet superpotential is deformed as, $\mathscr{W} = x^3 \to x^3 - t\,x$

$$\left(\frac{\partial}{\partial t}\right)^r \Bigg|_{t=0} \langle V_n \,|\, l \rangle = \int_{V_n} x^{r+l-1} e^{-x^3} dx = \frac{1}{3}\Gamma\left(\frac{r+l}{3}\right)\left(1 - \omega^{r+l}\right)\omega^{(r+l)n}$$

What?

- GVW superpotential exists in these LG orbifold models as well.

- The superpotential is in fact exact! *[Becker, Becker, Vafa Walcher '06]*

What?

- GVW superpotential exists in these LG orbifold models as well.

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$$\frac{1}{\tau - \bar{\tau}} \int G_3 \wedge \bar{G}_3 = \int F_3 \wedge H_3 = 12$$

Moduli Stabilization

- Finding SUSY Minkowski vacua -

1. Pick fluxes $\Omega_{l_1,l_2 \ldots l_9} \in H^{(2,1)}$ $\left(\sum_i l_i = 12\right)$

2. Ensure flux quantization and tadpole cancellation

- They generically have massless directions (maximal mass matrix rank of 26). *[Becker, Gonzalo, Walcher, Wrase '22]*

- We would like to expand the superpotential around the critical points,

$$W_{expand} = \frac{1}{2!}\partial_i\partial_j W\left(t^i t^j\right) + \frac{1}{3!}\partial_i\partial_j\partial_k W\left(t^i t^j t^k\right) + \ldots$$

t^i , $i = 1,2\ldots,64$ are the deformations around the critical point.

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Moduli Stabilization

Tadpole conjecture target $= 12 \times 3 = 36$ moduli

Moduli Stabilization

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Moduli Stabilization

$$\left(\frac{\partial}{\partial t}\right)^r \bigg|_{t=0} \langle V_n | l \rangle = \int_{V_n} x^{r+l-1} e^{-x^3} dx = \frac{1}{3}\Gamma\left(\frac{r+l}{3}\right)\left(1 - \omega^{r+l}\right)\omega^{(r+l)n}$$

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$$\frac{\partial}{\partial t^{\mathbf{k}_1}}\frac{\partial}{\partial t^{\mathbf{k}_2}}\cdots\frac{\partial}{\partial t^{\mathbf{k}_r}}\int\Omega_{\mathbf{l}}\wedge\Omega\Big|_{t^{\mathbf{k}}=0} = \delta_{\mathbf{l}+\mathbf{L}}\frac{1}{3^9}\prod_{i=1}^{9}(1-\omega^{L_i})\Gamma\left(\frac{L_i}{3}\right).$$

where, $\mathbf{L} = \sum_{\alpha=1}^{r}\mathbf{k}_\alpha + \mathbf{1}$

$$W = \frac{1}{2}(\phi - \psi^2)^2$$

- Now going upto cubic order in the fields, $W_2 + W_3 = \frac{1}{2}\phi^2 - \phi\psi^2$

$$\partial_\phi\Big(W_2 + W_3\Big) = \phi - \psi^2 = 0\,,\ \partial_\psi\Big(W_2 + W_3\Big) = -2\phi\psi = 0$$

$$\implies \phi = \psi = 0$$

- The correct thing to do would be,

$$\partial_\phi W_2 + \Big(\partial_\phi W_3\Big)\Big|_{\phi=\phi_{(1)}=0} = \phi - \psi^2 = 0 \qquad \partial_\psi W_2 + \Big(\partial_\psi W_3\Big)\Big|_{\phi=\phi_{(1)}=0} = 0$$

Moduli Stabilization

- A vast classification of these possible flux choices was pursed recently.

[Becker, Brady, Sengupta '23]

- The fluxes are classified in terms of number of Ω's "turned on".
- Consider 1 Ω, $G_3 = A\Omega_{\mathbf{l}}$

63 choices of $\mathbf{l}$ $\left(\sum_i l_i = 12\right)$

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Moduli Stabilization

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[Becker, Brady, Sengupta '23]

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- Consider 1 Ω,

$$G_3 = A\Omega_{\mathbf{l}} \qquad N_{flux} = 27$$

~~63~~ 2 choices of $\mathbf{l}$ $\left(\sum_i l_i = 12\right)$

Moduli Stabilization

- Consider 2 Ω's, $G_3 = A_1 \Omega_{\mathbf{l}_1} + A_2 \Omega_{\mathbf{l}_2}$ $N_{flux} = 18$

  6 choices of $\mathbf{l}_1, \mathbf{l}_2$ $\left(\sum_i l_i = 12 \right)$

- 4 Ω's can give rise to physical solutions with $N_{flux} = 12$
- Physical solutions are only possible upto 12 Ω's. *[Becker, Brady, Sengupta '23]*

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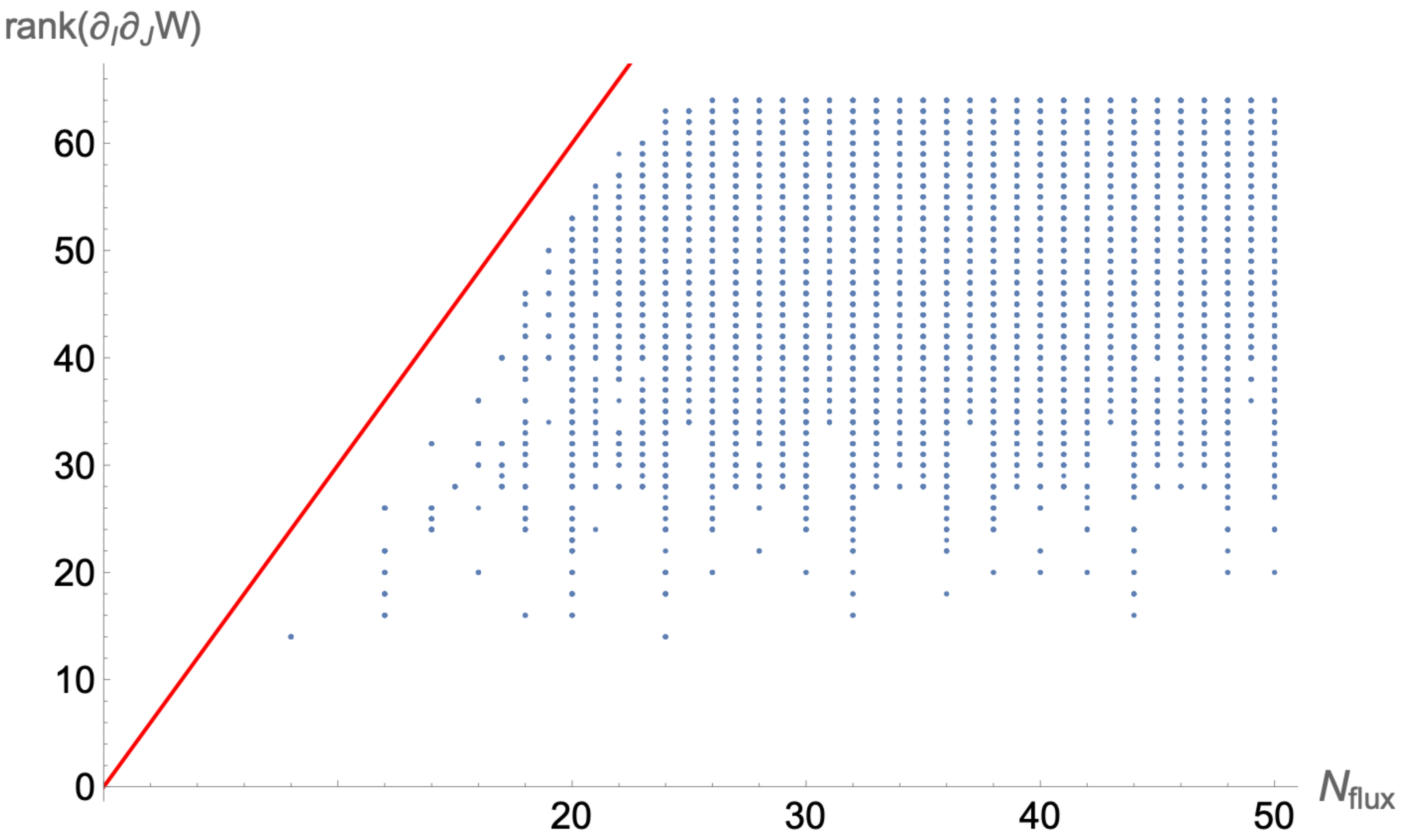

Moduli Stabilization

Model	massive	3rd power	4th power	5th power	6th power
$G^{[8,8]}_{(1)}$	14	0	0	0	0
$G^{[12,12]}_{(1)}$	22	0	0	0	0
$G^{[12,12]}_{(2)}$	26	0	0	0	0
$G^{[12,12]}_{(3)}$	26	0	0	0	0
$G^{[12,4]}_{(1)}$	22	0	0	0	0
$G^{[12,4]}_{(2)}$	26	0	0	0	0
$G^{[12,4]}_{(3)}$	16	6	0	0	0
	16	6	0	0	?
	16	6	4	0	0
	16	7	1	0	0
	16	7	4	0	0
$G^{[12,12]}_{(4)}$	20	2	0	4	1
	20	2	0	0	0

Summary

- Non-geometric LG Models are promising tools for the Swampland program.
- Moduli stabilization is possible with higher order terms in the superpotential.
- Tadpole Conjecture appears to hold in non-geometric models (for now) in the interiors of moduli space.
- Stay tuned!

Thank you!

Deformations

$$(c,c)\ \text{ring} \qquad \mapsto \qquad \mathscr{R} = \left[\frac{\mathbb{C}[x_1,\ldots,x_9]}{\partial_{x_i}\mathscr{W}(x_1,\ldots,x_9)}\right]$$

- The above ring is spanned by,

$$\mathbf{x}^{\mathbf{k}} = x_1^{k_1} \cdot x_2^{k_2} \cdots x_9^{k_9}$$

with $\mathbf{k} = (k_1, \ldots, k_9)$ such that $k_i \in \{0,1\}$ and $\sum_i k_i = 0 \bmod 3$.

- The monomials of the kind $x_i x_j x_k$ with $i \neq j \neq k \neq i$ form a basis of the allowed marginal deformations of the superpotential.

GKP vs BBVW

- How is this different from GKP? [Giddings, Kachru, Polchinski '01]

$$K_{GKP} = K_{CS} - 3log[-(T-\bar{T})] - log[-(\tau-\bar{\tau})]$$

- Solving the SUSY equations, $D_\tau W = D_i W = 0 \implies$ ISD fluxes

$$K_{BBVW} = K_{CS} - 4log[-(\tau-\bar{\tau})]$$

[Becker, Becker, Walcher '07]

- SUSY equations do not require ISD fluxes unlike in GKP.
- For SUSY Minkowski solutions GKP and BBVW are almost identical.

Explicit Example

$$G_3 = \frac{\mathrm{i}}{3\sqrt{3}}\left(\Omega_{1,1,1,1,2,1,2,1,2} - \Omega_{1,1,1,1,2,1,2,2,1} - \Omega_{1,1,1,1,2,2,1,1,2} - \Omega_{1,1,1,1,2,2,1,2,1}\right)$$

- Mass matrix rank = 16 *[Becker et al '22]*

- The already massive fields can be fixed order by order with no ambiguity. That is,

$$\partial_{\tilde{a}} W = 0$$

where $\tilde{a}$ runs over the 16 massive fields can be solved to get,

$$t_a = t_{a(1)} + t_{a(2)} + t_{a(3)} + \dots$$

Explicit Example

- Solving the quadratic order constraints from the cubic order terms for the massless fields leads to six new stabilized directions.

$$t_{20} = t_{20(1)} + t_{20(2)} + \dots$$

- Several branches of solutions. Need to be careful to not overfix.
- An exhaustive search is cumbersome and maybe even impossible.
- Progress towards classifying the various solutions. *[Becker et al '23]*
- General patterns and symmetry arguments?

$2^6/\mathbb{Z}_4$

- Similarly we can indentify the cohomology and homology bases starting from the building block of the $2^6/\mathbb{Z}_4$ model, $W_{ws} = x^4$.

- A cohomology basis is given by the RR ground states of the minimal model $|\, l\rangle$with $l = 1,2,3$. A homology basis is given by V_0, V_1, V_2, V_3 with $V_0 + V_1 + V_2 + V_3 = 0$.

- The overlap integral between the cycles and RR ground states is then calculable,

$$\langle V_n \,|\, l \rangle = \int_{V_n} x^{l-1} e^{-x^4} dx = \frac{1}{4}\Gamma\left(\frac{l}{4}\right)(1-\omega^l)\omega^{ln}$$

[Hori et al '00]

with $l = 1,2,3$, $n = 0,1,2,3$ and $\omega = e^{\frac{2\pi i}{4}}$

$2^6/\mathbb{Z}_4$

- The $2^6/\mathbb{Z}_4$ model has $h^{(2,1)} = 90$ and $h^{(1,1)} = 0$. $\left(W_{2^6} = \sum_{i=1}^{6} x_i^4, g : x_i \to e^{\frac{2\pi i}{4}} x_i\right)$
- The RR ground states of the model are labelled by $\Omega_{\mathbf{l}}$ where $\mathbf{l} = (l_1, l_2 \ldots, l_6)$ with $l_i = 1,2,3$ -

1. For $\Omega_{l_1,l_2,\ldots,l_6} \in H^{(2,1)}$, $\sum_i l_i = 10$.
2. For $\Omega_{l_1,l_2,\ldots,l_6} \in H^{(3,0)}$, $\sum_i l_i = 6$

- The orientifold involution we will work with is,

$$\sigma : (x_1, x_2 \ldots, x_6) \to e^{\frac{2\pi i}{4}}(x_1, x_2 \ldots, x_6)$$

which has an orientifold charge of 40 that has to be canceled by fluxes. *[Becker et al '06]*

$2^6/\mathbb{Z}_4$

- The $2^6/\mathbb{Z}_4$ orientifold with tadpole charge 40 could give a way out.
- This model has 91 moduli including the axio-dilation.
- The tadpole conjecture does not imply that all 91 moduli cannot be stabilized ($40 \times 3 = 120 > 91$).
- For example, we find solutions with mass matrix rank of 84 (out of 91) moduli.

$2^6/\mathbb{Z}_4$

- A flux choice that gives 84 massive fields,

$$\begin{aligned}
G_3 = &-\frac{1}{2}\Omega_{1,1,3,3,1,1} + \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{1,2,1,1,3,2} - \left(\frac{1}{4}-\frac{\mathrm{i}}{4}\right)\Omega_{1,2,2,3,1,1} - \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{1,2,3,1,1,2} \\
&+ \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{1,3,1,1,2,2} + \frac{1}{2}\mathrm{i}\Omega_{1,2,1,1,3,2} - \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{1,3,2,1,1,2} + \left(\frac{1}{4}-\frac{\mathrm{i}}{4}\right)\Omega_{1,3,2,2,1,1} \\
&+ \left(\frac{1}{2}-\frac{\mathrm{i}}{2}\right)\Omega_{1,3,3,1,1,1} + \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{2,1,1,1,3,2} - \frac{1}{2}\Omega_{2,1,2,3,1,1} - \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{2,1,3,1,1,2} \\
&- \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{2,1,3,2,1,1} + \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{2,2,1,1,2,2} + \frac{1}{2}\mathrm{i}\Omega_{2,2,1,1,3,1} - \left(\frac{1}{4}-\frac{\mathrm{i}}{4}\right)\Omega_{2,2,1,3,1,1} \\
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&- \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{3,1,2,1,1,2} - \left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\Omega_{3,1,2,2,1,1} - \frac{1}{2}\mathrm{i}\Omega_{3,1,3,1,1,1} + \frac{1}{2}\mathrm{i}\Omega_{3,2,1,1,2,1} \\
&+ \left(\frac{1}{4}-\frac{\mathrm{i}}{4}\right)\Omega_{3,2,2,1,1,1} + \frac{1}{2}\Omega_{3,3,1,1,1,1}
\end{aligned}$$