Conditions for O-plane unsmearing from second-order perturbation theory

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Smeared sources

String theory supplements supergravity with localized sources.

Vacua = Dimensional reduction + truncation to zero-modes + ...

Generally not a consistent truncation of the theory:
localized sources couple to every mode

Equivalent to solving EOM with smeared source density

If a solution with localized sources exists, its smeared variant is expected to approximate its zero-mode dynamics

O6 in massive type IIA

$\mathbb{R}^7 \times T^3/\mathbb{Z}_2$, smeared and localized solutions similar properties.

Baines, Van Riet '05

$AdS_7 \times S^3$, D6/O6, smeared solution does not resemble localized.

Blåbäck et al '11, Apruzzi et al '15

DGKT: e.g. $AdS_4 \times T^6/{\mathbb{Z}_3}^2$,
only smeared solution fully constructed

De Wolfe et al '05, Acharya et al '06

Scale-separated AdS, interesting holographic properties,
Swampland testing ground
Perturbative unsmearing of O-planes to leading order

Junghans '03, Marchesano et al '03

Interlude: spurious solutions in perturbation theory

Leading order perturbation theory is linear in the perturbation. Solutions are "tangent vectors" to a family of configurations.

At intersection of branches, spurious solutions from linear combinations.

Eliminated through constraints appearing at second order.

"Linearization instability" in GR : e.g. Einstein gravity on $\mathbb{R} \times T^3$
Linearized gravity waves solve linearized equations, but can't be lifted to full non-linear solution.

Deser et al '73

DGKT review

Massive type IIA with intersecting O6-planes

$$ds^2 = w^2 ds^2_{AdS_4} + g_{mn} dy^m dy^n$$

O6 charge cancelled (globally) by $F_0 H_3$ term.

Unfixed 4-form flux, can be taken large: $F_4 \sim n$

$$w \sim n^{3/4} \,, \quad g_{mn} \sim n^{1/2} \,, \quad \tau = e^{-\phi} \sim n^{3/4}$$
$$H_3 \sim n^0 \quad F_2 \sim n^{1/2}$$

Smeared solution has constant warp factor and dilaton, Ricci-flat internal metric, harmonic fluxes.

Leading order unsmearing

Large n simultaneously results in large internal volume, weak coupling and separation between AdS and KK scales.

Treat smeared solution as leading order in a $1/n$ expansion

Expand fields as

$$X = n^{\alpha_X} \sum_k X^{(k)} n^{-\beta_X k}$$

with α_X the scaling of the smeared solution, β_X is choice of step in the expansion, which can differ between fields.

Equations of motion

Zeroth order dilaton and metric equations

$$0 = \tau^{(0)} R^{(0)} - 8\tau^{(0)} \frac{\Box w^{(0)}}{w^{(0)}} - 12\tau^{(0)} \frac{\partial_m w^{(0)} \partial^m w^{(0)}}{w^{(0)2}} - 4\Box\tau^{(0)}$$

$$0 = R^{(0)}_{mn} - 4\frac{\nabla_m \partial_n w^{(0)}}{w^{(0)}} - 2\frac{\nabla_m \partial_n \tau^{(0)}}{\tau^{(0)}} + 2\frac{\partial_m \tau^{(0)} \partial_n \tau^{(0)}}{\tau^{(0)2}}$$

$$0 = w^{(0)} \Box w^{(0)} + 3\ \partial_m w^{(0)} \partial^m w^{(0)} - 2\frac{w^{(0)}}{\tau^{(0)}} \partial_m \tau^{(0)} \partial^m w^{(0)}$$

Source terms do not enter leading order! Constant/harmonic fields.

Equations of motion

First order

$$2\tau^{(0)}\nabla^m\nabla^n g^{(1)}_{mn} - 2\tau^{(0)}\Box g^{(1)m}{}_m - 8\Box\tau^{(1)} - 16\frac{\tau^{(0)}}{w^{(0)}}\Box w^{(1)} = \sum_i \mu(j^{(i)} - \rho^{(i)})$$

$$4\frac{\tau^{(0)}}{w^{(0)}}\Box w^{(1)} = \sum_i \mu(j^{(i)} - \rho^{(i)})$$

$$R^{(1)}_{mn} - 4\tau^{(0)}\frac{\nabla_m\partial_n w^{(1)}}{w^{(0)}} - 2\nabla_m\partial_n\tau^{(1)} = \sum_i \frac{\mu}{2}\left(\Pi^{(i)}_{mn} - \frac{1}{2}g_{mn}\right)(j^{(i)} - \rho^{(i)})$$

and in particular imply

$$\frac{\Box w^{(1)}}{w^{(0)}} = -\frac{1}{3}\frac{\Box\tau^{(1)}}{\tau^{(0)}} = -\frac{1}{2}\left(\nabla^m\nabla^n g^{(1)}_{mn} - \Box g^{(1)m}{}_m\right) \equiv \frac{1}{4}\Box\Phi$$

Singular terms at second order

Second order equations contain non-integrable divergent terms near O-plane locus

$$
\begin{aligned}
0 = & -2\tau^{(0)2}\frac{\Box w^{(2)}}{w^{(0)}} + \frac{4\tau^{(0)2}}{w^{(0)}}\left(\nabla^m w^{(1)}\nabla^n g^{(1)}_{mn} + g^{(1)}_{mn}\nabla^m\nabla^n w^{(1)}\right) \\
& + 4\tau^{(0)2}\frac{w^{(1)}}{w^{(0)}}\frac{\Box w^{(1)}}{w^{(0)}} - 8\tau^{(0)2}\frac{\tau^{(1)}}{\tau^{(0)}}\frac{\Box w^{(1)}}{w^{(0)}} - 8\tau^{(0)2}\frac{\nabla^m\tau^{(1)}}{\tau^{(0)}}\frac{\nabla_m w^{(1)}}{w^{(0)}} \\
& + |F_2^{(1)}|^2 - 12\tau^{(0)2}\frac{\nabla^m w^{(1)}\nabla_m w^{(1)}}{w^{(0)2}} - 2\tau^{(0)2}\frac{\nabla^m w^{(1)}\nabla_m g^{(1)n}{}_n}{w^{(0)2}} \\
& - \sum_i \mu\ \rho^{(i)}\ \left(\tau^{(1)} - \frac{1}{2}(g^{(1)}_\perp)^m_m\right) + \ldots
\end{aligned}
$$

must hold locally, but also integral must vanish, so singular contributions must cancel out.
All sources are identical, so cancellation must be local as well!

Near-source behavior

Expressing the first order solutions in terms of the single function Φ with near-source behavior $\frac{\mu}{r}$, the singular terms become

$$-2\frac{\Box w^{(2)}}{w^{(0)}} + \frac{5}{4}\nabla^m\Phi\nabla_m\Phi + \frac{5}{4}\Phi\Box\Phi + ... = 0$$

$w^{(2)}$ behaves as $\frac{1}{r^2}$ around the source, but contains a singularity at $r = 0$.

$\Box w^{(2)}$ integrates to zero over the whole manifold as expected from a total derivative, despite singular nature!

The $\frac{5}{4}$ coefficient leads to a match with the $\frac{1}{r}$ expansion of an $O6$ in flat space!

Similar for other fields...

Summary and Outlook

Perturbative solutions can be spurious due to linearity.
These are weeded out by constraints appearing at higher orders.

The leading order unsmearing of O6-planes passes a non-trivial consistency check! Good news for DGKT!

Not specific to DGKT. Same second order equation appears for O6 on $\mathbb{R}^7 \times M_3$.

Need to verify constraint for non-singular parts for a complete check.

Higher derivative corrections enter at the same order. Does this consistency constrain them in any way?

Other dimensions, source types etc... failure cases?