

(A)dS solutions from type II Scherk-Schwarz orbifolds

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In a warped compactification of D -dimensional gravity, de Sitter spacetime is obstructed by a **Strong Energy Condition** Maldacena, Nunez '00

$$\int_{D-d} \mathcal{R}_d + e^{2A} \tilde{T} = 0, \quad \tilde{T} := -T_{\mu}^{\mu} + \frac{d}{D-2} T_L^L,$$

SEC: at tree-level in g_s and α' , w/out localised sources, non-perturbative effects etc..

$$\tilde{T} \geq 0 \implies \mathcal{R}_d \leq 0$$

~~SEC~~ is a necessary condition for dS

classical de Sitter?

~~SEC~~ from $p + 1$ -dim localised sources, still classical corner

$$\tilde{T}_{\text{loc}} \sim T_p \delta(\Sigma) < 0 \quad \text{if} \quad T_p < 0 \implies O_p\text{-planes}$$

Very intensive research conjectured **3 open problems** [Andriot '19](#),
[references therein](#)

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1. No classical de Sitter solution with parallel sources
2. Classical de Sitter solutions with intersecting sources are **pert. unstable**
3. **No small g_s and large \mathcal{V}** within string-theory origin
(quantised fluxes, bounded $N_{O_p, \dots}$)

Refined dS conjecture: No (metastable) de Sitter in the
asymptotics of moduli space

SUSY-breaking at string scale

Let's move to the **non-supersymmetric** corner

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10d tachyon-free **non-susy** strings: $SO(16) \times SO(16)$, Sugimoto
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There are **no de Sitter solutions** Basile, Lanza '20

... still very interesting Mink, AdS solutions: "Dudas-Mourad" ,
 $AdS_3 \times S^7$, $AdS_7 \times S^3$... Dudas, Mourad '00 Mourad, Sagnotti '17

SUSY-breaking at compactification scale

Our work: Strings with SSB $M_{SUSY} \sim M_{KK}$: Scherk-Schwarz toroidal orbifolds

SEC from 1-loop (negative) "Casimir energy"

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 $g_s \sim n_{H_3} \mathcal{V}^\alpha, \alpha > 0$

Scherk-Schwarz orbifolds

IIB on a Torus $T_{ss}^n(R_{ss}) \rightarrow$ orbifold $g = (-1)^F \delta_{kk}$

F : spacetime fermion number

$$\delta_{kk} : (x_L, x_R) \rightarrow (x_L + \pi R_{ss}/2, x_R + \pi R_{ss}/2) \implies n_{kk} \rightarrow n_{kk} + \frac{F}{2}$$

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1-loop effective potential of **runaway** type:

$$V_{SS}(R_{SS}) \sim - \int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2^2} \mathcal{T}_{\text{IIB}/g} \stackrel{R_{SS} \gg 1}{\sim} \underbrace{(n_F^0 - n_B^0)}_{\Delta \text{ massless dof}} \times \text{const} \times \frac{1}{R_{SS}^{10-n}}$$

fermions massed up $\mathbf{n}_F^0 = \mathbf{0}$, bosons still massless $\mathbf{n}_B^0 = \mathbf{64}$

Can we stabilise R_{SS} within a de Sitter solution using fluxes?

Unwarped product compactifications, w/out localised sources

$$M_{1,d-1} \times \underbrace{Y^m \times T_{ss}^{10-d-m}}_{\text{internal space}}$$

Y^m : m-dimensional **curved** Euclidean manifold

NSNS H_3 , H_7 and RR F_q fluxes

$$\Phi = \Phi_0 = \text{Log}(g_s)$$

Compactification ansatz

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Useful to decompose fluxes as

$$H_3 = \sum_{p_3=0}^3 H_3^{(p_3)}, F_q = \sum_{s_q=0}^q F_q^{(s_q)}$$

with p_3 , s_q legs on Y^m and remaining $3 - p_3$, $(q - s_q)$ legs on T_{ss}^n

10d dS No-gos

Strategy:

dilaton eom + traces Einstein eqs $\implies \mathcal{R}_d \leq 0 \implies$ **dS No-gos**

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in $d \leq 5$ **dS solutions could exist, must minimally have**

$$\underbrace{\text{Negative Casimir}}_{\text{SEC if } n_b^0 > n_f^0}, \quad \mathcal{R}_\gamma > 0, \quad \mathbf{H}_3^{(0)}, \quad F_q^{(s_q)}|_{q < 5 - 2s_q}$$

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$H_3^{(0)}$: H_3 flux component with 3 legs on the SS-torus (**will spoil control!!**)

$$(s_q) = \# \text{flux legs on } Y$$

Bianchi identity and parametric control

W/out localised sources and for non-trivial background fluxes,
Bianchi identities require **tadpole-free fluxes**

$$dF_q = 0 = H_3 \wedge F_{q-2}$$

can be satisfied by distributing appropriately the flux legs in the
internal space

If so, flux numbers are **unbounded** DeWolfe, Giryavets, Kachru, Taylor'05:

Can we have

$$g_s \sim n^{-\alpha} \quad \mathcal{V} \sim n^\beta \quad \alpha, \beta > 0$$

for a (A)dS solution?

We now go in the EFT to discuss **stability** and **control** on putative dS solutions

In lower d -dimensional Einstein frame $\varphi^i = \{\phi_d, \omega, \chi\}$ **universal moduli**

$$ds_{10}^2 = e^{\frac{4}{d-2}(\phi_d - \langle \phi_d \rangle)} ds_{M_{1,d-1}}^2 + e^{2\chi} ds_{Y^m(R)}^2 + e^{2\omega} ds_{T_{SS}^n(R_{SS})}^2$$

ω, χ : string-frame volume moduli $e^{\langle \chi \rangle} = R, e^{\langle \omega \rangle} = R_{SS}$

$\phi_d := \Phi - \frac{n}{2}\omega - \frac{m}{2}\chi$ lower-dim dilaton

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Scalar potential V from dim. reduction

$$S_d = \frac{1}{2\kappa_d^2} \int d^d x \sqrt{-g_d} \left(\mathcal{R}_d - K_{ij} \partial_\mu \varphi^i \partial^\mu \varphi^j - V(\varphi^i) \right)$$

$$V = \underbrace{V_{\text{flux}} + V_{\text{curv}}}_{\text{tree-level}} + \underbrace{V_{SS}}_{\text{1-loop}}$$

Assumption: $g_s \ll 1$ and $R, R_{SS} \gg 1$ higher order corrections negligible

EFT analysis: stability

de Sitter: $\nabla V|_{\text{crit}} = 0$, $V|_{\text{crit}} > 0$

For **consistent truncations**, stability can be addressed in the EFT by inspecting Hessian/Mass-matrix $M^i_j = K^{ik} \nabla_k \partial_j V|_{\text{crit}}$

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we find for $d \geq 4$ (K_{ij} is positive def)

$$\partial_{\phi_d}^2 V|_{\text{crit}} = -2d V|_{\text{crit}} \quad \text{universal tachyon}$$

Hence, by *Sylvester's criterion*:

Any de Sitter solution ($V|_{\text{crit}} > 0$) in $d \geq 4$ is pert. unstable.

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$$\eta_V := \frac{\min(\text{Eigen}(M^i_j))}{V}|_{\text{crit}} \leq -\frac{d}{2}(d-2) \lesssim -\mathcal{O}(1)$$

EFT analysis: dS under (parametric) control?

$$M_{1,d-1} \times Y^m(R) \times T_{ss}^{10-m-d}(R_{ss})$$

Notice that:

1. Crucially $V_{ss} \sim \frac{g_s^2}{R_{ss}^{10}}$ is only suppressed in g_s and $\frac{1}{R_{ss}}$ but not in $\frac{1}{R}$
2. for any solution, terms in the potential must be comparable

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but for **dS** $H_3^{(0)}$ term i.e **$p_3 = 0$ should be leading!** \implies No control on putative dS sol ($g_s \sim n_{H_3} R_{\text{ss}}^2$)

An example of *AdS* solution under control: In IIA

$$AdS_7 \times S^2 \times S_{ss}^1$$

with $F_2^{(2)} \sim \text{vol}_{S^2}$, $H_3^{(2)} \sim \text{vol}_{S^2} \wedge \text{vol}_{S^1}$

We solved all the 10d eoms, including BI

Flux numbers $\int_{S^2} F_2 \sim n_2$, $\int_{S^2 \times S^1} H_3 \sim n_3$ are unbounded

Solution is perfectly under control ($\delta \ll 1$):

$$g_s \sim n_3^{\frac{5}{7}} n_2^{-\frac{6}{7}}, \quad R \sim n_3^{\frac{5}{7}} n_2^{\frac{1}{7}}, \quad R_{ss} \sim n_3^{\frac{2}{7}} n_2^{-\frac{1}{7}}$$

Conclusions

1. type II Scherk-Schwarz orbifolds come with a one-loop negative runaway potential that breaks the SEC
2. We looked for dS in product compactifications where the runaway could be stabilised with fluxes
3. We individuated dS no-gos as well as minimally required ingredients for dS solutions
4. for dS , we found a **tachyon** in the set of universal moduli \implies **instability** of consistent truncations
5. **dS solutions cannot be under control.** In contrast, we provide an **AdS solution** to the 10d eoms that shows **parametric control**.

...Thank you!