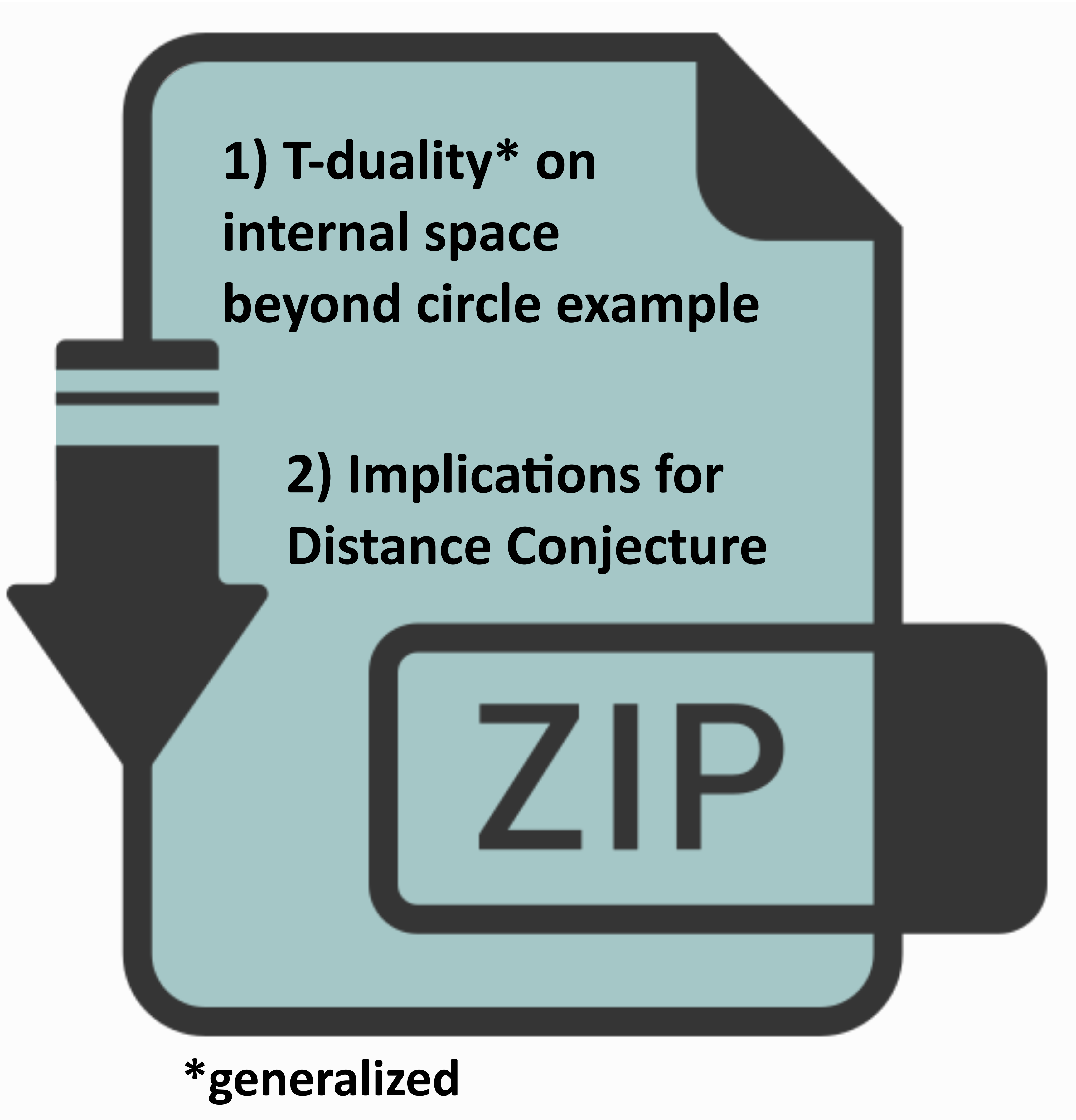
MAX PLANCK INSTITUTE
FOR PHYSICS

Topology change and non-geometry at infinite distance

[Saskia Demulder, Dieter Lüst, **TR**; **2312.07674**]

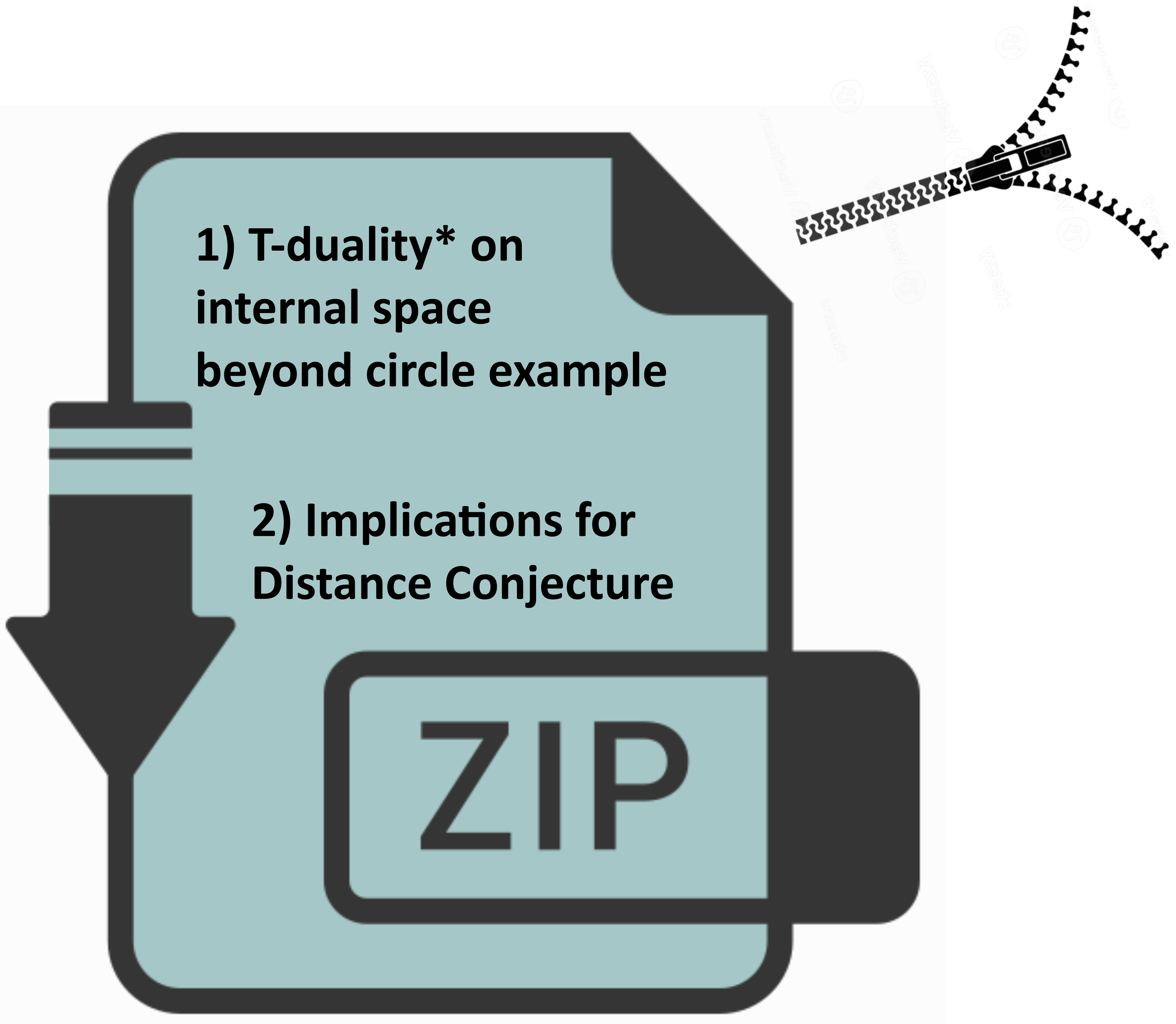
String Phenomenology 24
Padova, 27.06.2024

Thomas Raml

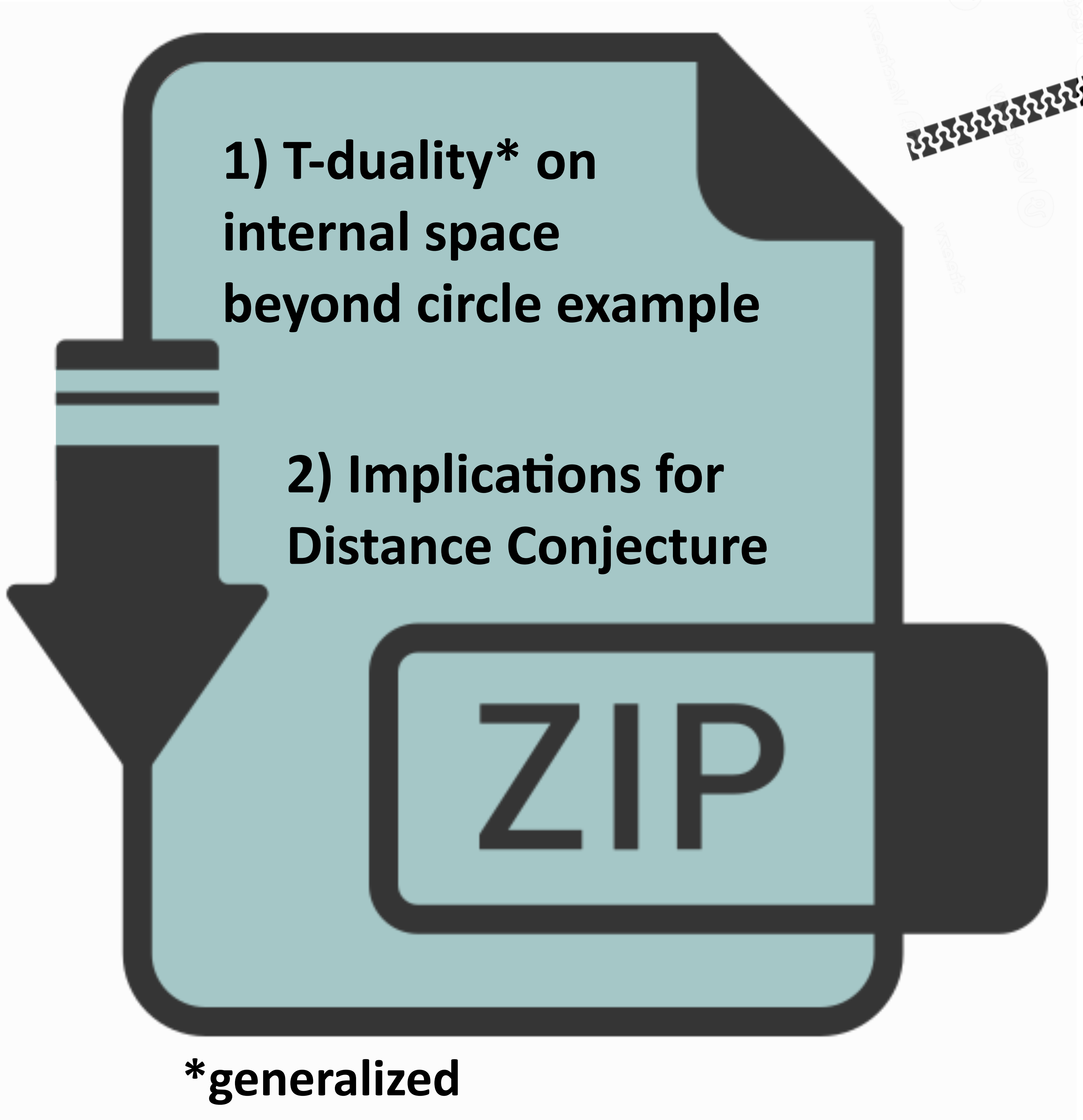

1) T-duality* on internal space beyond circle example
2) Implications for Distance Conjecture
ZIP
*generalized

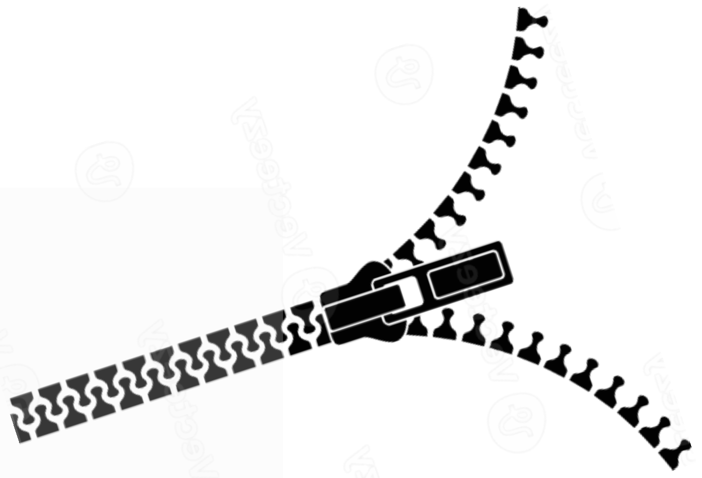
1) T-duality* on internal space beyond circle example

2) Implications for Distance Conjecture



*generalized

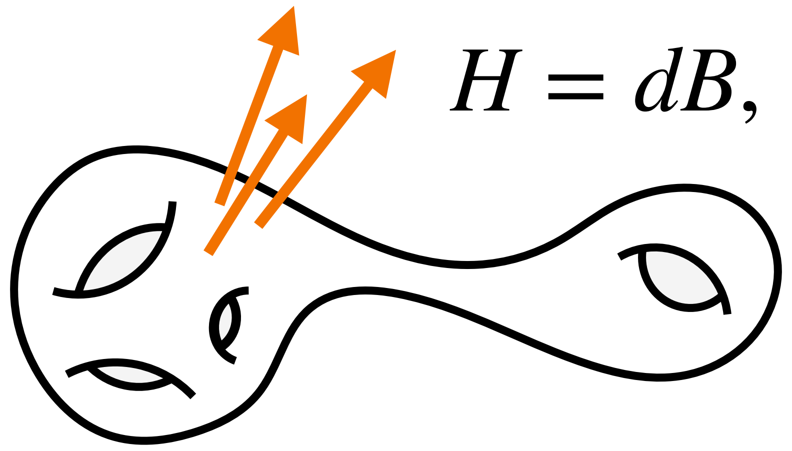




- Non-trivial fibrations / Curved manifolds
- Fluxes

T-duality

$H = dB, \quad F = dC$



1) T-duality* on internal space beyond circle example

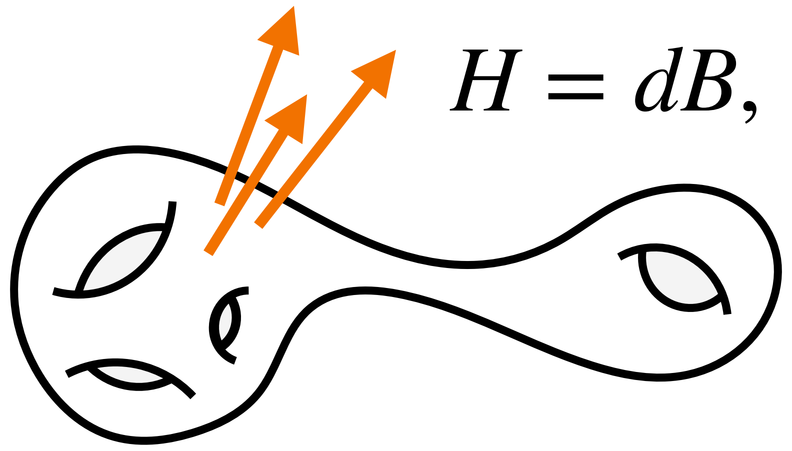
2) Implications for Distance Conjecture

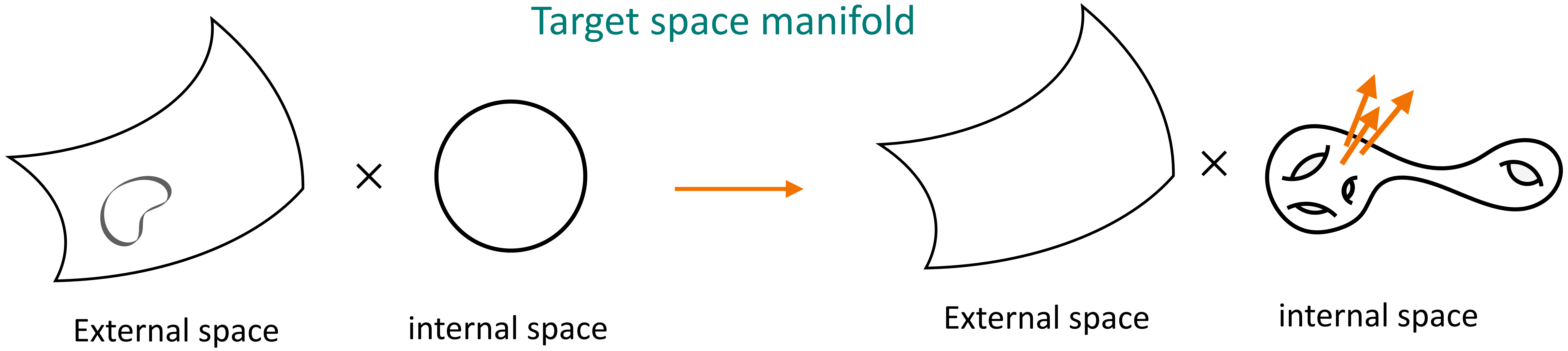
*generalized

- ▷ Non-trivial fibrations / Curved manifolds
- ▷ Fluxes

T-duality

$$H = dB, \quad F = dC$$





1) T-duality* on internal space beyond circle example

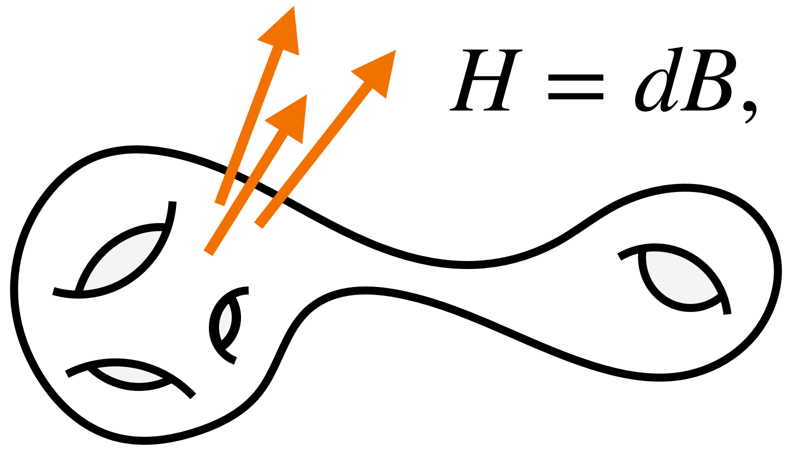
2) Implications for Distance Conjecture

*generalized

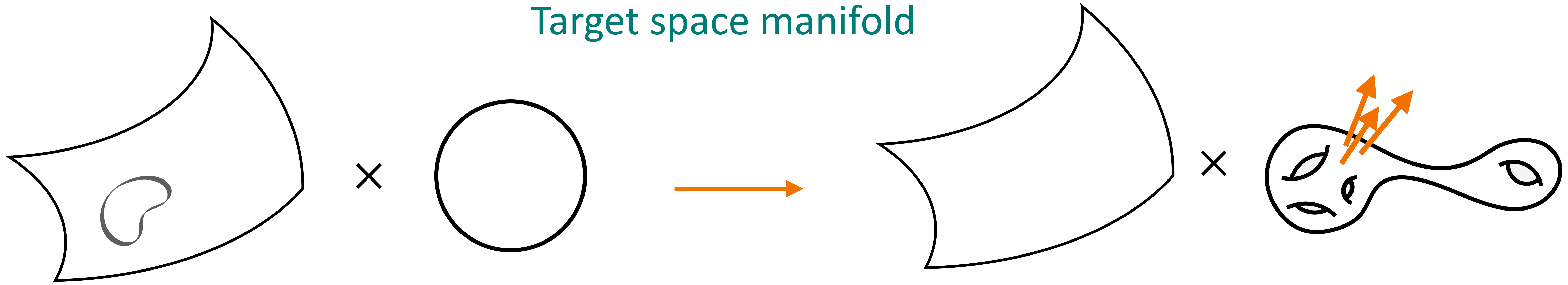
- ▷ Non-trivial fibrations / Curved manifolds
- ▷ Fluxes

T-duality

$$H = dB, \quad F = dC$$



Target space manifold



- ▷ Non-trivial momentum-winding exchange
- ▷ Moduli spaces with (NSNS) flux contributions
- ▷ Scalar potential on moduli space
- ▷ Non-geometric backgrounds

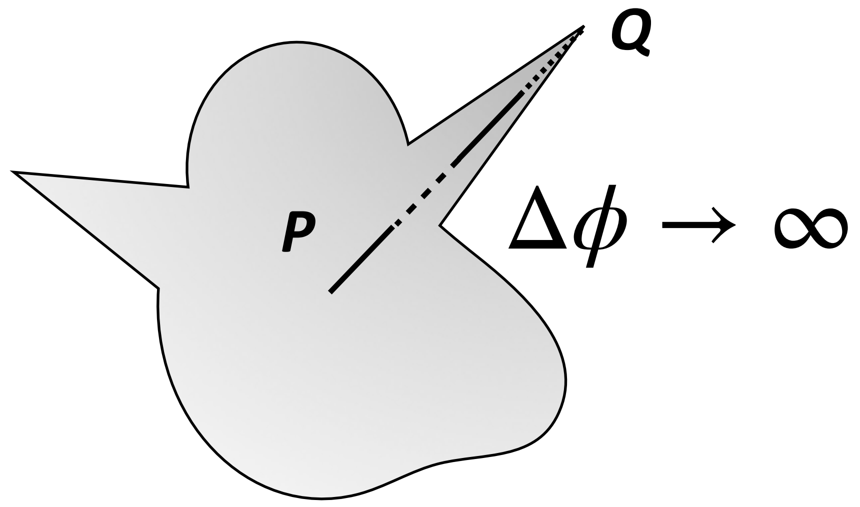
Recap: Distance Conjecture & S^1

In any consistent theory of quantum gravity: [Ooguri, Vafa '06]

When going to **large distances in its moduli space**,
encounter an **infinite tower of states** which **become light** exponentially

$$M(Q) \sim M(P)e^{-\lambda\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty\,, \quad \Delta\phi \equiv d(P,Q)$$

Moduli space



describes the parameters
of the internal space

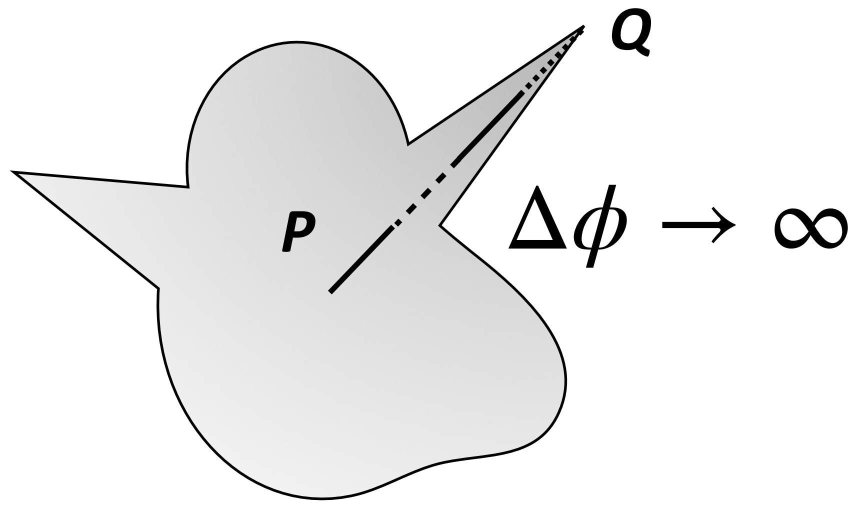
Recap: Distance Conjecture & S^1

In any consistent theory of quantum gravity: [Ooguri, Vafa '06]

When going to **large distances in its moduli space**,
encounter an **infinite tower of states** which **become light** exponentially

$$M(Q) \sim M(P)e^{-\lambda\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty\,, \qquad \Delta\phi \equiv d(P,Q)$$

Moduli space



describes the parameters
of the internal space

Example: Circle compactification

$$S_{\mathrm{EH}} \sim \int \mathrm{d}^{D-1}x\,\sqrt{-g}\left(\mathscr{R}(g) - \frac{c}{R^2}(\partial R)^2\right)$$

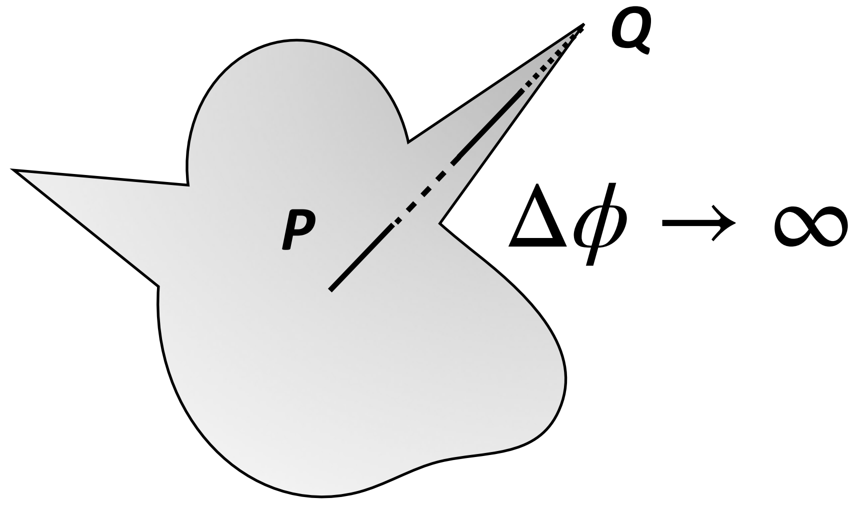
Moduli space:

$R = 0$
infinite distance point

$R \to \infty$
infinite distance point

Recap: Distance Conjecture & S^1

Moduli space



describes the parameters
of the internal space

In any consistent theory of quantum gravity: [Ooguri, Vafa '06]

When going to **large distances in its moduli space**,
encounter an **infinite tower of states** which **become light** exponentially

$$M(Q) \sim M(P)e^{-\lambda\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty\,, \quad \Delta\phi \equiv d(P,Q)$$

Example: Circle compactification

$$S_{\mathrm{EH}} \sim \int \mathrm{d}^{D-1}x\sqrt{-g}\left(\mathcal{R}(g) - \frac{c}{R^2}(\partial R)^2\right)$$

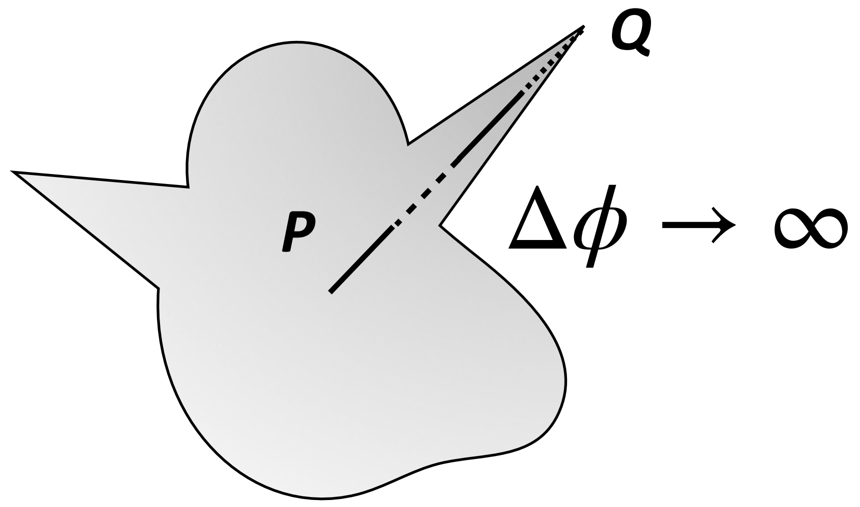
For $R \to 0$
Infinite tower of
massless **KK**-modes
$$m_{KK}^2 \sim \frac{1}{R^2}$$

&

For $R \to \infty$
Infinite tower of
massless **winding**-modes
$$m_w^2 \sim R^2$$

Recap: Distance Conjecture & S^1

Moduli space



describes the parameters
of the internal space

In any consistent theory of quantum gravity: [Ooguri, Vafa '06]

When going to **large distances in its moduli space**,
encounter an **infinite tower of states** which **become light** exponentially

$$M(Q) \sim M(P)e^{-\lambda\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty\,, \qquad \Delta\phi \equiv d(P,Q)$$

Example: Circle compactification

$$S_{\mathrm{EH}} \sim \int \mathrm{d}^{D-1}x\sqrt{-g}\left(\mathscr{R}(g) - \frac{c}{R^2}(\partial R)^2\right)$$

Moduli space:

$R = 0$
infinite distance point

$R \to \infty$
infinite distance point

For $R \to 0$
Infinite tower of
massless **KK**-modes

$$m_{KK}^2 \sim \frac{1}{R^2}$$

&

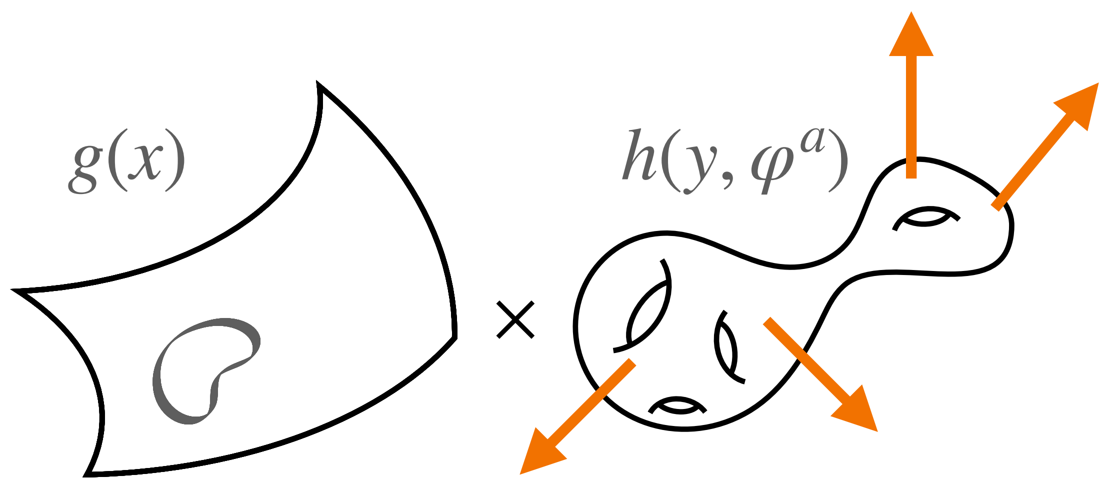
For $R \to \infty$
Infinite tower of
massless **winding**-modes

$$m_w^2 \sim R^2$$

T-duality

What about more complicated compact geometries?

What about more complicated compact geometries?



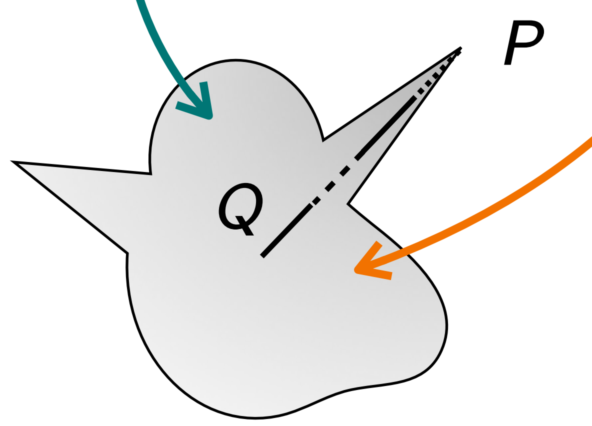
$$S = \frac{1}{2\kappa_0^2} \int \mathrm{d}^D X \sqrt{-G} e^{-2\Phi} \left(\mathscr{R}(G) - \frac{1}{12} H_{IJK} H^{IJK} + 4 \partial_I \Phi \partial^I \Phi \right)$$

$$G(x, y) = g(x) \oplus h(y, \varphi^a(x))$$

$$S \sim \int \mathrm{d}^{D-n} x \sqrt{-g} \left(\mathscr{R}(g) - \gamma_{ab} \partial_\mu \varphi^a \partial^\mu \varphi^b - V(\varphi^a) \right)$$

metric

potential



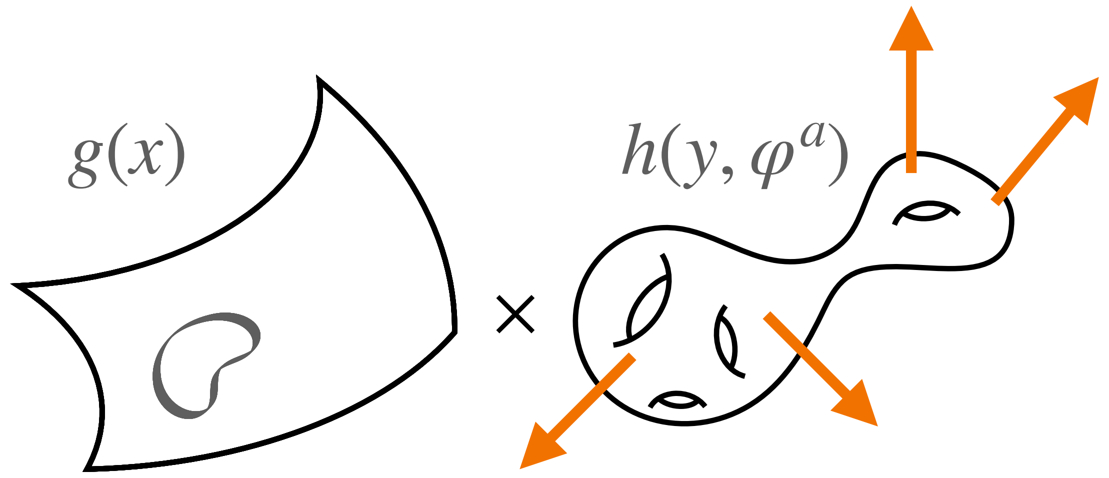
$$\gamma_{ab} \sim \int \mathrm{d}^n y \sqrt{h} \left(\mathrm{tr}\left(h^{-1} \partial_{\varphi_a} h \; h^{-1} \partial_{\varphi_b} h\right) - \mathrm{tr}\left(h^{-1} \partial_{\varphi_a} B \; h^{-1} \partial_{\varphi_b} B\right) \right)$$

$$V(\varphi^i) \sim \int \mathrm{d}^n y \sqrt{h} \left(\mathscr{R}(h) - \frac{1}{12} H_{ijk} H^{ijk} + 4 \partial_i \Phi \partial^i \Phi \right)$$

Moduli space

What about more complicated compact geometries?

A much more challenging question…



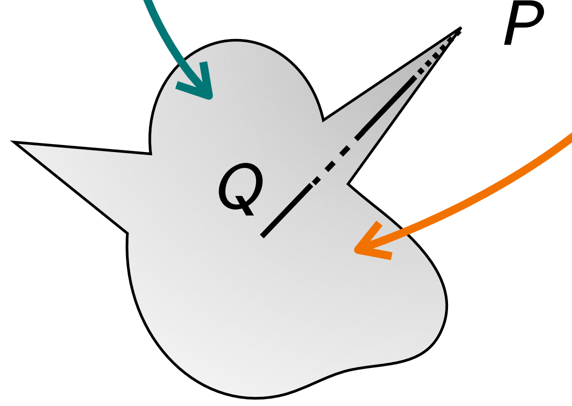
$$S = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D X \sqrt{-G}\, e^{-2\Phi}\left(\mathscr{R}(G) - \frac{1}{12}H_{IJK}H^{IJK} + 4\partial_I\Phi\partial^I\Phi\right)$$

$$G(x,y) = g(x) \oplus h(y, \varphi^a(x))$$

$$S \sim \int \mathrm{d}^{D-n}x\,\sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b - V(\varphi^a)\right)$$

metric

potential



$$\gamma_{ab} \sim \int \mathrm{d}^n y\sqrt{h}\left(\mathrm{tr}\left(h^{-1}\partial_{\varphi_a}h\; h^{-1}\partial_{\varphi_b}h\right) - \mathrm{tr}\left(h^{-1}\partial_{\varphi_a}B\; h^{-1}\partial_{\varphi_b}B\right)\right)$$

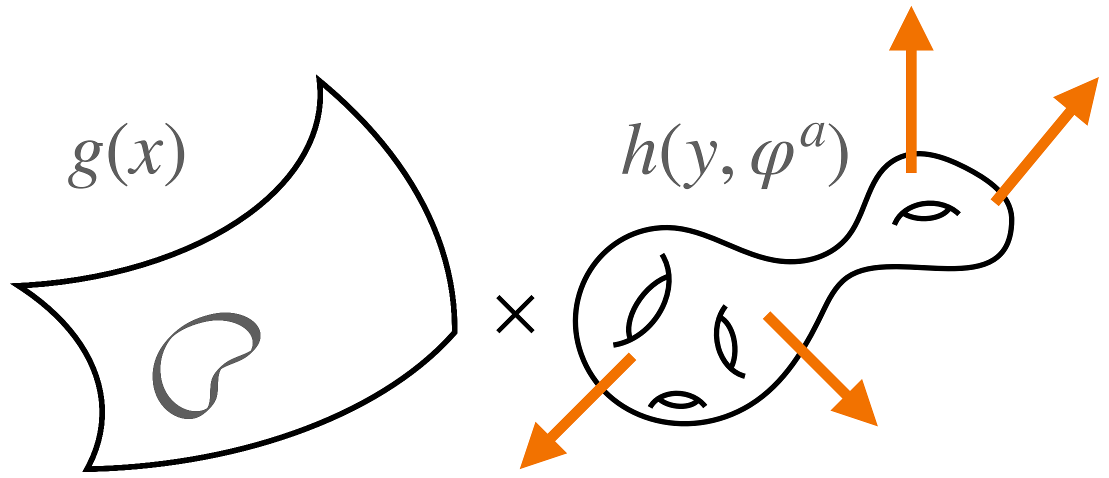
$$V(\varphi^i) \sim \int \mathrm{d}^n y\sqrt{h}\left(\mathscr{R}(h) - \frac{1}{12}H_{ijk}H^{ijk} + 4\partial_i\Phi\partial^i\Phi\right)$$

Moduli space

What about more complicated compact geometries?

A much more challenging question…

▷ Backgrounds display **curvature and/or fluxes**: sources a **scalar potential**



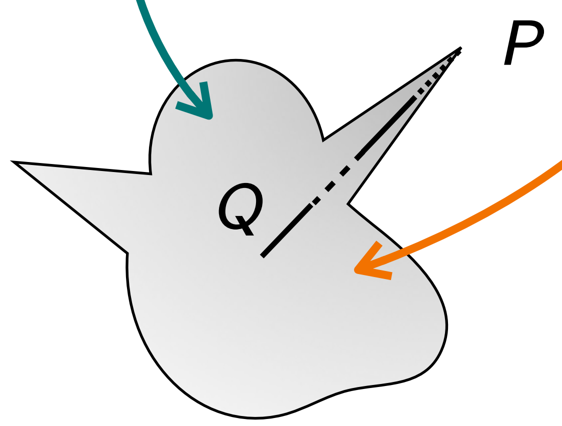
$$S = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D X \sqrt{-G}\, e^{-2\Phi}\left(\mathscr{R}(G) - \frac{1}{12}H_{IJK}H^{IJK} + 4\partial_I \Phi \partial^I \Phi\right)$$

$$G(x,y) = g(x) \oplus h(y, \varphi^a(x))$$

$$S \sim \int \mathrm{d}^{D-n}x \sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu \varphi^a \partial^\mu \varphi^b - V(\varphi^a)\right)$$

metric

potential



$$\gamma_{ab} \sim \int \mathrm{d}^n y \sqrt{h}\left(\mathrm{tr}\left(h^{-1}\partial_{\varphi_a} h\; h^{-1}\partial_{\varphi_b} h\right) - \mathrm{tr}\left(h^{-1}\partial_{\varphi_a} B\; h^{-1}\partial_{\varphi_b} B\right)\right)$$

$$V(\varphi^i) \sim \int \mathrm{d}^n y \sqrt{h}\left(\mathscr{R}(h) - \frac{1}{12}H_{ijk}H^{ijk} + 4\partial_i \Phi \partial^i \Phi\right)$$

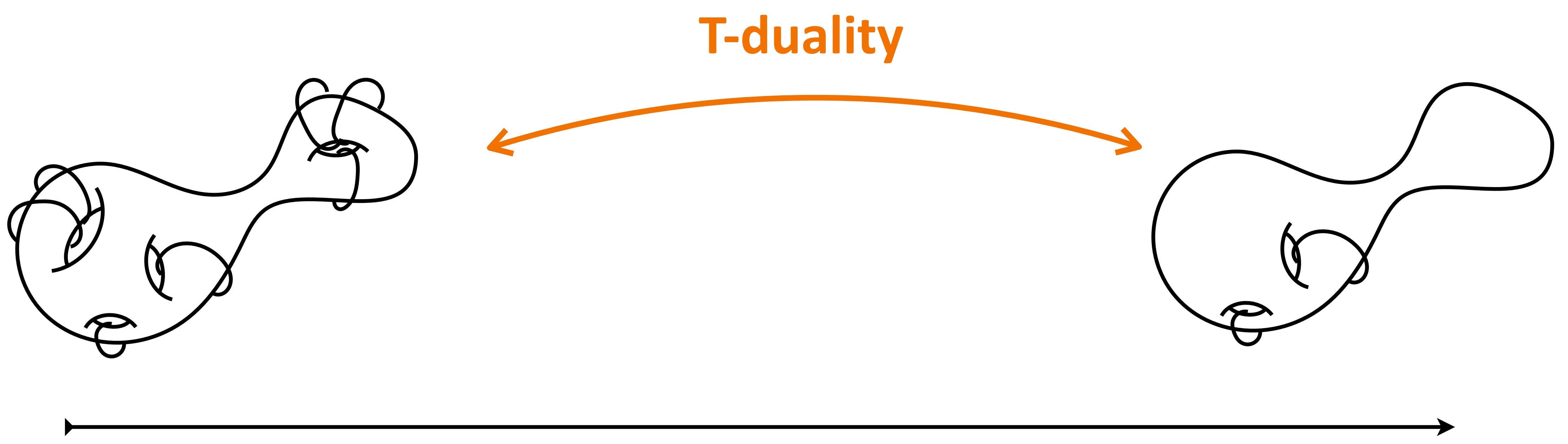
Moduli space

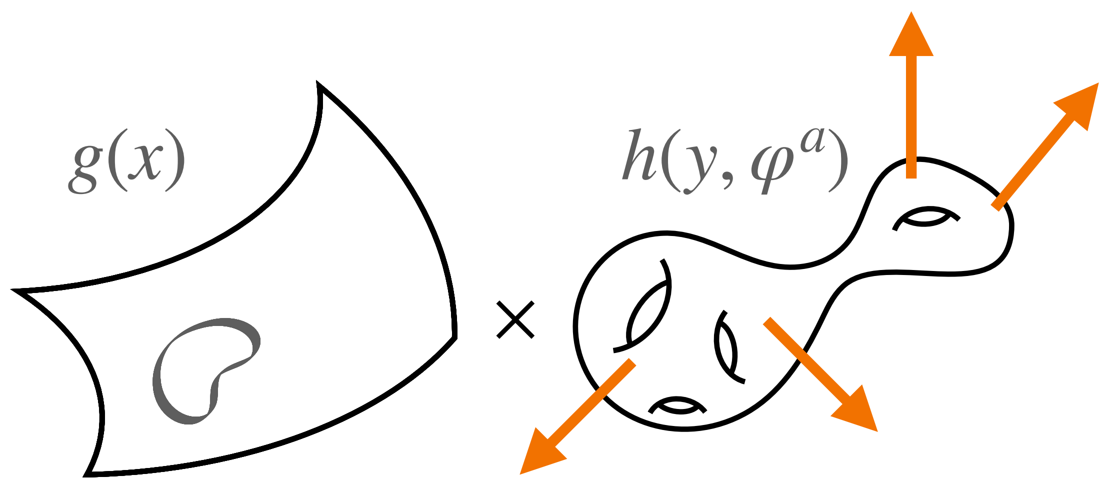
What about more complicated compact geometries?

A much more challenging question...

▷ Backgrounds display **curvature and/or fluxes**: sources a **scalar potential**

▷ Under T-duality may display **changes in topology**

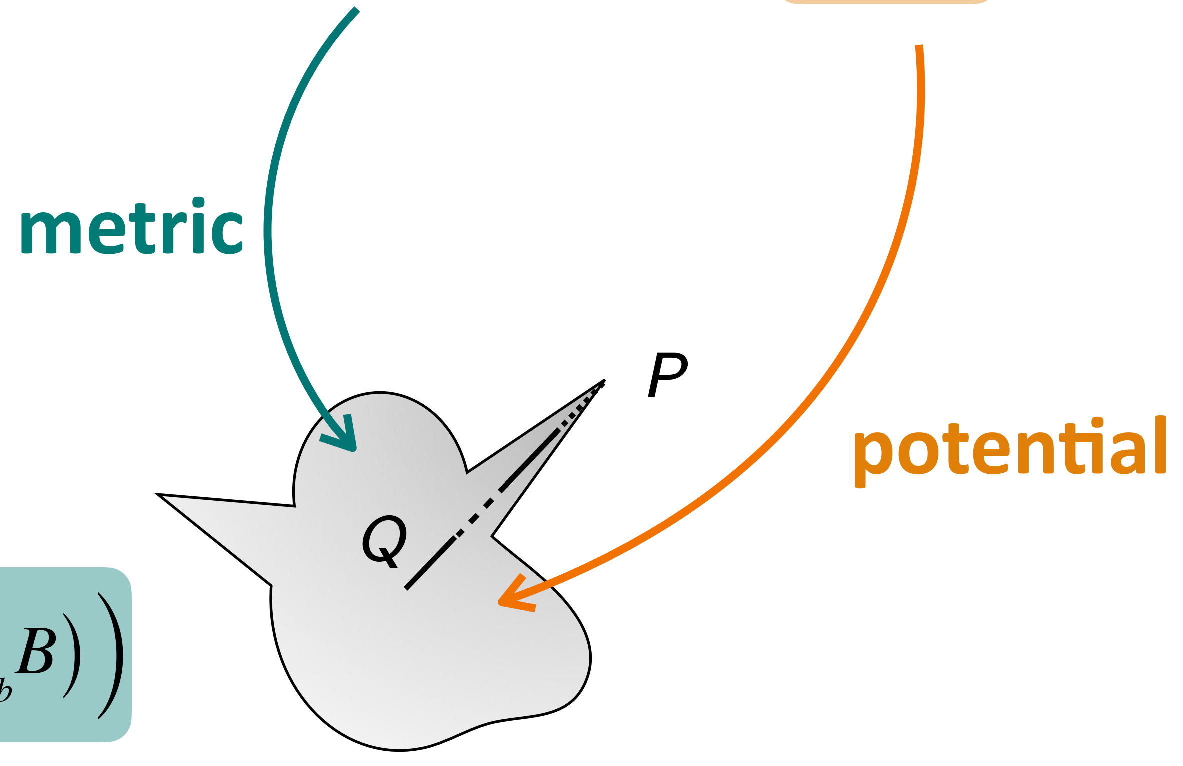




$$S = \frac{1}{2\kappa_0^2} \int \mathrm{d}^D X \sqrt{-G} e^{-2\Phi} \left(\mathscr{R}(G) - \frac{1}{12} H_{IJK} H^{IJK} + 4 \partial_I \Phi \partial^I \Phi \right)$$

$$G(x, y) = g(x) \oplus h(y, \varphi^a(x))$$

$$S \sim \int \mathrm{d}^{D-n} x \sqrt{-g} \left(\mathscr{R}(g) - \gamma_{ab} \partial_\mu \varphi^a \partial^\mu \varphi^b - V(\varphi^a) \right)$$



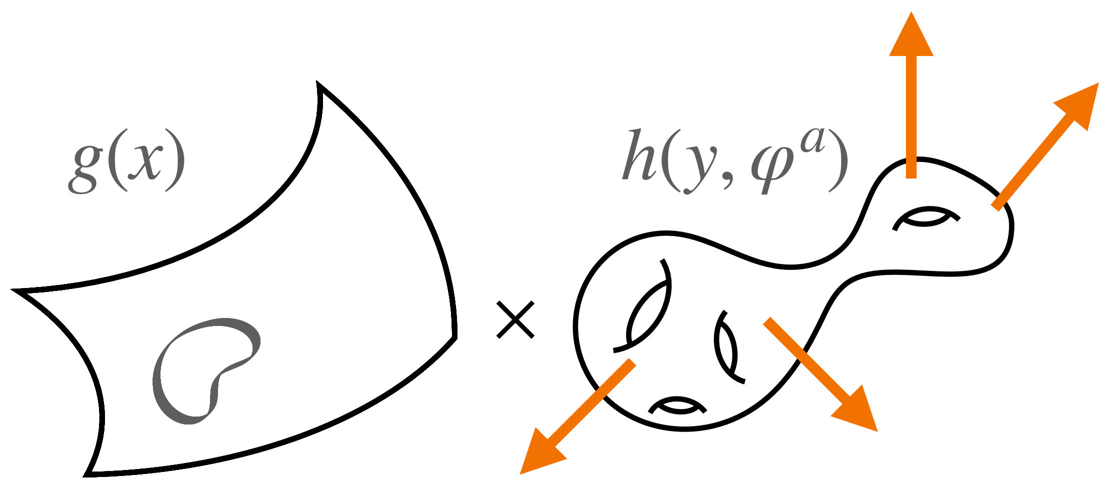
Moduli space

$$\gamma_{ab} \sim \int \mathrm{d}^n y \sqrt{h} \left(\mathrm{tr}\left(h^{-1} \partial_{\varphi_a} h \; h^{-1} \partial_{\varphi_b} h\right) - \mathrm{tr}\left(h^{-1} \partial_{\varphi_a} B \; h^{-1} \partial_{\varphi_b} B\right) \right)$$

$$V(\varphi^i) \sim \int \mathrm{d}^n y \sqrt{h} \left(\mathscr{R}(h) - \frac{1}{12} H_{ijk} H^{ijk} + 4 \partial_i \Phi \partial^i \Phi \right)$$

What about more complicated compact geometries?

A much more challenging question…

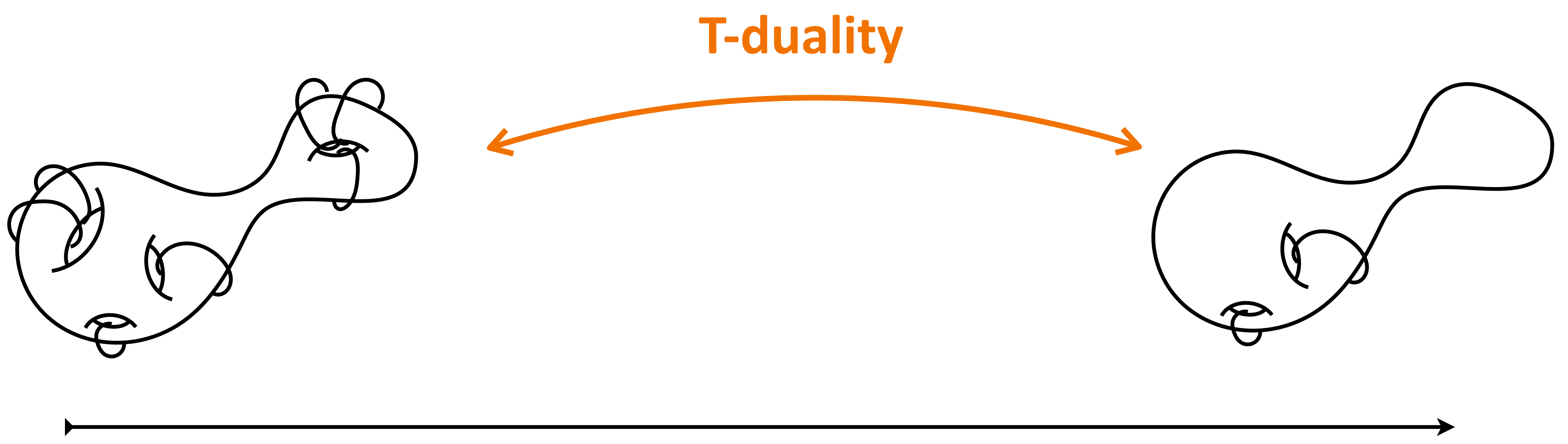


$$S = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D X \sqrt{-G}\, e^{-2\Phi}\left(\mathscr{R}(G) - \frac{1}{12}H_{IJK}H^{IJK} + 4\partial_I\Phi\partial^I\Phi\right)$$

- ▷ Backgrounds display **curvature and/or fluxes**: sources a **scalar potential**

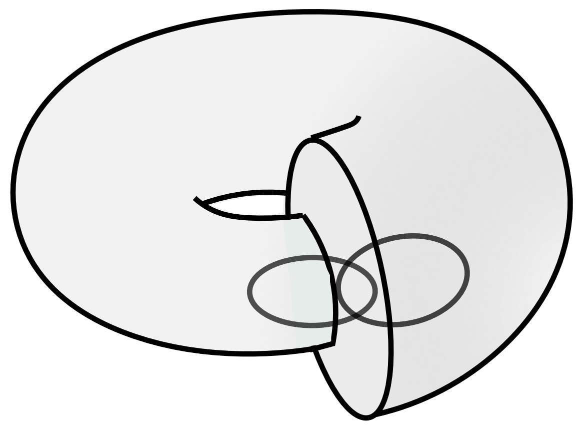
$$G(x,y) = g(x) \oplus h(y, \varphi^a(x))$$

- ▷ Under T-duality may display **changes in topology**



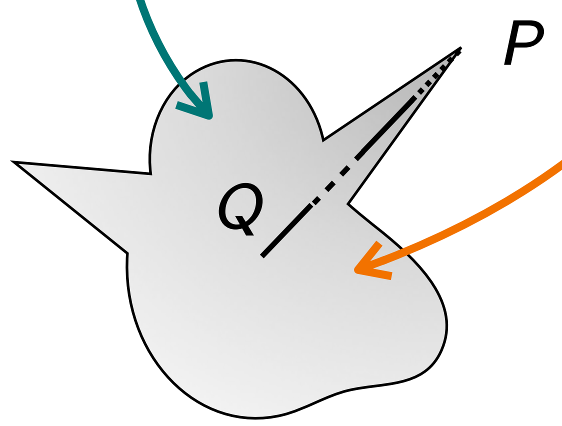
$$S \sim \int \mathrm{d}^{D-n}x\sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b - V(\varphi^a)\right)$$

- ▷ **Non-geometric backgrounds**



metric

potential



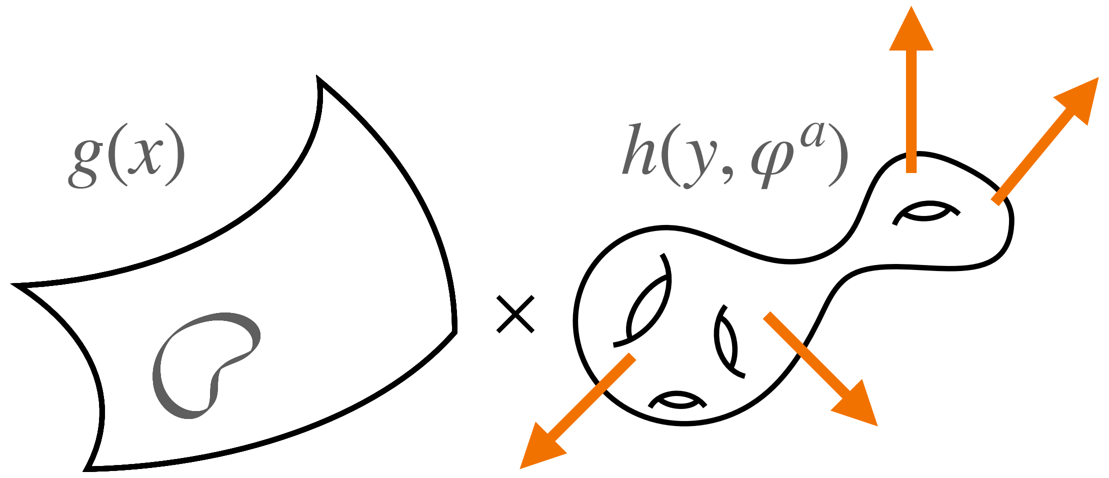
$$\gamma_{ab} \sim \int \mathrm{d}^n y \sqrt{h}\left(\mathrm{tr}\left(h^{-1}\partial_{\varphi_a}h\; h^{-1}\partial_{\varphi_b}h\right) - \mathrm{tr}\left(h^{-1}\partial_{\varphi_a}B\; h^{-1}\partial_{\varphi_b}B\right)\right)$$

$$V(\varphi^i) \sim \int \mathrm{d}^n y\sqrt{h}\left(\mathscr{R}(h) - \frac{1}{12}H_{ijk}H^{ijk} + 4\partial_i\Phi\partial^i\Phi\right)$$

Moduli space

What about more complicated compact geometries?

A much more challenging question…

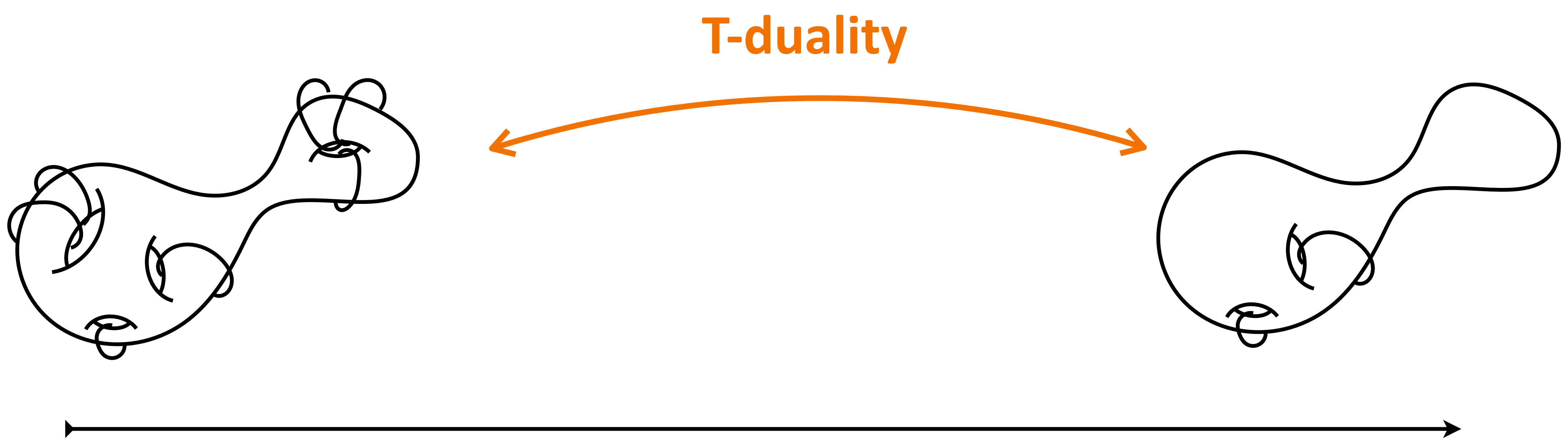


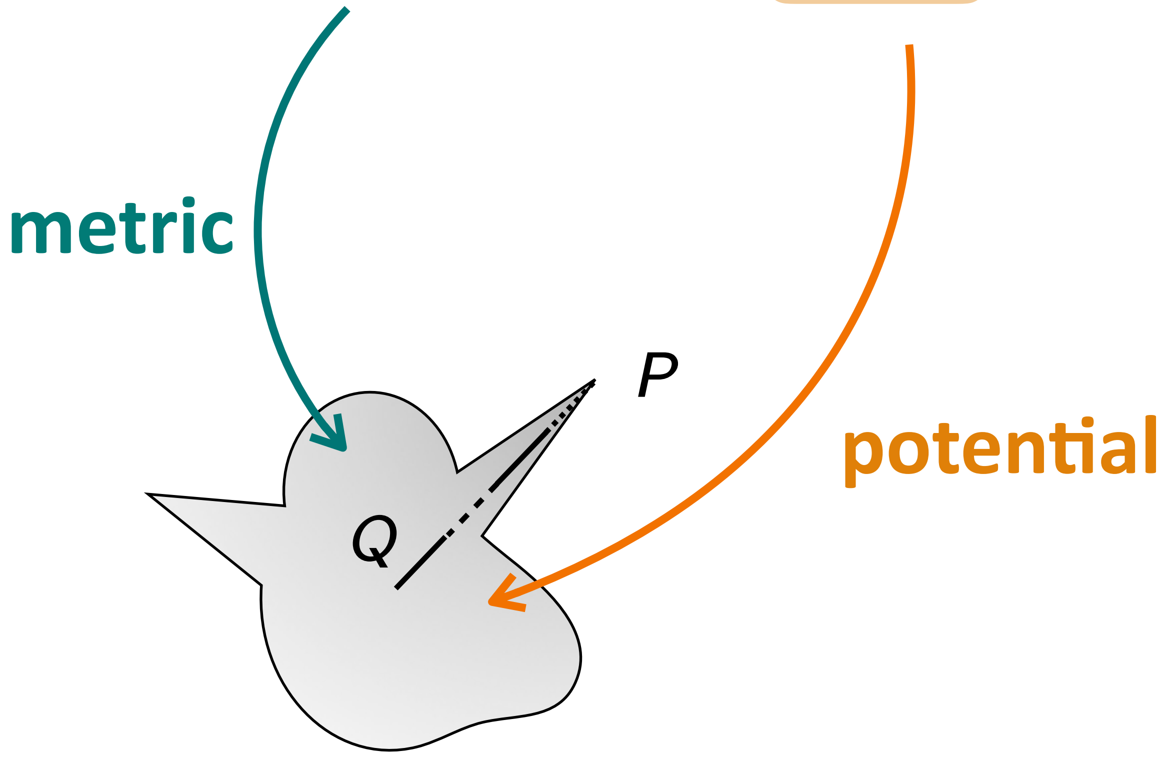
$$S = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D X \sqrt{-G}\, e^{-2\Phi}\left(\mathcal{R}(G) - \frac{1}{12}H_{IJK}H^{IJK} + 4\partial_I\Phi\partial^I\Phi\right)$$

$$G(x,y) = g(x) \oplus h(y, \varphi^a(x))$$

$$S \sim \int \mathrm{d}^{D-n}x\sqrt{-g}\left(\mathcal{R}(g) - \gamma_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b - V(\varphi^a)\right)$$

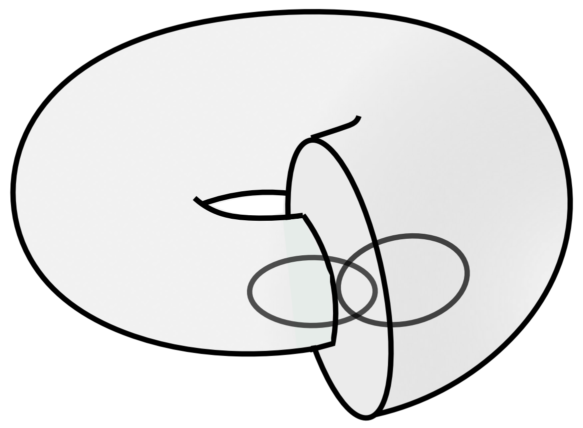
- ▷ Backgrounds display **curvature and/or fluxes**: sources a **scalar potential**
- ▷ Under T-duality may display **changes in topology**





Moduli space

- ▷ **Non-geometric backgrounds**



Do these properties modify the Swampland Distance Conjecture?

Example: S^3 with H-flux

$$S_{\text{EH}} \sim \int \mathrm{d}^d x \sqrt{-g} \left(\mathscr{R}(g) - \gamma_{ab} \partial_\mu \varphi^a \partial^\mu \varphi^b - V(\varphi^a) \right)$$

$$ds^2 = R^2 (\mathrm{d}\eta^2 + \mathrm{d}\xi_1^2 + \mathrm{d}\xi_2^2 + 2\cos(\eta) \mathrm{d}\xi_1^2 \mathrm{d}\xi_2^2)$$

$$H = k \sin(\eta) \mathrm{d}\eta \wedge \mathrm{d}\xi_1 \wedge \mathrm{d}\xi_2$$

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R; k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

Example: S^3 with H-flux

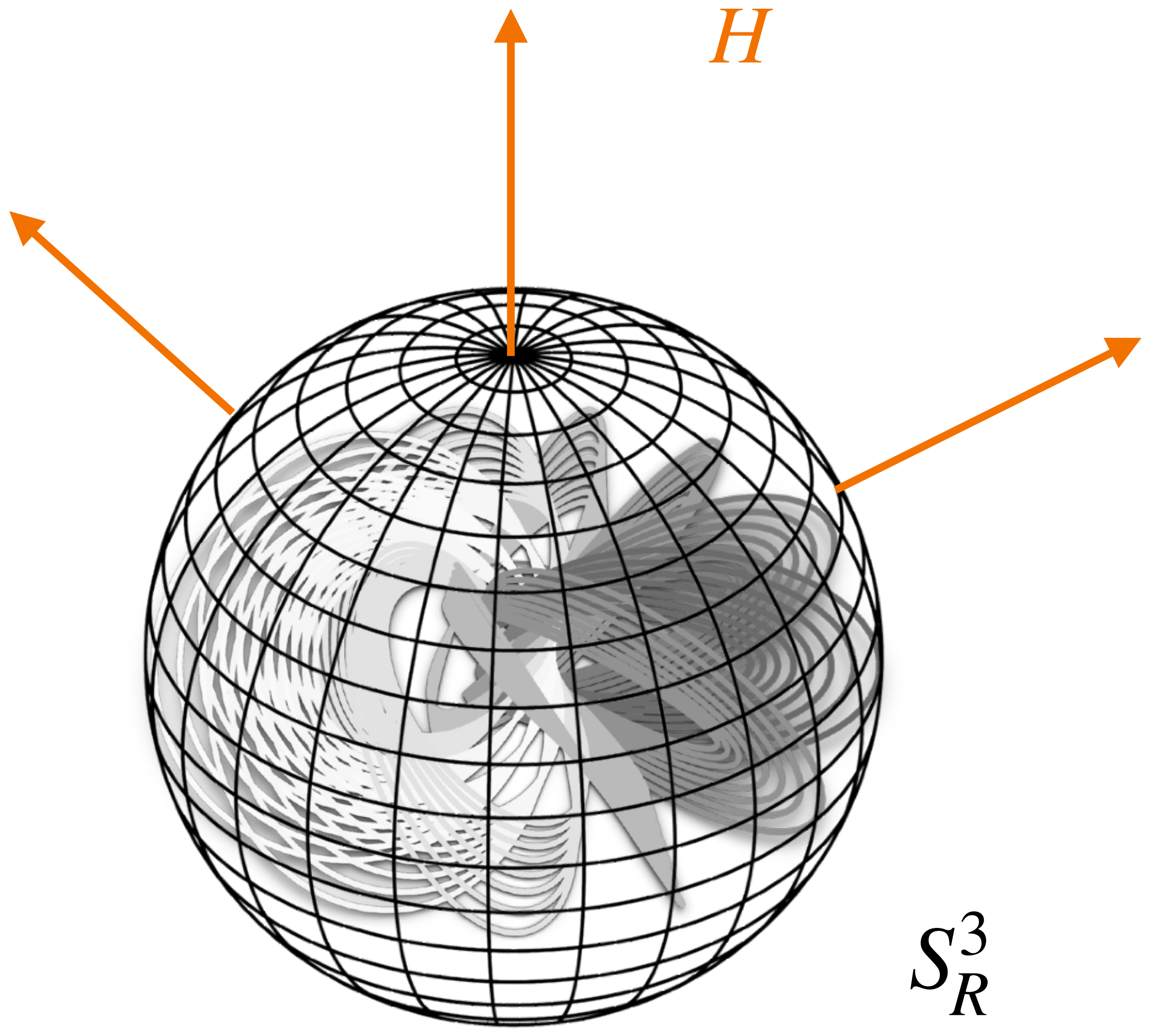
$$S_{\text{EH}} \sim \int \mathrm{d}^d x \sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b - V(\varphi^a)\right)$$

$$ds^2 = R^2(\mathrm{d}\eta^2 + \mathrm{d}\xi_1^2 + \mathrm{d}\xi_2^2 + 2\cos(\eta)\mathrm{d}\xi_1^2\mathrm{d}\xi_2^2)$$

$$H = k\sin(\eta)\mathrm{d}\eta \wedge \mathrm{d}\xi_1 \wedge \mathrm{d}\xi_2$$

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



~~winding~~

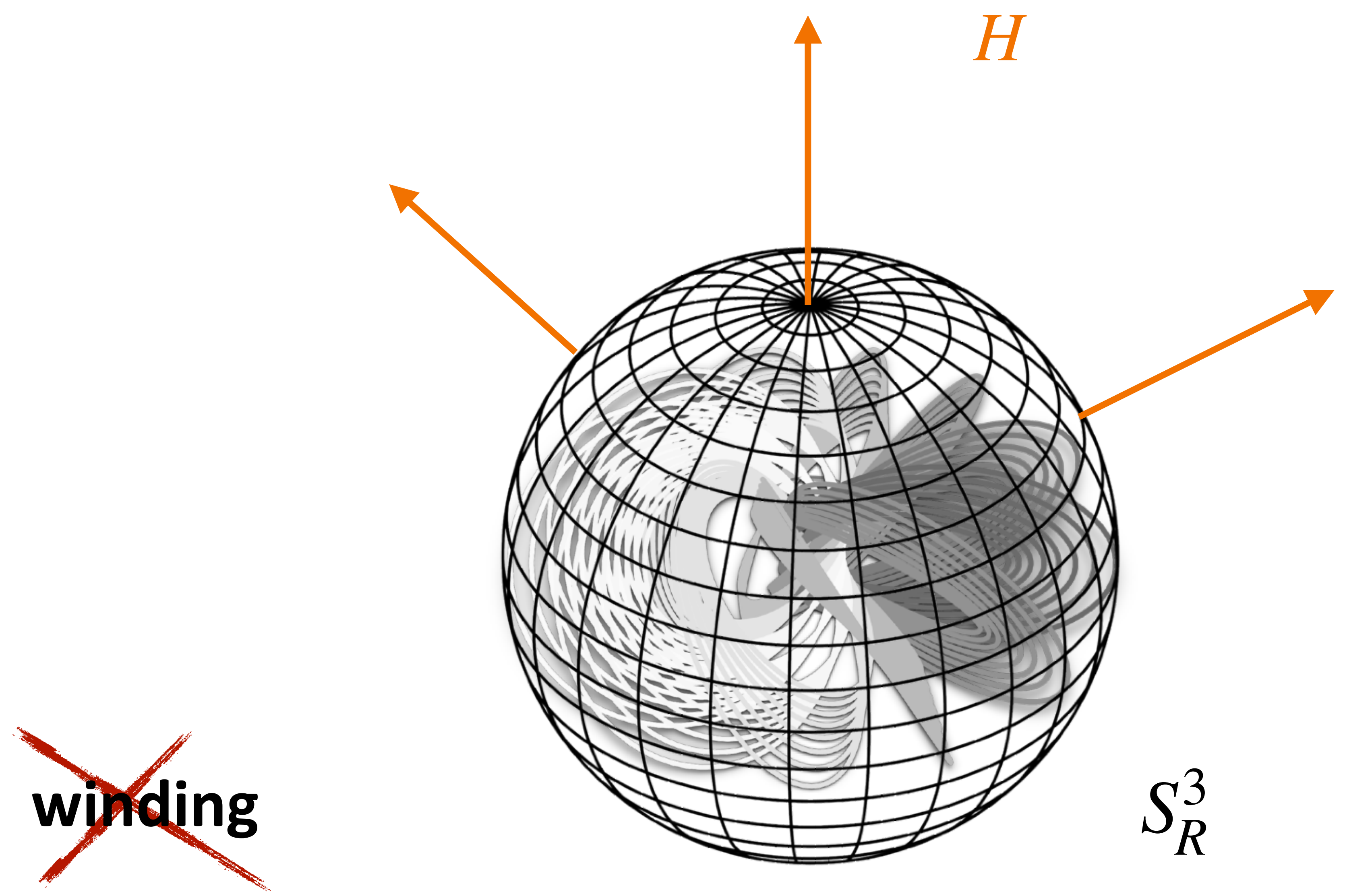
$\pi_1(S^3) = 0$

Example: S^3 with H-flux

$$S_{\text{EH}} \sim \int \mathrm{d}^d x \sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b - V(\varphi^a)\right)$$

$$ds^2 = R^2(\mathrm{d}\eta^2 + \mathrm{d}\xi_1^2 + \mathrm{d}\xi_2^2 + 2\cos(\eta)\mathrm{d}\xi_1^2\mathrm{d}\xi_2^2)$$
$$H = k\sin(\eta)\mathrm{d}\eta \wedge \mathrm{d}\xi_1 \wedge \mathrm{d}\xi_2$$

$$\gamma_{RR} = \frac{3}{R^2}$$
$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



$\pi_1(S^3) = 0$

How is **absence of winding modes** compatible with **T-duality**?

What does this mean for the **Swampland Distance Conjecture**?

▸ T-duality:

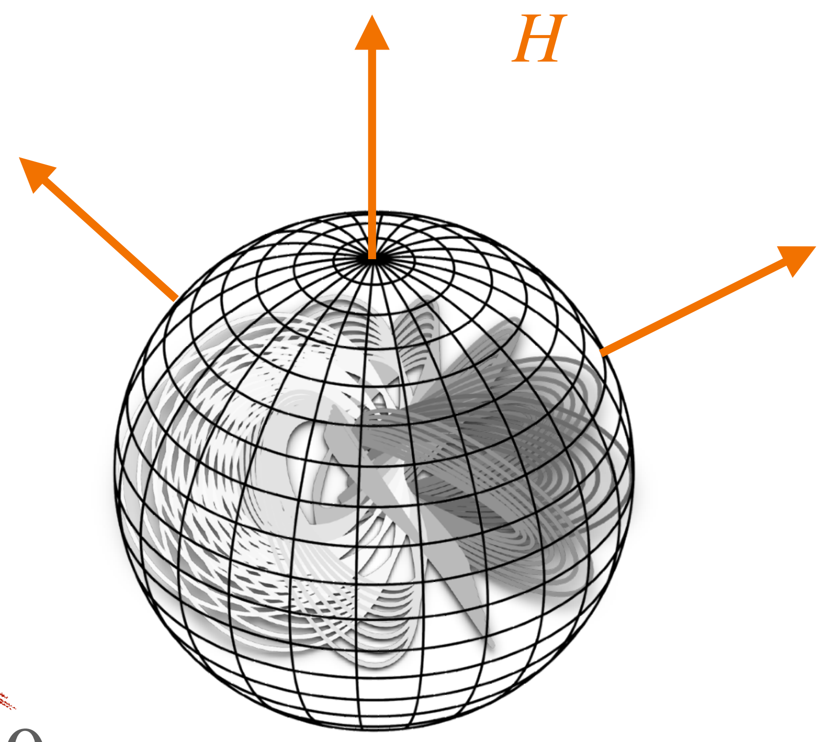
$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$

T-duality ⟷ along Hopf-fiber

$\mathbf{S^1_{1/R}} \hookrightarrow \tilde{\mathbf{E}}$ with $[\tilde{\mathbf{H}}] = \mathbf{1}$

$\downarrow$

$\mathbf{S^2_R}$



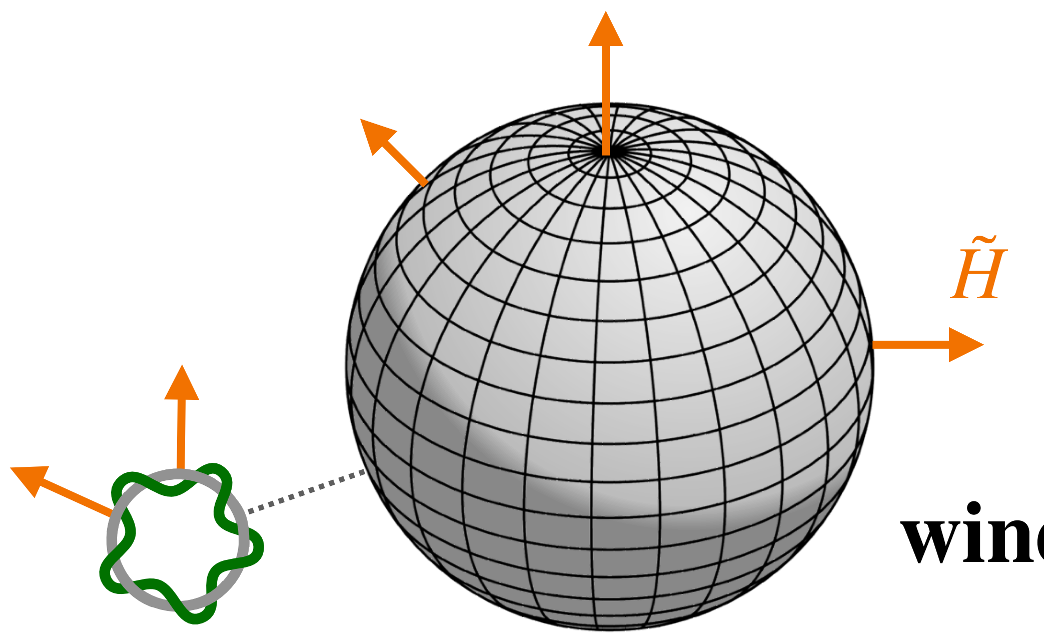
~~winding~~

$\pi_1(S^3) = 0$

$c_1(S^3) = 1$

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



winding $\in \mathbb{Z}_k$

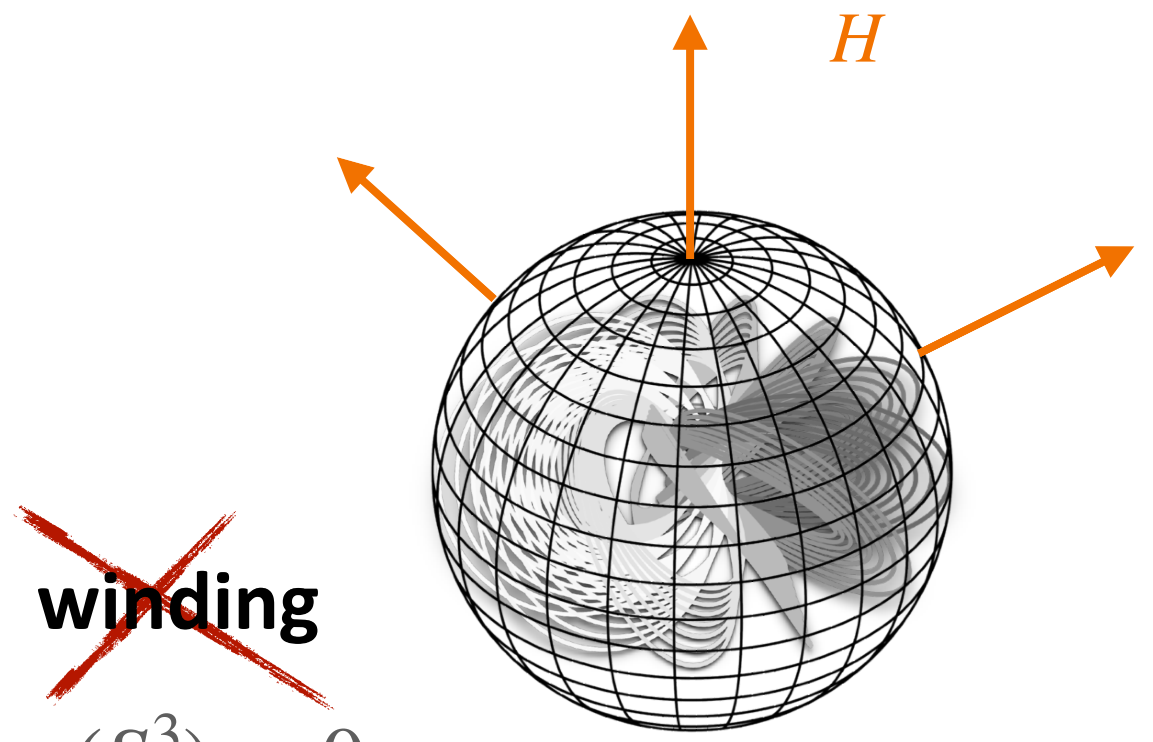
$\pi_1(\tilde{E}) = \mathbb{Z}_k$

$S^1_{1/R}$ S^2_R

$c_1(\tilde{E}) = k$

▸ T-duality:

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$ ⟷ T-duality along Hopf-fiber ⟷ $\mathbf{S^1_{1/R}} \hookrightarrow \tilde{\mathbf{E}} \to \mathbf{S^2_R}$ with $[\tilde{\mathbf{H}}] = \mathbf{1}$

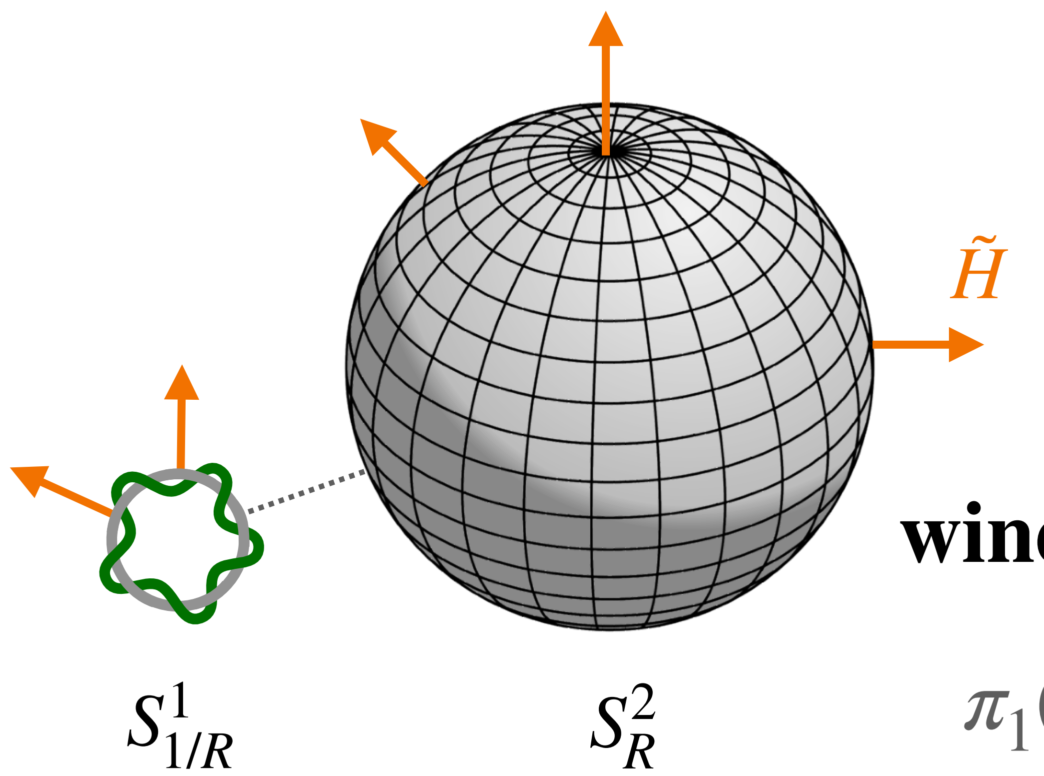


$\pi_1(S^3) = 0$

$c_1(S^3) = 1$

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



winding $\in \mathbb{Z}_k$

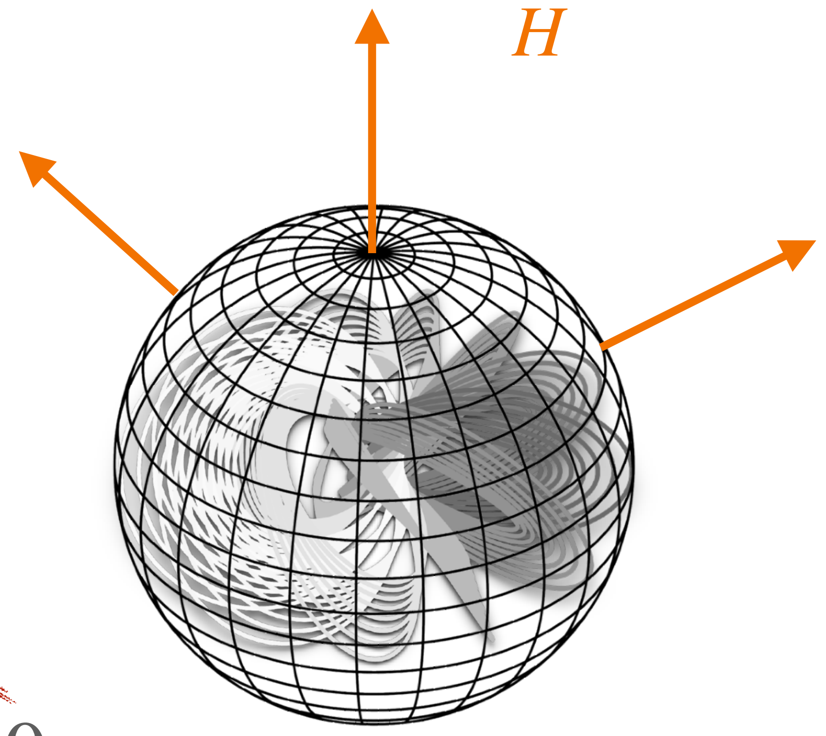
$\pi_1(\tilde{E}) = \mathbb{Z}_k$

$c_1(\tilde{E}) = k$

		$R = 0$	$R \to \infty$
S^3	winding	$\varnothing$	$\varnothing$
	momentum	heavy	light
$\tilde{E}$	winding	$\mathbb{Z}_k$ (heavy)	$\mathbb{Z}_k$ (light)
	momentum	heavy/ non-conserved	light
		no modes becoming light	tower of light states

▸ T-duality:

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$ ⟷ (T-duality along Hopf-fiber) $\mathbf{S^1_{1/R}} \hookrightarrow \tilde{\mathbf{E}} \to \mathbf{S^2_R}$ with $[\tilde{\mathbf{H}}] = \mathbf{1}$

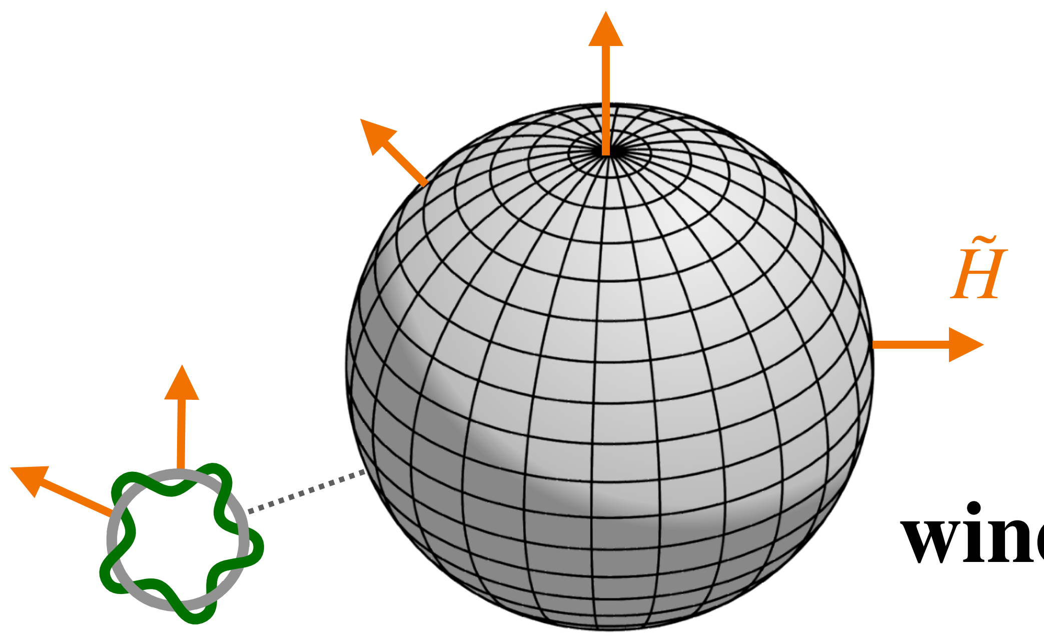


~~winding~~

$\pi_1(S^3) = 0$

$c_1(S^3) = 1$

$$\gamma_{RR} = \frac{3}{R^2}$$
$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



$S^1_{1/R}$ S^2_R

winding $\in \mathbb{Z}_k$

$\pi_1(\tilde{E}) = \mathbb{Z}_k$

$c_1(\tilde{E}) = k$

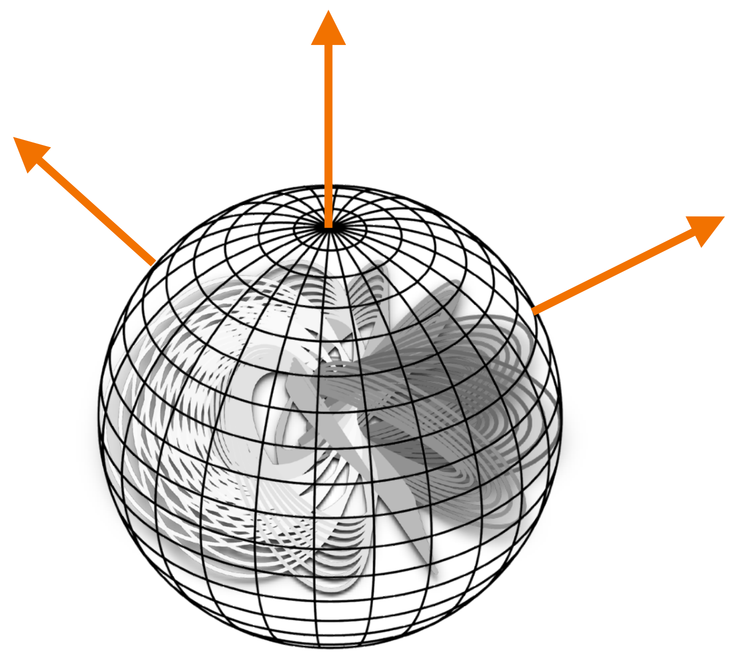
		$R = 0$	$R \to \infty$
S^3	winding	∅	∅
	momentum	heavy	light
$\tilde{E}$	winding	$\mathbb{Z}_k$ (heavy)	$\mathbb{Z}_k$ (light)
	momentum	heavy/ non-conserved	light
		no modes becoming light	tower of light states

▸ **Distance Conjecture:**

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$



Apparent inconsistency: S^3 with appropriately tuned H-flux is **valid string background** and therefore should be in the **Landscape**

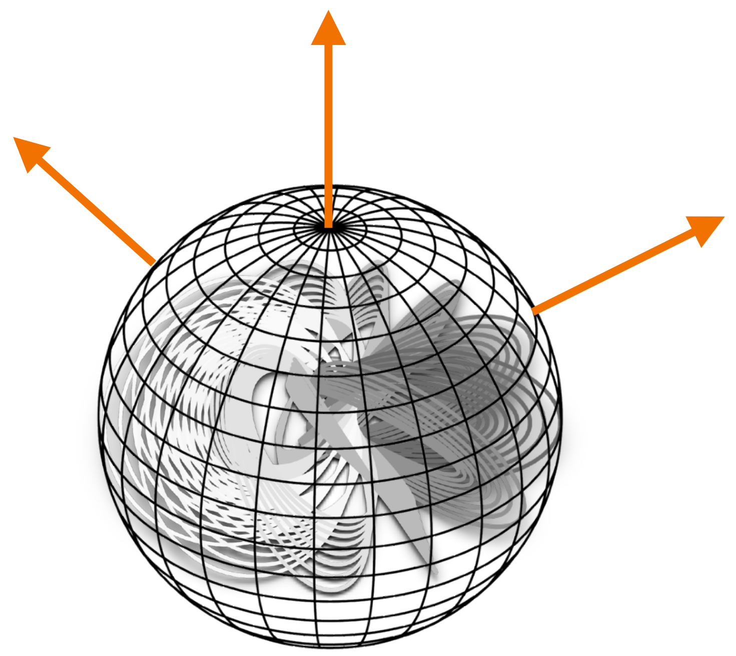
However there is **no tower of light states** for R → 0 which is an infinite distance limit

▸ **Distance Conjecture:**

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$



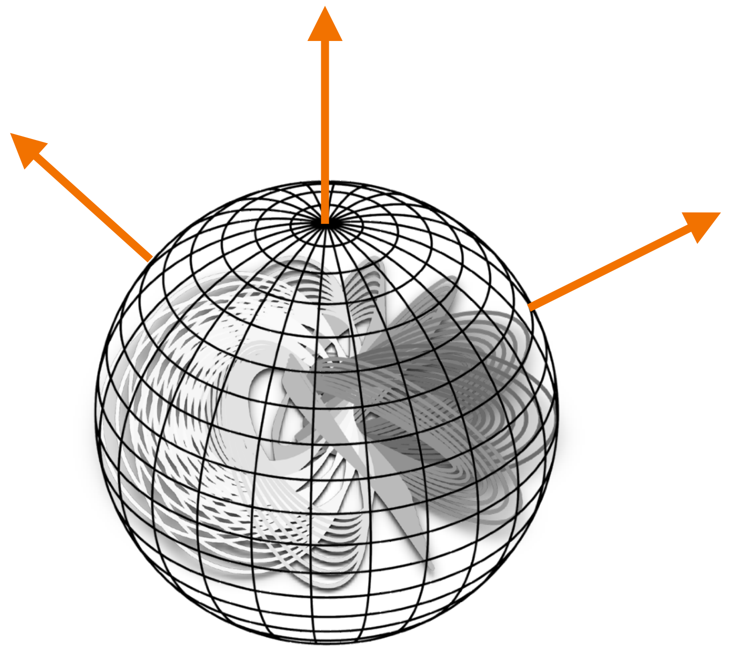
Apparent inconsistency: S^3 with appropriately tuned H-flux is **valid string background** and therefore should be in the **Landscape**

However there is **no tower of light states** for R → 0 which is an infinite distance limit

…need to **take into account scalar potential**

▸ **Distance Conjecture:**

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$



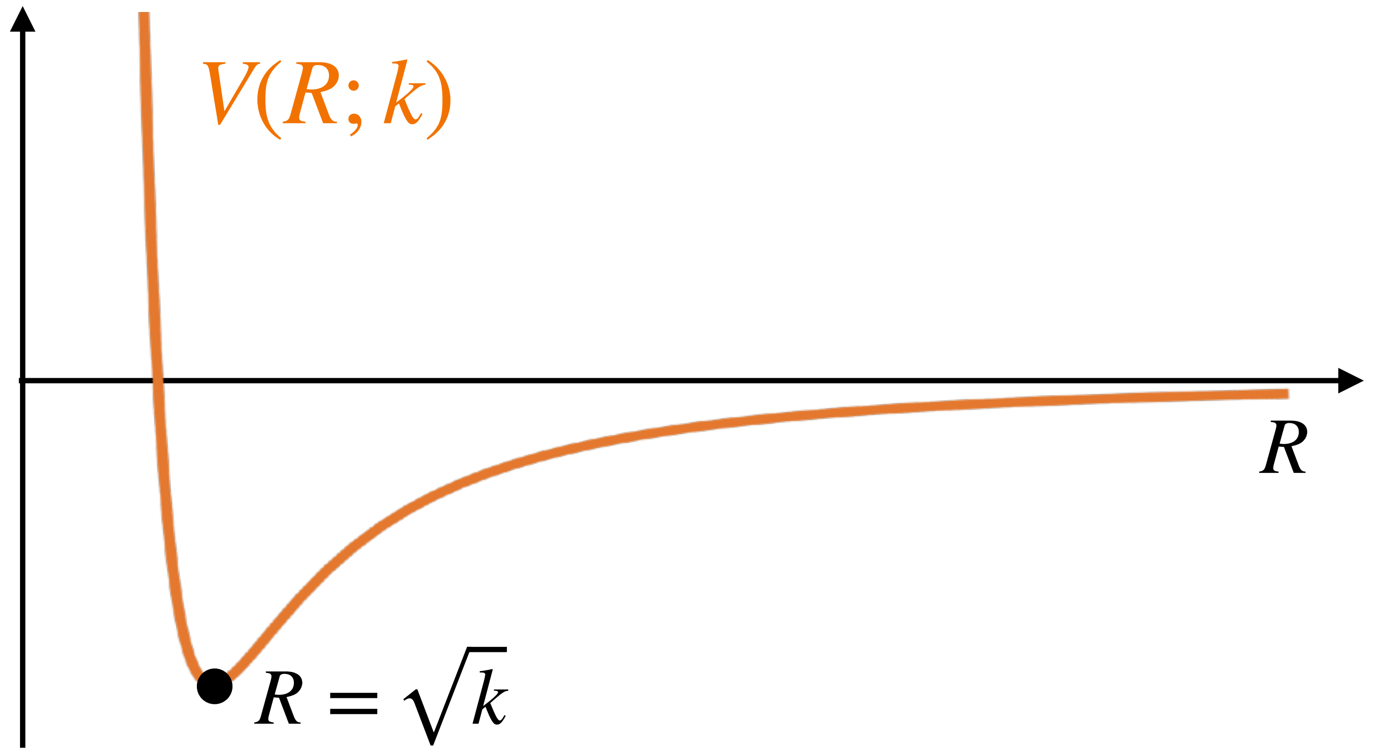
$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

Apparent inconsistency: S^3 with appropriately tuned H-flux is **valid string background** and therefore should be in the **Landscape**

However there is **no tower of light states** for R → 0 which is an infinite distance limit

…need to **take into account scalar potential**



[SD, Lüst, TR ’23]

In **effective field theories** that **can be lifted to a theory of quantum gravity** in the UV, a **divergence in the scalar potential** emerges when approaching an **infinite locus point** for which the target space geometry **cannot give rise to a light tower of states**.

That is, the **potential signals pathological infinite distance loci** in the scalar field space.

▸ Invariance of metric & flux variations:

Metric on moduli space given by

$$\gamma_{ab} \sim \int \mathrm{d}^n y \sqrt{h} \Big(\underbrace{\mathrm{tr}\big(h^{-1}\partial_{\varphi_a} h \; h^{-1}\partial_{\varphi_b} h\big) - \mathrm{tr}\big(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\big)}_{= \frac{1}{2}\mathrm{tr}[(\mathscr{H}^{-1}\partial_{\varphi_a}\mathscr{H})^2]} \Big)$$

$$O(d,d) \ni \mathscr{H} = \begin{pmatrix} h - Bh^{-1}B & Bh^{-1} \\ -h^{-1}B & h^{-1} \end{pmatrix}$$

So by $O(d,d)$ invariance γ_{ab} **is invariant** under (abelian) **T-duality**.

[SD, Lüst, TR ’23]

▸ Invariance of metric & flux variations:

Metric on moduli space given by

$$\gamma_{ab} \sim \int d^n y \sqrt{h} \underbrace{\left(\mathrm{tr}\left(h^{-1}\partial_{\varphi_a} h \; h^{-1}\partial_{\varphi_b} h\right) - \mathrm{tr}\left(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\right) \right)}_{= \frac{1}{2}\mathrm{tr}[(\mathscr{H}^{-1}\partial_{\varphi_a}\mathscr{H})^2]}$$

$$O(d,d) \ni \mathscr{H} = \begin{pmatrix} h - Bh^{-1}B & Bh^{-1} \\ -h^{-1}B & h^{-1} \end{pmatrix}$$

> So by $O(d,d)$ invariance γ_{ab} **is invariant** under (abelian) **T-duality**.

[SD, Lüst, TR '23]

▸ Example: S^3 with k(x)=$R^2(x)$ $\xleftrightarrow{\text{T-duality}}$ $\tilde{E}$

$\tilde{\mathbf{E}}$ ...modulus only in spacetime metric h

γ_{RR} obtained in standard way from "deWitt" metric

▸ Invariance of metric & flux variations:

Metric on moduli space given by

$$\gamma_{ab} \sim \int \mathrm{d}^n y \sqrt{h} \underbrace{\left(\mathrm{tr}\left(h^{-1}\partial_{\varphi_a} h \; h^{-1}\partial_{\varphi_b} h\right) - \mathrm{tr}\left(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\right)\right)}_{= \frac{1}{2}\mathrm{tr}[(\mathcal{H}^{-1}\partial_{\varphi_a}\mathcal{H})^2]}$$

$$O(d,d) \ni \mathcal{H} = \begin{pmatrix} h - Bh^{-1}B & Bh^{-1} \\ -h^{-1}B & h^{-1} \end{pmatrix}$$

So by $O(d,d)$ invariance $\gamma_{\mathbf{ab}}$ **is invariant** under (abelian) **T-duality**.

[SD, Lüst, TR '23]

▸ Example: S^3 with k(x)=$R^2(x)$ $\xleftrightarrow{\text{T-duality}}$ $\tilde{E}$

$$H = R^2 \sin(\eta)\mathrm{d}\eta \wedge \mathrm{d}\xi_1 \wedge \mathrm{d}\xi_2$$
$$B = -R^2 \cos(\eta)\mathrm{d}\xi_1 \wedge \mathrm{d}\xi_2$$

$\tilde{\mathbf{E}}$ …modulus only in spacetime metric h

γ_{RR} obtained in standard way from "deWitt" metric

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k} = \mathbf{R^2}$ …modulus in h **and B**

also contribution $\mathrm{tr}\left(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\right) \neq 0 \subset \gamma_{RR}$

▸ Invariance of metric & flux variations:

Metric on moduli space given by

$$\gamma_{ab} \sim \int d^n y \sqrt{h} \underbrace{\Big(\mathrm{tr}\big(h^{-1}\partial_{\varphi_a} h \; h^{-1}\partial_{\varphi_b} h\big) - \mathrm{tr}\big(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\big) \Big)}_{= \frac{1}{2}\mathrm{tr}[(\mathscr{H}^{-1}\partial_{\varphi_a}\mathscr{H})^2]}$$

$$O(d,d) \ni \mathscr{H} = \begin{pmatrix} h - Bh^{-1}B & Bh^{-1} \\ -h^{-1}B & h^{-1} \end{pmatrix}$$

> So by $O(d,d)$ invariance γ_{ab} **is invariant** under (abelian) **T-duality**.

[SD, Lüst, TR '23]

▸ Example: S^3 with k(x)=$R^2(x)$ $\xleftrightarrow{\text{T-duality}}$ $\tilde{E}$

$$H = R^2 \sin(\eta) d\eta \wedge d\xi_1 \wedge d\xi_2$$
$$B = -R^2 \cos(\eta) d\xi_1 \wedge d\xi_2$$

$\tilde{\mathbf{E}}$ …modulus only in spacetime metric h

γ_{RR} obtained in standard way from "deWitt" metric

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k} = \mathbf{R^2}$ …modulus in h **and B**

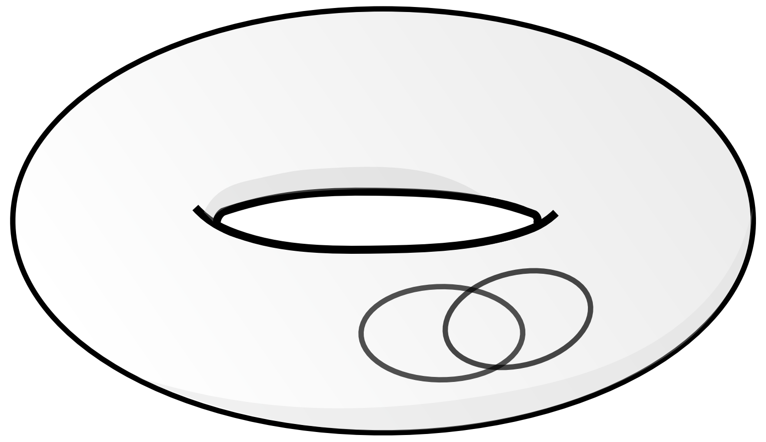
also contribution $\mathrm{tr}\big(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\big) \neq 0 \subset \gamma_{RR}$

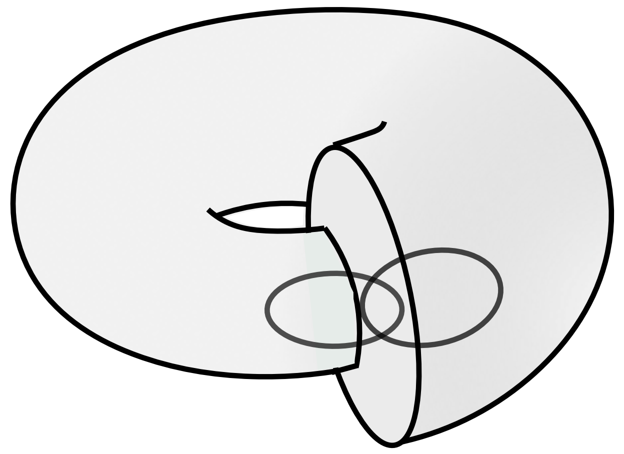
$\tilde{\gamma}_{RR} = \gamma_{RR}$ only if **flux variation** are taken into account

c.f also [Li,Palti,Petri '23] & [Palti,Petri '24]

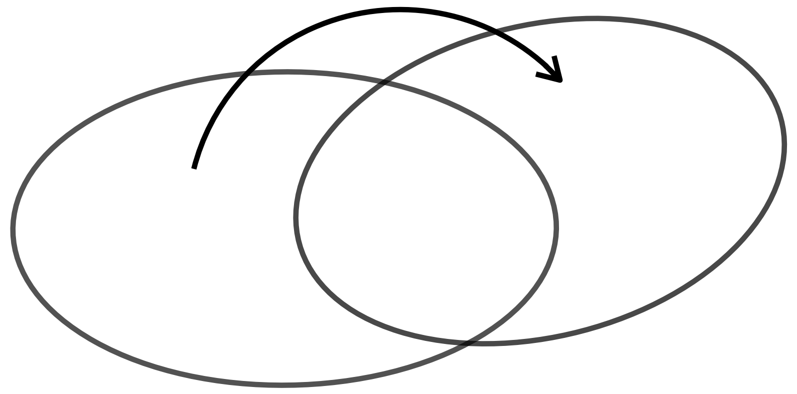
Non geometric backgrounds

Manifold, e.g. a torus vs. non-geometric background

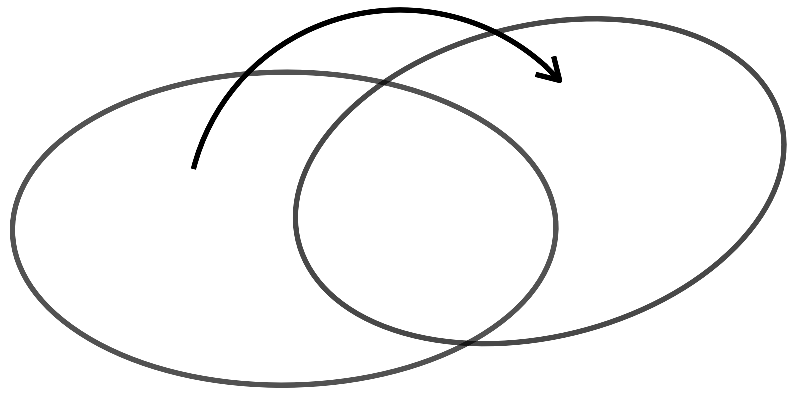




diffeomorphism

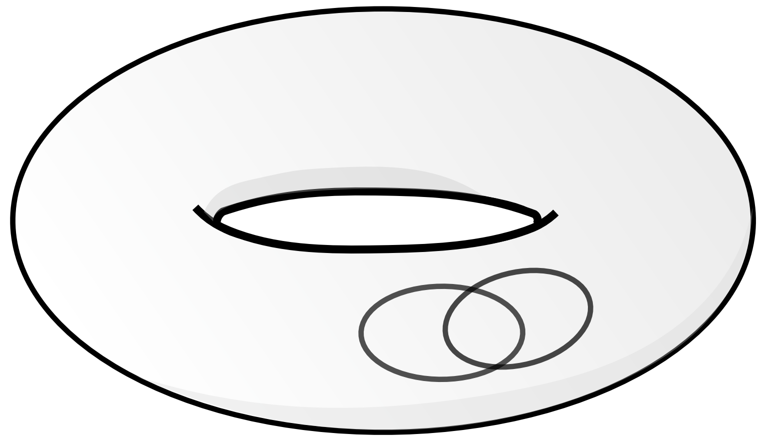


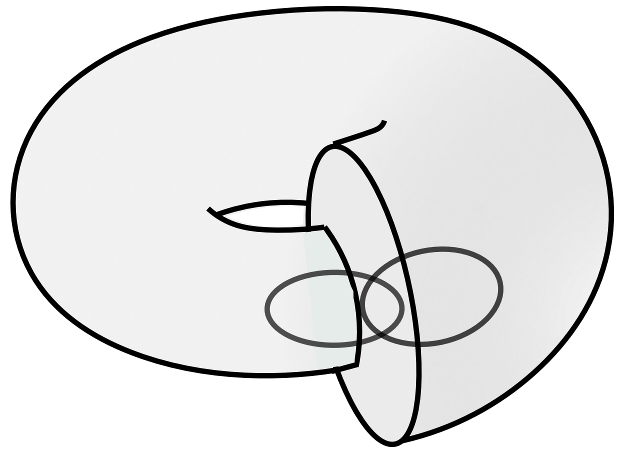
... and T-duality



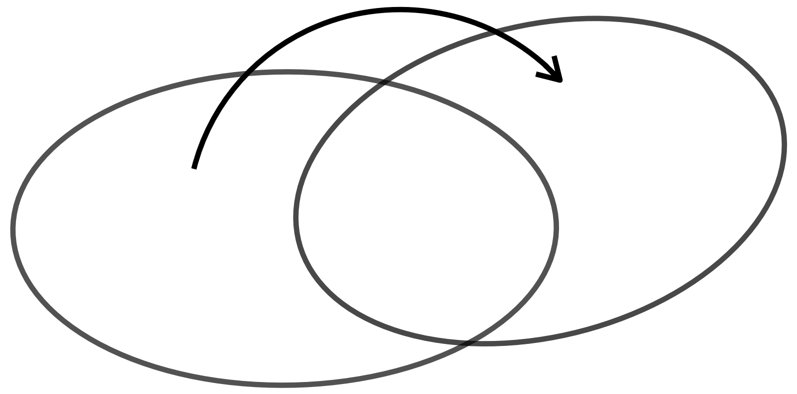
Non geometric backgrounds

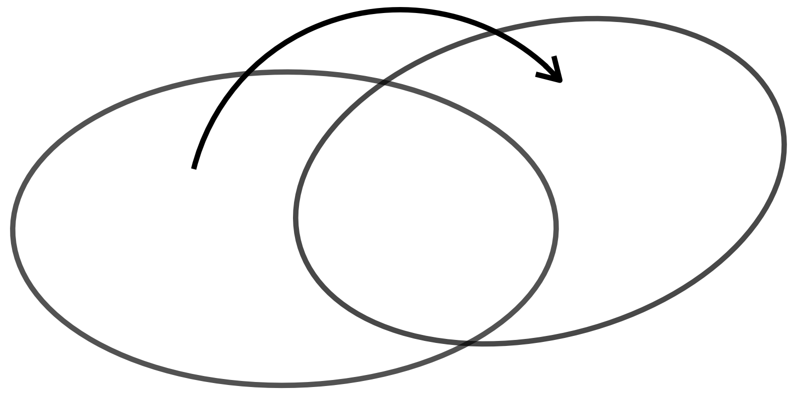
Manifold, e.g. a torus vs. non-geometric background

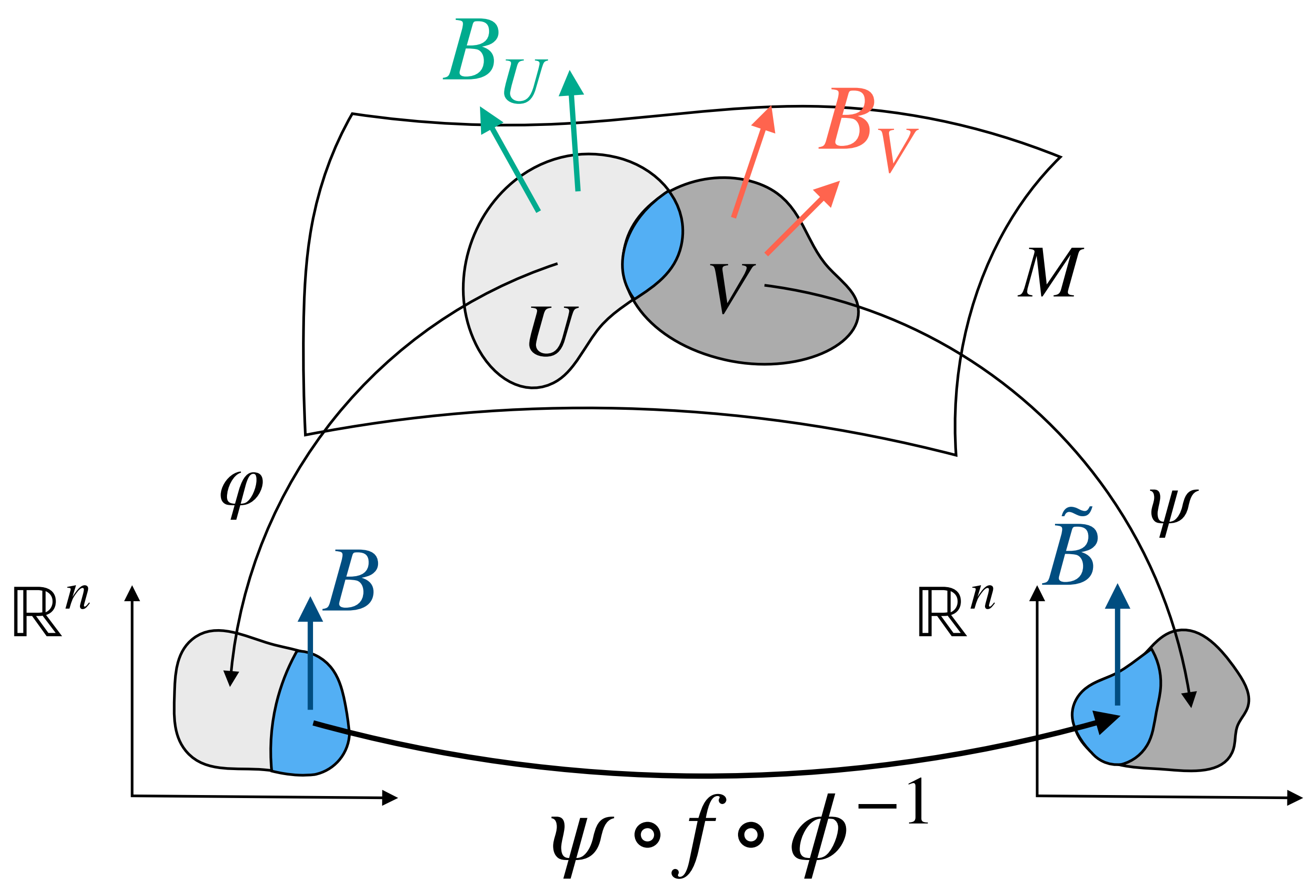




diffeomorphism ... and T-duality







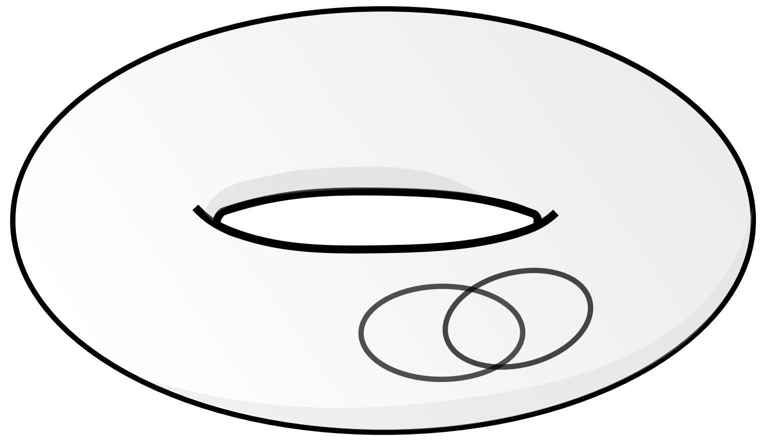
$$f \in \begin{cases} Diff(M) & \text{: Riemannian} \\ Diff(M) \cup \text{T-duality} & \text{: non-geometric} \end{cases}$$

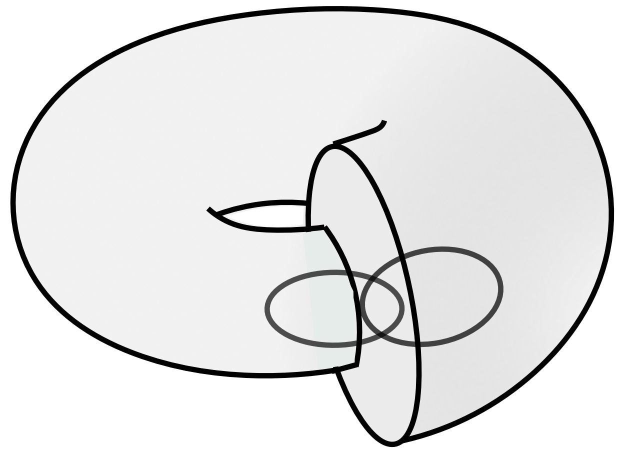
... described locally by Riemannian geometry with fluxes.

However, **transition functions** are allowed to be **T-dualities**.

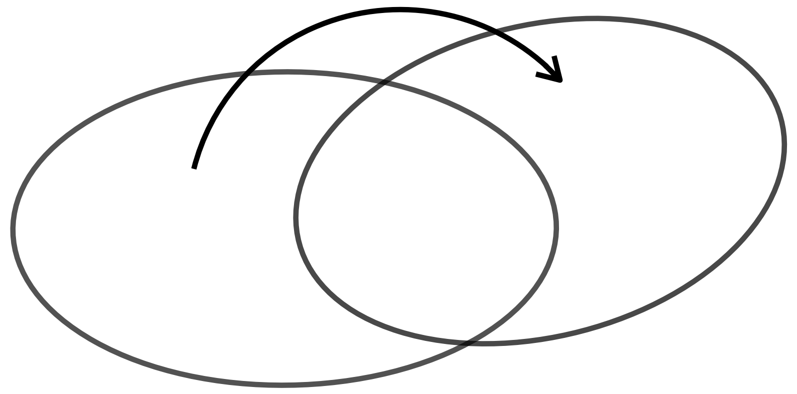
Non geometric backgrounds

Manifold, e.g. a torus vs. non-geometric background

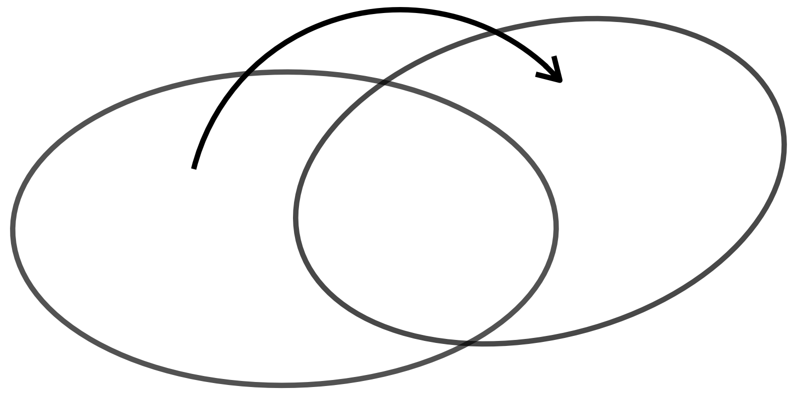


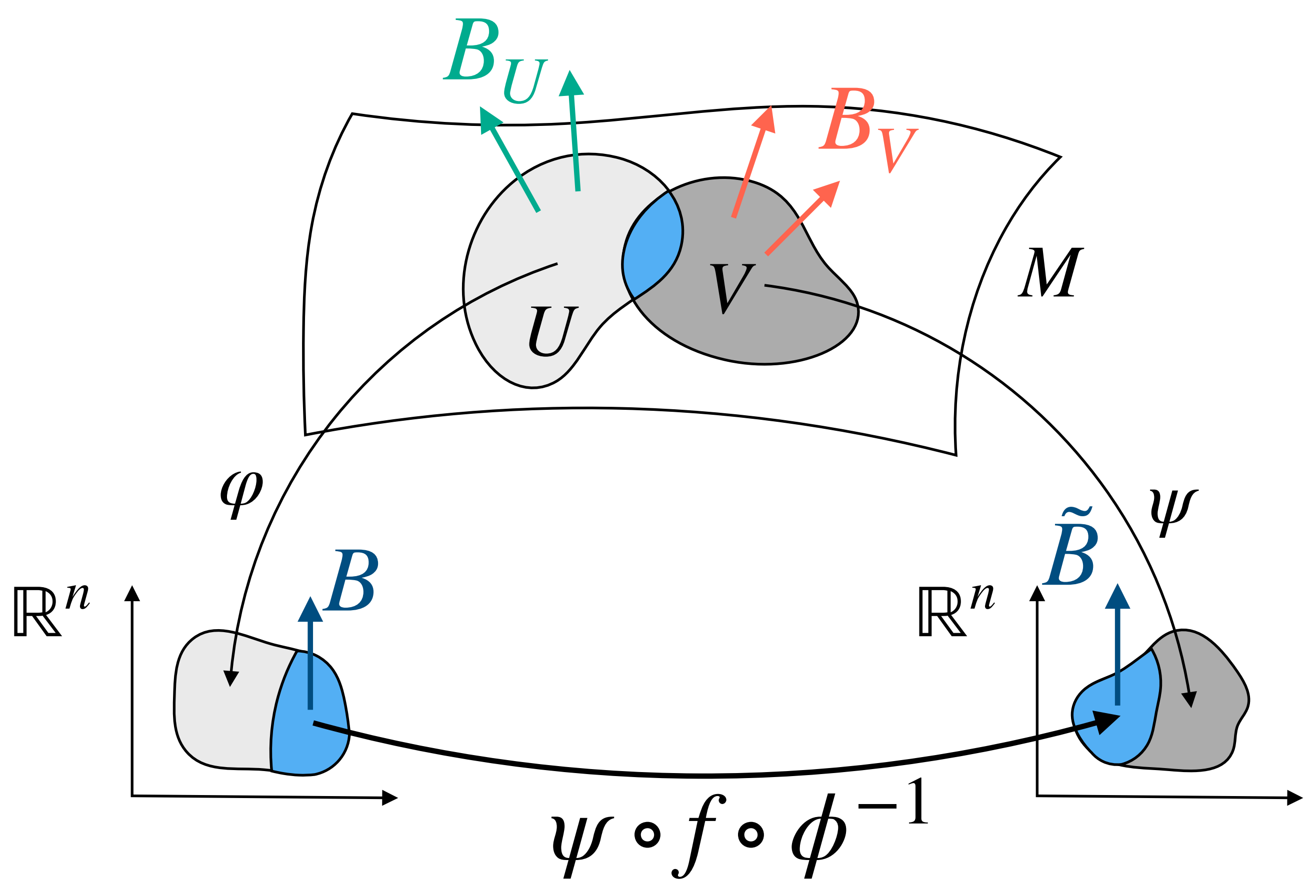


diffeomorphism



... and T-duality





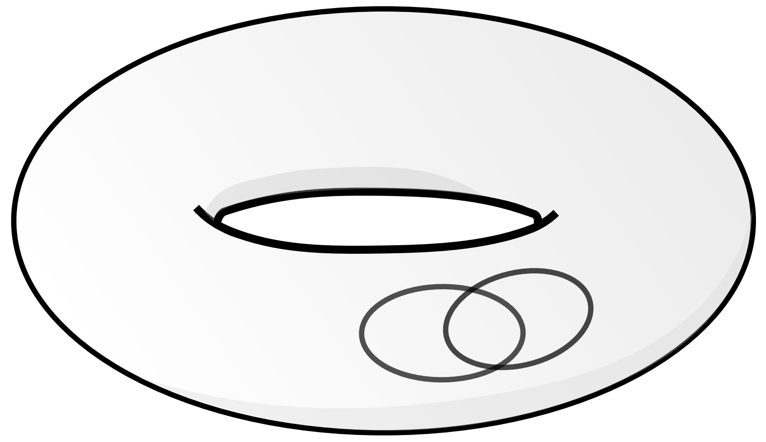
$$f \in \begin{cases} Diff(M) & : \text{Riemannian} \\ Diff(M) \cup \text{T-duality} & : \text{non-geometric} \end{cases}$$

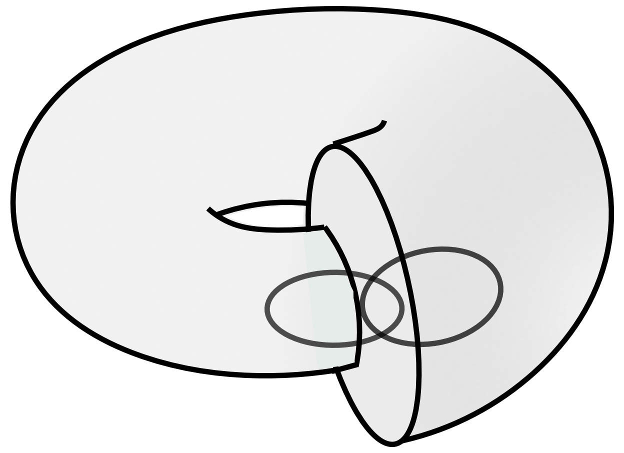
... described locally by Riemannian geometry with fluxes.
However, **transition functions** are allowed to be **T-dualities**.

Perform the **field redefinition:** $(g + B)^{-1} = (\tilde{g}^{-1} + \beta)$ ⟶ $\mathscr{L}_\beta = \mathscr{L}_{NSNS} + \partial(\ldots)$ ... β-supergravity action

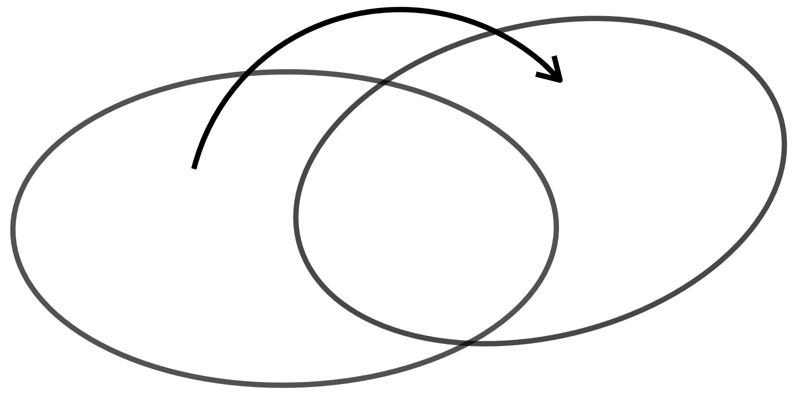
Non geometric backgrounds

Manifold, e.g. a torus vs. non-geometric background

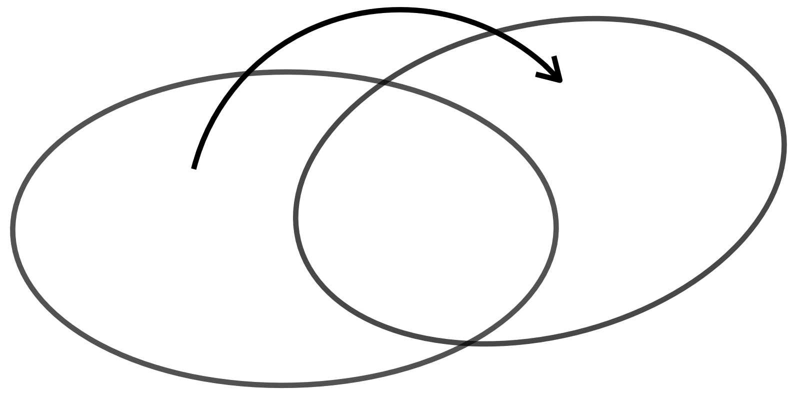


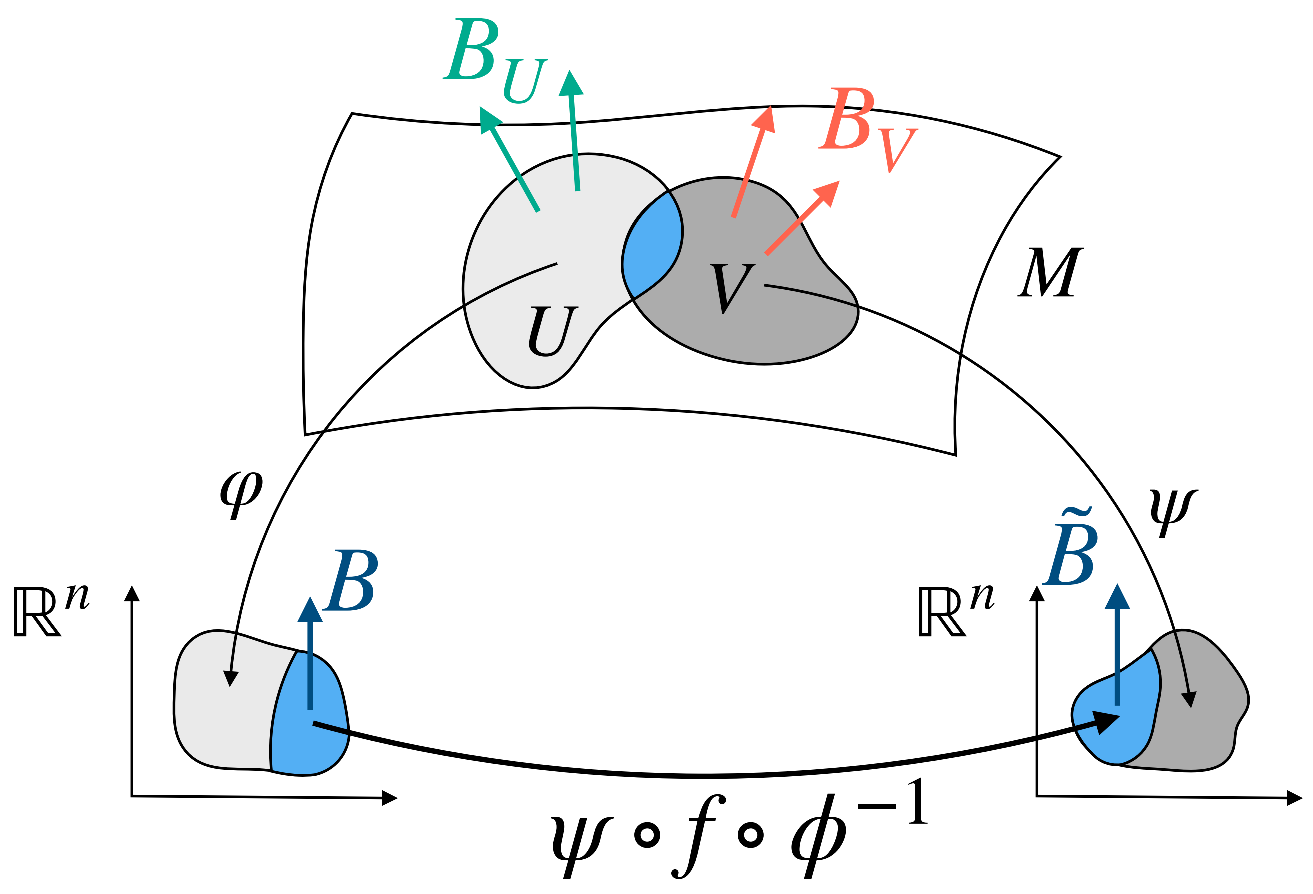


diffeomorphism



... and T-duality





$$f \in \begin{cases} Diff(M) & \text{: Riemannian} \\ Diff(M) \cup \text{T-duality} & \text{: non-geometric} \end{cases}$$

... described locally by Riemannian geometry with fluxes.

However, **transition functions** are allowed to be **T-dualities**.

Perform the **field redefinition:** $(g + B)^{-1} = (\tilde{g}^{-1} + \beta)$ ⟶ $\mathscr{L}_\beta = \mathscr{L}_{NSNS} + \partial(\ldots)$ ... β-supergravity action

Crucial to **use β-supergravity** for consitency of **non-geometry backgrounds & geometric duals:**

A **consistent picture** between a (globally) non-geometric space and its geometric dual
- i.e. matching moduli spaces, potentials and towers of states -
can be established **only after moving to the β-frame**, where the background is well-defined.

[SD, Lüst, TR '23]

Summary & Conclusions

- ▷ Studied **Distance Conjecture** for **curved compact spaces (with fluxes)**
- ▷ **Invariance** of the metric on moduli space **under (abelian) T-duality**
- ▷ Interplay of **scalar potential** and Distance Conjecture & **absence of tower of states**
- ▷ First step towards **non-geometric backgrounds** and associated distance on moduli space

Summary & Conclusions

- ▷ Studied **Distance Conjecture** for **curved compact spaces (with fluxes)**
- ▷ **Invariance** of the metric on moduli space **under (abelian) T-duality**
- ▷ Interplay of **scalar potential** and Distance Conjecture & **absence of tower of states**
- ▷ First step towards **non-geometric backgrounds** and associated distance on moduli space

?

- ▷ More **realistic setups: full 10d backgrounds**, e.g. $AdS_5 \times S^5, AdS_4 \times T^6$ with fluxes,...
- ▷ Flux variations & potential: **on-shell vs off-shell** [Li,Palti,Petri '23] & [Palti,Petri '24]
- ▷ **Deformations and generalized T-duality** (Poisson-Lie T-duality)
- ▷ **Truly non-geometric spaces** and the Swampland?

– work in progress –

Summary & Conclusions

- ▷ Studied **Distance Conjecture** for **curved compact spaces (with fluxes)**
- ▷ **Invariance** of the metric on moduli space **under (abelian) T-duality**
- ▷ Interplay of **scalar potential** and Distance Conjecture & **absence of tower of states**
- ▷ First step towards **non-geometric backgrounds** and associated distance on moduli space

?

- ▷ More **realistic setups: full 10d backgrounds**, e.g. $AdS_5 \times S^5, AdS_4 \times T^6$ with fluxes,...
- ▷ Flux variations & potential: **on-shell vs off-shell** [Li,Palti,Petri '23] & [Palti,Petri '24]
- ▷ **Deformations and generalized T-duality** (Poisson-Lie T-duality)
- ▷ **Truly non-geometric spaces** and the Swampland?

– work in progress –

Thank you!