Lorenzo Paoloni
June 27th, 2024

On the Moduli Space Curvature at Infinity

based on 2311.07979 with Luca Melotti & Fernando Marchesano
and WIP with Alberto Castellano & Fernando Marchesano

"la Caixa" Foundation
UAM Universidad Autónoma de Madrid
CSIC Consejo Superior de Investigaciones Científicas
ift Instituto de Física Teórica UAM-CSIC

Motivation: Swampland

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Swampland Conjectures

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- There will always exist trajectories of **infinite geodesic length** in the moduli space

Ooguri & Vafa '06

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- The scalar curvature of the moduli space near points at infinity is **non-positive** (strictly negative for a moduli space of dimension d>1)

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Ooguri & Vafa '06

COUNTEREXAMPLE:
POSITIVE DIVERGING CURVATURE
type IIA on (a specific) CY 3-fold
Trenner & Wilson '09

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❖ There will always exist trajectories of **infinite geodesic length** in the moduli space

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Ooguri & Vafa '06

¿WHY?

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Setup:
type IIA on a Calabi-Yau

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Action for the bosonic part of the vector multiplets:

$$S_{4\text{d}}^{\text{VM}} = \frac{1}{2\kappa_4^2}\int_{\mathbb{R}^{1,3}} R * 1 - 2g_{ab}dT^a \wedge *d\bar{T}^{\bar{b}} + \frac{1}{2}\int_{\mathbb{R}^{1,3}} I_{AB}F^A \wedge *_4 F^B$$

Setup: Metric

Action for the bosonic part of the vector multiplets:

$$S^{\text{VM}}_{\text{4d}} = \frac{1}{2\kappa_4^2}\int_{\mathbb{R}^{1,3}} R * 1 - 2 g_{a\bar{b}}\, dT^a \wedge * d\bar{T}^{\bar{b}} + \frac{1}{2}\int_{\mathbb{R}^{1,3}} I_{AB} F^A \wedge *_4 F^B$$

$$g_{ab} \overset{\text{LV}}{=} -\frac{\partial}{\partial T^a}\frac{\partial}{\partial \bar{T}^{\bar{b}}} \log\left(\frac{4}{3}\mathcal{K}\right)$$

Special Kähler manifold of the **local** type

$$I_{AB} \overset{\text{LV}}{=} -\frac{\mathcal{K}}{6}\begin{pmatrix} 1 + 4 g_{ab} b^a b^b & 4 g_{ab} b^b \\ 4 g_{ab} b^b & 4 g_{ab} \end{pmatrix}$$

$$A = 0, a$$

Setup:
EFT String Limits

Lanza et al '21

Class of infinite distance trajectories such that

$$t^a = t_0^a + e^a \phi, \quad \text{with} \quad \phi \to \infty$$

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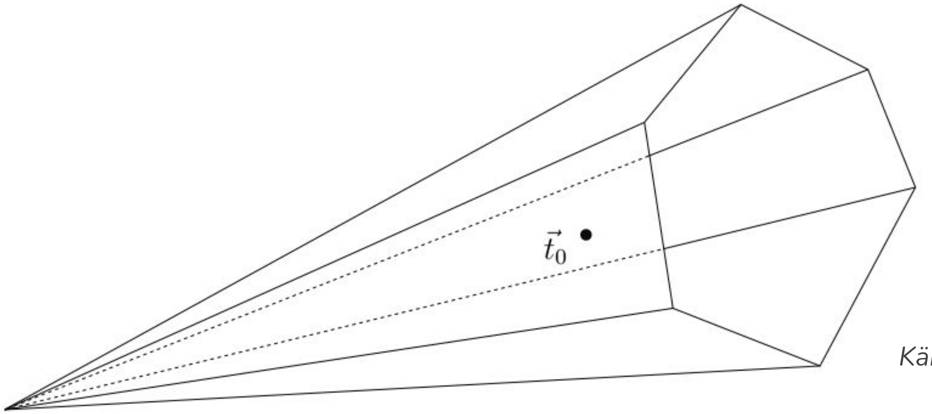

Kähler cone

Setup: EFT String Limits

Lanza et al ‘21

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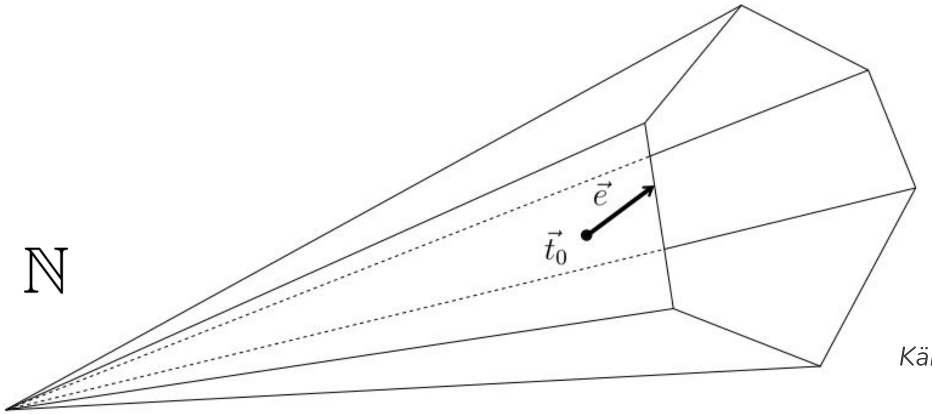
$$e^a \in \mathbb{N}$$



Kähler cone

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Lanza et al '21

Class of infinite distance trajectories such that

$$t^a = t_0^a + e^a \phi, \quad \text{with} \quad \phi \to \infty$$

In order not to move in the hypermultiplet moduli space, also

$$g_s(\phi) = g_{s,0} \frac{\mathcal{V}_{CY}^{1/2}(\phi)}{\mathcal{V}_{CY,0}^{1/2}} \to \infty$$

Scalar Curvature of the Moduli Space

Strominger '90

For a Special Kähler manifold the scalar curvature is

$$R^{\text{cl}} = -2n_V(n_V+1) + \frac{9}{8\mathcal{K}^2} g^{ab} g^{cd} g^{ef} \mathcal{K}_{ace} \mathcal{K}_{bdf}$$

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Bounded from below but not from above

Origin of the Divergence

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GEOMETRY

Origin of the Divergence

Luca Melotti's parallel session

GEOMETRY

- **M-th variables** perspective
- In terms of **boundaries** of the 5d M-th **moduli space**
- **Shrinking divisors**

Origin of the Divergence

Luca Melotti's parallel session

GEOMETRY

PHYSICS

- M-th variables perspective
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- **4d EFT** perspective
- In terms of the **d.o.f.'s** that remain dynamical **below the cutoff**
- **Rigid theory**

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Origin of the Divergence: the EFT picture

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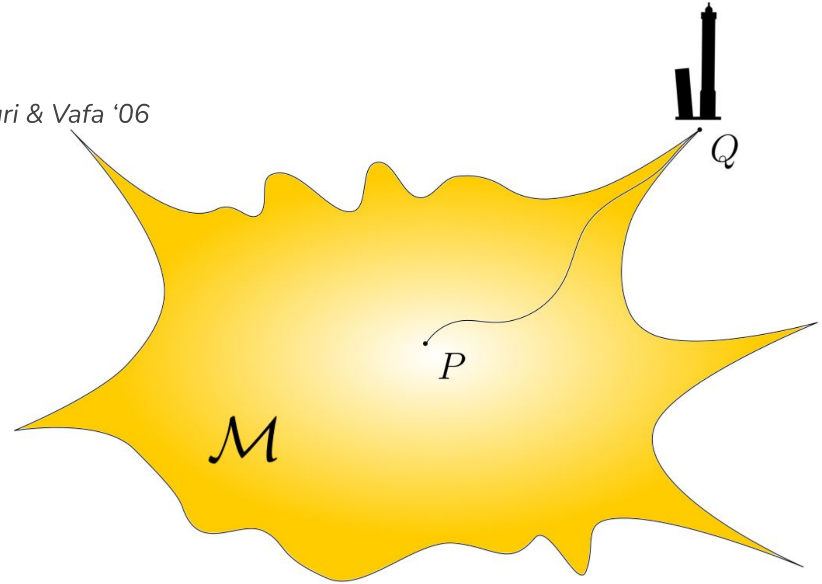
Swampland Distance Conjecture

Ooguri & Vafa '06

$$\frac{m_*(Q)}{M_P} \sim \frac{m_*(P)}{M_P}\, e^{-\alpha d(P,Q)}$$

$$d(P,Q) \longrightarrow \infty$$



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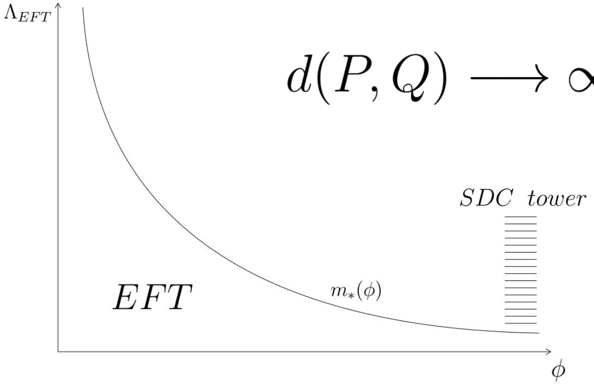

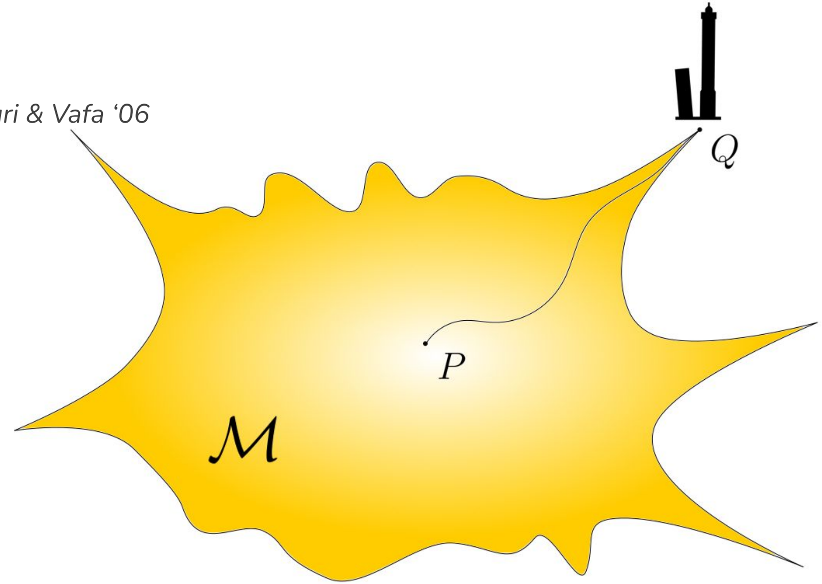

$$\Lambda_{EFT} = m_*(\phi)$$

Origin of the Divergence: the EFT picture

For our limit

$$t^a \to \infty$$

$$g_s(\phi) \to \infty$$

the lightest tower is the one of D0-branes: $m_{D0} \sim g_s^{-1} \to 0$

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$$\Lambda_{EFT} = m_*(\phi) \sim m_{D0}$$

Origin of the Divergence: the EFT picture

$$S_{\rm 4d}^{\rm VM} \supset M_P^2 \int g_{ab}\, dT^a \wedge *d\bar{T}^{\bar{b}}$$

$$= m_*^2 \int \tilde{g}_{ab}\, dT^a \wedge *d\bar{T}^{\bar{b}}$$

$$\stackrel{!}{=} m_*^2 \int I_{ab}\, dT^a \wedge *d\bar{T}^{\bar{b}}$$

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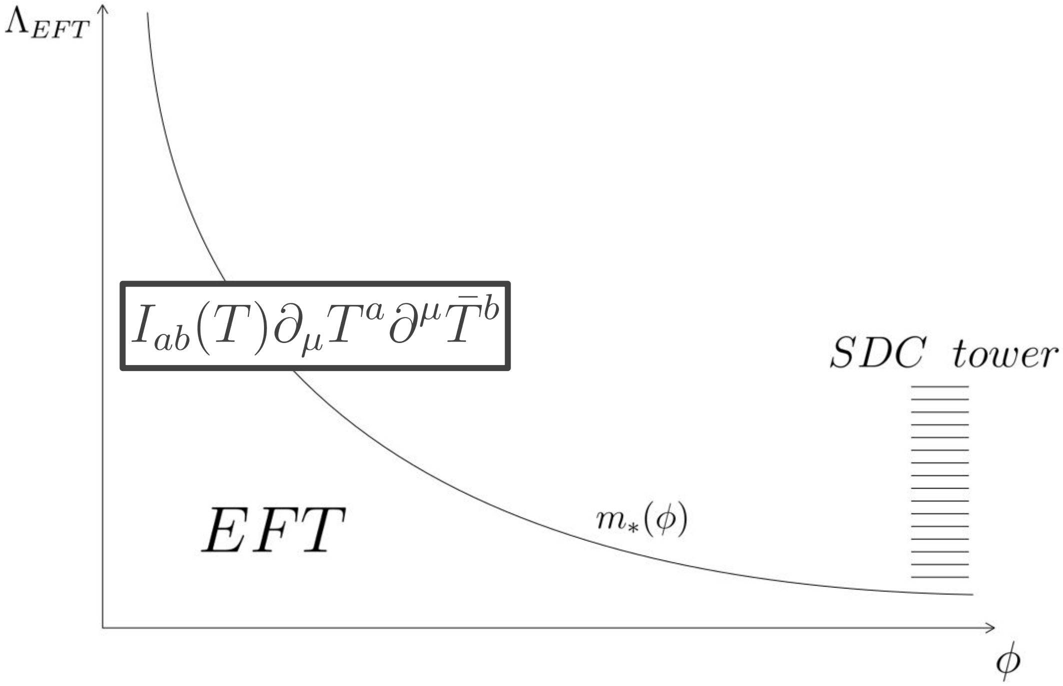

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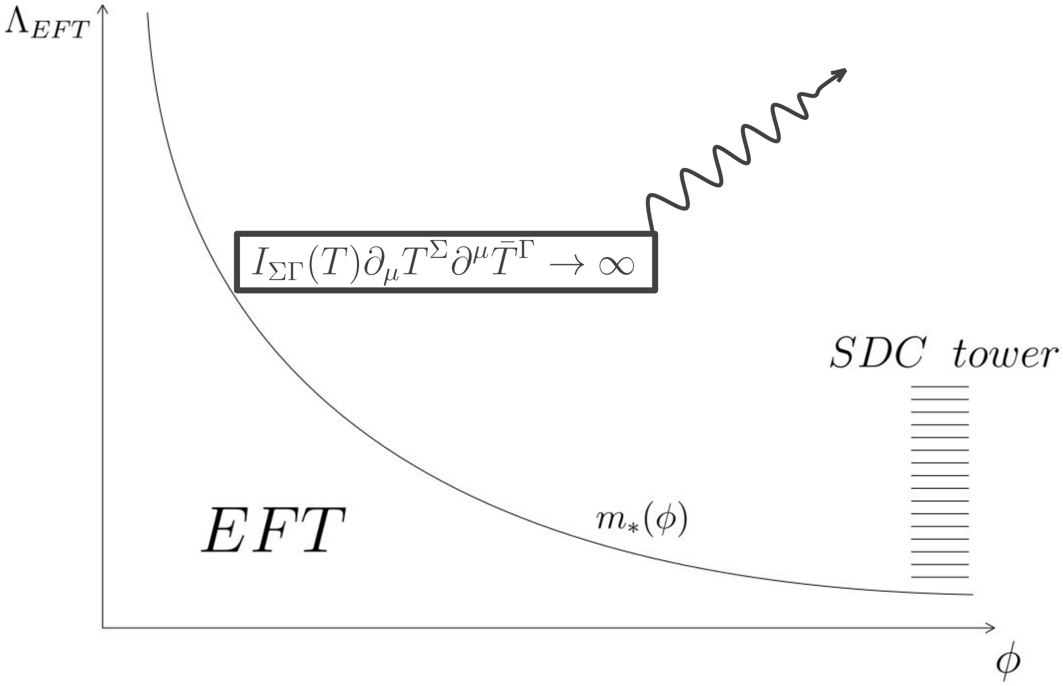

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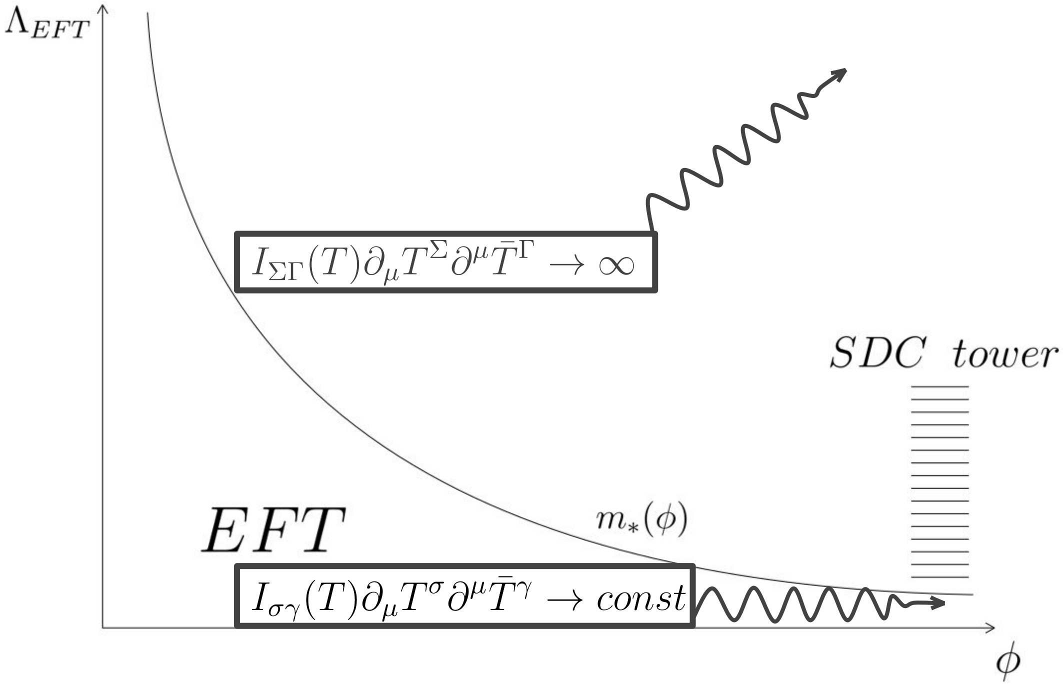

Origin of the Divergence: the EFT picture

Below the SDC scale, we recover a **4d N=2** **rigid** **field theory**

$$S^{\rm VM}_{\rm 4d,rigid} = m_*^2 \int_{\mathbb{R}^{1,3}} I_{\sigma\rho} dT^\sigma \wedge * d\bar{T}^{\bar{\rho}} + \frac{1}{2}\int_{\mathbb{R}^{1,3}} I_{\sigma\rho} F^\sigma \wedge *_4 F^\rho$$

Special Kähler manifold
of the **local** type

$$K \sim \log \mathcal{K}$$

Special Kähler manifold
of the **rigid** type

$$K \sim \mathcal{K}$$

How to select the directions:

$$\boxed{\vec{t}_\sigma \in \ker \mathbf{K}}$$

$$\boxed{\mathbf{K}_{ab} = \mathcal{K}_{abc} e^c}$$

Origin of the Divergence: the EFT picture

For its moduli space, the scalar curvature is

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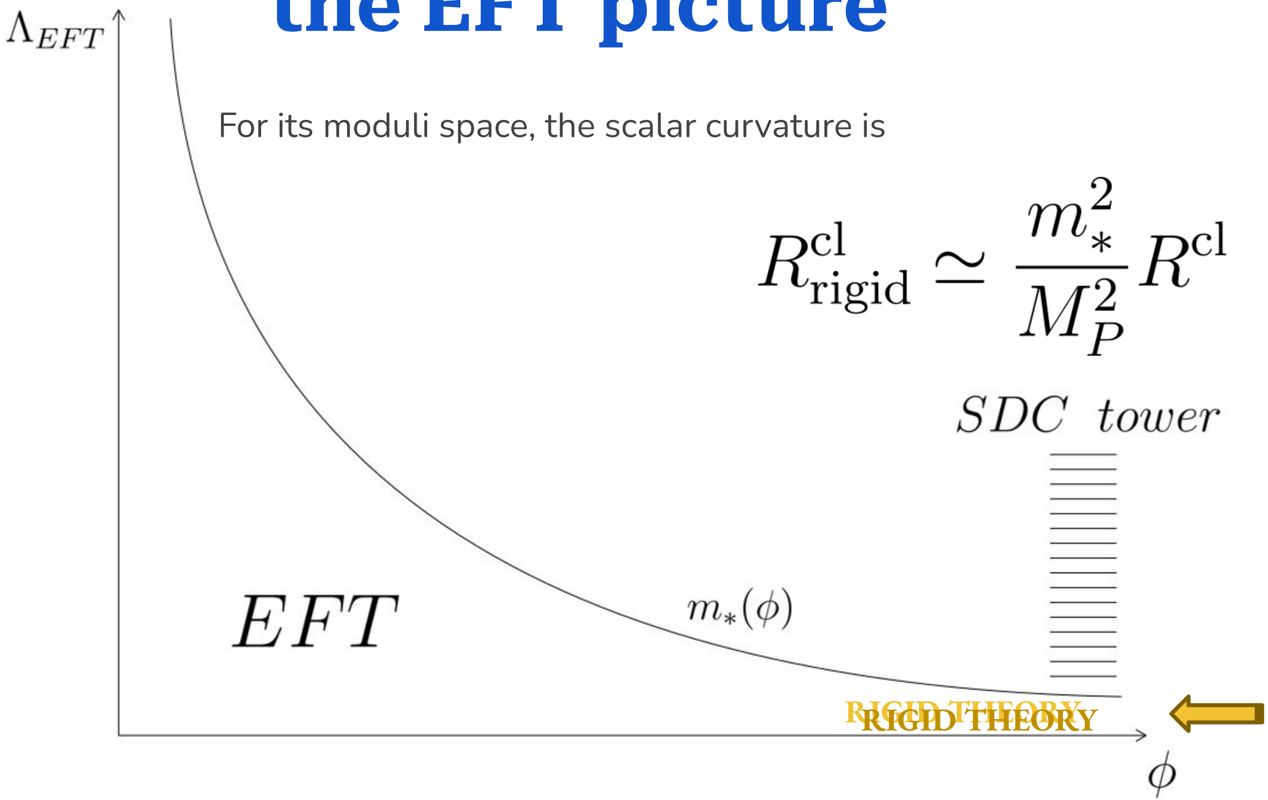

Curvature divergence sourced by a gauge theory that dominates over gravity

The Curvature Criterion

Along a geodesic trajectory of infinite distance, moduli space scalar curvature that diverges asymptotically implies the presence of a field theory sector that is decoupled from gravity.

Conclusions

- We have analysed the asymptotic behaviour of the scalar curvature in **4d N=2 moduli spaces**, focusing on **type IIA CY VM sector** at **large volume**, which provide a huge set of limits, recently classified in light of the SDC.
- In our case, the **SDC tower always involved D0-branes**, and so there is an M-theory description.
- **Take-home message**: **curvature divergences** appear where there is a **non-trivial EFT below the SDC scale** that decouples from gravity.
- **Work in progress**: test this picture in **more general setups**, i.e. type IIB, conifold points, SW points, etc.

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Thank you!

Quantitative Analysis

Case		$R^{\mathrm{cl}}_{\mathrm{IIA}}(\phi \to \infty)$
w=3	$r = n_V$	$-2n_V^2 + n_V + \mathfrak{C}$
	$r < n_V$	$R^{\mathrm{cl}}_{\mathrm{rigid}} \frac{2}{3}\mathbf{k}\phi^3$
w=2	$r = n_V$	$-2n_V^2 + n_V$
	$r < n_V$, smooth	$-2(n_V^2 - 2n_V + 3)$
	$r < n_V$, non-smooth	$\left[-2n_V^2 + 4n_V - 3r\right]^*$
w=1	$r = n_V - 1$	$-2(n_V^2 - 2n_V + 3)$
	$r < n_V - 1$	$R^{\mathrm{cl}}_{\mathrm{rigid}} 2\mathbf{K}_{\Sigma\Lambda} t_0^{\Sigma} t_0^{\Lambda} \phi$

$$\mathfrak{C} = -\frac{\mathbf{k}}{6}\mathbf{K}^{ab}\mathbf{K}^{cd}\mathbf{K}^{ef}\mathcal{K}_{ace}\mathcal{K}_{bdf}$$

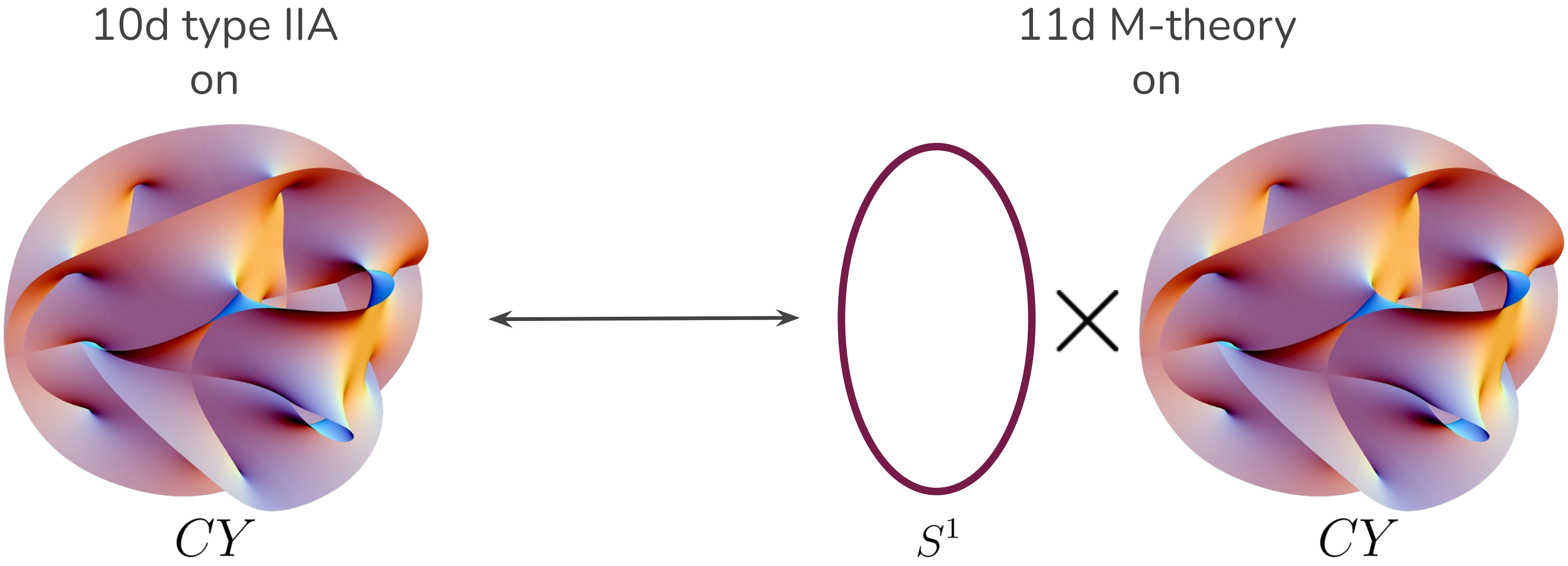
$$\mathbf{k} = \mathcal{K}_{abc}e^a e^b e^c$$

$R^{cl}_{rigid} = 0$ but there can be divergence due to worldsheet instanton corrections

$r = \mathrm{rank}(\mathbf{K})$ $\quad$ $\mathbf{K}_{ab} = \mathcal{K}_{abc}e^c$

$n_V = h^{1,1}(CY)$

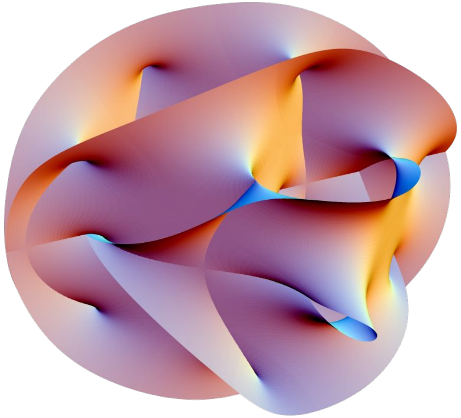
Origin of the Divergence: the Geometric picture



Origin of the Divergence: the Geometric picture

10d type IIA
on

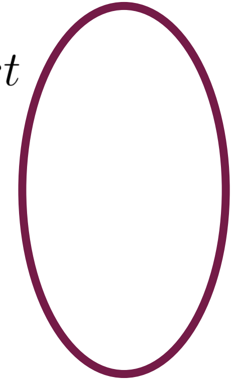
11d M-theory
on



CY

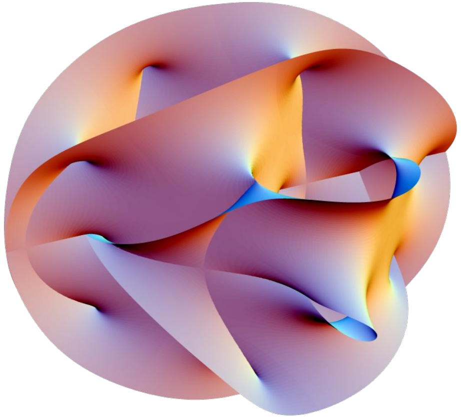
$$\mathcal{V}_{11} = \frac{\mathcal{V}_{CY}}{g_s^2} \to const$$

$\longleftrightarrow$



S^1

$\times$



CY

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10d type IIA
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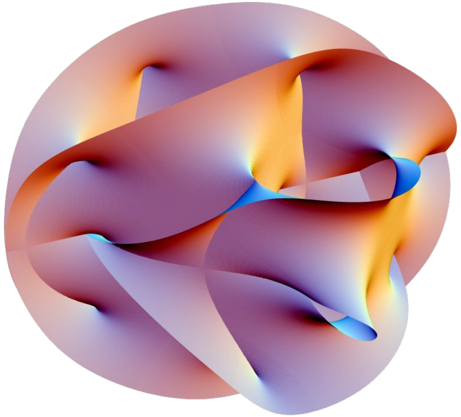


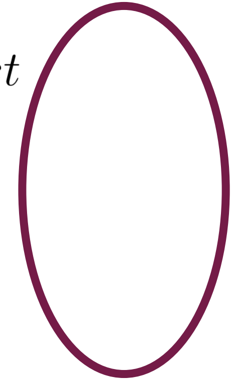
CY

$$\mathcal{V}_{11} = \frac{\mathcal{V}_{CY}}{g_s^2} \rightarrow const$$

$\longleftrightarrow$

$$R_{11} = \frac{\mathcal{V}_{CY}^{\frac{1}{3}}}{\mathcal{V}_{11}^{\frac{1}{3}}} \rightarrow \infty$$

11d M-theory
on



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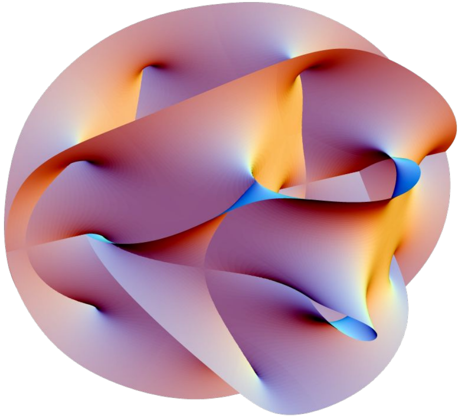

CY

Origin of the Divergence: the Geometric picture

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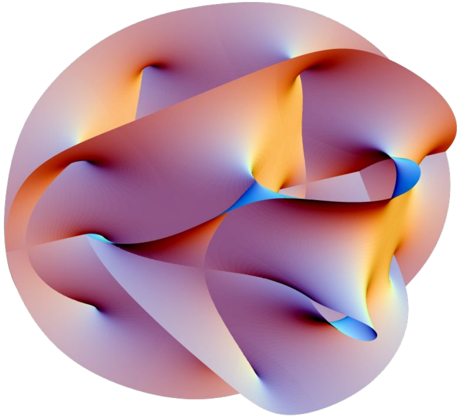


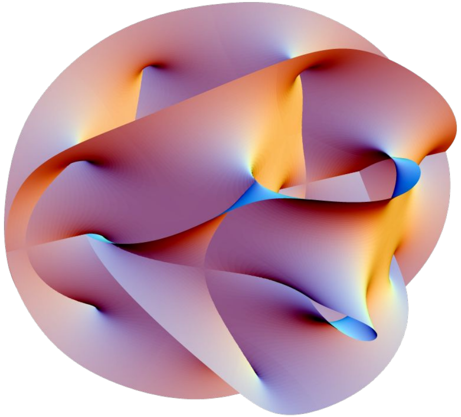
CY

$\equiv$ 5d N=2 SUGRA

Origin of the Divergence: the Geometric picture

$w = 3$

11d M-theory
on



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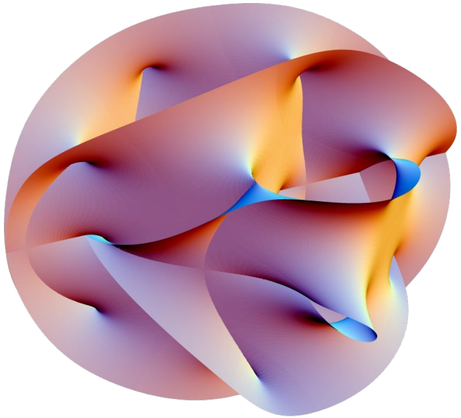

CY

$w = 3$ 

$w = 2$ 

$w = 1$

$\equiv$ 5d N=2 SUGRA

Origin of the Divergence: the Geometric picture

11d M-theory
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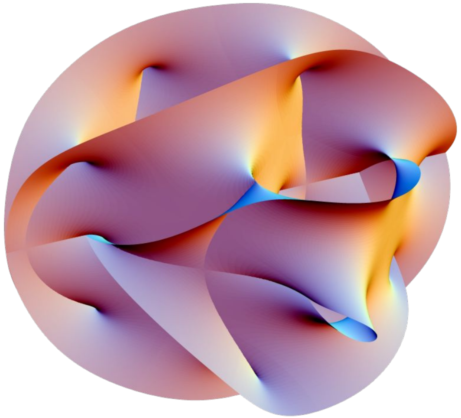

CY

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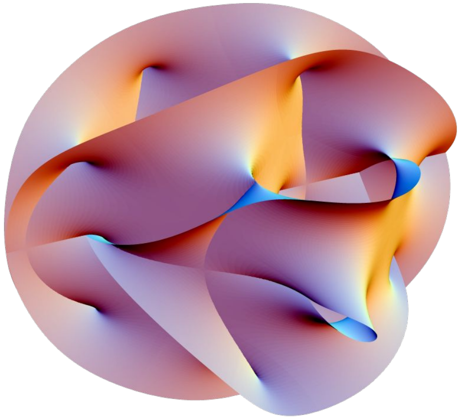
Origin of the Divergence: the Geometric picture

FEATURES:

- $\mathcal{M}_{\text{5d}}^{\text{VM}}$
- $M^a = \dfrac{t^a}{\mathcal{V}_{CY}^{\frac{1}{3}}}$
- $\mathcal{K}_{abc} M^a M^b M^c = 6$

11d M-theory
on



CY

$\equiv$ 5d *N*=2 SUGRA

Origin of the Divergence: the Geometric picture

- Map the **trajectory** of the limit in terms of the **5d variables**
- Write the **curvature** in terms of the **5d gauge kinetic function**

THE CURVATURE CAN ONLY DIVERGE AT THE BOUNDARIES OF THE 5D MODULI SPACE

Origin of the Divergence: the Geometric picture

Types of Calabi-Yau 3-fold (M-th) Kähler boundaries:

Witten '95

- **Finite distance boundaries** (w=3)

  1. **Curve** collapsing to a **point** → NO DIVERGENCE
  2. **Divisor** collapsing to a **curve** → NO DIVERGENCE
  3. **Divisor** collapsing to a **point** → DIVERGENCE

- **Infinite distance boundaries** (w=1,2) → UNCERTAIN

Origin of the Divergence: the Geometric picture

Types of Calabi-Yau 3-fold (M-th) Kähler boundaries:

- **Finite distance boundaries** (w=3)
  1. **Curve** collapsing to a **point**
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$$\mathcal{D}_\sigma = t^a_\sigma \mathcal{D}_a$$
$$\vec{t}_\sigma \in \ker \mathbf{K}$$

- **Infinite distance boundaries** (w=1,2)