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# EFT strings & the Convex Hull Distance Conjecture

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String Phenomenology 2024, Padova, 27th June 2024

Based on WIP with A.G., Ruiz, Valenzuela





1) Introduction

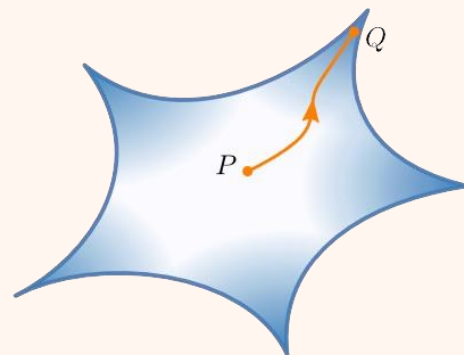
2) Reconstructing the Convex Hull

3) Outlook



## Swampland Distance Conjecture (SDC) [Ooguri, Vafa '06]

$$M(d(P, Q)) \sim M(P)e^{-\alpha \cdot d(P, Q)} \quad \text{as } d(P, Q) \rightarrow \infty \\ \text{with } \alpha = \mathcal{O}(1)$$

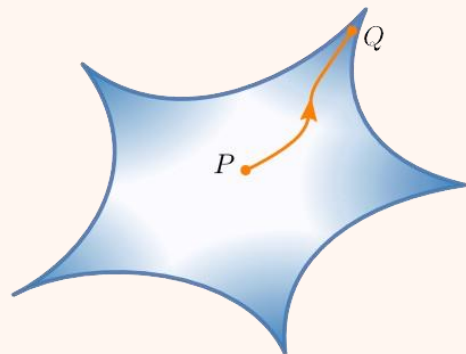


[van Beest, Calderón-Infante, Mirfendereski, Valenzuela, '20]

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[van Beest, Calderón-Infante, Mirfendereski, Valenzuela, '20]

## Emergent String Conjecture [Lee, Lerche, Weigand, '19]

Any infinite distance limit is either

- a decompactification limit (KK states)
- a limit in which a weakly coupled string becomes tensionless (string oscillator modes)

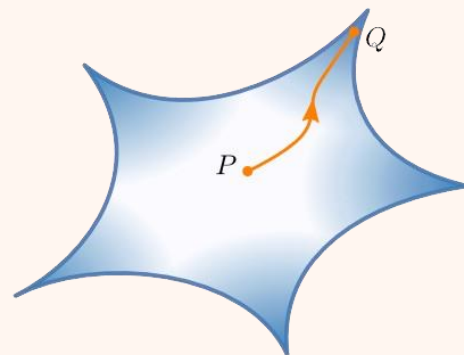
## Swampland Distance Conjecture (SDC) [Ooguri, Vafa '06]

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**Sharpened Distance Conjecture :**  $\alpha \geq \frac{1}{\sqrt{d-2}}$  [Etheredge, Heidenreich, Kaya, Qiu, Rudelius '20]

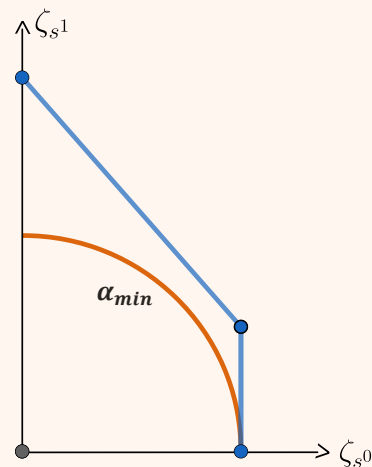


[van Beest, Calderón-Infante, Mirfendereski, Valenzuela, '20]

## Scalar charge-to mass ratio vectors

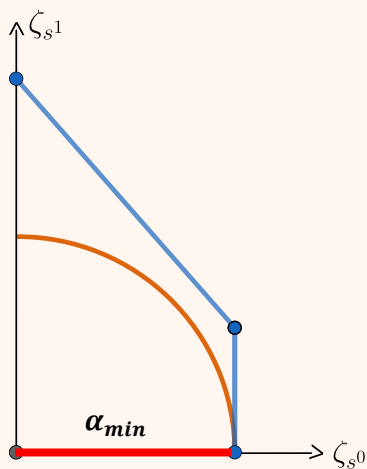
$$\vec{\zeta}_a = -\vec{\nabla}_\phi \log \frac{m_a}{M_{Pl;d}}, \quad m_a = m_a(\phi^i)$$

- Defined locally on moduli space  $\mathcal{M}$
- In each asymptotic limit we can draw a **convex hull** of light towers
- For an infinite distance limit with tangent vector  $\hat{t}$ ,  $\alpha = \vec{\zeta}_a \cdot \hat{t}$
- $|\vec{\zeta}_a| \geq \alpha_{min} = \frac{1}{\sqrt{d-2}}$  («Scalar WGC»)



## Convex Hull Distance Conjecture [Calderón-Infante, Uranga, Valenzuela]

In any given asymptotic region of a quantum gravity theory, the convex hull generated by the  $\vec{\zeta}$ -vectors of all light towers must remain outside the ball of radius  $\alpha_{min}$ .



Different convex hulls in different asymptotic regions, how do they combine?

[Etheredge, Heidenreich, Rudelius, Ruiz, Valenzuela '24]

$$\vec{\zeta}_a = -\vec{\nabla}_\phi \log \frac{m_a}{M_{Pl;d}}$$

## **Convex Hull Distance Conjecture** [Calderón-Infante, Uranga, Valenzuela]

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## **Convex Hull Distance Conjecture** [Calderón-Infante, Uranga, Valenzuela]

This formulation is very useful when asymptotic limits involve multiple moduli fields.

Can we use it to say something more about the Distance Conjecture in 4D, just from EFT perspective?

There is an EFT object which helps us in this.

# EFT strings

[Lanza, Marchesano, Martucci, Valenzuela '21]

- Fundamental axionic BPS strings which probe the asymptotic region of **4D N=1** moduli space
- To avoid the EFT breakdown, the tension  $\mathcal{J}_{EFT}$  of the string has to satisfy

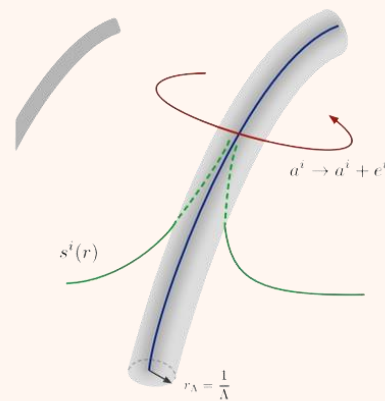
$$\Lambda^2 \leq \mathcal{J}_{EFT} \leq M_{Pl;4}^2, \quad \Lambda \text{ is the EFT cutoff}$$

- **As we approach any infinite distance limit in moduli space, there is always an EFT string becoming tensionless**

$$\mathcal{J}_{EFT} \rightarrow 0 \text{ as } s^i \rightarrow \infty$$

$$\mathcal{J}_{EFT} = M_{Pl;d}^2 e^{l_i} l_i$$

$$t^i = a^i + i s^i, \quad l_i = -\frac{1}{2} \frac{\partial K}{\partial s^i}$$



# EFT strings

[Lanza, Marchesano, Martucci, Valenzuela '21]

The behaviour of the leading tower scale  $m_*$  in any limit is dictated by the asymptotic behaviour of the EFT string which describes the limit

$$m_*^2 \sim (\mathcal{J}_{EFT})^\omega$$

where  $\omega \in \mathbb{Z}_{>0}$  is the **scaling weight**. In 4D N=1 examples, we only observe  $\omega = 1, 2, 3$ .

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**What can we say about the EFT strings  $\vec{\zeta}_{EFT}$ -vectors in asymptotic regions? How do they fit in the Convex Hull Distance Conjecture formulation?**



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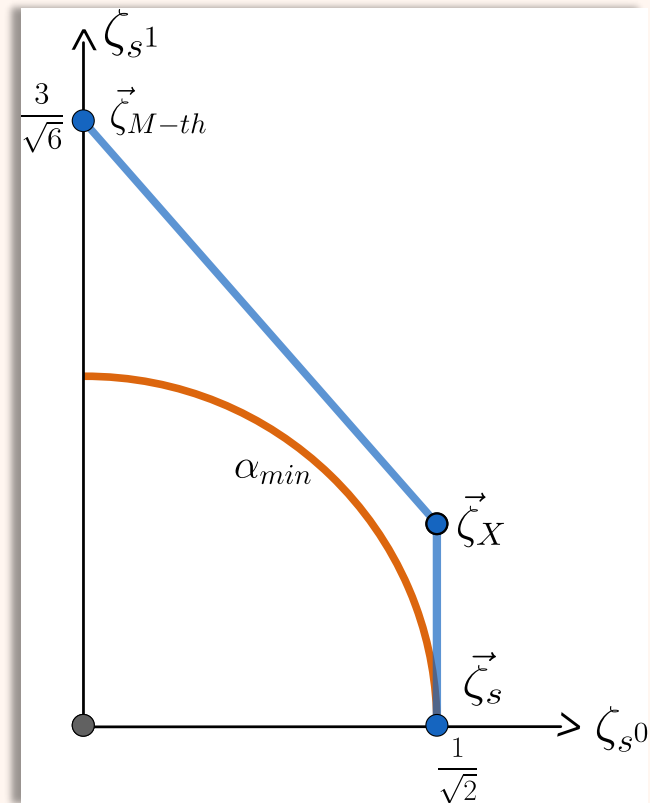
## Example 1 - Heterotic theory on a CY3

$$(s^0 = e^{-2\phi} V_X, s^1), \quad l^i = -\frac{1}{2} \frac{\partial K}{\partial s^i}$$

$$V_X \sim (s^1)^3, \quad K = -\log s^0 - \log V_X$$

$$G_{ij} = \frac{1}{2} \frac{\partial^2 K}{\partial s^i \partial s^j} = e_i^a e_j^b \delta_{ab}, \quad \zeta_a = -e_a^i \frac{\partial \log M/M_{Pl;4}}{\partial s^i}$$

| Tower             | Mass  | $\vec{\zeta}$   |
|-------------------|---|---|
| String oscillator | $m_s \sim M_{Pl;4} (s^0)^{-1/2}$            | $\left(\frac{1}{\sqrt{2}}, 0\right)$                  |
| Total X decomp.   | $m_X \sim M_{Pl;4} (s^0)^{-1/2} V_X^{-1/6}$ | $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}\right)$ |
| M-theory          | $m_{M-th} \sim M_{Pl;4} V_X^{-1/2}$         | $\left(0, \frac{3}{\sqrt{6}}\right)$                  |



## Integer weight reconstruction

Can we say something about the convex hull from a bottom-up, EFT perspective?



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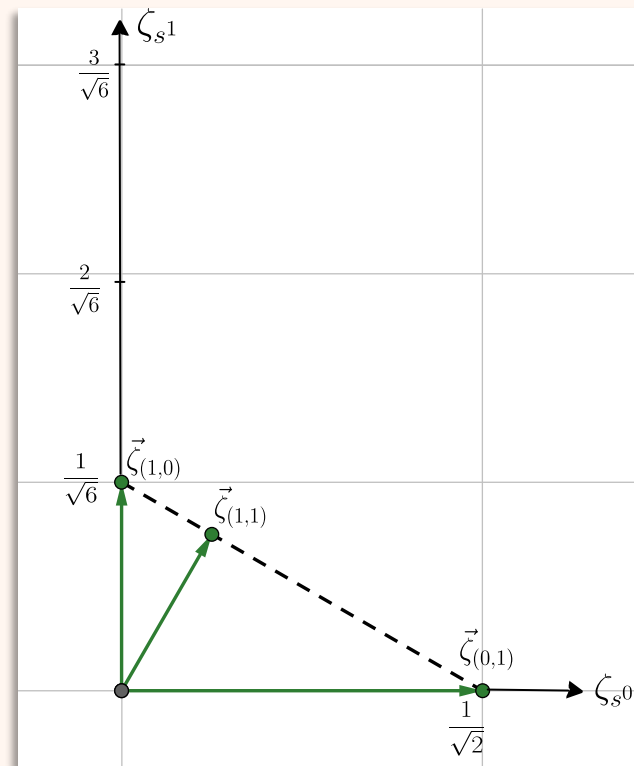
Consider EFT string limits and associate  $\vec{\zeta}$ -vectors defined as  $\zeta_{\vec{e},a} = \lim_{s^i \rightarrow \infty} \left( -\frac{1}{2} e_a^i \frac{\partial \log \mathcal{J}_{EFT}}{\partial s^i} \right)$ , where  $\vec{e} = (e^0, e^1)$  are the EFT charges.

| $\vec{e}$ | $\mathcal{J}_{EFT}$           | $\vec{\zeta}_{(e^0, e^1)}$                                |
|-----------|-------------------------------|---|
| (1,0)     | $\sim M_{Pl,4}^2 l_0$         | $\left( \frac{1}{\sqrt{2}}, 0 \right)$                    |
| (0,1)     | $\sim M_{Pl,4}^2 l_1$         | $\left( 0, \frac{1}{\sqrt{6}} \right)$                    |
| (1,1)     | $\sim M_{Pl,4}^2 (l_0 + l_1)$ | $\left( \frac{1}{4\sqrt{2}}, \frac{3}{4\sqrt{6}} \right)$ |

## Integer weight reconstruction

**a)** Plot the EFT vectors in the  $\zeta$ -plane and identify the elementary/primitive ones

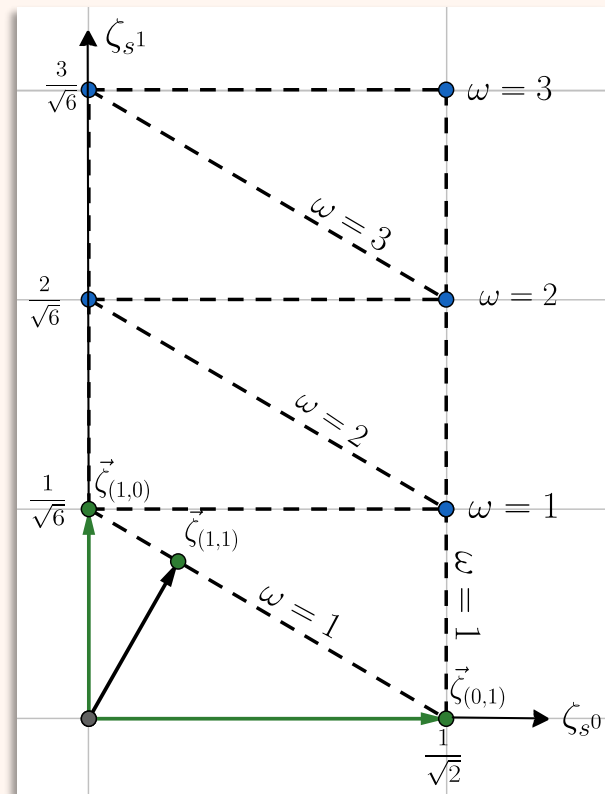
| $\vec{e}$ | $\mathcal{J}_{EFT}$           | $\vec{\zeta}_{(e^0, e^1)}$                              |
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## Integer weight reconstruction

**b)** Draw the lattice of points generated by the primitive EFT vectors with integer coefficients  $\omega_i \leq 3$ . This is motivated by the top-down observation that  **$\zeta$ -vectors for towers always have an integer projection on EFT limits, regardless of the tower being leading or not in that limit**

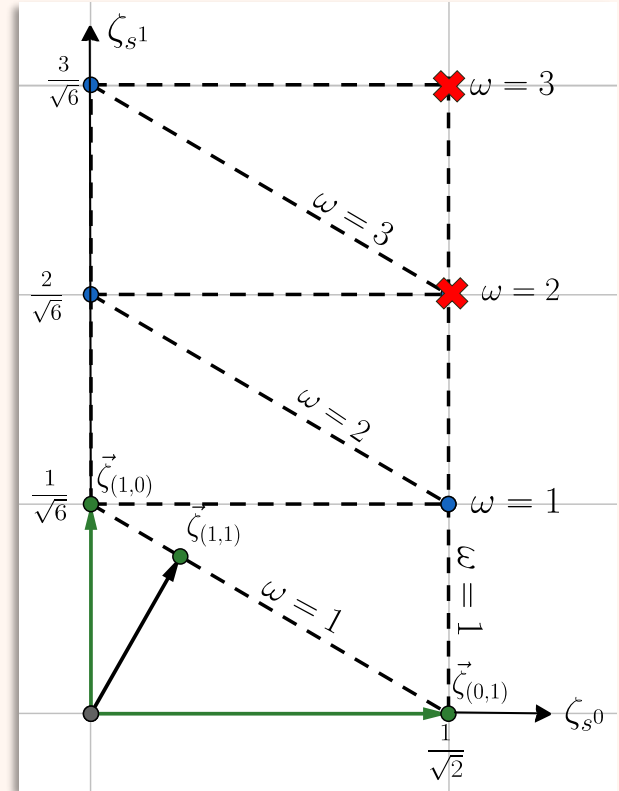
$$\vec{\zeta}_a \cdot \vec{\zeta}_{EFT} = \omega |\vec{\zeta}_{EFT}| \quad \omega \in \{0, 1, 2, 3\}$$



## Integer weight reconstruction

**c)** A limit where string oscillator modes dominate will always correspond to an EFT string, hence  $\omega = 1$  for the leading tower in the  $s^0$  direction. Also eliminate  $\zeta$ -vectors which would correspond to KK towers of non-integer number of decompactification dimensions  $n_I$ . For this, impose that their norm is

$$|\vec{\zeta}| = \sqrt{\frac{2+n_I}{2n_I}} \text{ with } n_I \text{ positive integer}$$





## Integer weight reconstruction - summary

1. Every tower generating the Distance Conjecture convex hull has to lie on the points of a lattice with elementary EFT strings as primitive vectors.

$$\vec{\zeta}_t = \omega_{1,0}^t \vec{\zeta}_{(1,0)} + \omega_{0,1}^t \vec{\zeta}_{(0,1)}$$

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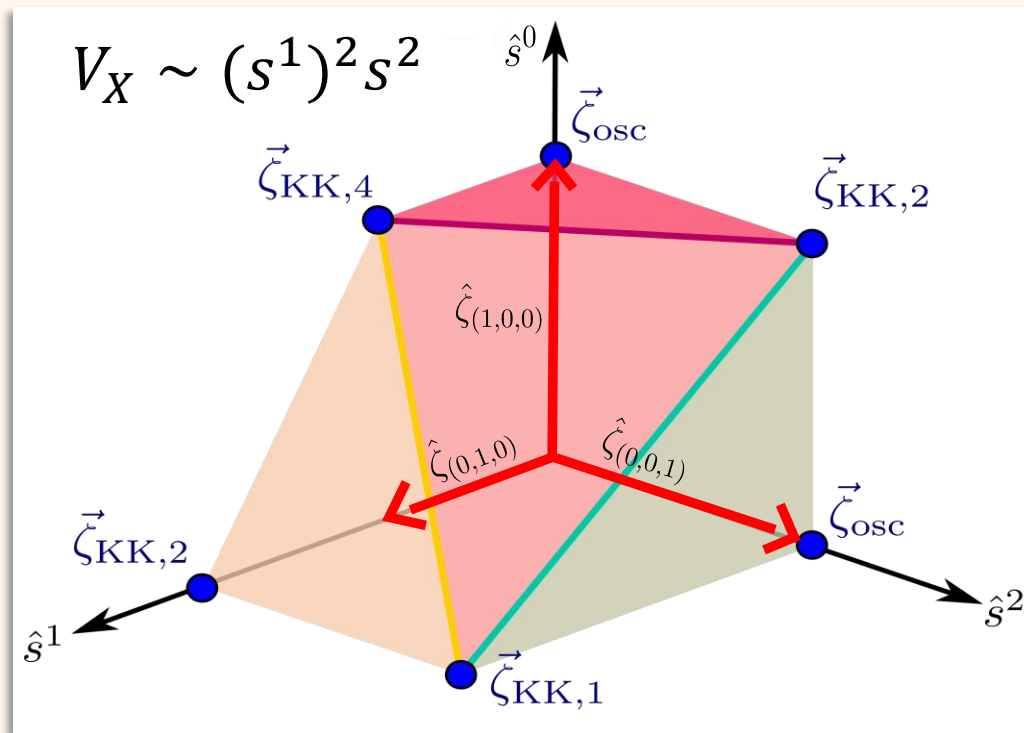
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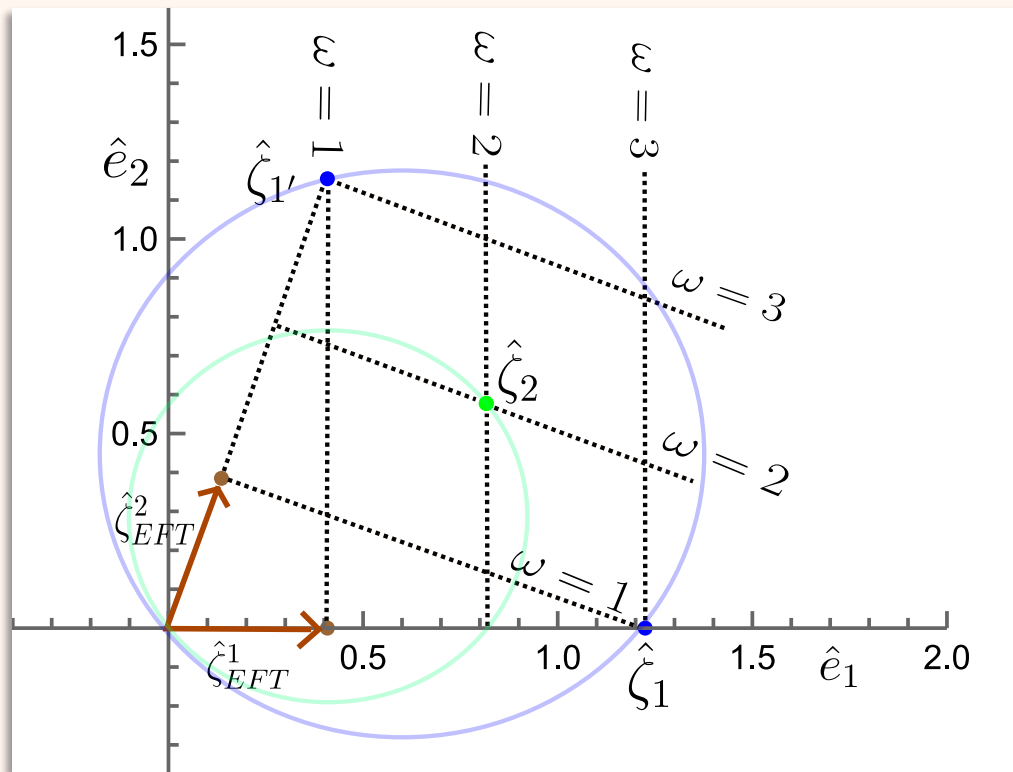
2. Leading emergent string limits  $\zeta$ -vectors, if present, always coincide with EFT string vectors and in these limits  $|\vec{\zeta}_{EFT}| = \frac{1}{\sqrt{2}} = |\vec{\zeta}_s|$ .
3. To obtain a unique convex hull, you have to impose a further consistency condition which amounts to discard non-integer numbers of decompactifying dimensions.



## Example 1 - Heterotic theory on a CY3



## Example 2 – M-theory on a G2 manifold





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- Check the reconstruction in type II compactifications
- What is the role of the species scale convex hull? Can we say something more about the duality frames, maybe using the pattern

$$\vec{\zeta}_t \cdot \vec{Z}_{sp} = \frac{1}{d-2} \quad [\text{Castellano, Ruiz, Valenzuela '23}]$$

- What is the meaning of the observed value of the scaling weight? It looks like

$\omega = 3$  M-theory limit

$\omega = 2$  F-theory/partial decomp. limit [Marchesano, Melotti '22]

$\omega = 1$  Emergent string limit

Is this always the case?



Thank you!

