M-theoretic Emergence

String Pheno 2024 in Padova - June 27 - Aleksandar Gligovic

Based on 2309.11551 and 2309.11554

in collaboration with R. Blumenhagen, N. Cribiori and A. Paraskevopoulou

Emergence in Quantum Gravity ?

Put very briefly, the Emergence Proposal states that

$\boxed{\text{kinetic terms arise in IR by integrating out towers of states down from a UV scale } \Lambda.}$

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Main quest :

- carry out an emergence computation where one integrates out the **full tower**

Sub-quests :

- define a limit, identify perturbative states
- find “simple” setup + amplitude
- find a suitable regularization

Plan for the talk

01 Framework and M-theory limit

02 Emergence of the one-loop free energy

Setting the stage

Setup : type IIA on CY_3 $(Y) \longrightarrow N = 2$ supersymmetry in 4D with $\mathscr{M} = \mathscr{M}_{VMP} \times \mathscr{M}_{HMP}$

$U(1)$ **gauge fields**
$h_{11}(Y)$ from C_3 (in VMP)
1 from C_1 (in gravity multiplet)

Scalars VMPs	**Scalars HMPs**
Kähler moduli $T^i = t^i + ib^i$	CS moduli z^k + $S = \rho + i\sigma$

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Higher-derivative corrections encoded in $\mathscr{F} = \sum_{g=0}^{\infty} \mathscr{F}_g W^{2g}$ with $\mathscr{F}_g$ at order g_s^{2g-2}

$\mathscr{F}_g$'s related to topological string amplitudes [BCOV '94] & are 1/2 BPS-saturated

Free energy at genus 0 and 1

We were particularly interested in the free energy at **genus 0** and **genus 1**

$$\mathcal{F}_0 = -\frac{1}{g_s^2}\left[\frac{1}{6}C_{ijk}\,T^iT^jT^k + \frac{\zeta(3)}{2}\chi(Y) - \sum_{\beta\in H_2(Y,\mathbb{Z})} \overset{\substack{\text{Gopakumar-Vafa Invariants}\\ \downarrow}}{a_0^\beta}\,\mathrm{Li}_3(e^{-\beta\cdot T})\right] \rightarrow \textbf{kinetic terms} \text{ of VMPs}$$

$$\mathcal{F}_1 = -\frac{1}{24}c_{2,i}\,T^i - \sum_{\beta\in H_2(Y,\mathbb{Z})}\left(\frac{\alpha_0^\beta}{12} + \alpha_1^\beta\right)\mathrm{Li}_1(e^{-\beta\cdot T})$$

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Emergence computation for $\mathcal{F}_0$ also tackled in [Hattab, Palti '23 & '24]

computation of $\mathcal{F}_g\,(g>1)$ already in [Gopakumar, Vafa '98] , albeit with different motivation

Limit and perturbative states

Study **M-theory** limit for Type IIA on $\mathbb{R}^{1,3} \times Y$. Take $g_s \to \lambda\, g_s$ with $\lambda \to \infty$

such that (i) $M_{\rm pl}^{(4)} = \text{const.}$ and (ii) $M_*^6\, V_6 = \text{const}\,.$

11D Planck scale

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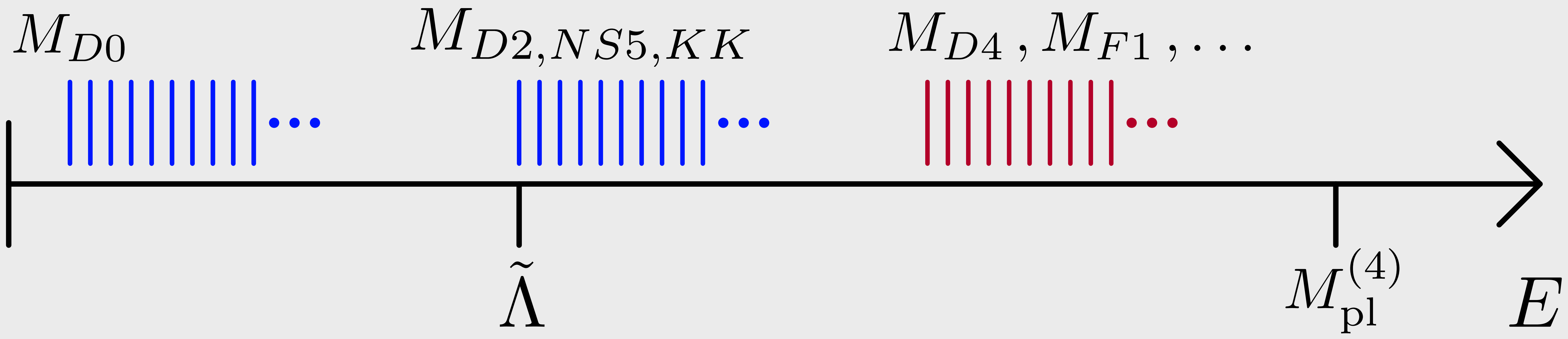

Species scale [Dvali ’07]

$$\tilde{\Lambda} = M_{\rm pl}^{(4)} \Big/ \sqrt{N_{\rm sp}} = M_*^{(5)}$$

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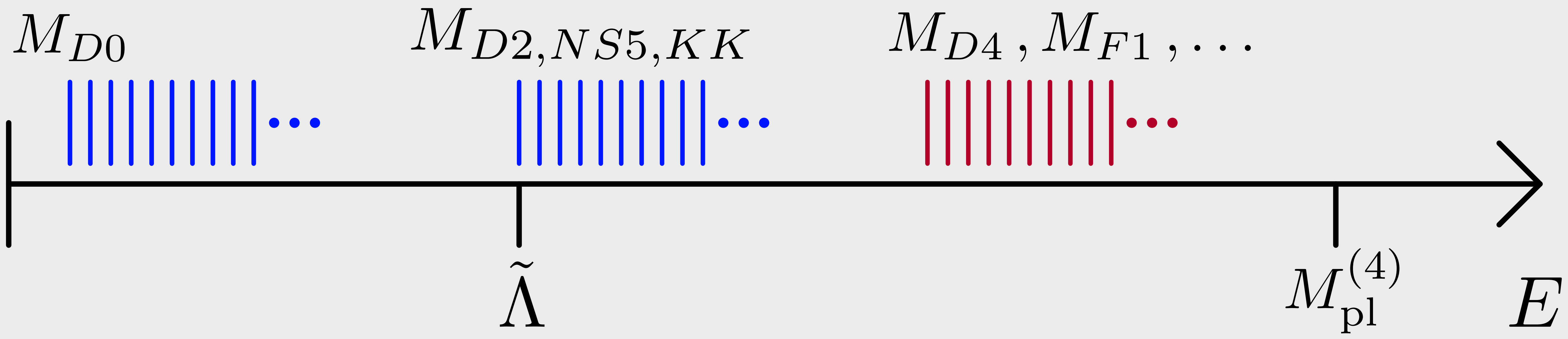

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Guiding principle: $m_{\rm pert} = g^{\alpha}\,\tilde\Lambda\,,\;\; m_{\rm non-pert} = \tilde\Lambda / g^{\beta}$ with $g_E = r_{11}^{-1} \ll 1$ and $\alpha \geq 0\,, \beta > 0$

fundamental

classical, soliton-like

The Gopakumar-Vafa (GV) formula

[Gopakumar, Vafa '98] : $\mathscr{F}_g$'s can be determined from **1-loop** calculation, with constant, self-dual background of the graviphoton

contributions for $r \leq g$

$$\sum_{g=0}^{\infty} \mathscr{F}_g W^{2g} = \sum_{\vec{\beta}, r, n} \alpha_r^{\vec{\beta}} \int_0^{\infty} \frac{ds}{s} \frac{W^2}{(2i \sinh(sW/2))^{2-2r}} e^{-s Z_{\vec{\beta},n}}$$

Integrate out **D2/D0's** with

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First observations:

- expect also $NS5$ and KK to contribute, but uncharged under VMP
- for $g = 0$ and $g = 1$ need a UV regulator

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The genus 1 free energy

We study the resolved conifold (one rigid 2-cycle, $\alpha_0^1 = 1$). The GV formula for $g = 1$ gives

$$\mathscr{F}_1 = -\frac{1}{12} \sum_{n \in \mathbb{Z}} \int_{\epsilon}^{\infty} \frac{ds}{s} e^{-s Z_n} \qquad \text{with} \qquad Z_n = \frac{M_{\text{pl}}}{\mathscr{V}^{1/2}} (T + in)$$

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"minimal subtraction"

1st we integrate and expand in ϵ : $\mathcal{F}_1^{D0,D2} = \frac{1}{12}\sum_{n\in\mathbb{Z}}\left(\gamma_E + \cancel{\log(\epsilon)} + \log(Z_n) + \mathcal{O}(\epsilon)\right)$

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We are left with: $\mathcal{F}_1^{D0,D2} = \dfrac{1}{12}\displaystyle\sum_{n\in\mathbb{Z}}\log\left(\frac{T+in}{\mu(\mathcal{V},\epsilon)}\right)$ … now use **analytic continuation** of $\zeta(s)$!

Result and interpretation

$$\boxed{\mathcal{F}_1^{D2-D0} = \frac{2\pi T}{24} - \frac{1}{12}\sum_{m\geq 1}\frac{1}{m}e^{-2\pi m T} = \frac{2\pi}{24}\left(\frac{t}{g_E} + ib\right) - \frac{1}{12}\sum_{m\geq 1}\frac{1}{m}e^{-2\pi m(\frac{t}{g_E}+ib)}}$$

… emergent theory has perturbative-like expansion in $g_E \simeq 1/\lambda \simeq 1/N_{\rm sp}$. Indeed,

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With the same techniques one can compute $\mathcal{F}_0 = \sum_{n\in\mathbb{Z}}\int_{\epsilon}^{\infty}\frac{ds}{s^3}e^{-s\,Z_n}$

Wrap-up and outlook

define M-theory limit → identify pert. states with $\tilde{\Lambda}$ → integrate out P states at 1-loop

$M_{\rm pl}^{(4)}$ constant + no decompact. of CY_3

$(M2, M5) \perp S^1$ + KK along S^1 are perturbative

here: GV formula + ζ - function regularization

Remaining challenge: how to regularize sum over GV Invariants?

e.g. for a compact CY with $h_{11} = 1$: $\kappa_{111} \overset{?}{\simeq} \sum_{n=1}^{\infty} n^3\, \mathrm{GV}_n$ with $\mathrm{GV}_{\rm n} \sim \exp(n)$

Extra slides

Details about M-theory limit

We look at M-theory limits of type IIA on X_k $(d + k = 10)$ to d dimensions

$$R_{11} \to \lambda R_{11} \quad , \quad M_* \to \frac{M_*}{\lambda^{\frac{1}{d-1}}} \quad , \quad R_I \to \lambda^{\frac{1}{d-1}} R_I \quad \text{with} \quad \lambda \to \infty$$

Can be used to compute mass scales of type IIA/M-theory objects

$$M_{D0} \sim \frac{M_s}{g_s} \sim \frac{M_{\rm pl}^{(d)}}{\lambda} \quad \longrightarrow \quad \tilde{\Lambda} \simeq \frac{M_{\rm pl}^{(d)}}{\lambda^{\frac{1}{d-1}}} \sim M_*$$

$$M_{D2,NS5} \sim \frac{M_s}{g_s^{1/3}} \sim \frac{M_{\rm pl}^{(d)}}{\lambda^{\frac{1}{d-1}}} \sim M_*$$