A Deeper Look at the Minimal Weak Gravity Conjecture

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based on arXiv:2312.04619 [hep-th]

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Tower Weak Gravity Conjecture

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Every U(1) gauge theory coupled to gravity has a **tower** with **infinitely** many super-extremal states such that

$$\frac{g^2_{\mathrm{U(1),\,D}} q^2}{m_D^2} \geq \gamma \frac{1}{M_{\mathrm{Pl,\,D}}^{D-2}} .$$ [1]

Motivations

1. Consistency under **dimensional reduction**.[1]
2. **Absence** of global symmetries in limit $g_{\mathrm{U(1),\,D}} \longrightarrow 0$.

How to satisfy the Weak Gravity Conjecture

1. Tower of super-extremal **particles**: $m_n \leq M_{\mathrm{BH,\,min.}}$ with charge nq.
2. Tower of super-extremal state **above** the BH threshold.

[1] B. Heidenreich, M. Reece, T. Rudelius, *JHEP* **02**, 140, arXiv: 1509.06374 (hep-th); M. Montero, G. Shiu, P. Soler, *JHEP* **10**, 159, arXiv: 1606.08438 (hep-th); S. Andriolo, D. Junghans, T. Noumi, G. Shiu, *Fortsch. Phys.* **66**, 1800020, arXiv: 1802.04287 (hep-th)

The Minimal Weak Gravity Conjecture

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Towers of (super-)extremal particle states exist **if and only if** they are required by the WGC under dimensional reduction:

1. Emergent string limits,
2. Kaluza–Klein reductions with KK gauge bosons.
3. Strongly coupled limits with exactly extremal states. (**?**)

Absent Towers

All known cases without established super-extremal tower are **consistent**:

1. Perturbative open string $\mathrm{U}(1)$s.
2. F-theory away from emergent string limits.
3. Conifold $\mathrm{U}(1)$ in M-theory.
4. ...

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Definition of Tower of Particles

The Tower

Super-extremal particle states **below** the black hole **threshold** of charge $n\,q$, for every charge q in the charge lattice and for any $n \in \mathcal{I}_q$, with $\mathcal{I}_q$ an infinite index set.

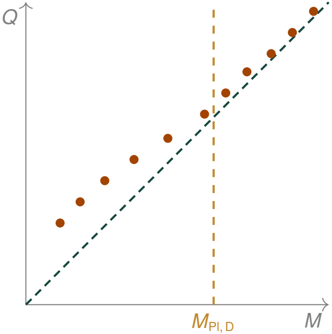
Particles below Black Hole Threshold

When

$$M^2_{\text{BH, ext.}} \sim Q^2 \frac{g^2_{\text{U(1), D}} M^{D-2}_{\text{Pl, D}}}{\gamma} \leq M^2_{\text{BH, min.}}$$

for arbitrary large Q?

1. **Weak coupling**: $g^2_{\text{U(1), D}} M^{D-2}_{\text{Pl, D}} \longrightarrow 0$ and $\gamma \sim \mathcal{O}(1)$.
2. **Strong coupling**: $\gamma \longrightarrow \infty$.



Weak Coupling Limits

Weak Coupling

$$g^2_{\mathrm{U(1),\,D}} M^{D-2}_{\mathrm{Pl,\,D}} \to 0 \qquad \text{and} \qquad \gamma \sim \mathcal{O}(1)$$

1. The mass of the extremal black hole is set by the WGC scale
$$\Lambda^2_{\mathrm{WGC,\,D}} = g^2_{\mathrm{U(1),\,D}} M^{D-2}_{\mathrm{Pl,\,D}}\,.$$
2. The weakly-coupled limits correspond to **infinite distance limits** in moduli space.
3. In decompactification limits, the states are below the **species scale**,[2] i.e. $M^2_{\mathrm{Pl},\,\infty}$.
4. In emergent string limits, the WGC scale should be **below** the Planck scale.

Conditions

$$g^2_{\mathrm{U(1),\,D}} M^{D-4}_{\mathrm{Pl,\,D}} = \frac{\Lambda^2_{\mathrm{WGC,\,D}}}{M^2_{\mathrm{Pl,\,D}}} \ll 1 \qquad \text{and} \qquad \frac{\Lambda^2_{\mathrm{WGC,\,D}}}{M^2_{\mathrm{Pl},\,\infty}} \ll 1$$

[2] G. Dvali, *Fortsch. Phys.* **58**, 528–536, arXiv: 0706.2050 (hep-th); G. Dvali, D. Lust, *Fortsch. Phys.* **58**, 505–527, arXiv: 0912.3167 (hep-th); G. Dvali, C. Gomez, arXiv: 1004.3744 (hep-th)

Strong Coupling Limits

Strong Coupling

$$\gamma \longrightarrow \infty$$

1. $\gamma \longrightarrow \infty$ implies $g^2_{\mathrm{U(1),\,D}} M_{\mathrm{Pl,\,D}}^{D-4} \longrightarrow \infty$, but the converse does **not** hold.
2. Strong coupling with $\gamma \sim \mathcal{O}(1)$ can be achieved by taking an infinite distance limit.
3. We need finite distance limits to have a tower.

Conditions

$$\left(g^2_{\mathrm{U(1),\,D}} M_{\mathrm{Pl,\,D}}^{D-4}\right)\Big|_{p_\varepsilon} > \varepsilon^{-1} \qquad \text{and} \qquad d(p_1, p_\varepsilon) < \delta_1\, M_{\mathrm{Pl,\,D}}^{\frac{D-2}{2}}$$

Existence of Towers of Particles

Existence of Towers of Particles

There **can** exist an infinite tower of super-extremal states that become particle-like excitations if either:

1. **Weak Coupling**:

$$\left(g^2_{\mathrm{U(1),\,D}} M^{D-4}_{\mathrm{Pl,\,D}}\right) < \varepsilon \qquad \text{and} \qquad \frac{g^2_{\mathrm{U(1),\,D}} M^{D-2}_{\mathrm{Pl,\,D}}}{M^2_{\mathrm{Pl},\,\infty}} < \varepsilon\,.$$

   - **Necessary** conditions for the existence of a tower of states.
   - Particle-like description also in the **highest-dimensional** description of the theory.

2. **Finite Distance Strong Coupling**:

$$\left(g^2_{\mathrm{U(1),\,D}} M^{D-4}_{\mathrm{Pl,\,D}}\right) > \varepsilon^{-1} \qquad \text{and} \qquad d(p_1\,, p_\varepsilon) < \delta_1\, M^{\frac{D-2}{2}}_{\mathrm{Pl,\,D}}\,.$$

   - **Necessary** conditions for the existence of a tower of states.
   - **Warning**: Strongly-coupled gauge theories at finite distance **without** an infinite tower of particle states.

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Arguing for the Minimal Radius

KK Particles

A KK tower of mass $M_{\text{KK}} \sim \frac{1}{2\pi r_{S^1}}$ can be **detected** from the EFT if

$$M_{\text{KK}} \sim \frac{1}{2\pi r_{S^1}} \leq M_{\text{BH, min.}}$$

Species Scale and Minimal Black Hole

$$\frac{M_{\text{BH, min.}}}{M_{\text{Pl, D}}} = \left(\frac{M_{\text{Pl, D}}}{\Lambda_{\text{QG}}}\right)^{D-3} \quad \text{with } r^{-1}_{\text{BH, min.}} \sim \Lambda_{\text{QG}}.$$ [3]

Minimal Radius

1. Suppose $\Lambda_{\text{QG}} = M_{\text{Pl, D}-1}$, then we have
$$2\pi r^{\text{min.}}_{S^1} = M^{-1}_{\text{Pl, D}-1} \; : \; \textbf{minimal radius}.$$
2. It exists every time the small radius limit is not associated with a weakly coupled tower.

[3] G. Dvali, *Fortsch. Phys.* **58**, 528–536, arXiv: 0706.2050 (hep-th); N. B. Agmon, A. Bedroya, M. J. Kang, C. Vafa, arXiv: 2212.06187 (hep-th)
See Max's talk at Strings and Geometry 2024.

F-theory/M-theory Duality

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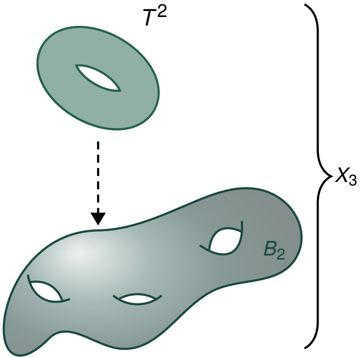
F-theory on $X_3 \times S^1$ is dual to M-theory on X_3 with

$$r_{S^1} M_{11d} = \frac{1}{2\pi \mathrm{vol}(T^2) M_{11d}^2} \equiv \frac{1}{2\pi\tau} \,.$$

Small Radius

1. Naively $r_{S^1} M_{11d} \to 0 \implies \tau \to \infty$.
2. Vector multiplet limit: $\mathrm{vol}(X_3) M_{11d}^6 \equiv \mathcal{V}_{X_3} \simeq \tau \mathcal{V}_{B_2} \simeq$ const.
3. Looking for **lower bound** on $r_{\text{min.}}$.



The Maximal Fiber

The **maximal** fiber is given by

$$\tau_{\text{max.}} = \max_{\tau \geq 0} \left(\tau \,\middle|\, \mathcal{V}_{X_3} = \alpha\tau^3 + \beta\tau^2 + \tau\mathcal{V}_{B_2} \overset{!}{=} 1 \right) .$$

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2. **Finite Distance Strong Coupling**:

$$\left(g^2_{\mathrm{U(1),\,D}} M^{D-4}_{\mathrm{Pl,\,D}}\right) > \varepsilon^{-1} \qquad \text{and} \qquad d(p_1\,, p_\varepsilon) < \delta_1\, M^{\frac{D-2}{2}}_{\mathrm{Pl,\,D}}\,.$$

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Thank you!

Genuine Gauge Theories

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A **genuine** gauge theory is a gauge theory that, within the validity of the EFT, cannot be resolved to be a **defect** or **higher-form** symmetry.

Length Scale for a Gauge Theory

It is possible to associate a length scale $\ell_{\text{perp.}}$ to defect or higher-form symmetries:

- **Defect theories**:
$$\ell_{\text{perp.}} = \text{diam}(\mathcal{S}_{\text{perp.}})\,,$$
where $\mathcal{S}_{\text{perp.}}$ is the $(D-d)$-dimensional space for a d-dimensional defect in a D-dimensional EFT.
- **Higher-form symmetries**:
$$\ell_{\text{perp.}} = \text{diam}(C)\,,$$
where C is the compact cycle over which an higher-form symmetry is reduced.

WGC for Genuine Gauge Theories

Constraints from Quantum Gravity

An EFT coupled to gravity has a quantum gravity cutoff Λ_{QG} that also set a **minimal length scale**

$$\ell_{\text{min.}} = \frac{1}{\Lambda_{\text{QG}}} \,.$$

WGC for Genuine Gauge Theories

A genuine gauge theory for the EFT coupled to gravity has

1. $\ell_{\text{perp.}} < \ell_{\text{min.}}$: EFT does **not** know the origin of the gauge theory.
2. $\Lambda_{\text{WGC}} \lesssim \Lambda_{\text{QG}}$: the gauge theory is **coupled** to gravity.