ETW brane networks for Calabi-Yau moduli

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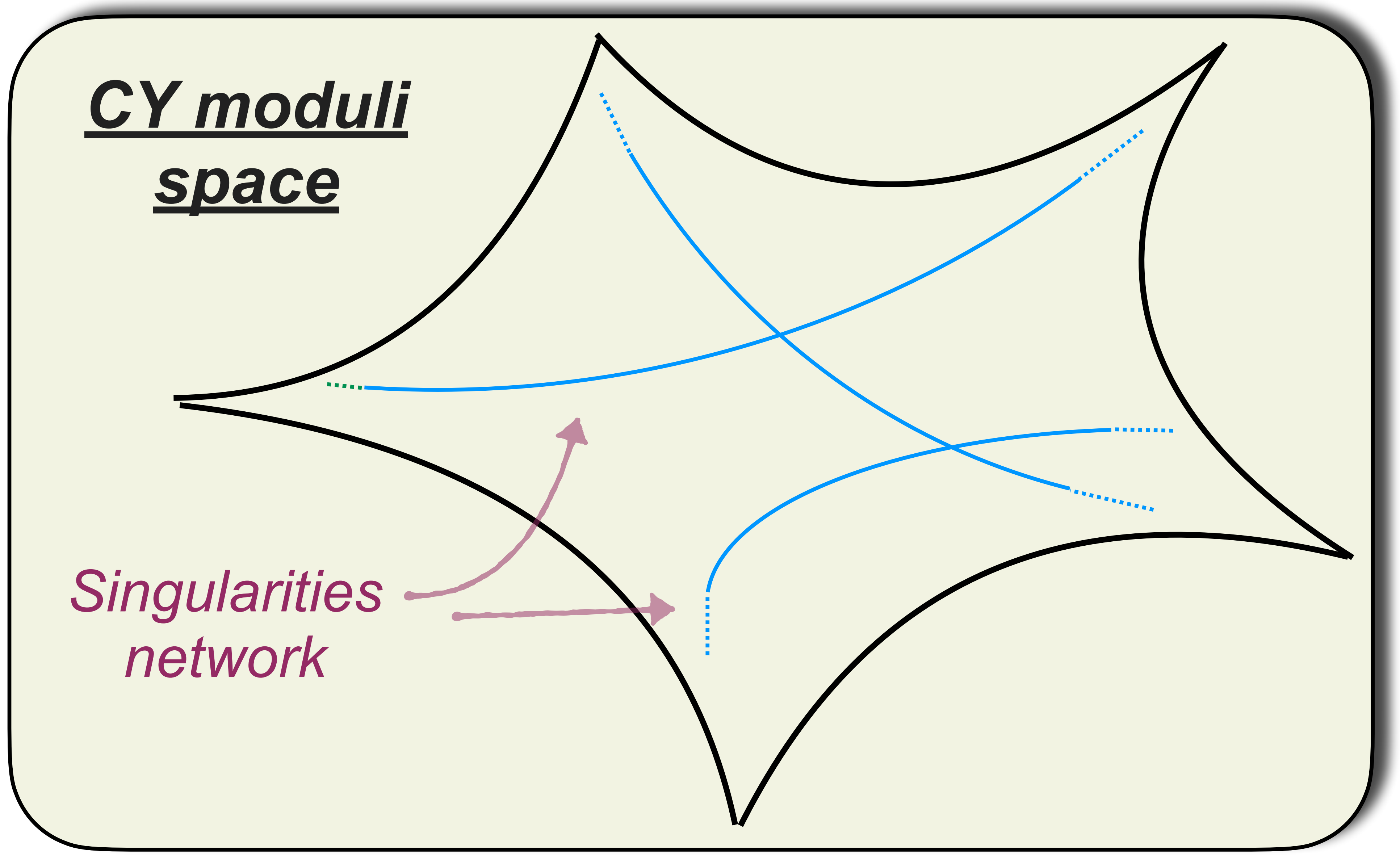
Based on: 2404.14486

and also 2312.16286 with A. Uranga and A. Makridou
2203.11240 with A. Uranga, J. Huertas, M. Delgado and J.Calderon-Infante

Motivations and techniques:

→ **To provide spacetime realizations for the network of infinite distance singularities in the complex structure moduli space of Calabi-Yau compactifications**



- Calabi-Yau compactifications provide a rich landscape of phenomenological interesting vacua;
- Infinite distance limits are the natural arena for testing many of the Swampland conjectures in a physically under control configuration;
- There exists a very powerful mathematical classification of these limits in terms of asymptotic Hodge theory

Techniques:

(a) Construction of the spacetime effective actions for the relevant moduli near the network of infinite distance singularities using the information encoded in the asymptotic Hodge structure of the moduli space;

(b) Computation of the Dynamical Cobordism solutions for these actions realizing networks of intersecting ETW branes in spacetime.

Dynamical Cobordism to Nothing

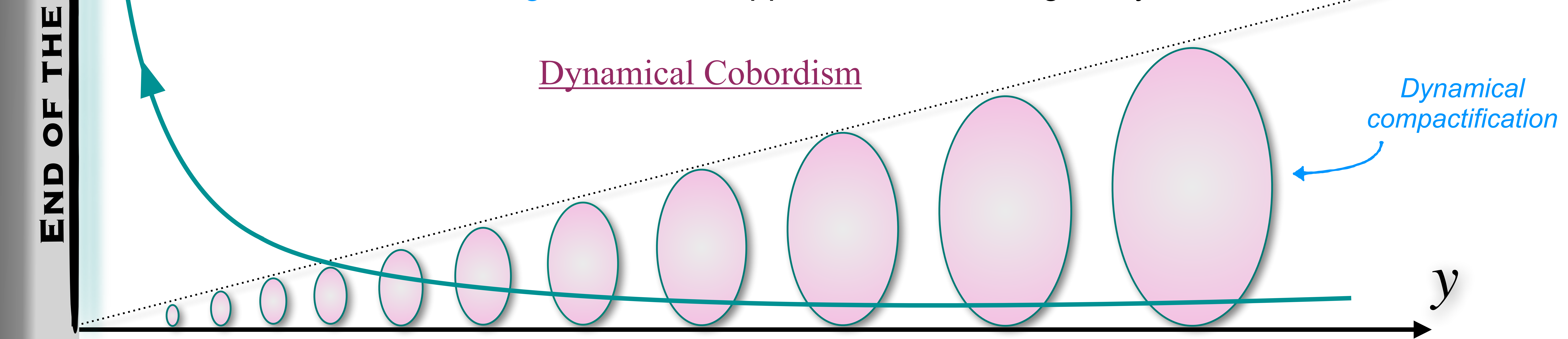
[Buratti,(Calderon-Infante),Delgado,Uranga '21],
[R.A.,(Calderon-Infante),(Delgado),(Huertas), Uranga '22, '23]
[Blumenhagen, (Cribiori), Kneissl, (Makridou), (Wang), '22,'23]

Let us consider the following *(n+1)-dim* action:

$$S = \int d^{n+1}x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}\partial\phi\partial\phi - V(\phi)\right\}$$

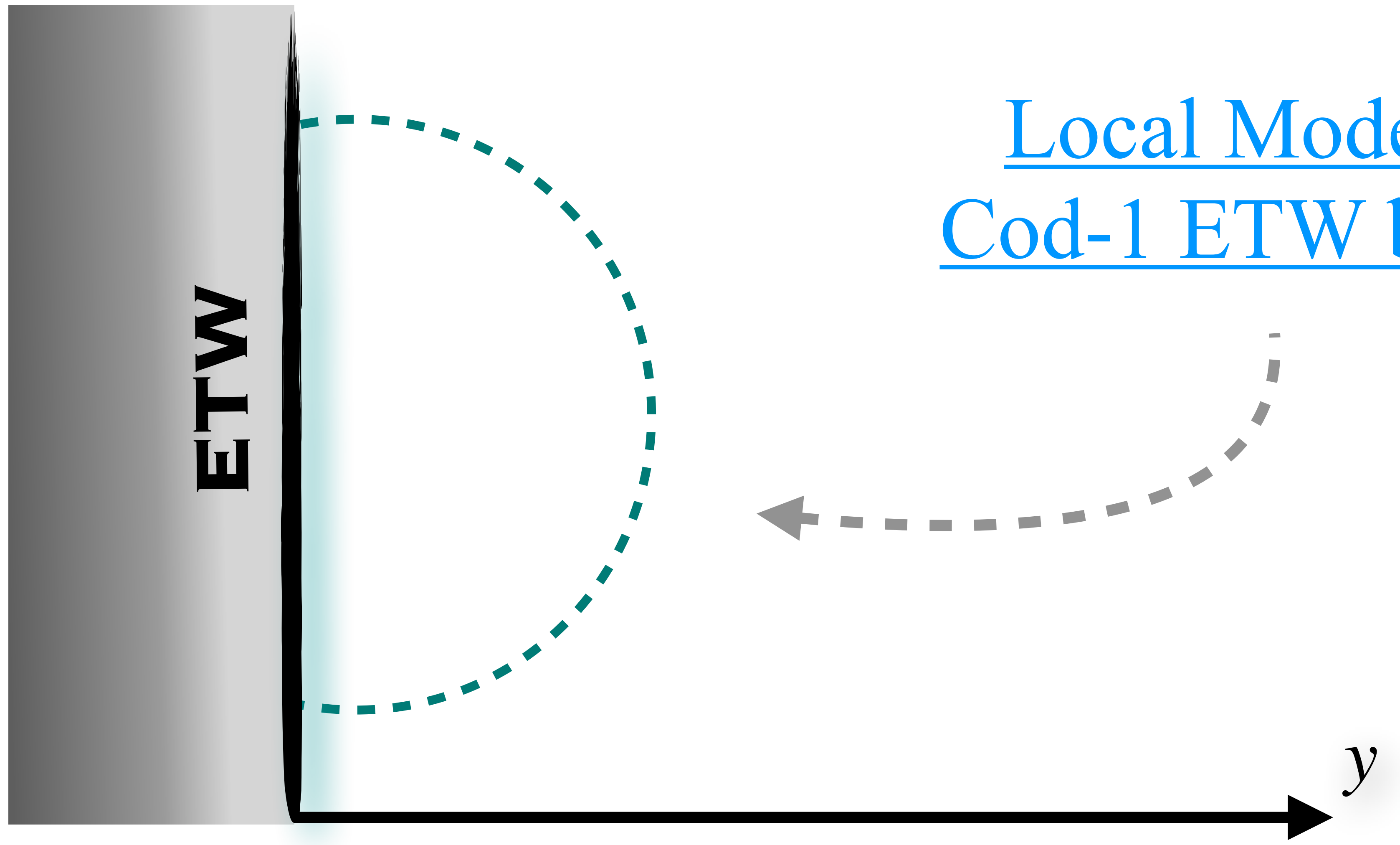
and look for solutions that:

- are spacetime dependent,
- show a Ricci singularity in the metric located at finite distance in spacetime,
- the scalar diverges when we approach to such singularity.



End of The World branes

Are different kind of ETW branes already distinguishable at the level of effective theory?



Local Model for Cod-1 ETW branes:

- $\varphi(y) \simeq -\frac{2}{\delta}\log y$
- $ds^2_{n+1} = e^{-2\sigma(y)}ds^2_n + dy^2$

with $\sigma(y) \simeq \pm \frac{4}{(n-1)\delta^2}\log y$

controlled by the following class of potentials: $\boxed{V(\phi) \simeq -cae^{\delta\phi}}$

Microscopic defect	$n+1$	δ	a
Bubble of Nothing	4	$\sqrt{6}$	0
D2 brane	4	$\sqrt{14/7}$	20/21
D2/D6 on $T^4 \times S^2$	4	$\sqrt{2}$	2/3

At fixed spacetime dimension, the only parameter specifying the kind of ETW-brane is the critical exponent:

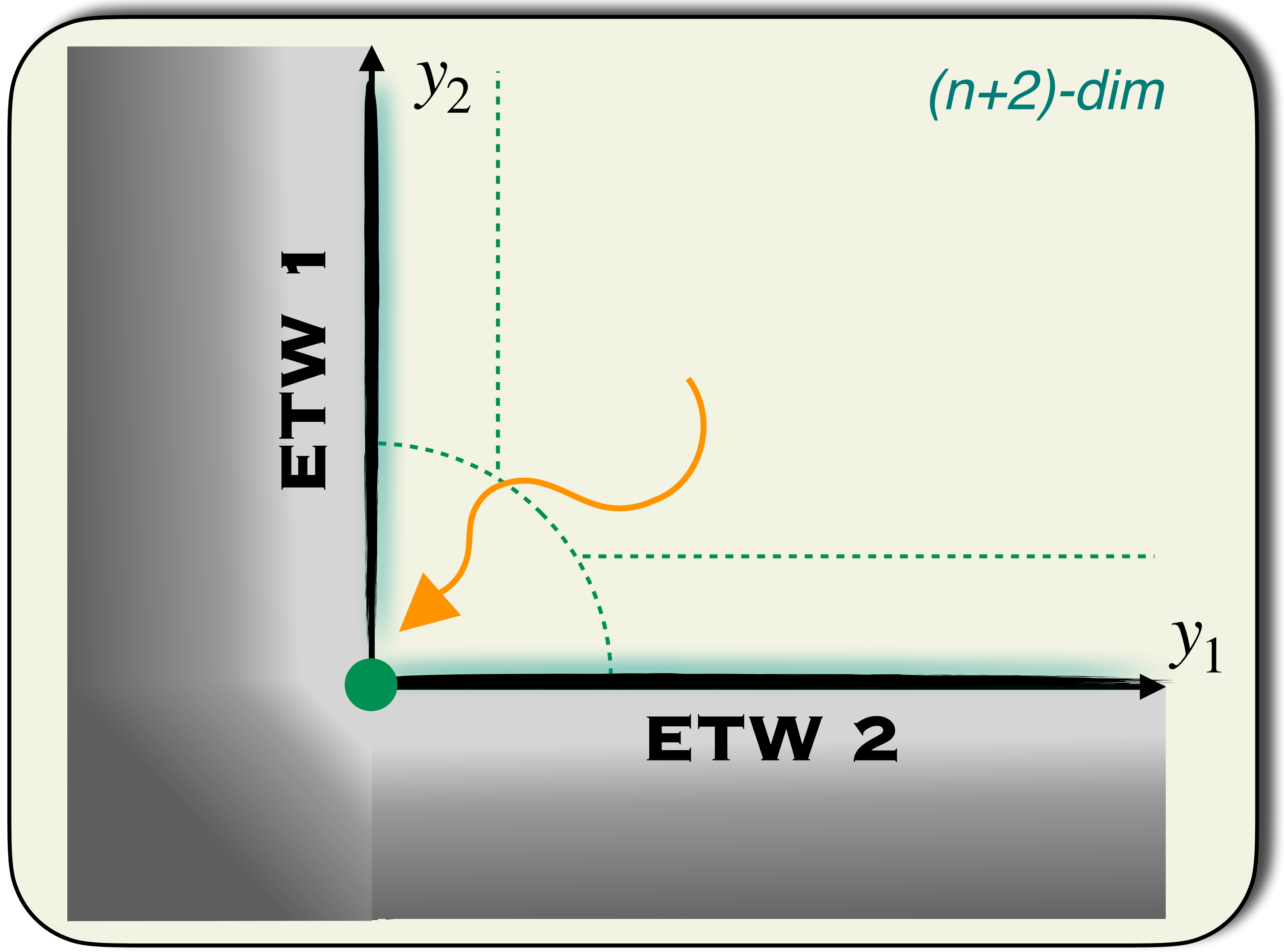
$$\delta = 2\sqrt{\frac{n}{n-1}(1-a)}$$

Intersecting ETW branes

[R.A.,Makridou,Uranga, '23]

→ Generalization of the previous analysis to explore corners of the moduli/field spaces parameterized by multiple scalars that explode simultaneously

$$S = \int d^{n+2}x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}\left(\partial\varphi_1\right)^2 - \frac{1}{2}\left(\partial\varphi_1\right)^2 - \frac{\alpha}{2}\partial\varphi_1\partial\varphi_2 - V(\varphi_1,\varphi_2)\right\}$$



Solutions describing two intersecting cod-1 ETW branes:

- $\varphi_1(y_1) = -\dfrac{2}{\delta_1}\log y_1$
- $\varphi_2(y_2) = -\dfrac{2}{\delta_2}\log y_2$
- $ds^2_{n+2} = e^{-2\sigma_1(y_1)-2\sigma_2(y_2)}ds_n^2 + e^{-2\sigma_2(y_2)}dy_1^2 + e^{-2\sigma_1(y_1)}dy_2^2$

with $\sigma_1(y_1) = \pm\dfrac{4}{n\delta_1^2}\log y_1$ and $\sigma_2(y_2) = \pm\dfrac{4}{n\delta_2^2}\log y_2$

controlled by the following class of potentials: $\boxed{V(\phi) \simeq -c_1v_1e^{\delta_1\varphi_1}e^{a_2\delta_2\varphi_2} - c_2v_2e^{a_1\delta_1\varphi_1}e^{\delta_2\varphi_2}}$

CY moduli space

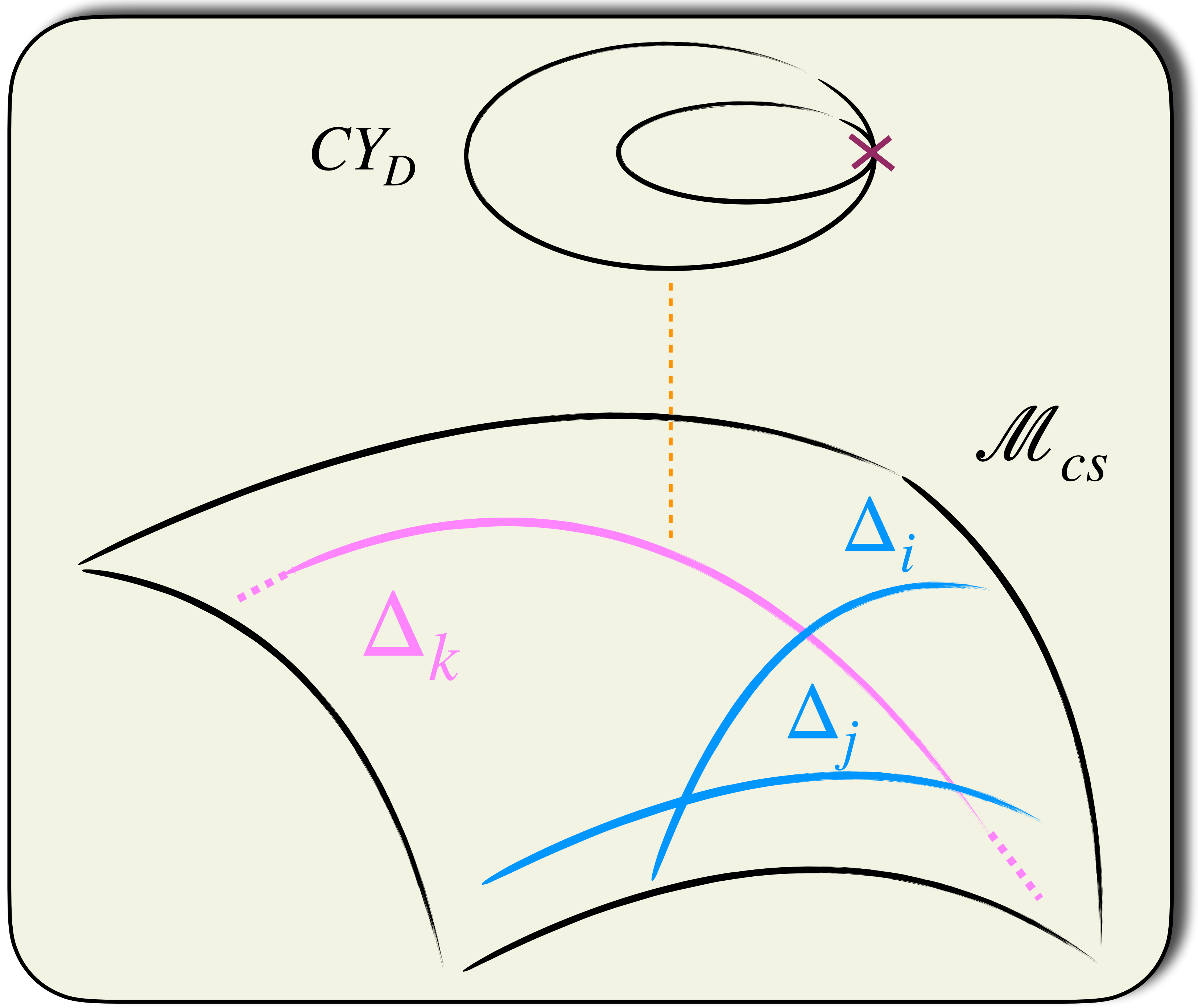
[Math: Cattani, Kaplan, Schmid '86; Deligne '91; Kerr, Pearlstein, Robles '19 …]
[Phys: (Bastian), Grimm, (Li), (Palti), (Ruehle), (Valenzuela), (van de Heisteeg),… '18, '19, '20, '21 …]

Let $\mathscr{M}_{cs}$ be the complex structure moduli space of CY_4:

- It has complex dimension $h^{3,1} = 2$;
- It is not smooth nor compact ;
- It is a quasi-projective manifold.

Discriminant locus

$$\Delta = \bigcup_k \Delta_k$$



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Local set of coordinates $\{\xi^i, t^k\}$ near the divisor Δ_k s.t.:

$$t^k = a^k + is^k \mapsto const + i\infty$$

Nilpotent matrix of local monodromy: $N_k = \log T_k^{(u)}$

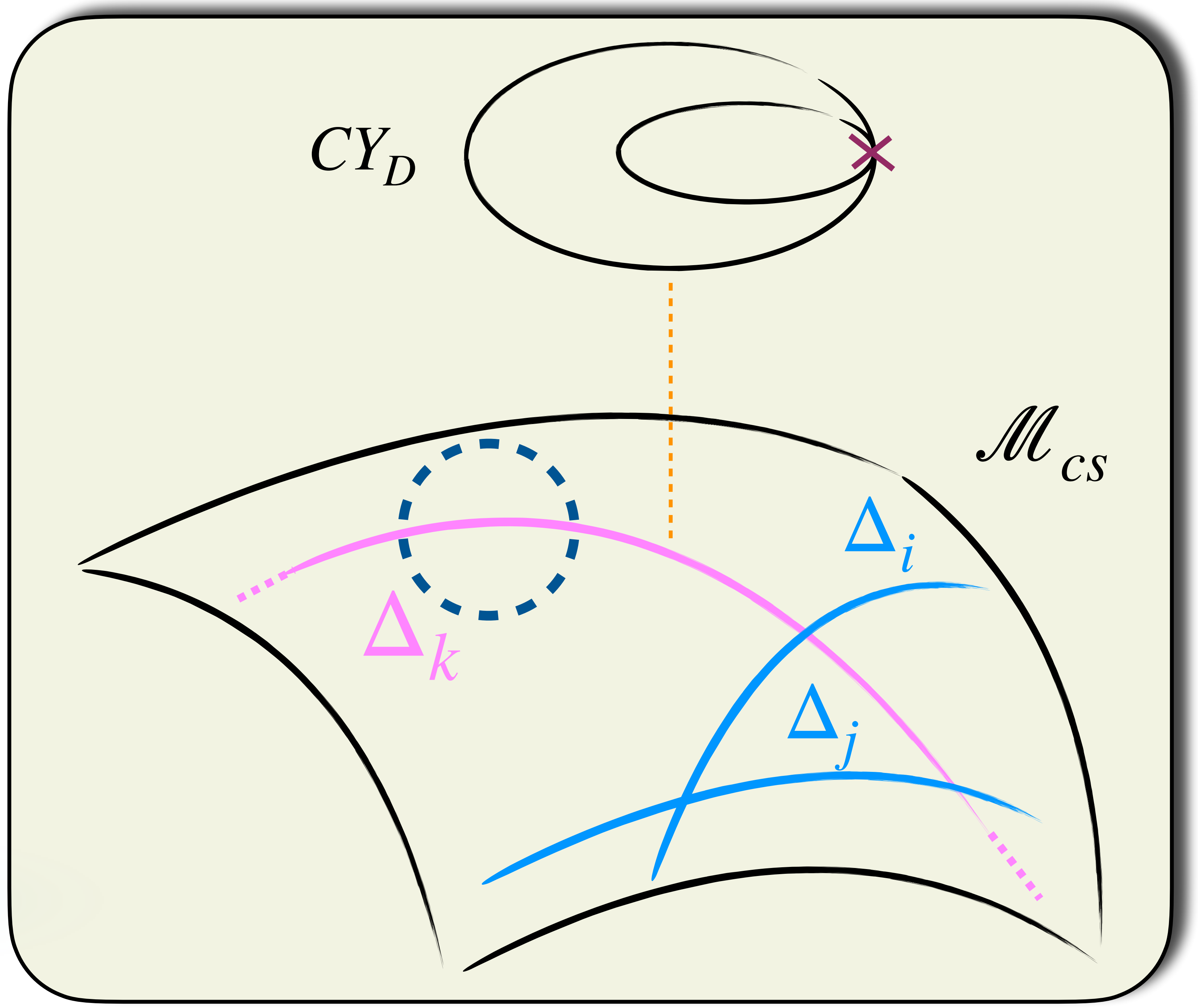
BULK → **ASYMPTOTIC SINGULARITIES**

$$H^4(Y_4, \mathbb{C}) = \oplus_{D=p+q} H^{p,q}$$

Hodge decomposition

$\{I^{p,q}\}$ with $0 \leq p+q \leq 8$

Hodge-Deligne splitting



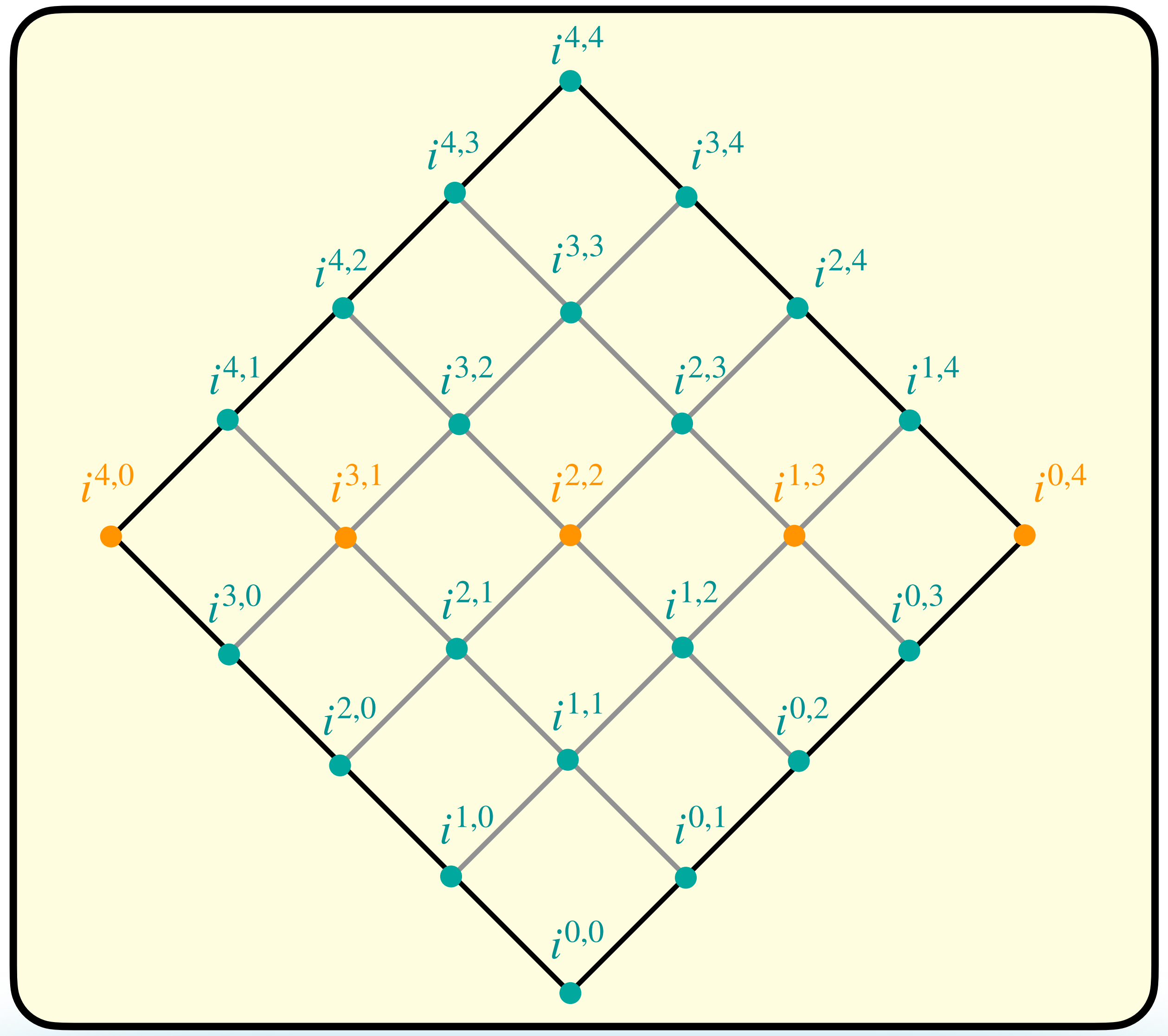
Singularities classification (for $h^{3,1} = 2$)

[Grimm, Li, Valenzuela, '19]

Classification of all the possible types of singularities in $\mathcal{M}_{cs}$ $\simeq$ Classification of all the possible Hodge-Deligne splittings associated to all the possible singular divisors Δ_k

- **Type I :** $I_{0,0}$, $I_{0,1}$, $I_{1,1}$, $I_{0,2}$, $I_{1,2}$, $I_{2,2}$
- **Type II :** $II_{0,0}$, $II_{0,1}$, $II_{1,1}$
- **Type III :** $III_{0,0}$, $III_{0,1}$, $III_{1,1}$
- **Type IV :** $IV_{0,1}$
- **Type V :** $V_{1,1}$, $V_{1,2}$, $V_{2,2}$

→ **Infinite distance in $\mathcal{M}_{cs}$**



Deligne diamond

Effective action

Let us consider M-theory compactified on a CY_4.

The classification of the singularities provides the information to write the action for s_k near Δ_k:

$$S = \int d^3x\sqrt{-g}\left\{\frac{1}{2}R - \frac{d_k}{4(s_k)^2}\left(\partial s^k\right)^2 + \dots\right\}$$

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Putting a G_4-flux in our CY_4 compactification we generate a 3d potetial:

$$V = \frac{1}{\mathcal{V}_4^3} \left(\int_{Y_4} G_4 \wedge \star \bar{G}_4 - \cancel{\int_{Y_4} G_4 \wedge G_4} \right)$$

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- Using the *Growth Theorem* for the Hodge norm we get the leading potential near the boundary Δ_k

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$$S = \int d^3x \sqrt{-g} \left\{ \frac{1}{2} R - \frac{1}{2} \left(\partial \phi_k \right)^2 - ||\rho||^2 e^{\sqrt{\frac{2}{d_k}}(l_k - 4)\phi_k} \right\}$$

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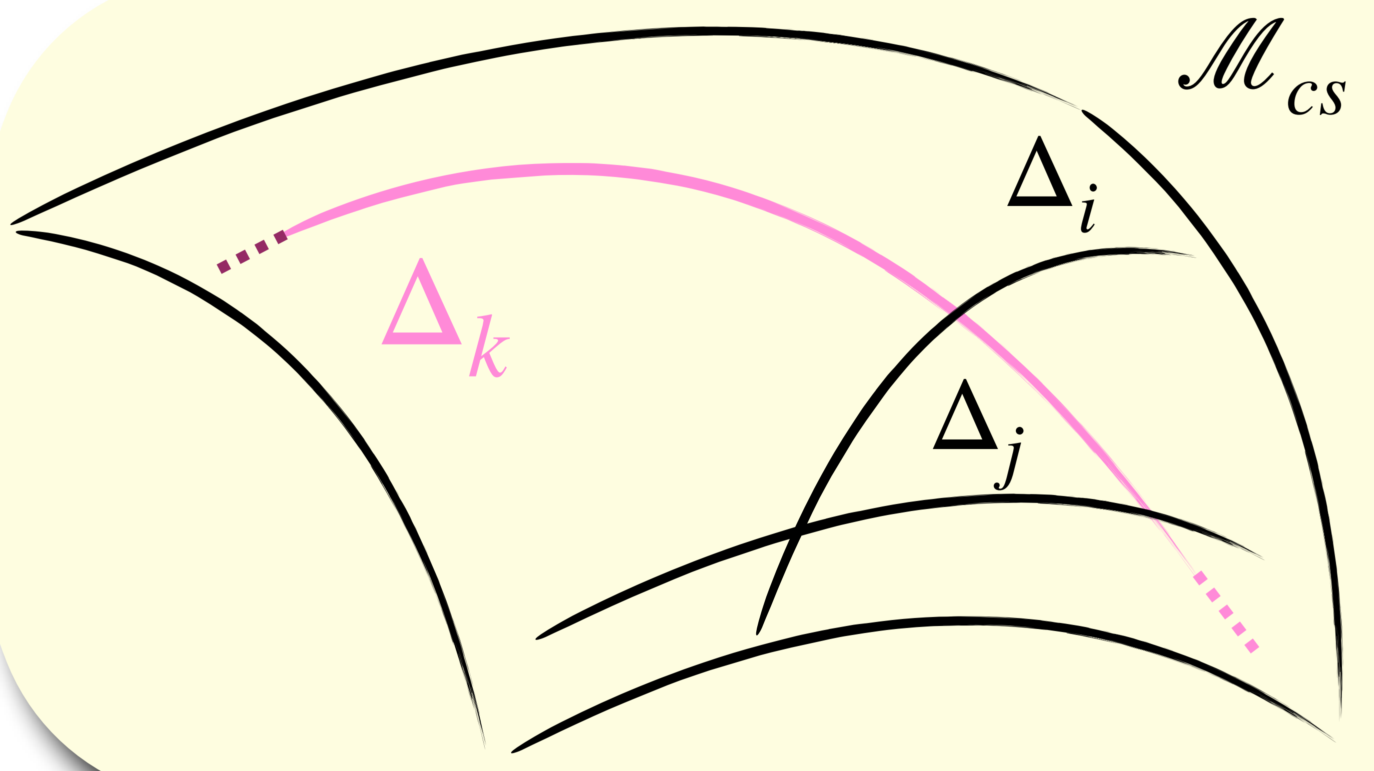
- Using the *Growth Theorem* for the Hodge norm we get the leading potential near the boundary Δ_k

- Redefining the field: $\phi_k = \sqrt{\dfrac{d_k}{2}} \log s_k$

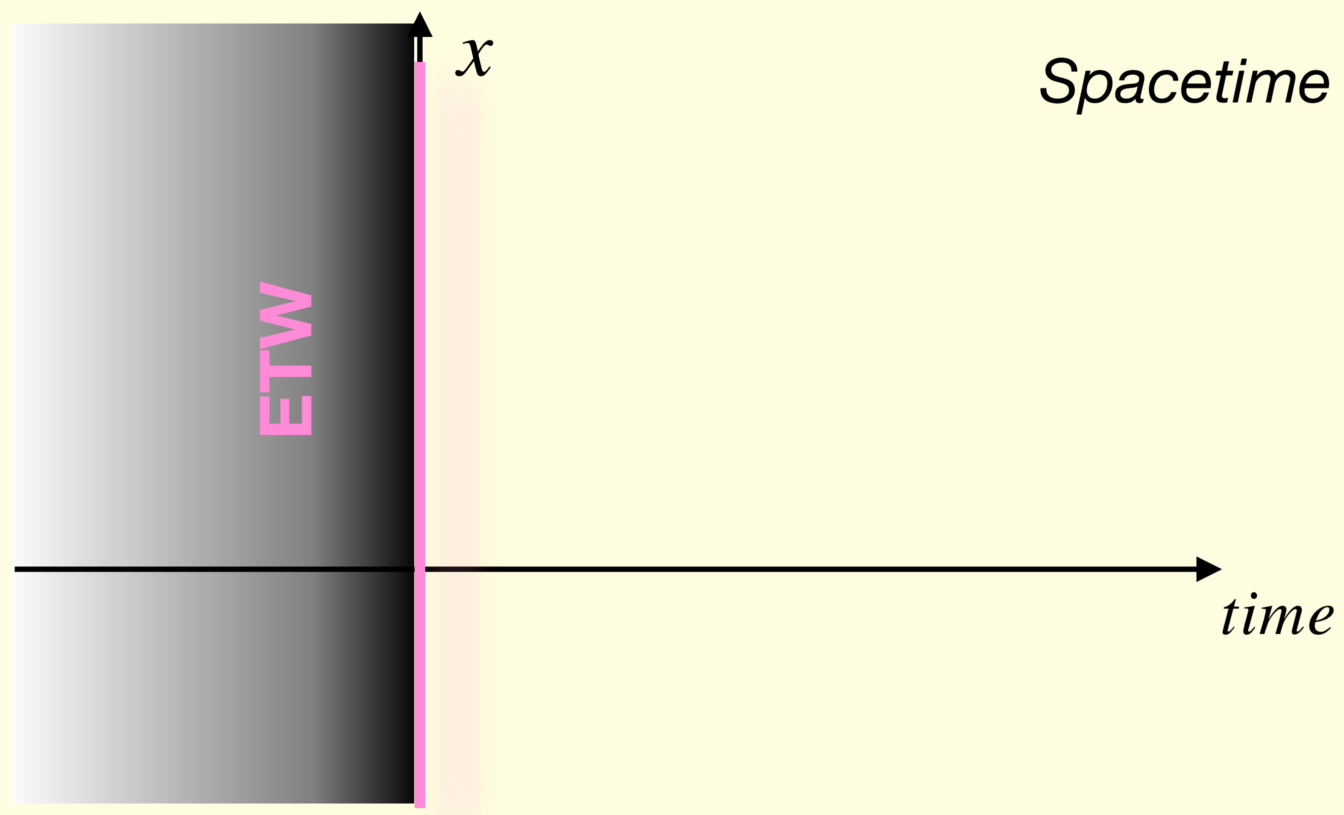
ETW branes for singular divisors

Applying Dynamical Cobordism thecniques in the resulting three-dimensional effective action:

→ We can associate to each singular divisor $\Delta_k \in \Delta$ a specific ETW brane in spacetime



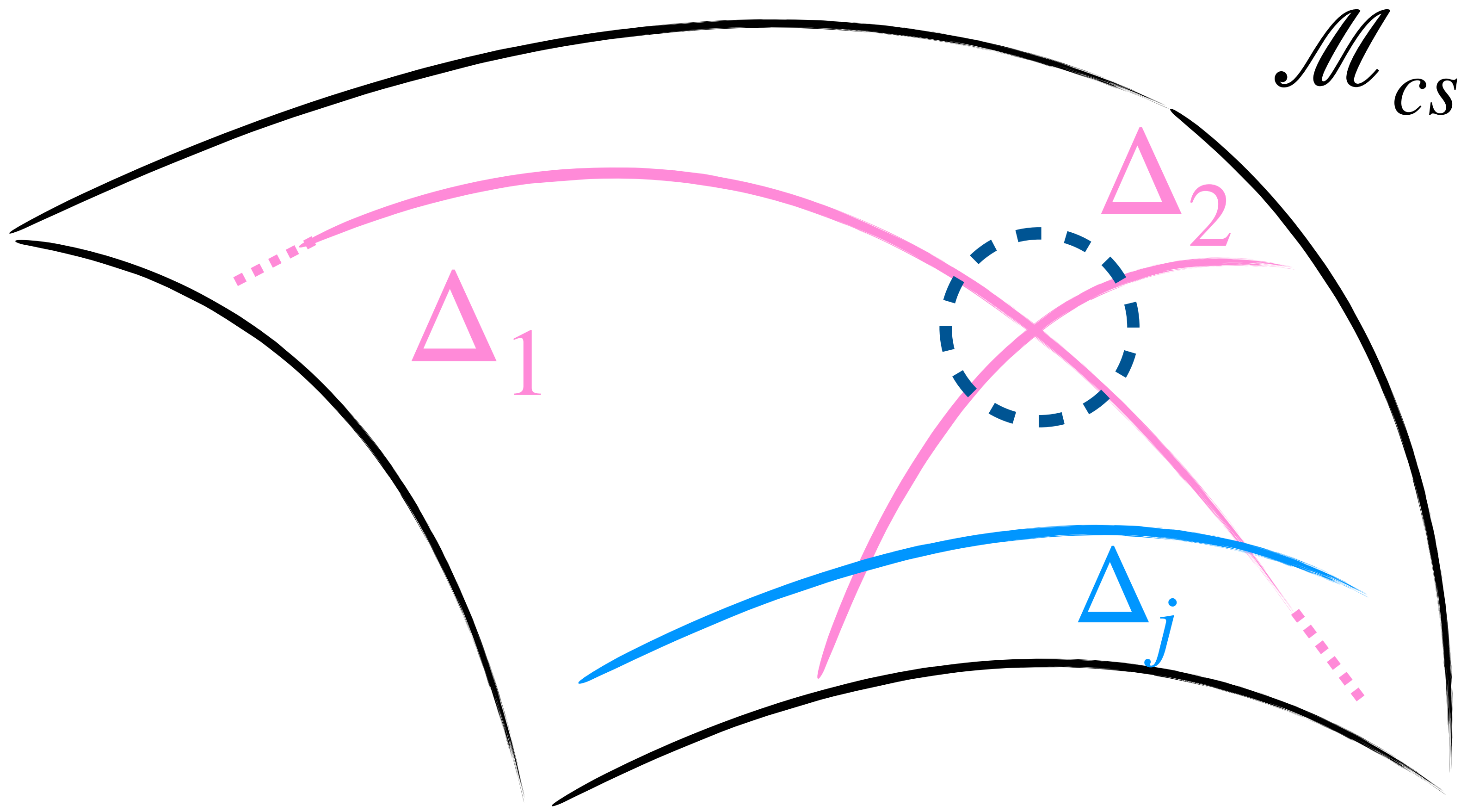
(F^p_{lim}, N_k)
+ flux



$$\delta = \sqrt{\frac{2}{d_k}(l_k - 4)}$$

Type	d_k	l_k	δ	Type	d_k	l_k	δ
$II_{0,0}$	1	5	$+\sqrt{2}$	$II_{0,1}$	1	5	$+\sqrt{2}$
$II_{1,1}$	1	5	$+\sqrt{2}$	$II_{1,1}$	1	6	$+2\sqrt{2}$
$III_{0,0}$	2	6	$+2$	$III_{1,1}$	2	6	$+2$
$III_{0,1}$	2	5	$+1$	$III_{0,1}$	2	6	$+2$
$IV_{0,1}$	3	5	$+\sqrt{6}/3$	$IV_{0,1}$	3	7	$+\sqrt{6}$
$V_{1,1}$	4	6	$+\sqrt{2}$	$V_{1,1}$	4	8	$+2\sqrt{2}$
$V_{1,2}$	4	5	$+\sqrt{2}/2$	$V_{1,2}$	4	6	$+\sqrt{2}$
$V_{1,2}$	4	8	$+2\sqrt{2}$	$V_{2,2}$	4	6	$+\sqrt{2}$
$V_{2,2}$	4	8	$+2\sqrt{2}$				

Effective action at the enhancements



Admissible enhancements:
[Grimm, Li, Valenzuela, '19]

$$Type\left(\Delta_1\right) \longrightarrow Type\left(\Delta_1 \cap \Delta_2\right)$$

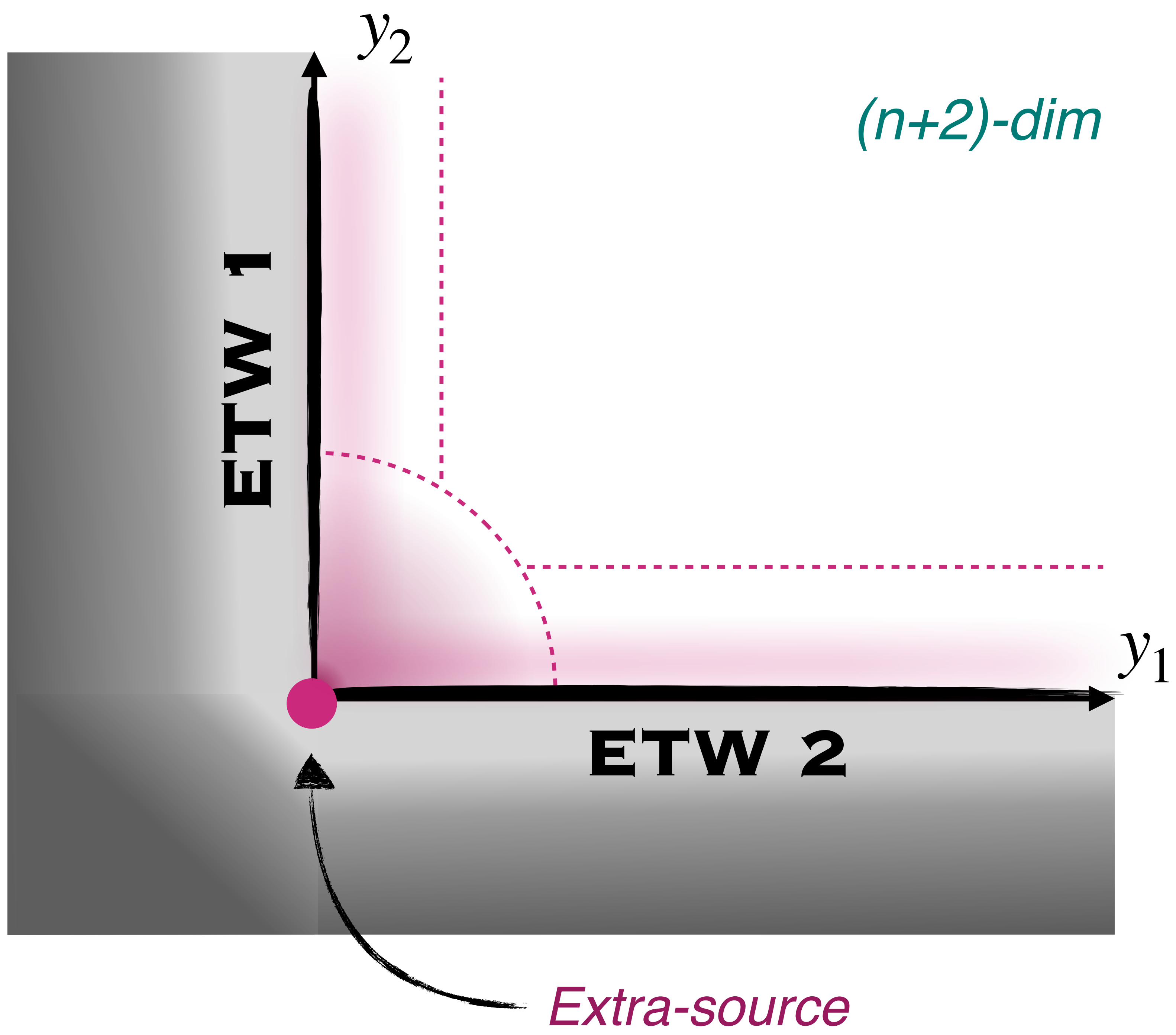
The effective action for the relevant scalars $s_1 \mapsto \psi_1$ and $s_2 \mapsto \psi_2$ is:

$$S = \int d^3x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\left(\partial\psi_1\right)^2 - \frac{1}{2}\left(\partial\psi_2\right)^2 - ||\rho||^2 e^{\sqrt{\frac{2}{d_1}}(l_1-4)\psi_1} e^{\sqrt{\frac{2}{d_e-d_1}}(l_e-l_1)\psi_2}\right]$$

- Diagonal kinetic metric
- Single potential term

$\longrightarrow$ The class of Dynamical Cobordism solutions we computed in [2312.16286] is not valid for this action.

Beyond the conformal flatness

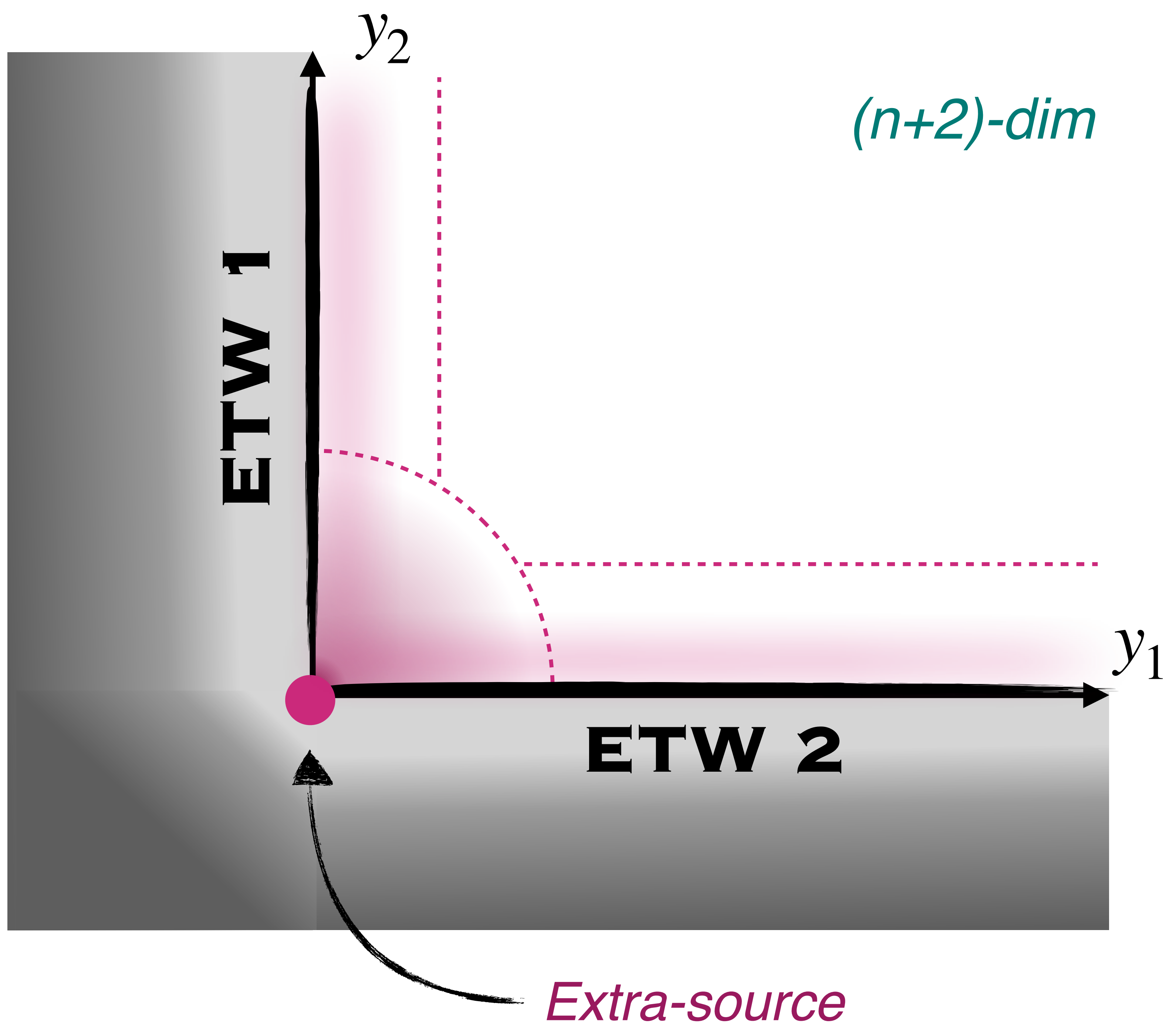


Solutions:

- $\psi_1 = -\sqrt{a_1 n(2-a_1-a_1 n)}\log y_1 - a_2(1-a_1 n)\sqrt{\frac{n}{a_1(2-a_1-a_1 n)}}\log y_2$
- $\psi_2 = -\sqrt{a_2 n\left(1+\frac{a_2(a_1 n-1)^2}{a_1(a_1+a_1 n-2)}\right)}\log y_2$
- $ds^2_{n+2} = y_1^{2(1-a_1 n)} y_2^{2a_2}\left[y_1^{4a_1 n-2} ds_n^2 + y_1^{-2(1-a_1 n)} dy_1^2 - y_2^{-2a_2 n} dy_2^2\right]$

Potential class: $\boxed{V(\phi) \simeq c \cdot v \cdot e^{A_1\psi_1} e^{A_2\psi_2}}$

Beyond the conformal flatness



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- $\psi_1 = -\sqrt{a_1 n(2-a_1-a_1 n)}\log y_1 - a_2(1-a_1 n)\sqrt{\dfrac{n}{a_1(2-a_1-a_1 n)}}\log y_2$
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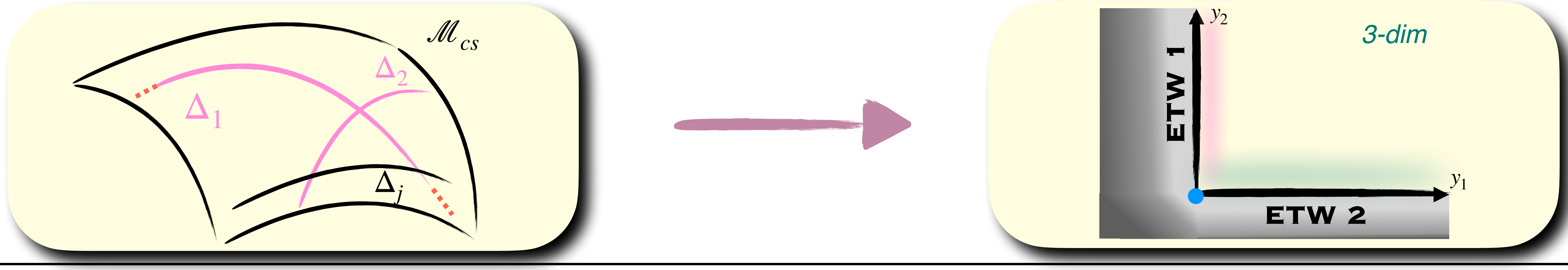
Potential class: $\boxed{V(\phi) \simeq c \cdot v \cdot e^{A_1\psi_1} e^{A_2\psi_2}}$

We can provide a spacetime realization for each enhancement as an intersection of two ETW branes: where the first brane represents the first singular divisor and the second brane represents the enhanced singularity



Conclusions

- I reviewed Dynamical Cobordism solutions describing codimension-1 boundaries in spacetime and the configurations realized through their intersection
- I constructed a new class of solution … the propriety of conformal flatness in the spacetime metric. The ne solution still admits an interpretation in terms of intersection of two codimension-1 ETW brane with a possible additional extra-source localized in the codimension-2 intersectin locus.
- I associated to each singular divisor $\Delta_k \in \Delta$ in $\mathscr{M}_{cs}$ classified in terms of asymptotic Hodge theory, a specific ETW brane in spacetime classified in terms of the critical exponent.
- I provided a spacetime realization for some of the admissible enhancements in Δ in terms of intersecting ETW branes.

The intersecting configurations of ETW branes are the building blocks able to probe the network of infinite distance singularities in the Calabi-Yau moduli space.

Thank you!