Symmetric moduli spaces: boundaries, geodesics and the Distance Conjecture

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IPhT CEA/Saclay

Work in progress with S. Baines, B. Fraiman, M. Graña and D. Waldram

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From string compactifications: universal patterns appearing at infinite distance in moduli spaces.

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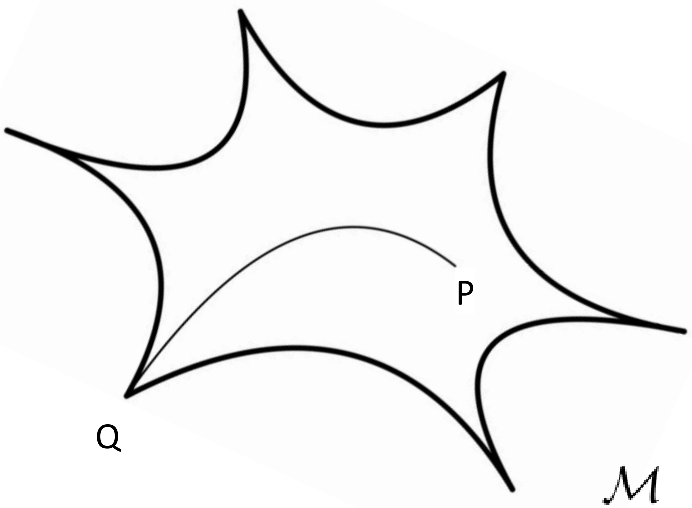
$$M(Q) \sim M(P) e^{-\alpha\, d_{P,Q}}$$

[Ooguri, Vafa, '06]



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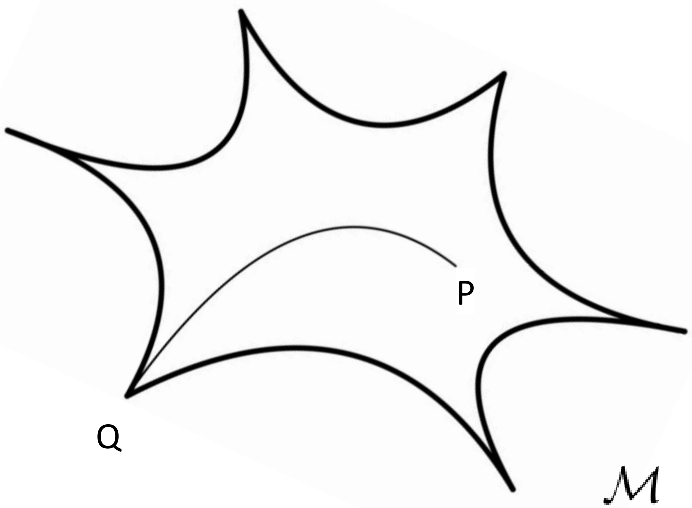

[Baume, Blumenhagen, Buratti, Calderon-Infante, Castellano, Cecotti, Corvilain, Cribiori, Etheredge, Font, Gendler, Grimm, Heidenreich, Herraez, Ibañez, Joshi, Kaya, Klaewer, Lee, Lerche, Li, Lockhart, Lust, McNamara, Marchesano, Martucci, Montella, Montero, Ooguri, Palti, Perlmutter, Qiu, Rastelli, Rudelius, Ruiz, Stout, Uranga, Vafa, Valenzuela, van de Heisteeg, Weigand, Wiesner, Wolf, …]

Symmetric spaces

Geometry of moduli spaces ⟷ **Spectrum** of the theory

- Geodesics
- Structure of the boundary

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Clear connection for symmetric moduli spaces

(Connected) group of isometries of $\mathcal{M}$

$$\mathcal{M} \sim G(\mathbb{Z}) \Big\backslash \frac{G(\mathbb{R})}{K}$$

Duality group

Subgroup of isometries fixing one point, o

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From string theory:

- M-theory on T^d $\quad G = E_{d(d)}$
- Heterotic on T^d $\quad G = O(d, d+16)$
- CHL on T^d $\quad G = O(d, d+8)$
- Bosonic on T^d $\quad G = O(d, d)$

Toy model: SL(2,R)

Moduli space of Type IIB in 10 dimensions, or of T^2 at fixed volume.

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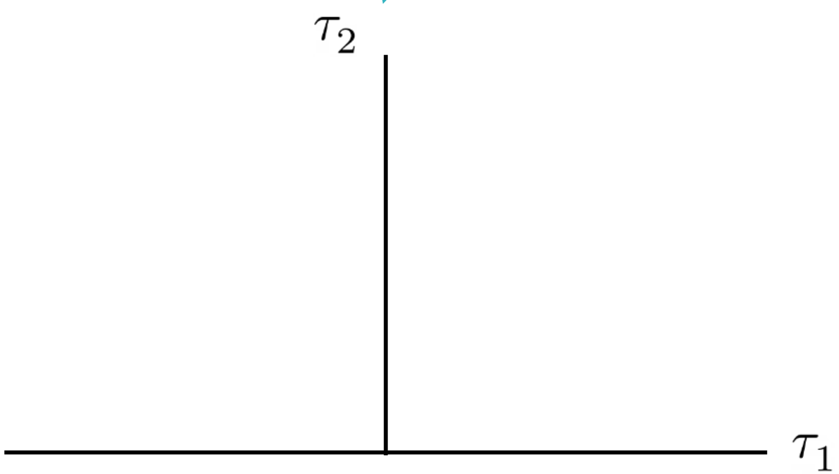

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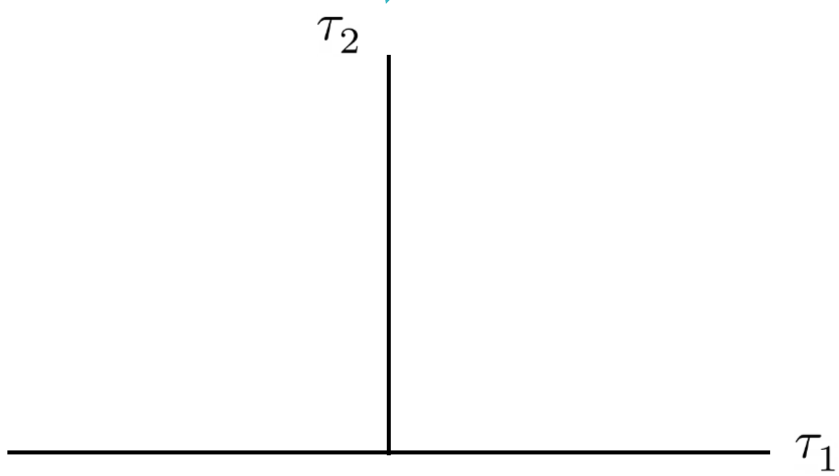

Boundary: $\mathbb{R} \cup \{i\infty\} \sim S^1$

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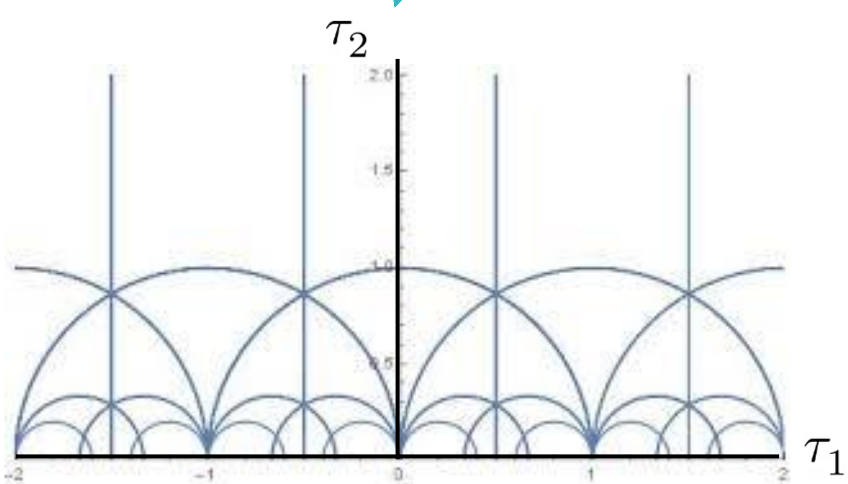

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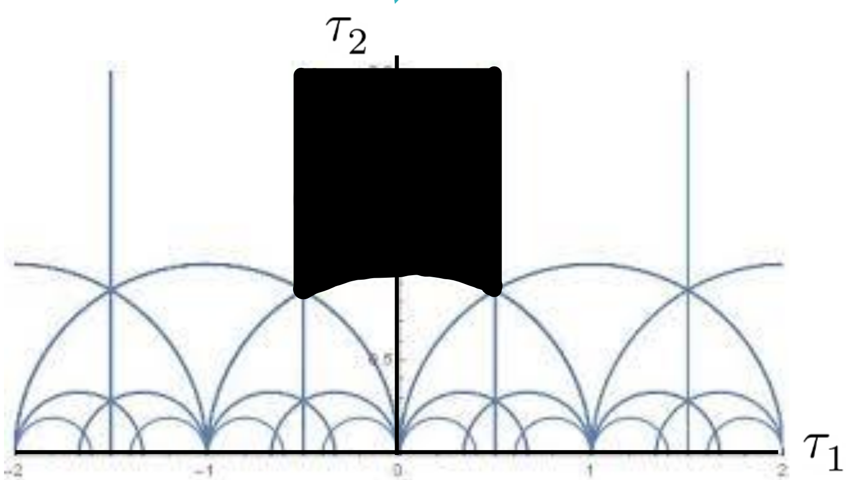

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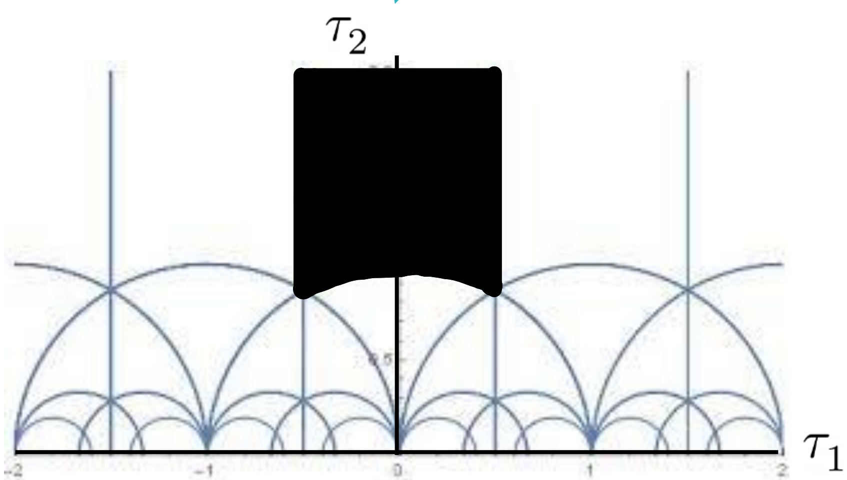

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$SL(2,\mathbb{Z})$: Restrict to one fundamental domain: one point at infinity

Geodesics on $\mathbb{H}^2$ either go to the boundary, or have an ergodic or periodic motion.

[Keurentjes, '06]

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[Borel, Ji '06]

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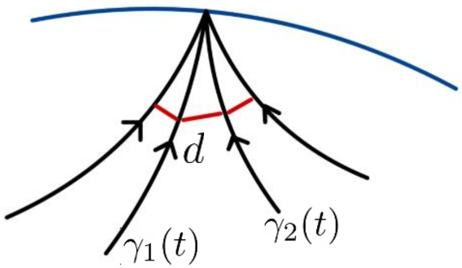

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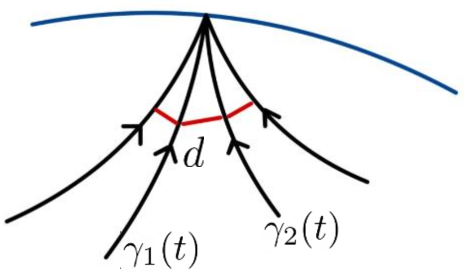

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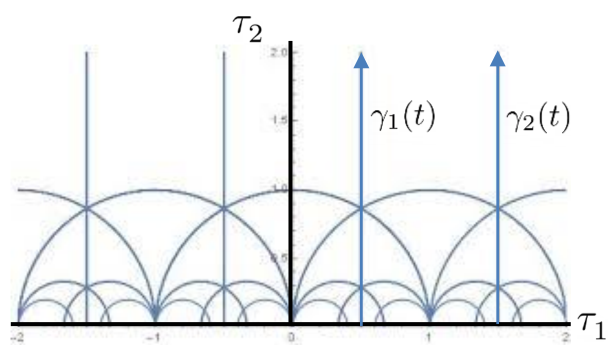

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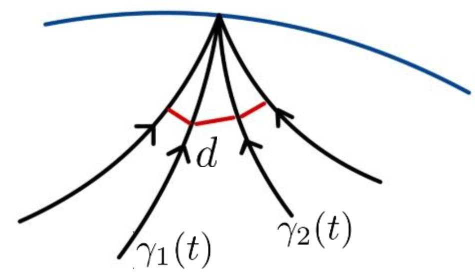

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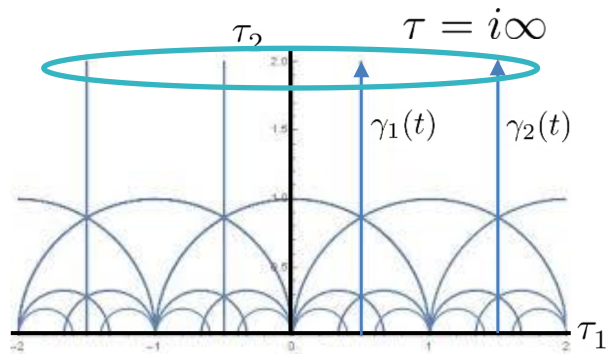

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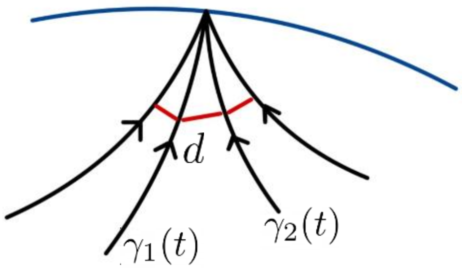

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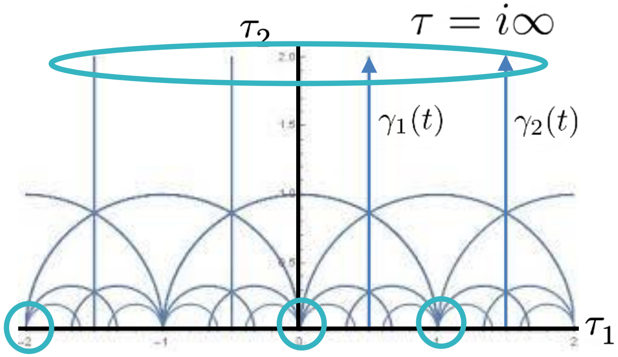

$d(\gamma_1(t), \gamma_2(t)) \to 0$

$\tau \in \mathbb{R}$

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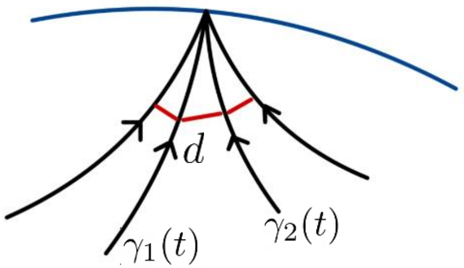

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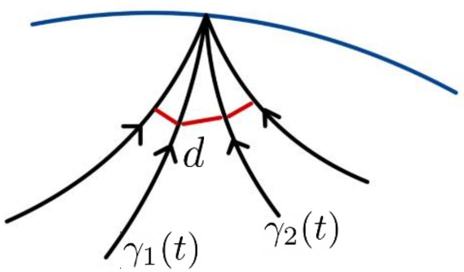

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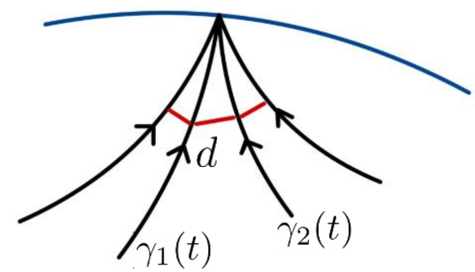

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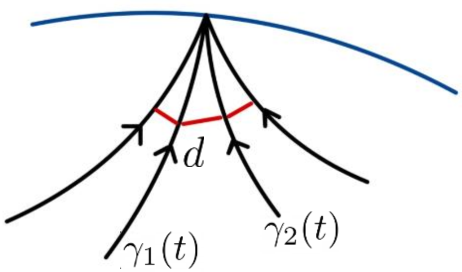

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- Discrete quotient: the boundary is encoded in $G(\mathbb{Q})$ and rational parabolic subgroups

Towards the Distance Conjecture

[Cecotti '15]

Assumptions: (motivated by string compactifications)

- Existence of a lattice of states $\Sigma \hookrightarrow V$ on which G acts

$$d_V(v,w) = v^T g^T g w, \quad v, w \in V,\, g \in \frac{G}{K}$$

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There is always a massless tower ✓

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Thank you!