Emergence of R^4-terms in M-theory

Testing the M-theoretic Emergence Proposal

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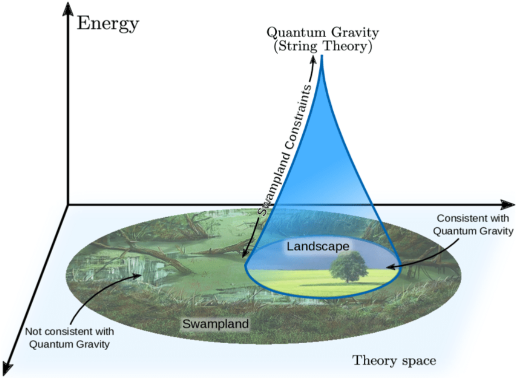
Review article: **2404.05801**

The Swampland Program

Conceptual framework

Consistent set of conjectures motivated mainly (but not exclusively) by string theory. Example reviews: [Palti ’18, van Beest, Calderón-Infante, Mirfendereski, Valenzuela ’22].



- No Global Symmetries Conjecture
- **Distance Conjecture**
- **Emergent String Conjecture**
- Weak Gravity Conjecture
- (A)dS Distance Conjecture
- Gravitino Conjecture

⋮

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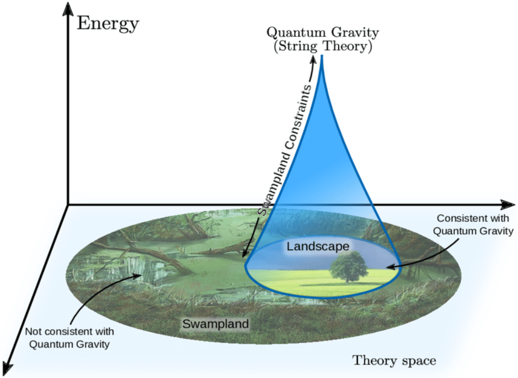

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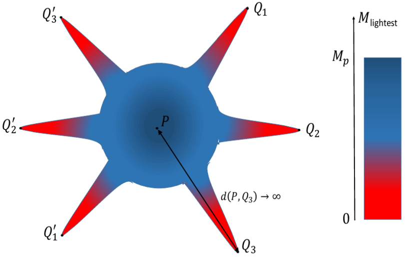
Introduction

Swampland Ingredients



Graphic taken from [Palti '18].

- **Distance Conjecture:** At an infinite distance in moduli space, a tower of exponentially light states appears [Ooguri, Vafa '06]

$$M(p) \sim M(p_0)\, e^{-\alpha d(p_0,p)}\,, \quad \alpha \sim \mathcal{O}(1)\,. \tag{1}$$

- **Emergent String Conjecture:** [Lee, Lerche, Weigand '18]

$$\to \begin{cases} \text{decompactification} \\ \text{emergent string limit} \end{cases}$$

Species Scale: The UV cut-off in the presence of many light fields is [Dvali '08]

$$\Lambda_{\text{sp}} = \frac{M_{\text{pl}}^{(d)}}{N_{\text{sp}}^{1/(d-2)}}\,. \tag{2}$$

Introduction

Emergence Basics

More recently, [van de Heisteeg, Vafa, Wiesner, Wu '22-'23]

$$S_{\mathrm{corr.},d} \subset \frac{M_{\mathrm{pl}}^{(d)\,d-2}}{2}\int d^dx\sqrt{-g}\left[\sum_n a_n(\phi_i)\frac{\mathcal{O}_n(\mathcal{R})}{M_{\mathrm{pl}}^{(d)\,2n-2}}\right], \quad \frac{1}{\Lambda_{\mathrm{sp}}(\phi_i)^{2n-2}} \simeq \frac{a_n(\phi_i)}{M_{\mathrm{pl}}^{(d)\,2n-2}}. \tag{3}$$

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Emergence Proposal (Strong): The dynamics for all fields are emergent in the infrared by integrating out towers of states down from an ultraviolet scale Λ, below the Planck scale. [Palti '19]

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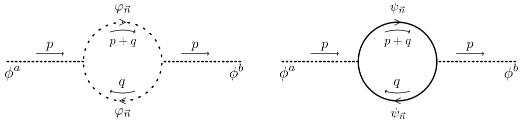
Toy models:



$$G_{\phi\phi}^{1-\text{loop}} \simeq \frac{\Lambda_{\text{sp}}^{d-1}}{M_{\text{pl}}^{(d)\,d-2}} \frac{\left(\partial_\phi \Delta m(\phi)\right)^2}{(\Delta m(\phi))^3} + \dots . \tag{4}$$

[Heidenreich, Reece, Rudelius ’18, Grimm, Palti, Valenzuela ’18, Lee, Lerche, Weigand ’21, Castellano, Herráez, Ibáñez ’22, Blumenhagen, Gligovic, AP ’23]

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An M-theoretic Emergence Proposal

Overview

Emergence Proposal (Strong): The dynamics for all fields are emergent in the infrared by integrating out towers of states down from a scale Λ [Palti '19].

Emergence Proposal (M-theory): In the **M-theory limit** $M_* R_{11} \gg 1$ with $M_{\mathrm{pl},d}$ kept fixed, a perturbative QG theory arises whose low energy effective description is emerging by integrating out the full **infinite** towers of states with a mass scale parametrically not larger than M_*. [Blumenhagen, Cribiori, Gligovic, AP '24]

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Light States: Transverse M2, M5 branes carrying KK momentum

$$R_{11} \to \lambda R_{11}\,, \quad M_* \to \frac{M_*}{\lambda^{\frac{1}{d-1}}}\,, \quad R_I \to \lambda^{\frac{1}{d-1}} R_I\,, \quad M_{D0} \sim \frac{M_{\mathrm{pl}}^{(d)}}{\lambda}\,, \quad M_{D2,NS5} \sim \frac{M_{\mathrm{pl}}^{(d)}}{\lambda^{1/(d-1)}}\,. \tag{5}$$

Similarities: BFSS matrix model [Banks, Fischler, Shenker, Susskind '97].

R^4 couplings

GGV approach

M-theory on a $(k+1)$ torus of volume $r_{11}\mathcal{V}_k$

$$S_{R^4} \simeq M_*^{d-8} \int d^d x \sqrt{-g}\, r_{11} \mathcal{V}_k\, a_d\, t_8 t_8\, R^4 \,, \quad k = 10 - d. \tag{6}$$

The **1/2 BPS saturated** coefficient a_d is known [Green,Gutperle,Vanhove '97]

$$a_{10-k} \simeq \frac{2\pi}{r_{11}\mathcal{V}_k} \sum_{m_I \in \mathbb{Z}} \int_0^\infty \frac{dt}{t^{\frac{4-k}{2}}} e^{-\pi t \sum_{I,J=1}^{k} m_I G^{IJ}_{(k+1)} m_J} \,. \tag{7}$$

Regularization: Poisson resummation over all integers and T-duality for $\hat{m}_I = 0$.

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What more can we learn?

Emergence of R^4 couplings

Testing the Emergence Proposal in M-theory

Extend the Ansatz of GGV to the **full spectrum** of **light** 1/2 BPS particle states

$$a_d \simeq \frac{2\pi}{r_{11}\mathcal{V}_k} \hat{\sum}_{N^A, m\in\mathbb{Z}} \int_0^\infty \frac{dt}{t^{\frac{d-6}{2}}}\, \delta(\mathrm{BPS})\, \exp\left(-\pi t\, N^A \mathcal{M}_{AB} N^B - \pi t\, \frac{m^2}{r_{11}^2}\right). \tag{8}$$

Regularization: UV cutoff + ζ-functions [Blumenhagen, Cribiori, Gligovic, AP '23].

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- $d = 10$: We can only have particle-like KK modes, just like GGV.

$$a_{10} \simeq \frac{2\pi}{r_{11}} \sum_{m\neq 0} \int_\epsilon^\infty \frac{dt}{t^2}\, e^{-\pi t \frac{m^2}{r_{11}^2}} \simeq \frac{2\zeta(3)}{r_{11}^3}\,, \tag{9}$$

using

$$\int_\epsilon^\infty \frac{dt}{t^2}\, e^{-\pi t A} = \frac{1}{\epsilon} + \pi A\left(\log(\pi A \epsilon) + \gamma_E - 1\right) + \mathcal{O}(\epsilon)\,. \tag{10}$$

Downside: The (constant) one-loop term is missing.

Emergence of R^4 couplings

Testing the Emergence Proposal in M-theory

- $d = 9$: Similar behavior (no **light** wrapped branes)

$$a_9 \simeq \frac{2\zeta(3)}{r_{11}^3} + \frac{8\pi}{r_{11}^2 r_1} \sum_{m\neq 0} \sum_{m_1>0} \left|\frac{m}{m_1}\right| K_1\left(2\pi|m|m_1 \frac{r_1}{r_{11}}\right) . \tag{11}$$

Emergence of R^4 couplings

Testing the Emergence Proposal in M-theory

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- $d = 8$: Also light wrapped $M2$-branes satisfying [Obers, Pioline '99]

$$\sum_J n_{IJ} m_J = 0 \rightarrow \mathcal{M}^2 = n_{12}^2 t_{12}^2 + \frac{m^2}{r_{11}^2} , \quad t_{12} = r_1 r_2 . \qquad (12)$$

Our ansatz implies that the contribution to the R^4 coupling is

$$a_{8,D2/D0} \simeq \frac{2\pi}{r_{11} t_{12}} \sum_{n_{12} \neq 0} \sum_{m \in \mathbb{Z}} \int_\epsilon^\infty \frac{dt}{t} e^{-\pi t \left(n_{12}^2 t_{12}^2 + \frac{m^2}{r_{11}^2} \right)} \rightarrow \qquad (13)$$

$$a_{8,D2/D0} \simeq \frac{2\pi}{r_{11} t_{12}} \left(\frac{\pi}{3} r_{11}\, t_{12} + 4 \sum_{n_{12}, m > 0} \frac{1}{n_{12}} e^{-2\pi n_{12} m r_{11} t_{12}} \right) = -\frac{2\pi}{r_{11} t_{12}} \log \left(|\eta(i r_{11} t_{12})|^4 \right) .$$

Emergence of R^4 couplings

Testing the Emergence Proposal in M-theory

Adding the KK contribution

$$a_{8,D0} \simeq \frac{2\zeta(3)}{r_{11}^3} - \frac{2\pi}{r_{11}t_{12}} \log\left(r_{11}r_2^2|\eta(iu)|^4\right) + \frac{8\pi}{r_{11}^2} \sum_{\substack{m>0 \\ (m_1,m_2)\neq(0,0)}} \frac{m\, K_1\left(2\pi \frac{m}{r_{11}}\sqrt{m_1^2 r_1^2 + m_2^2 r_2^2}\right)}{\sqrt{m_1^2 r_1^2 + m_2^2 r_2^2}} \tag{15}$$

where we have used

$$\int_\epsilon^\infty \frac{dt}{t} e^{-tA} = -\gamma_E - \log(\epsilon A) + \mathcal{O}(\epsilon)\,. \tag{16}$$

We thus obtain the **complete** result in 8d!

Can this be checked further?

Emergence of R^4 couplings

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- The pattern: **constant terms** $\leftrightarrow$ **extended objects** persists in $d = 6, 7$ and in the emergent string limit.

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Testing the Emergence Proposal in M-theory

- The instanton contributions can be generally determined.

Particle states	Instantons
$(D0, \mathrm{KK}_{(K)})$	$ED0_{(K)}$
$(D2_{(IJ)}, \mathrm{KK}_{(K)})$	$ED2_{(IJK)}$
$(NS5_{(IJKLM)}, \mathrm{KK}_{(N)})$	$ENS5_{(IJKLMN)}$
$(D2_{(IJ)}, D0)$	$EF1_{(IJ)}$
$(NS5_{(IJKLM)}, D0)$	$ED4_{(IJKLM)}$
$(NS5_{(IJKLM)}, D2_{(LM)})$	$ED2_{(IJK)}$

$$M_{\mathrm{pl}} = \mathrm{const}$$

Our result are consistent with more formal results [Obers, Pioline '99, [Green, Russo,Vanhove '10, Bossard, Kleinschmidt '15, Bossard, Pioline '16].

pert. string theory	desert	M-theory
$g_s \ll 1$	$g_s = \mathcal{O}(1)$	$g_s \gg 1$

$$a_d = \frac{c_0}{g_s^2} + \underbrace{\left(c_1 + \mathcal{O}\left(e^{-S_{\mathrm{ws}}}\right)\right)}_{\text{one-loop}} + \mathcal{O}\left(e^{-S_{\mathrm{st}}}\right) \quad = \quad \mathcal{E}^{E_{k+1(k+1)}}_{\Lambda_{E_{k+1}}, s=\frac{k}{2}-1} \quad = \quad \mathcal{E}^{E_{k(k)}}_{\Lambda_{E_k}\oplus 1, s=\frac{k}{2}-1}$$

Summary and Outlook

Testing the **M-theoretic Emergence Proposal**, we extended the ansatz of [Green, Gutperle, Vanhove '97] to the **full spectrum** of **light** 1/2 BPS particle states

$$a_d \simeq \frac{2\pi}{r_{11}\mathcal{V}_k} \hat{\sum}_{N^A, m\in\mathbb{Z}} \int_0^\infty \frac{dt}{t^{\frac{d-6}{2}}}\, \delta(\mathrm{BPS})\, \exp\left(-\pi t\, N^A \mathcal{M}_{AB} N^B - \pi t\, \frac{m^2}{r_{11}^2}\right). \qquad (17)$$

Regularization: UV cutoff, minimal subtraction and ζ-function regularization.

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Upshots:

- Self-consistently getting full results in $d = 7, 8$. Precise instanton predictions.
- Providing physical interpretation for previously ambiguous terms.
- Connecting the work of [Obers, Pioline '99] to the swampland framework.

Future directions:

- Non 1/2 BPS quantities?
- Connections with other approaches e.g. [Hattab, Palti '23-24]?
- M(-atrix) model implications?

Thank you

Back-up Slide: 8d calculation in perturbative string theory

The Kaluza-Klein contribution is

$$a_8^{1-\text{loop},(1)} \simeq \frac{2\pi}{\vartheta_{12}} \sum_{(m_1,m_2)\neq(0,0)} \int_0^\infty \frac{dt}{t} e^{-\pi t\left(\frac{m_1^2}{\rho_1^2}+\frac{m_2^2}{\rho_2^2}\right)}, \tag{18}$$

$$\int_0^\infty \frac{dx}{x^{1-\nu}} e^{-\frac{b}{x}-cx} = 2\left|\frac{b}{c}\right|^{\frac{\nu}{2}} K_\nu\left(2\sqrt{|b\,c|}\right), \qquad \int_\epsilon^\infty \frac{dt}{t} e^{-tA} = -\gamma_E - \log(\epsilon A) + \mathcal{O}(\epsilon)\,. \tag{19}$$

With $\epsilon \to \tilde{\epsilon} = \epsilon\, 4\pi e^{-\gamma_E}$, this gives rise to

$$a_8^{1-\text{loop},(1)} \simeq -\frac{2\pi}{\vartheta_{12}} \log\left(\rho_2^2 |\eta(iu)|^4\right). \tag{20}$$

Meanwhile, defining $\alpha = (1,0), (0,1), (1,1)$, the winding sector contributes as

$$a_8^{1-\text{loop},(2)} = -\frac{2\pi}{\vartheta_{12}} \log\left(|\eta(i\vartheta_{12})|^4\right). \tag{21}$$

We get the full **one-loop** result!

Back-up Slide: Example of an $NS5$-brane contribution

The full set of BPS conditions is [Obers, Pioline '99]

$$\sum_J n_{IJ} m_J = 0\,, \quad n_{[IJ}\, n_{KL]} + \sum_P m_P\, n_{PIJKL} = 0\,, \quad n_{I[J}\, n_{KLMNP]} = 0\,. \tag{22}$$

An example of such a solution is the configuration

$$(n_{45}, m_1) = P(-\tilde{\nu}_5, \tilde{n}_{23})\,, \quad (n_{15}, m_4) = Q(\tilde{\nu}_5, \tilde{n}_{23})\,, \quad (n_{14}, m_5) = R(-\tilde{\nu}_5, \tilde{n}_{23})\,, \tag{23}$$

where $P, Q, R \in \mathbb{Z}$. This contributes as

$$a_5^{\mathrm{typ}} \simeq \frac{2\pi}{r_{11} t_{12345}} \sum_{\tilde{n}_{23}, \tilde{\nu}_5 \in \mathbb{Z}} \sum_{N>0} \sum_{P,Q,R,m \in \mathbb{Z}} \int_0^\infty dt\, t^{\frac{1}{2}}\, e^{-\pi t \left(N^2 t_{23}^2 L^2 + \frac{m^2}{r_{11}^2} + \left(\frac{P^2}{r_1^2} + \frac{Q^2}{r_4^2} + \frac{R^2}{r_5^2} \right) L^2 \right)} \tag{24}$$

$$\simeq 2\pi \sum_{\tilde{n}_{23}, \tilde{\nu}_5 \in \mathbb{Z}} \sum_{N>0} \sum_{(P,Q,R,m) \neq (0,0,0,0)} \frac{1}{S\, L^2}\, e^{-2\pi N S}\,, \quad L = \sqrt{\tilde{\nu}_5^2 t_{145}^2 + \tilde{n}_{23}^2}\,, \tag{25}$$

$$S = \sqrt{P^2 t_{123}^2 + Q^2 t_{234}^2 + R^2 t_{235}^2 + m^2 \left(\tilde{n}_{23}^2 (r_{11} t_{23})^2 + \tilde{\nu}_5^2 (r_{11} t_{12345})^2 \right)}\,.$$