

ASPECTS OF DYNAMICAL COBORDISM IN ADS/CFT

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Instituto de Física Teórica (IFT), Madrid

String Pheno 2024, Padova, 25 June 2024

Based on [JH, Uranga, 2306.07335]

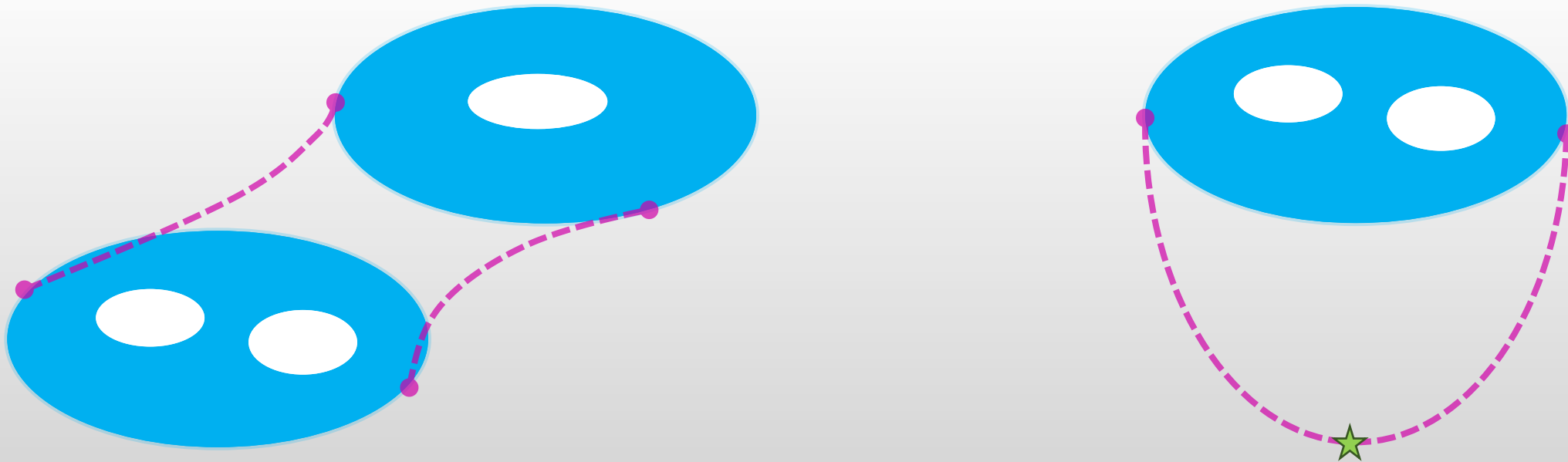


INTRODUCTION

COBORDISM CONJECTURE

[McNamara, Vafa, '19]

The cobordism classes of any solution of
Quantum Gravity have to be trivial



DYNAMICAL COBORDISM

[Angius, Buratti, Calderón-Infante,
Delgado, JH, Uranga, '21-22]

[Blumenhagen, Cribiori, Kneissl,
Makridou, Wang, '22-23]

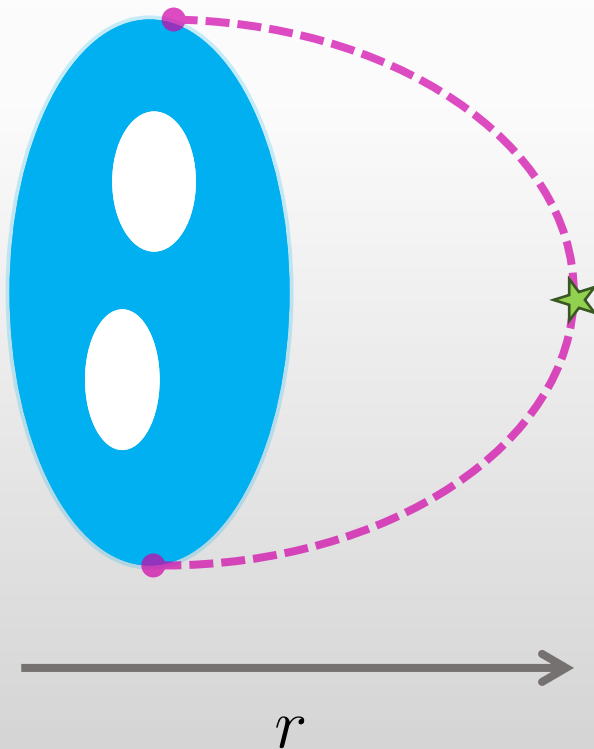
- “Dynamical”: The cobordism is a spacetime solution

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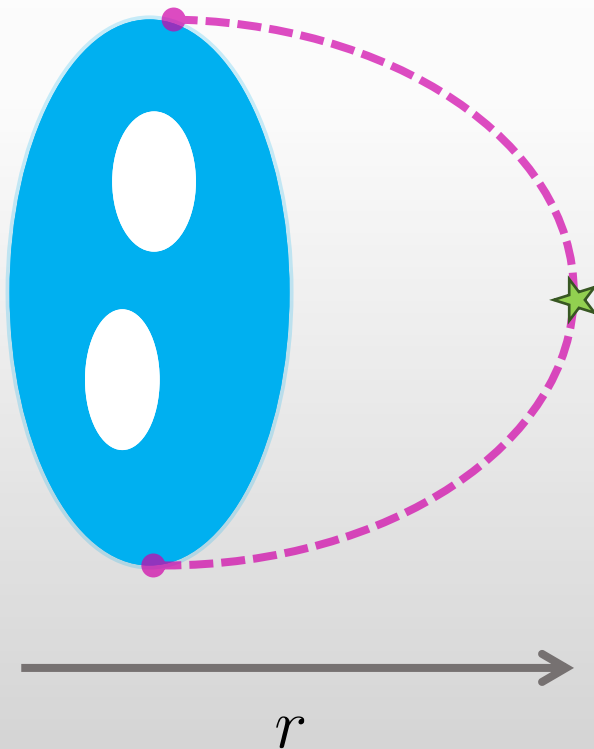


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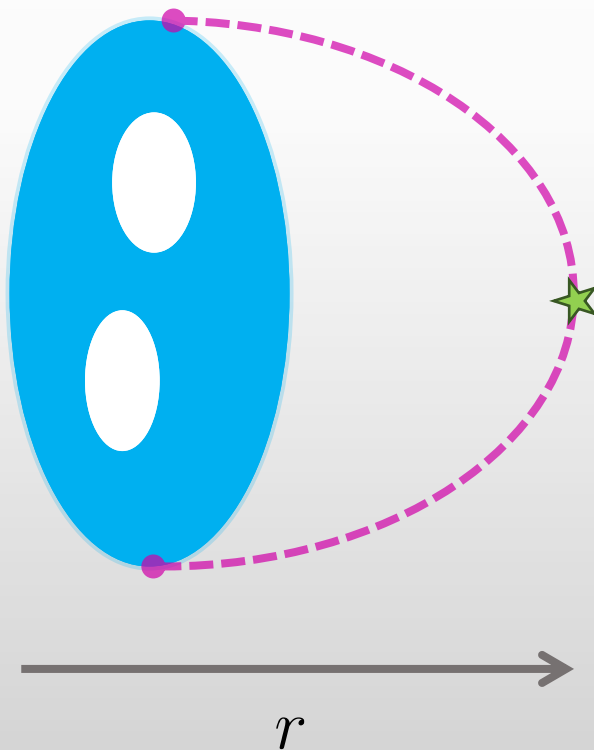
See Andriana, Bjoern,
Christian and Roberta's talks!!

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- “Dynamical”: The cobordism is a spacetime solution



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These solutions have
some issues...

$$V \rightarrow \infty$$

Sometimes the EFT breaks
down near the ETW brane

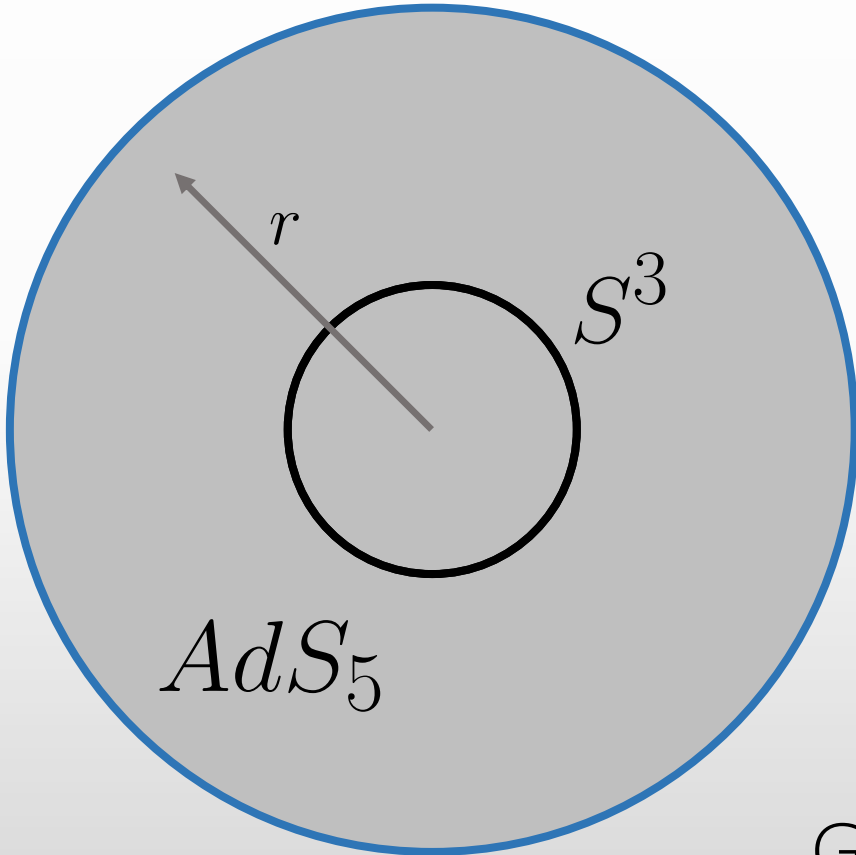
THIS TALK: COBORDISM IN ADS/CFT

- We will present and explain the solution that interpolates between $AdS_5 \times S^5$ and nothing.

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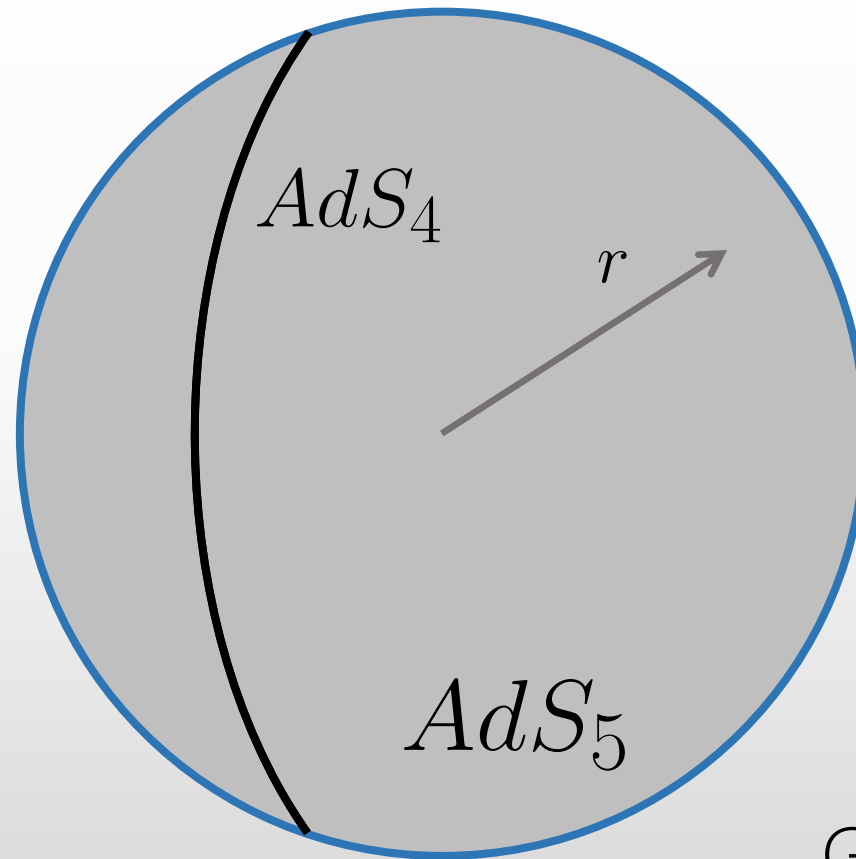
- We will present and explain the solution that interpolates between $AdS_5 \times S^5$ and nothing.
- We will study this setup in holography, the ETW brane (the cobordism to nothing), will have a holographic dual, so it is defined in the full quantum gravity, not only in the EFT.

A NOTE OF CAUTION



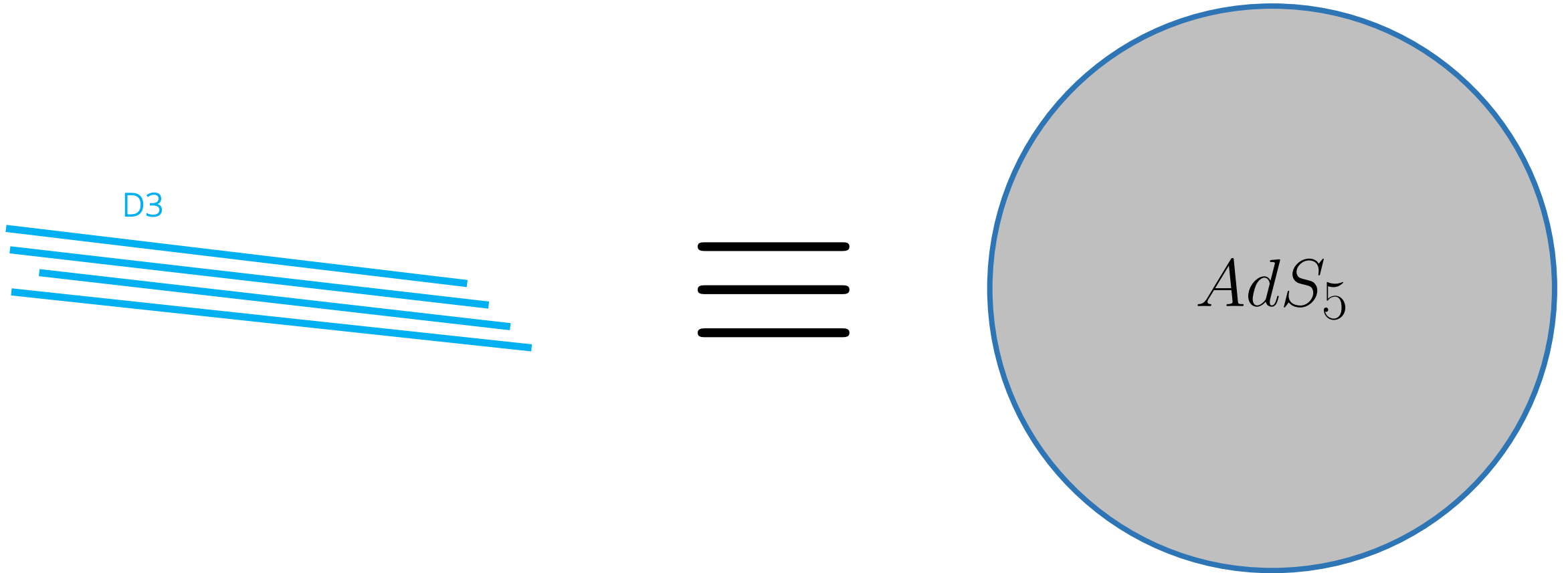
Global Coordinates

A NOTE OF CAUTION

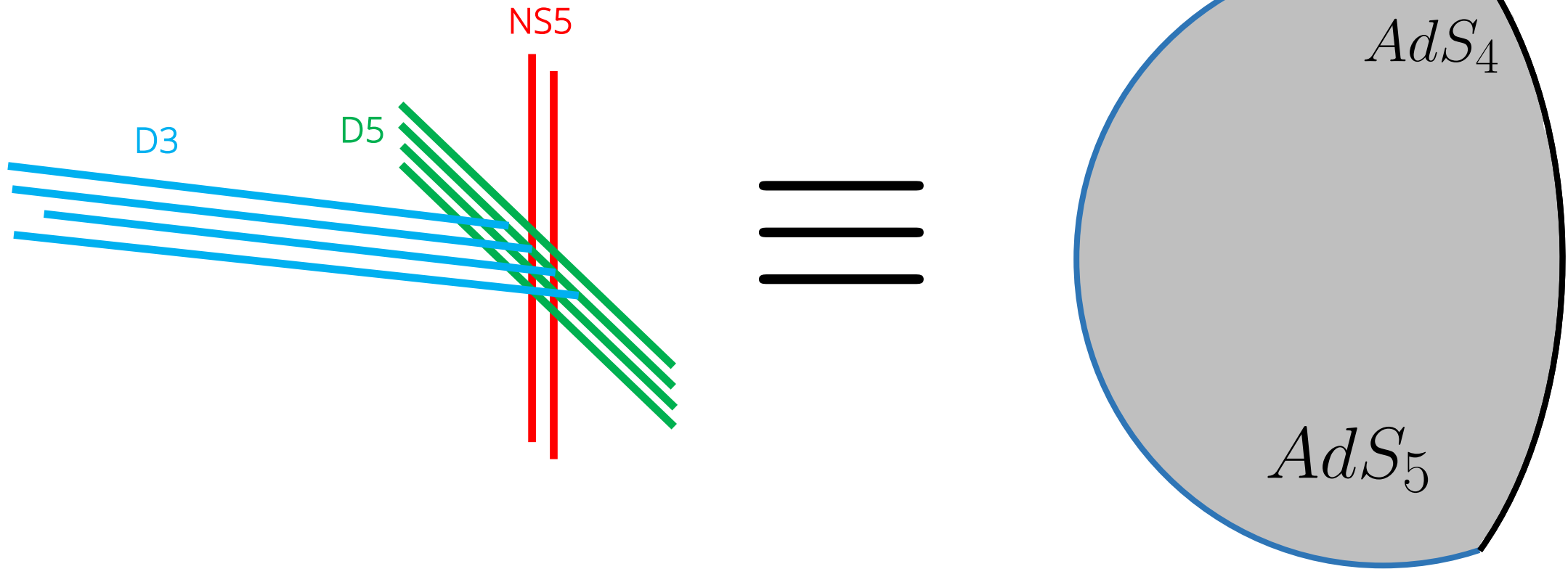


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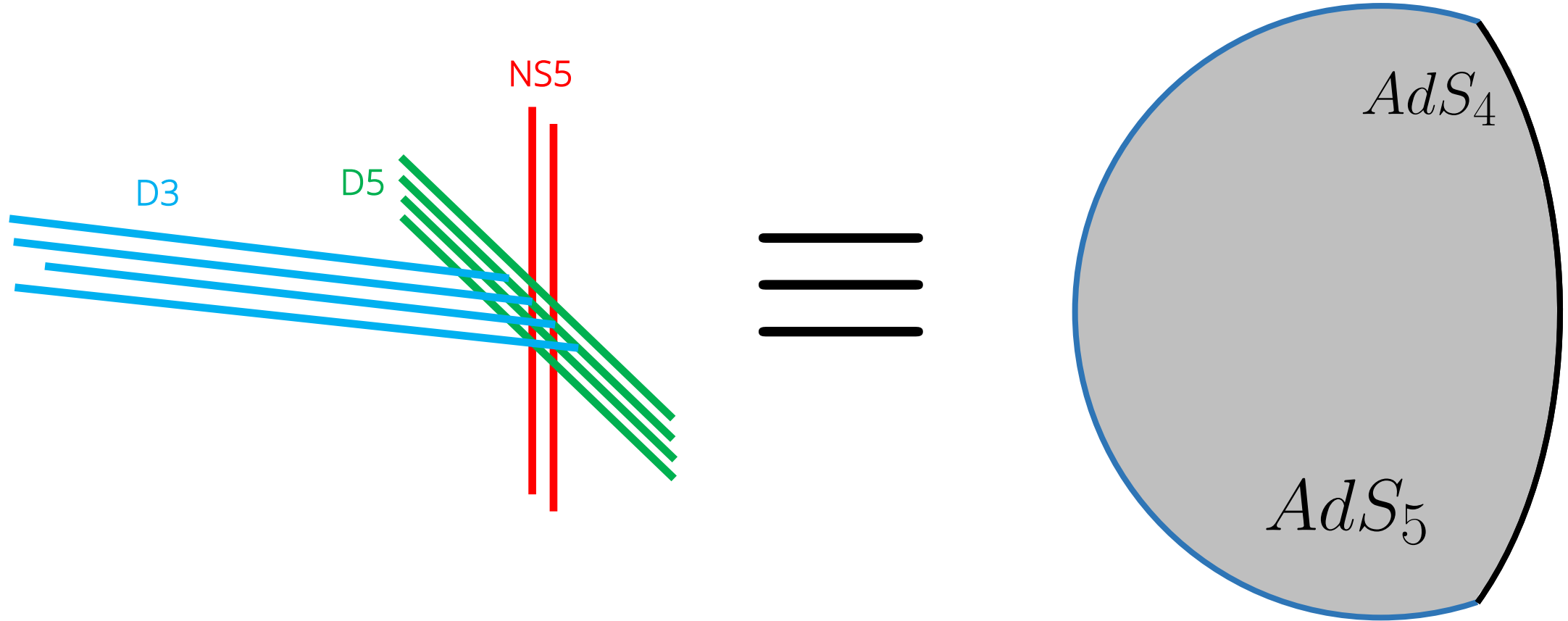
WHAT IS THE STRING THEORY INTERPRETATION OF THE ETW?



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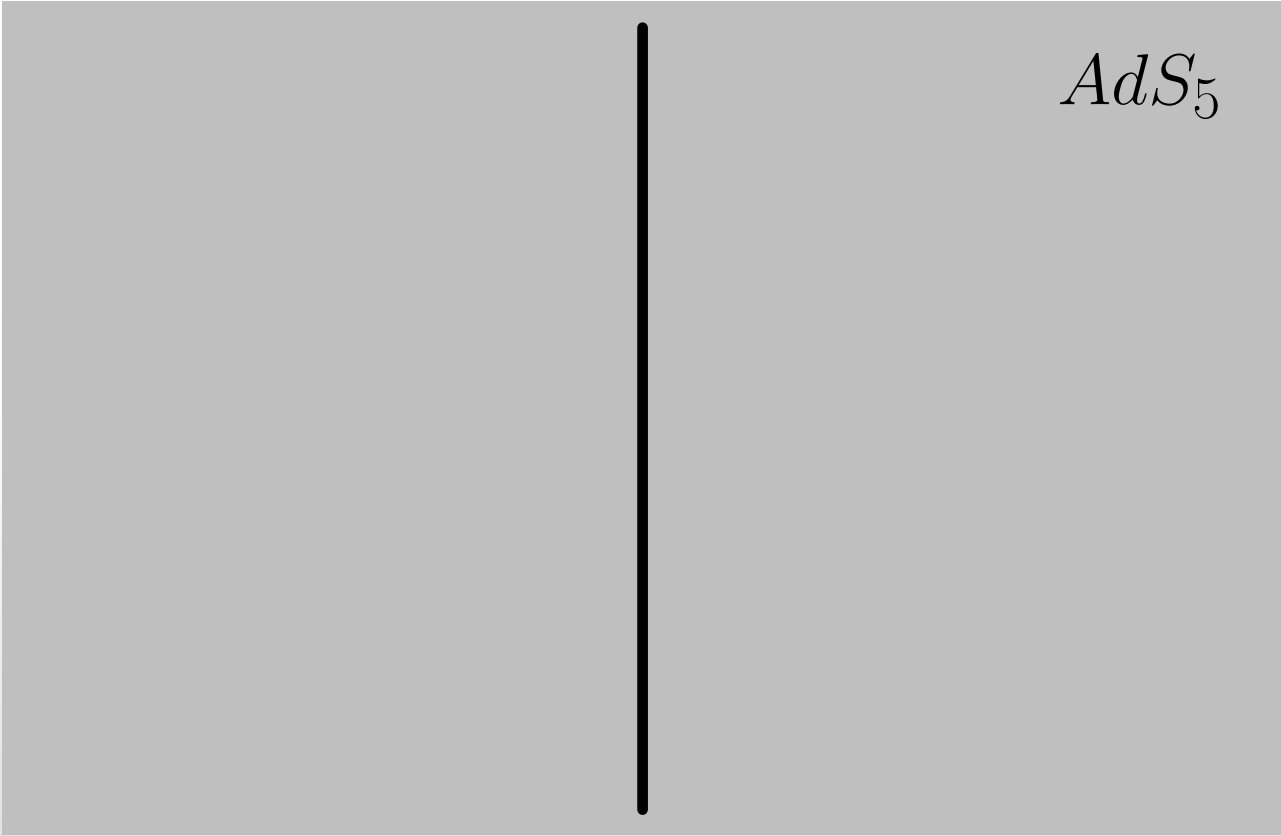


WHAT IS THE STRING THEORY INTERPRETATION OF THE ETW?



Holographic dual:





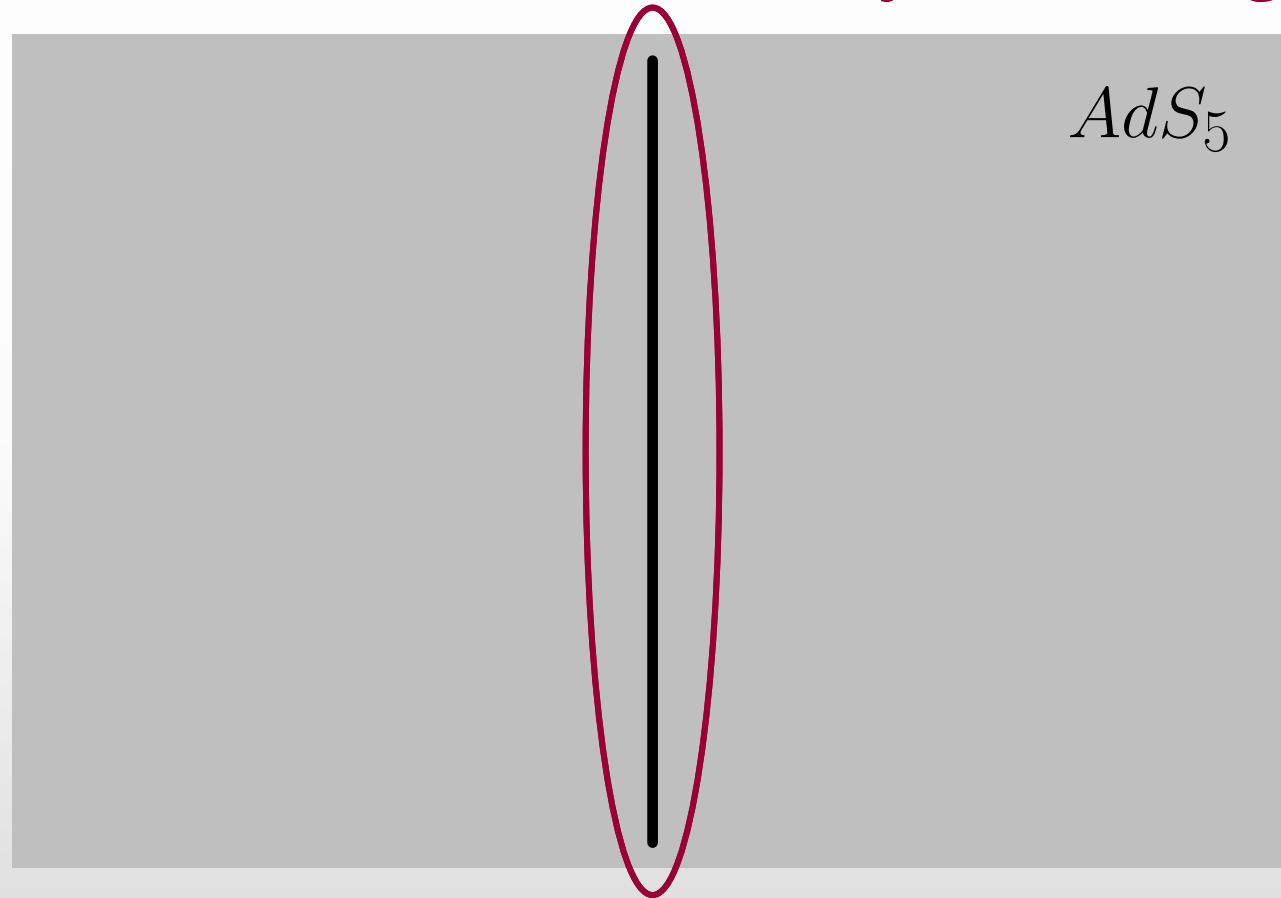
[Karch, Randall, '00]

AdS_4

AdS_5

**KARCH-
RANDALL
BRANE**

Locally localized gravity!



AdS_5

**KARCH-
RANDALL
BRANE**

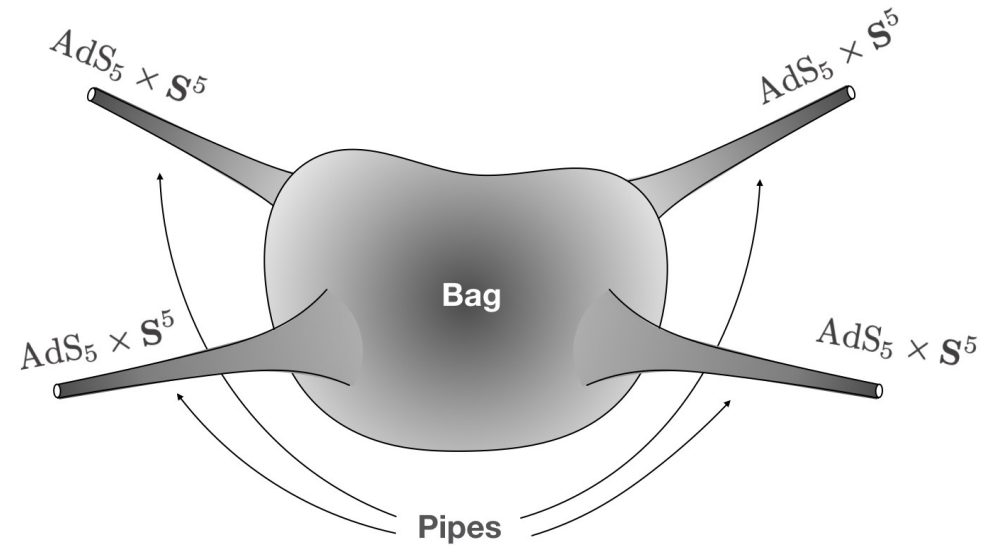
AdS_4

[Karch, Randall, '00]

THE SOLUTION

BAGPIPE SOLUTIONS

[D'Hoker, Estes, Gutperle, '07]

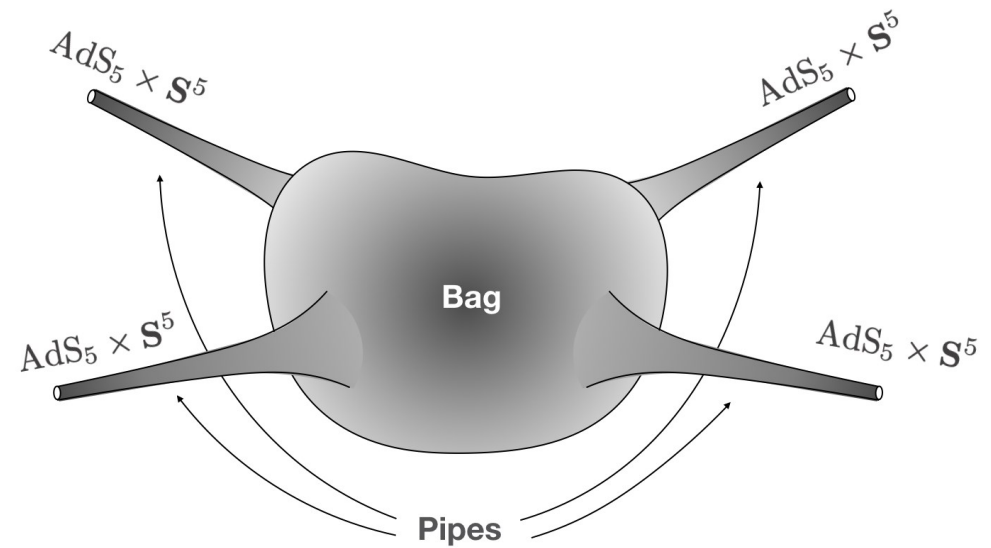


BAGPIPE SOLUTIONS

[D'Hoker, Estes, Gutperle, '07]

The theory has the symmetries

$$SO(2,3) \times SO(3) \times SO(3)$$



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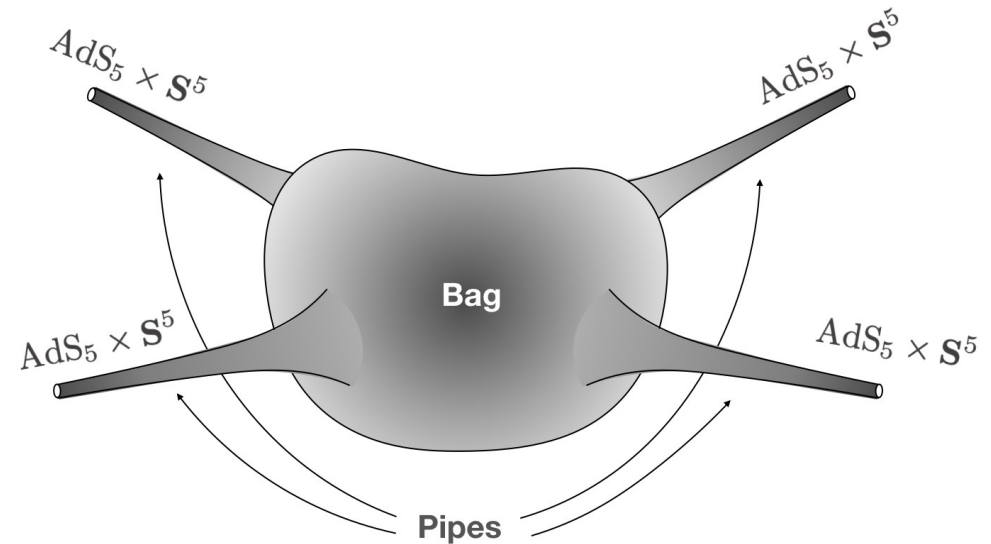
The theory has the symmetries

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which in spacetimes translate to the manifold

$$AdS_4 \times \mathbf{S}_1^2 \times \mathbf{S}_2^2 \times \Sigma$$

where everything is warped over the oriented Riemann surface Σ . The metric is:



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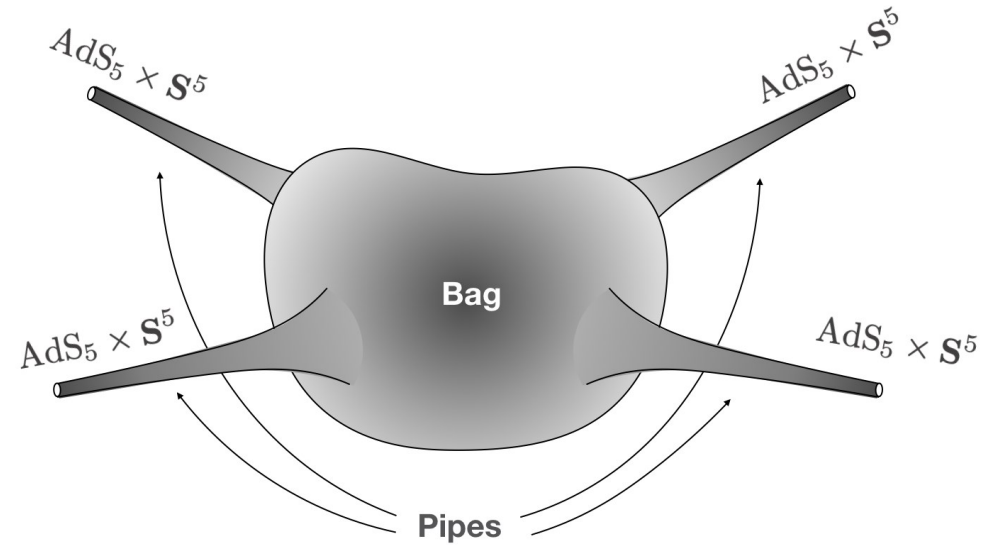
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$$ds^2 = f_4^2 ds_{AdS_4}^2 + f_1^2 ds_{\mathbf{S}_1^2}^2 + f_2^2 ds_{\mathbf{S}_2^2}^2 + ds_{\Sigma}^2$$

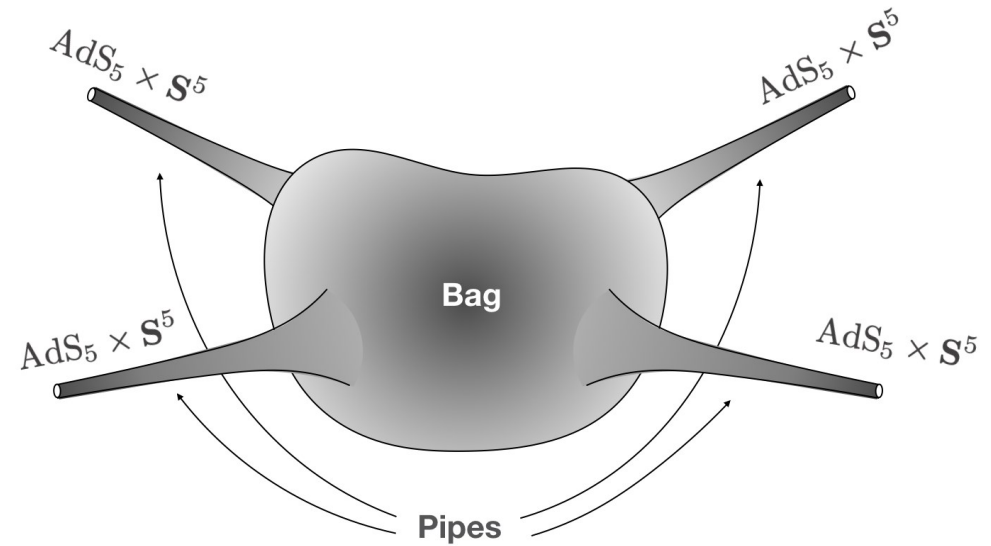


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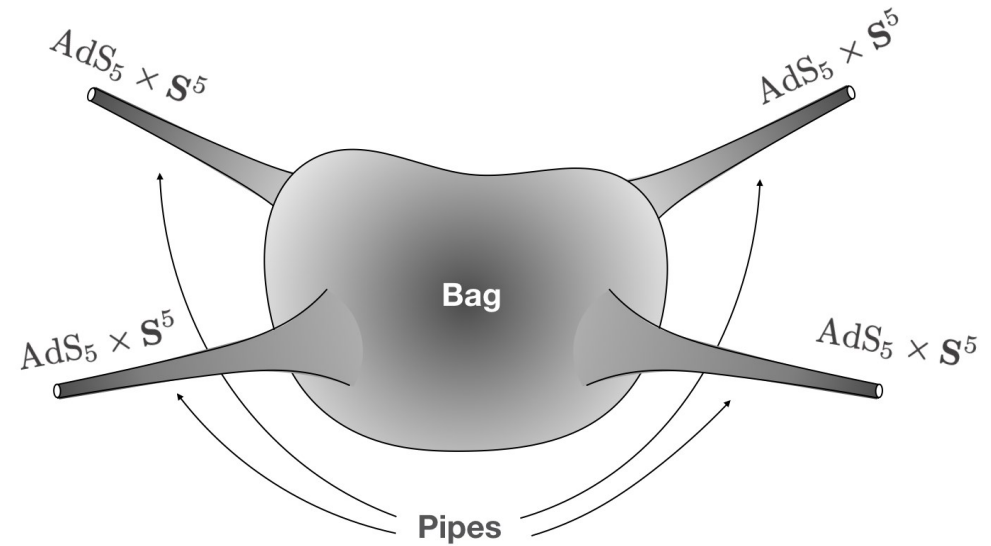
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$$f_4 \equiv f_4(r, \varphi)$$

$$f_1 \equiv f_1(r, \varphi)$$

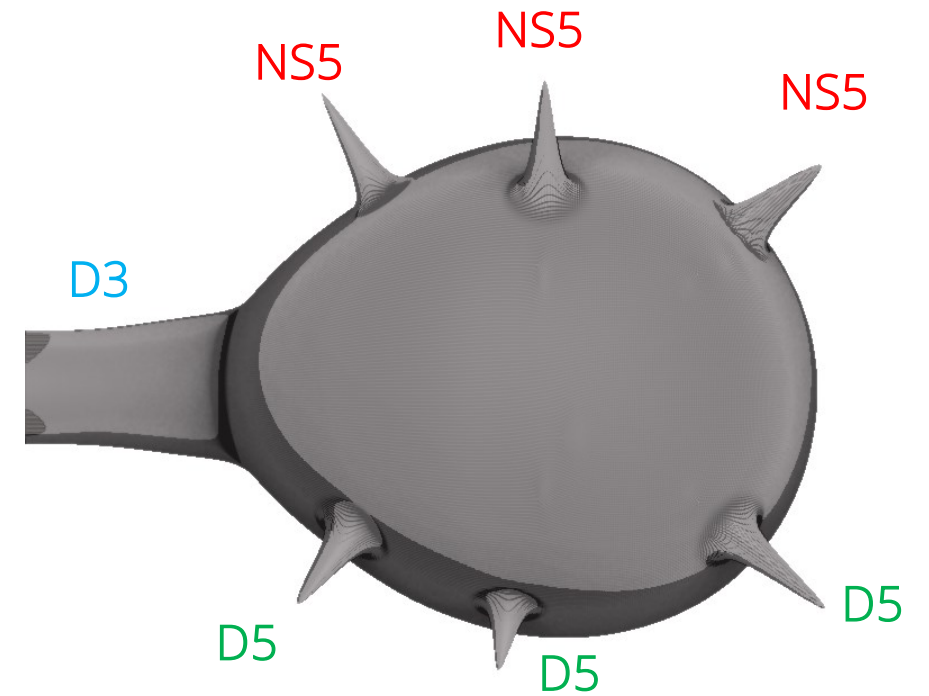
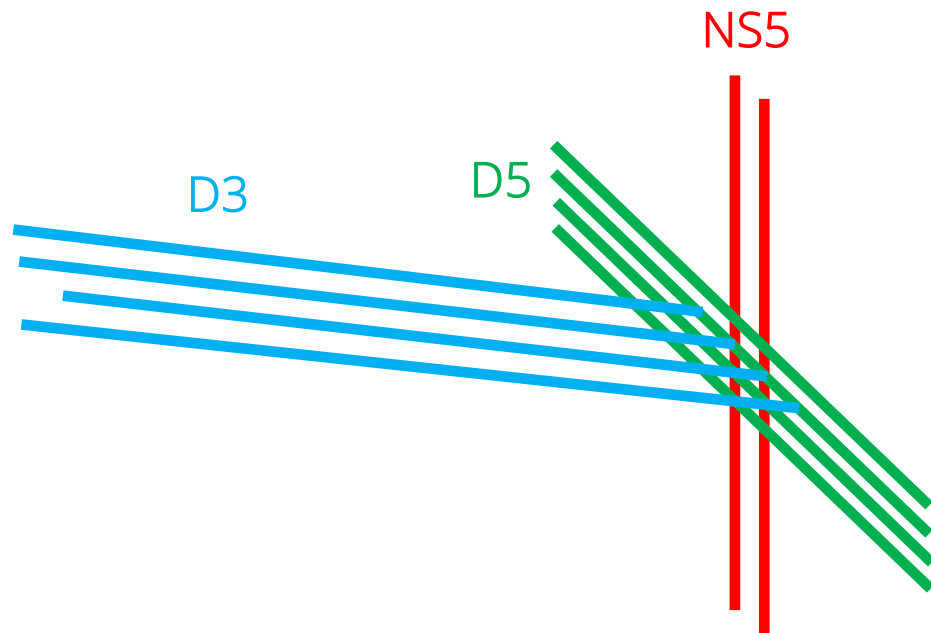
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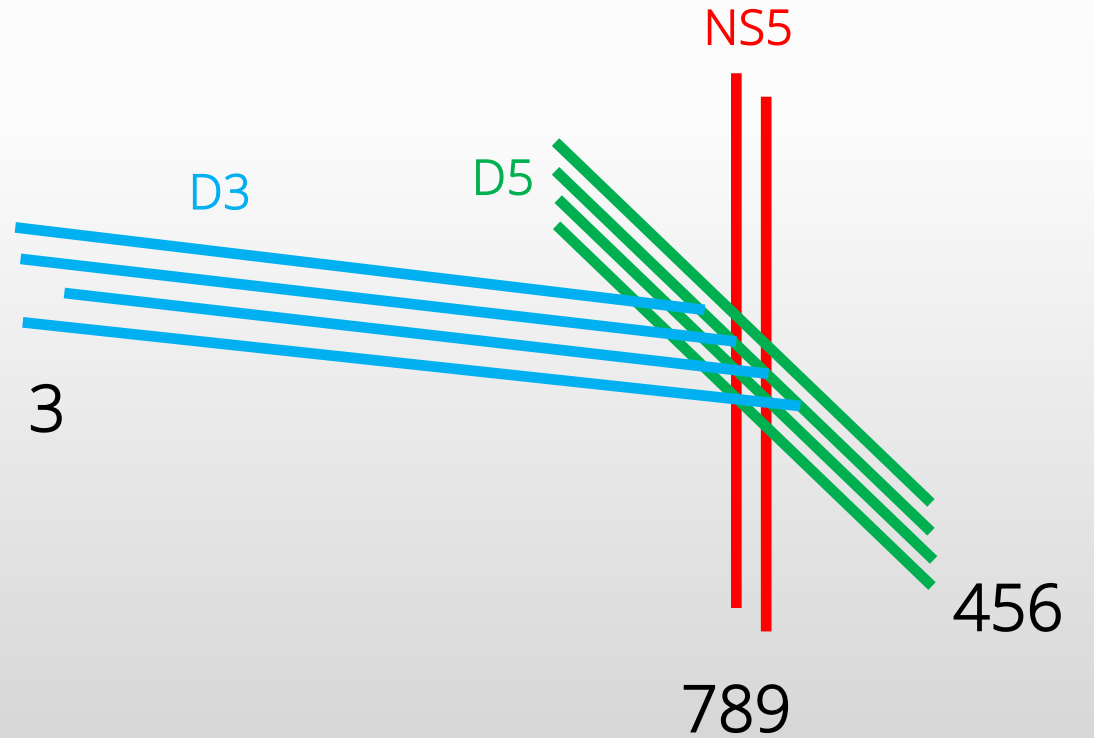
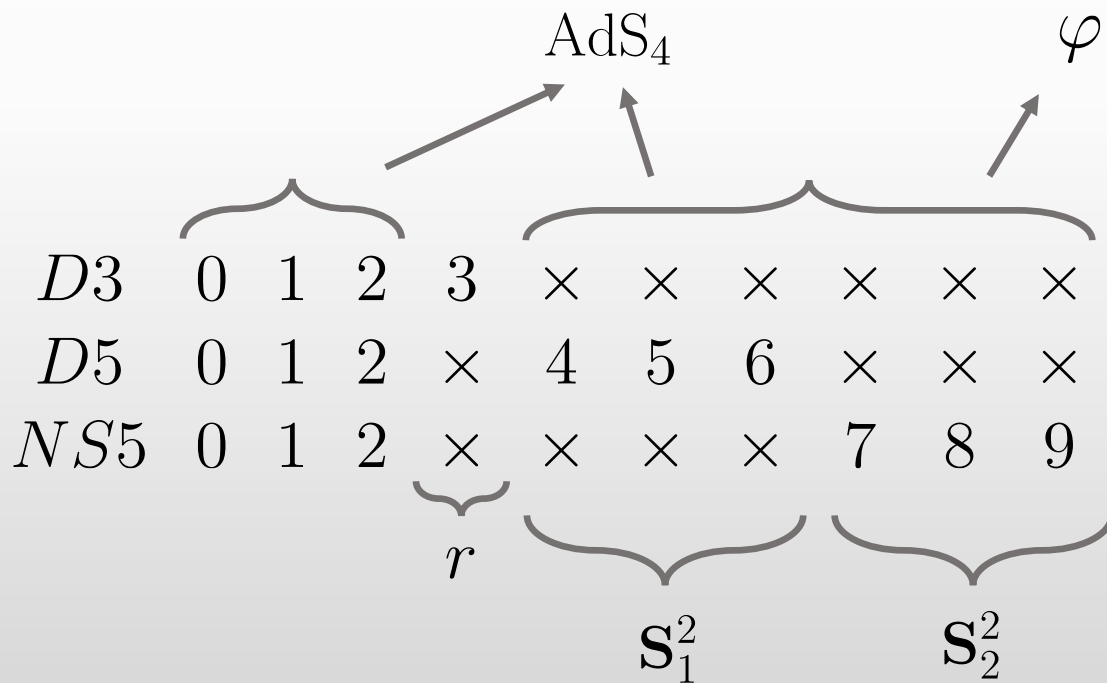
D3- BRANES ENDING ON 5-BRANES

[Aharony, Berdichevsky, Berkooz, Shamir, '11]

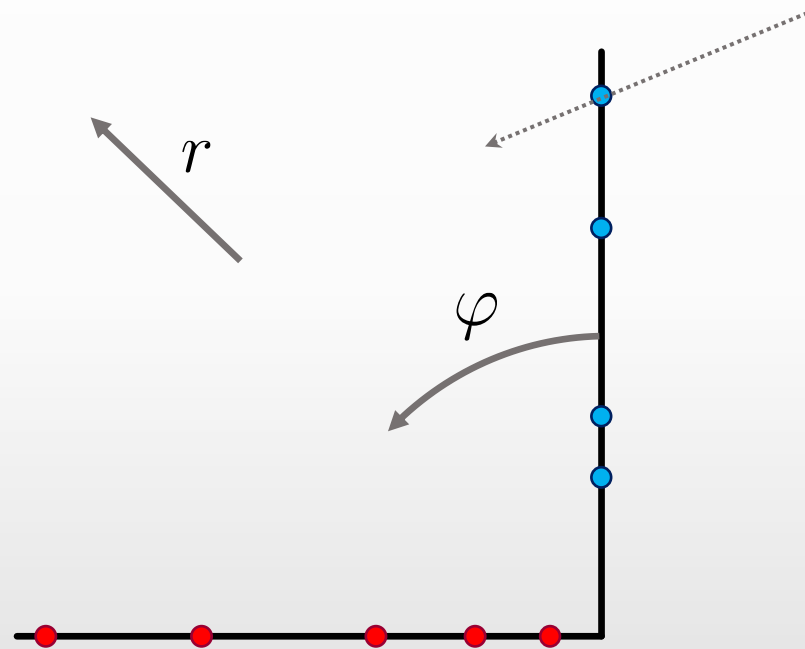


BRANE CONFIGURATION

$$ds^2 = f_4^2 ds_{AdS_4}^2 + f_1^2 ds_{S_1^2}^2 + f_2^2 ds_{S_2^2}^2 + 4\rho^2 (dr^2 + r^2 d\varphi^2)$$

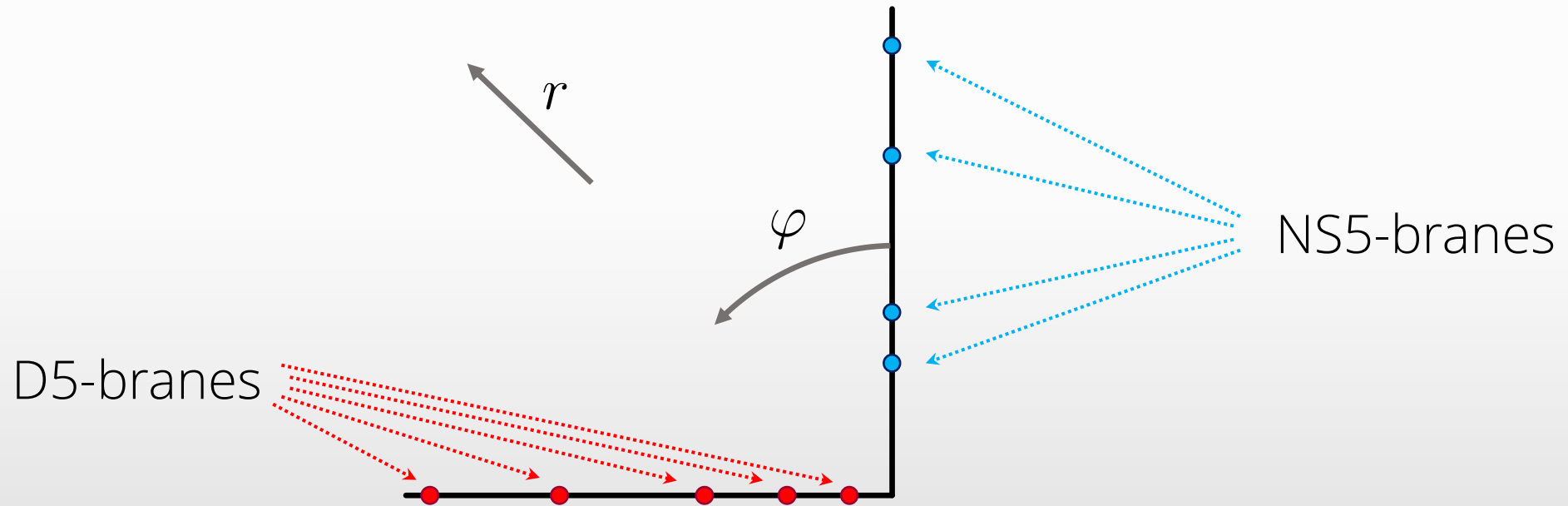


ETW PICTURE

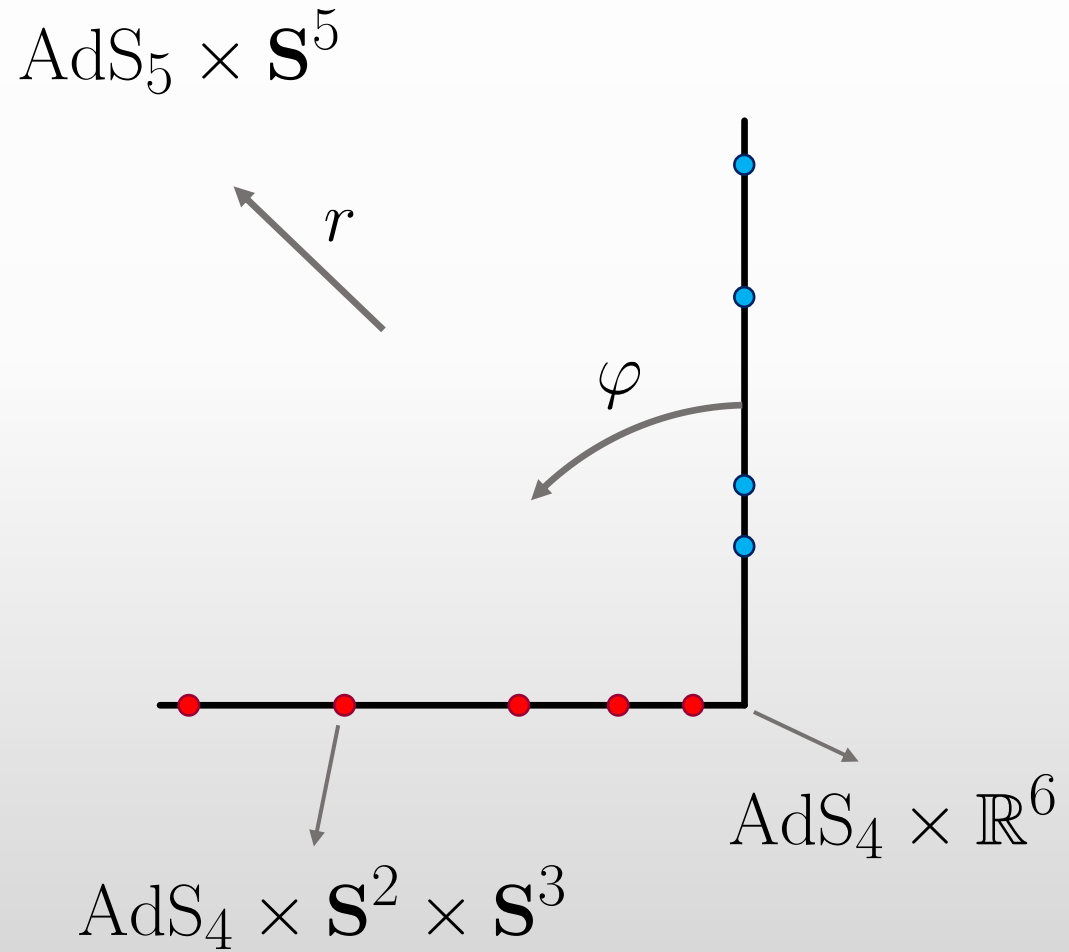


At each point on Σ ,
we have $\text{AdS}_4 \times \mathbf{S}_1^2 \times \mathbf{S}_2^2$

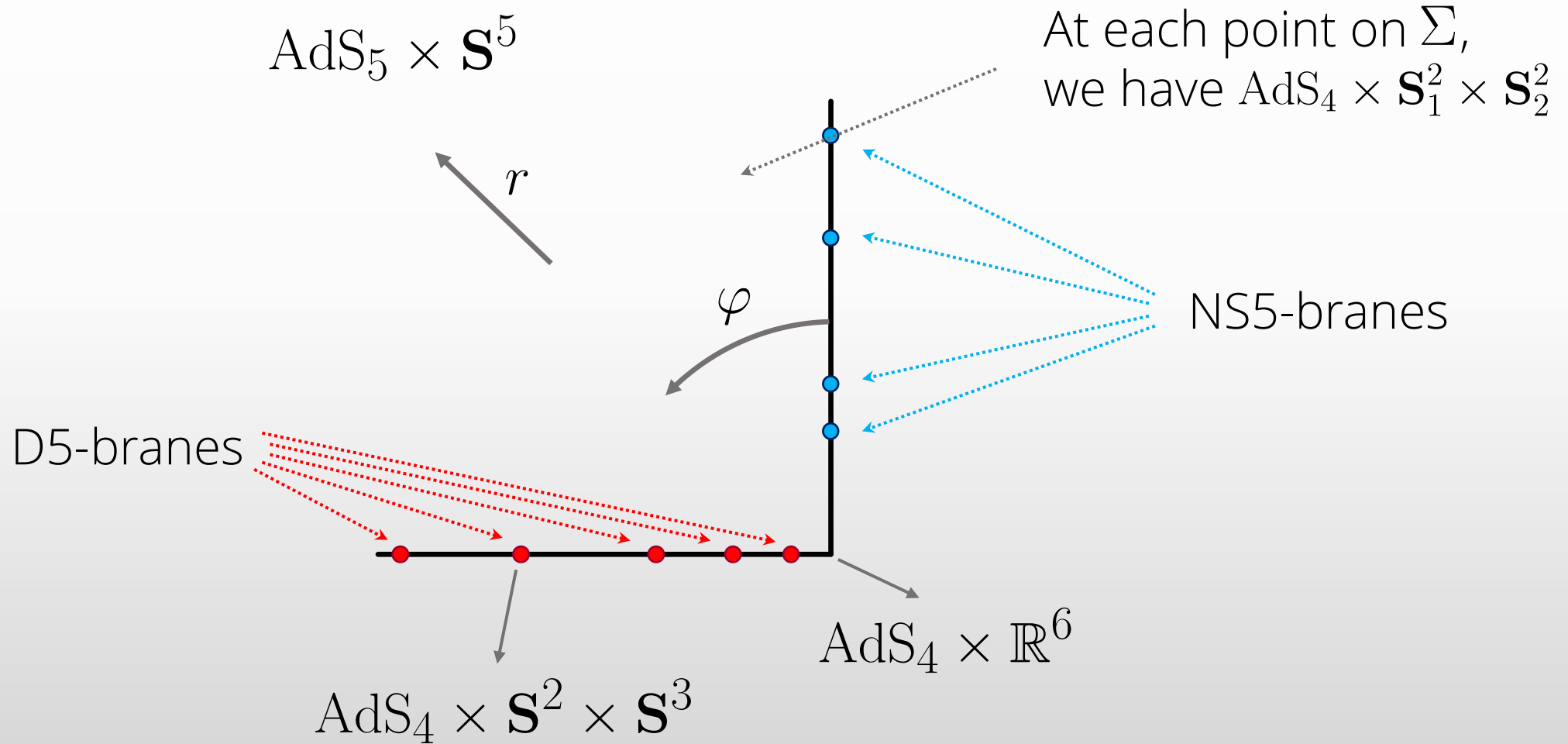
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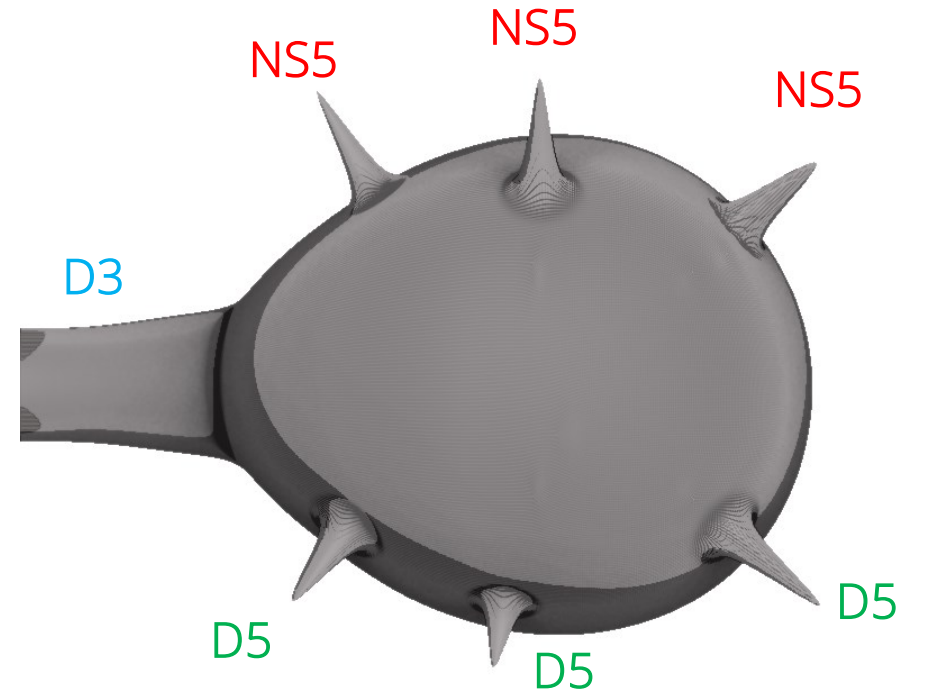
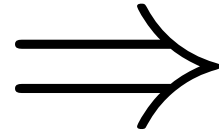
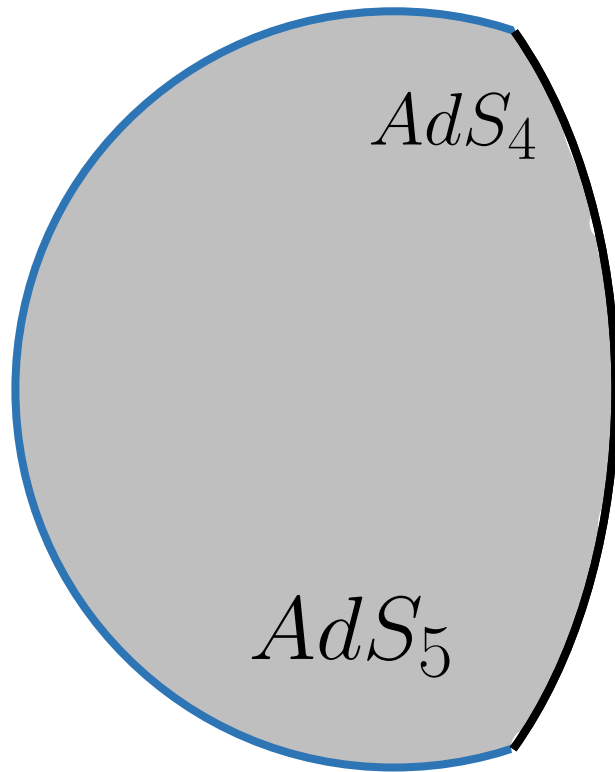


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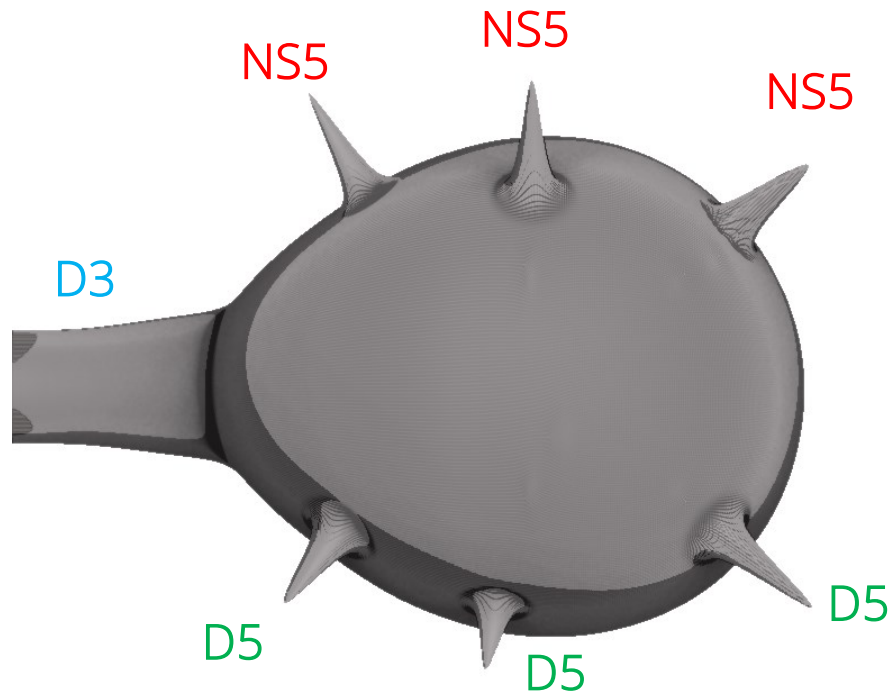


OUR RESULTS

A STRINGY ETW-BRANE FOR ADS/CFT



SMOOTHING THE SOLUTION



But this is a complicated solution!

The 5-branes break the $SO(6)$ rotational invariance of the S^5

We need to smooth the solution to compare with the Dynamical Cobordism results

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The $SO(6)$ symmetry is broken down to $SO(3) \times SO(3)$ by the 5-branes

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This mode parametrizes the overall volume of the S^5

I: COMPACTIFY THE 2-SPHERES

$$ds^2 = f_4^2 ds_{AdS_4}^2 + f_1^2 ds_{\mathbf{S}_1^2}^2 + f_2^2 ds_{\mathbf{S}_2^2}^2 + 4\rho^2(dr^2 + r^2 d\varphi^2)$$

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We reduce the 2-spheres parametrizing their volumes by radions

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and we get a 6d theory + radions:

$$ds_6^2 = \sqrt{f_1^2 f_2^2} [f_4^2 ds_{AdS_4}^2 + 4\rho^2 (dr^2 + r^2 d\varphi^2)]$$

II: TRUNCATE TO THE ZERO MODE

We do not yet have cod-1 manifold $ds_6^2 = \sqrt{f_1^2 f_2^2} [f_4^2 ds_{AdS_4}^2 + 4\rho^2 (dr^2 + r^2 d\varphi^2)]$

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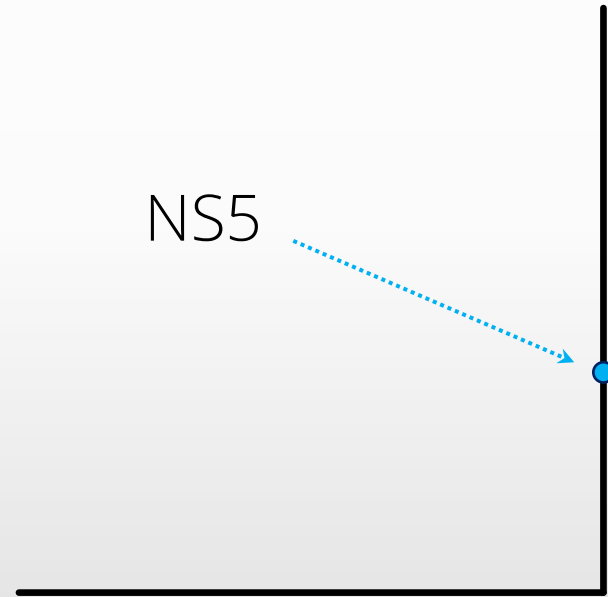
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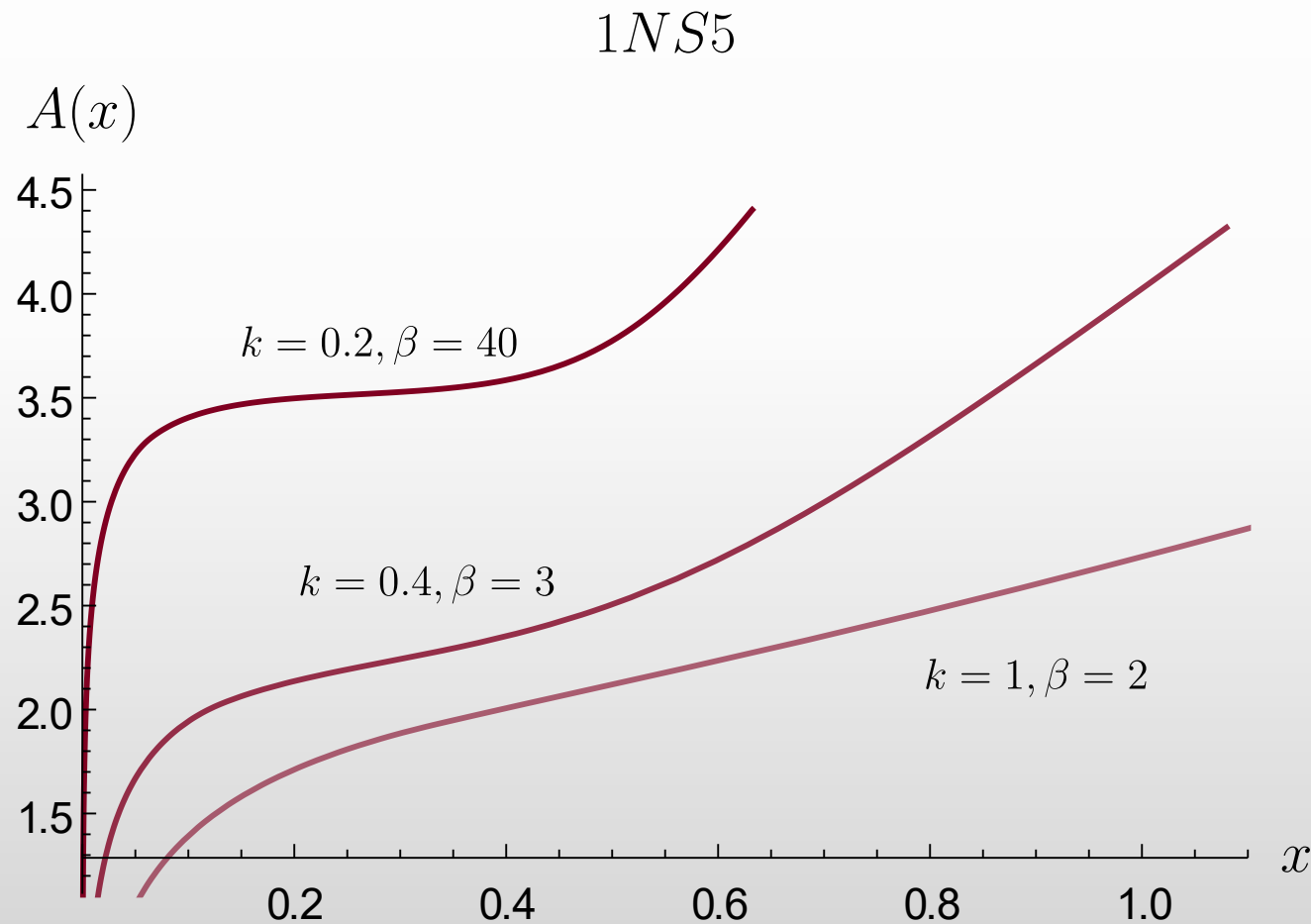
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$$ds_5^2 = e^{2A(x)} ds_{AdS_4}^2 + dx^2$$

EXAMPLE: THE GENUS ONE CASE



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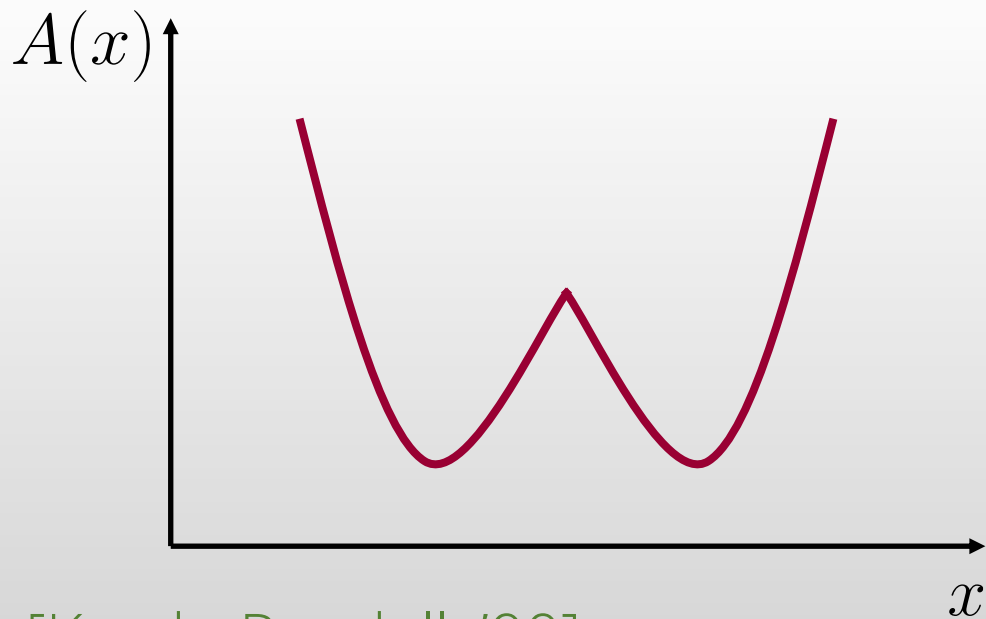


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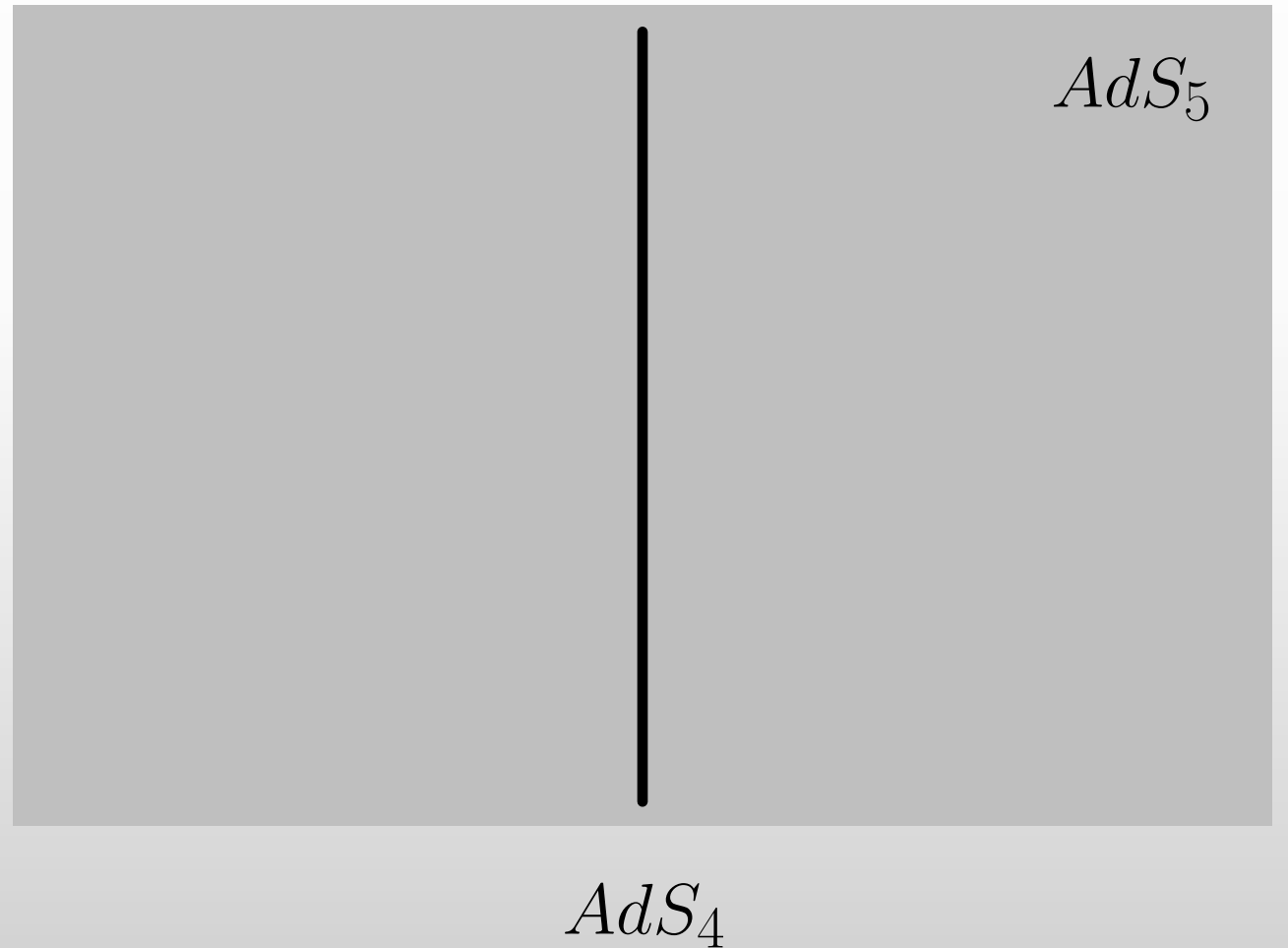
Too simple, no bump!

LOCALIZATION OF GRAVITY IN KR

$$ds_5^2 = e^{2A(x)} ds_{AdS_4}^2 + dx^2$$

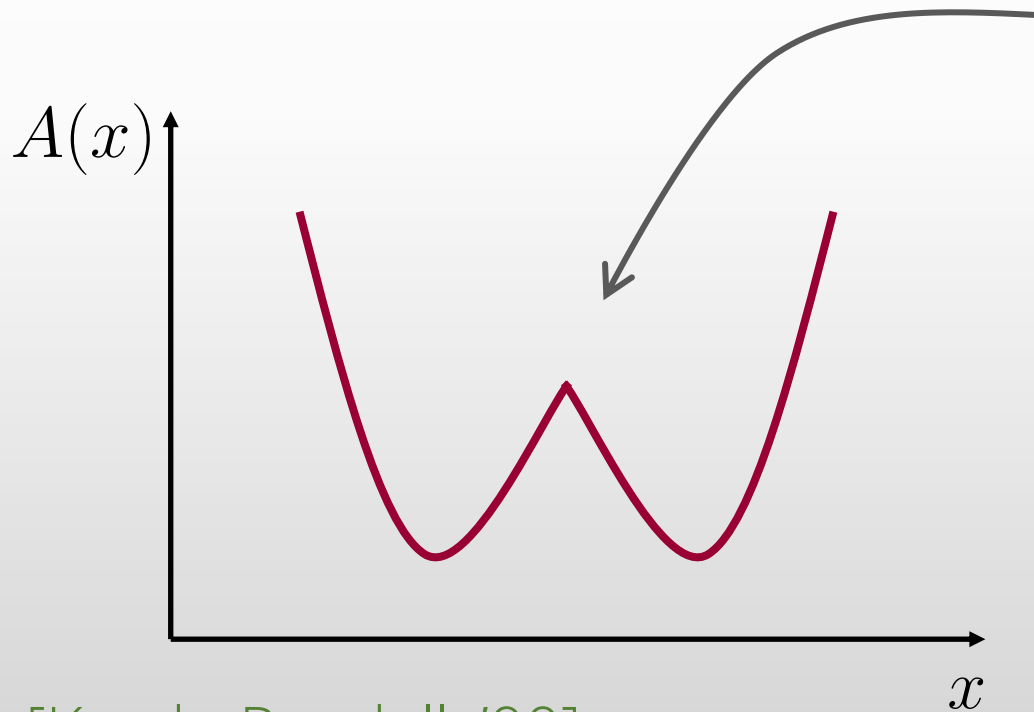


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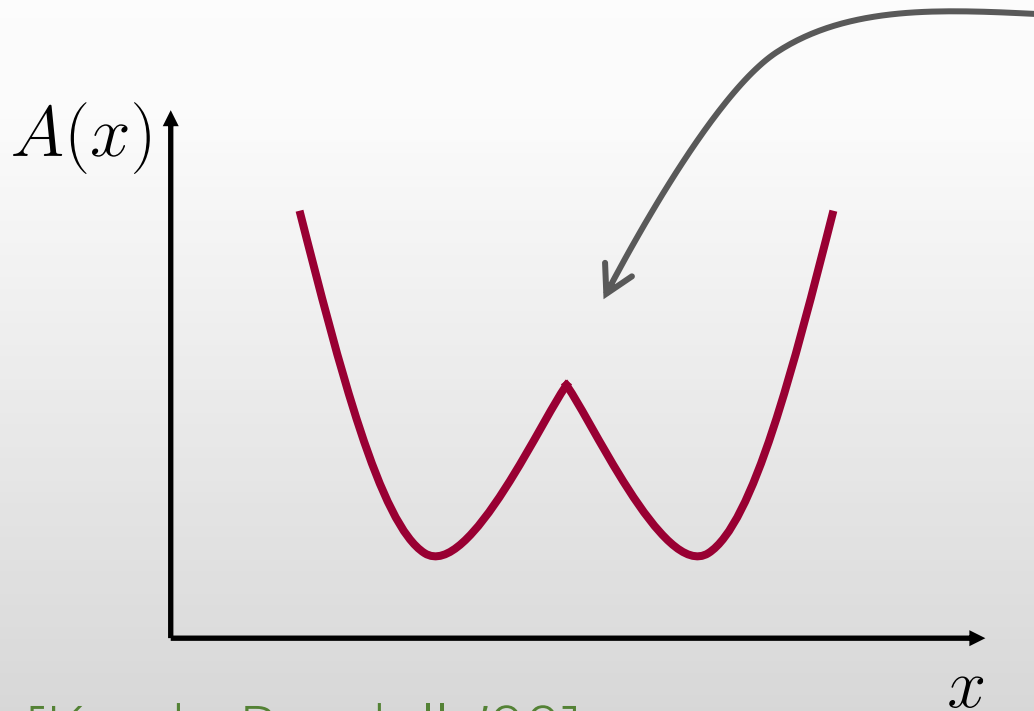
The graviton is bound on the brane!

We get a 4d massive graviton that couples to the 5d massless one

The mass is proportional to the coupling to the 5d space

LOCALIZATION OF GRAVITY IN KR

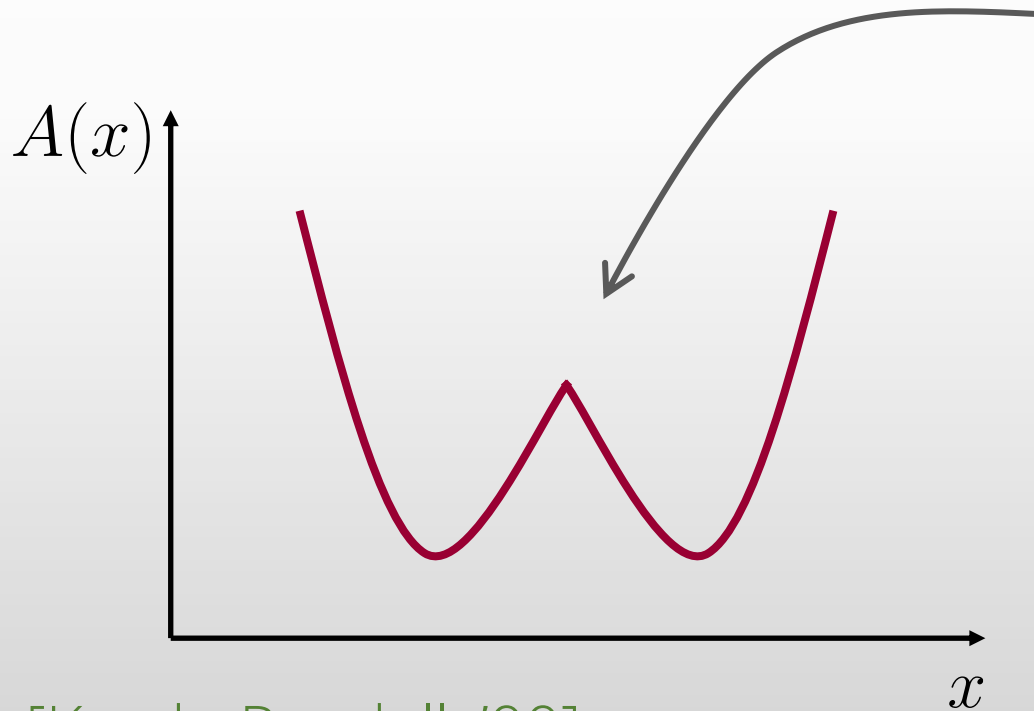
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Having a 4d massive graviton, means that holographically, we have a localized energy momentum tensor in our BCFT3 that couples to the CFT4

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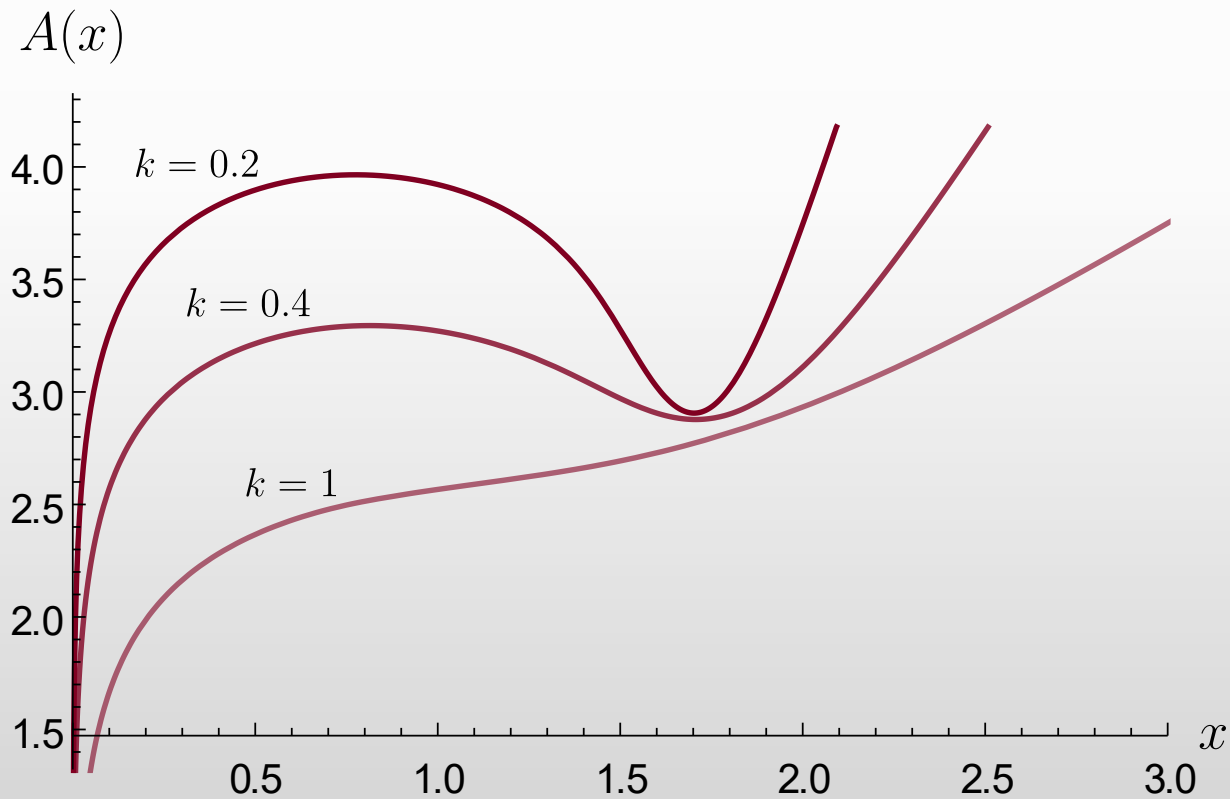


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→ Holographic interpretation of the ETW brane!!

LOCALIZATION OF GRAVITY IN OUR SETUP

$1NS5 + 1D5; \beta = -\alpha = 3, l = k$



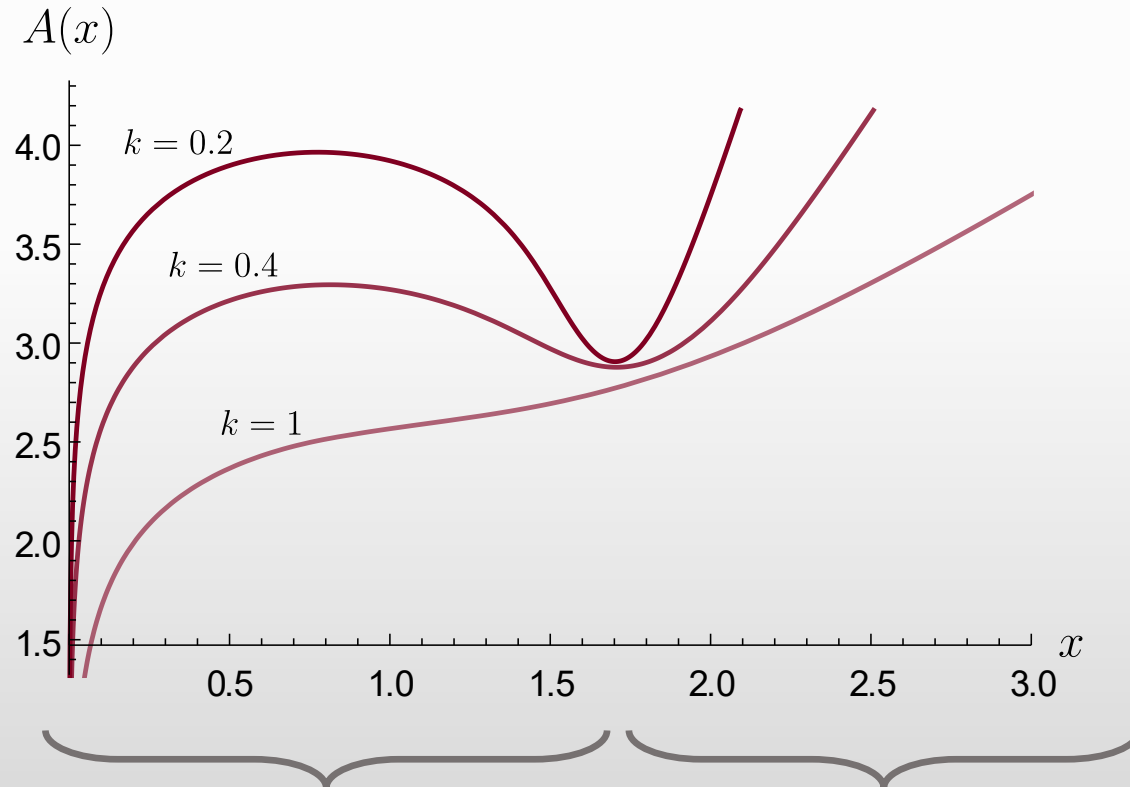
$$ds_5^2 = \hat{f}_4^2(r) ds_{AdS_4}^2 + \hat{\rho}^2(r) dr^2$$

$$ds_5^2 = e^{2A(x)} ds_{AdS_4}^2 + dx^2$$

This is an explicit realization
of a KR brane in string theory

LOCALIZATION OF GRAVITY IN OUR SETUP

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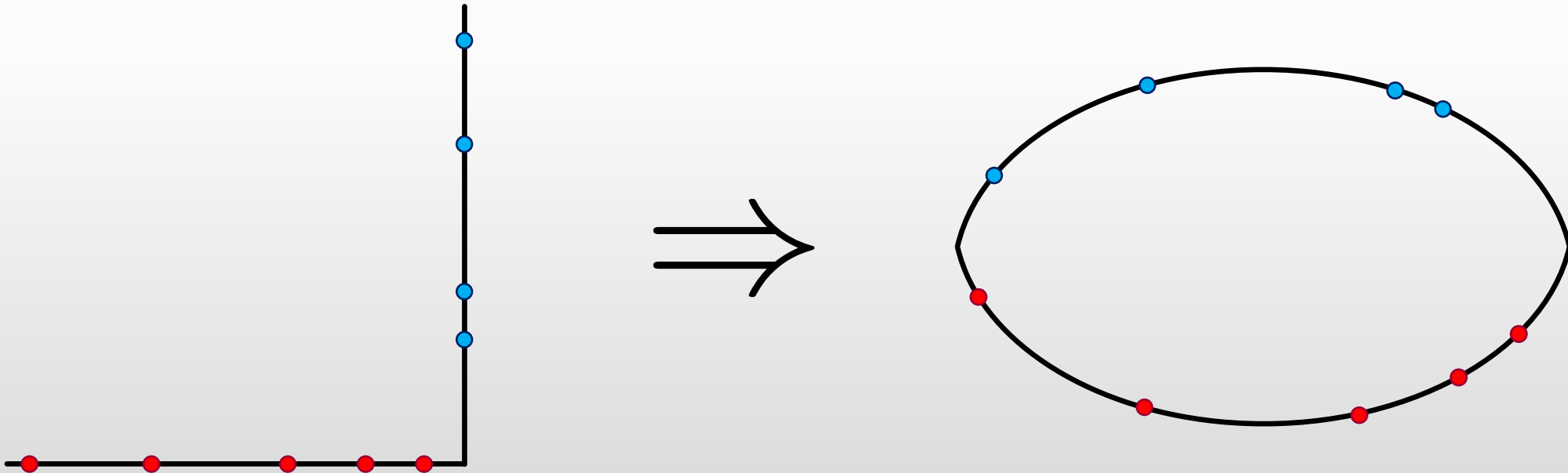
The Bag

The asymptotic AdS_5 throat

DOUBLE SCALING LIMIT

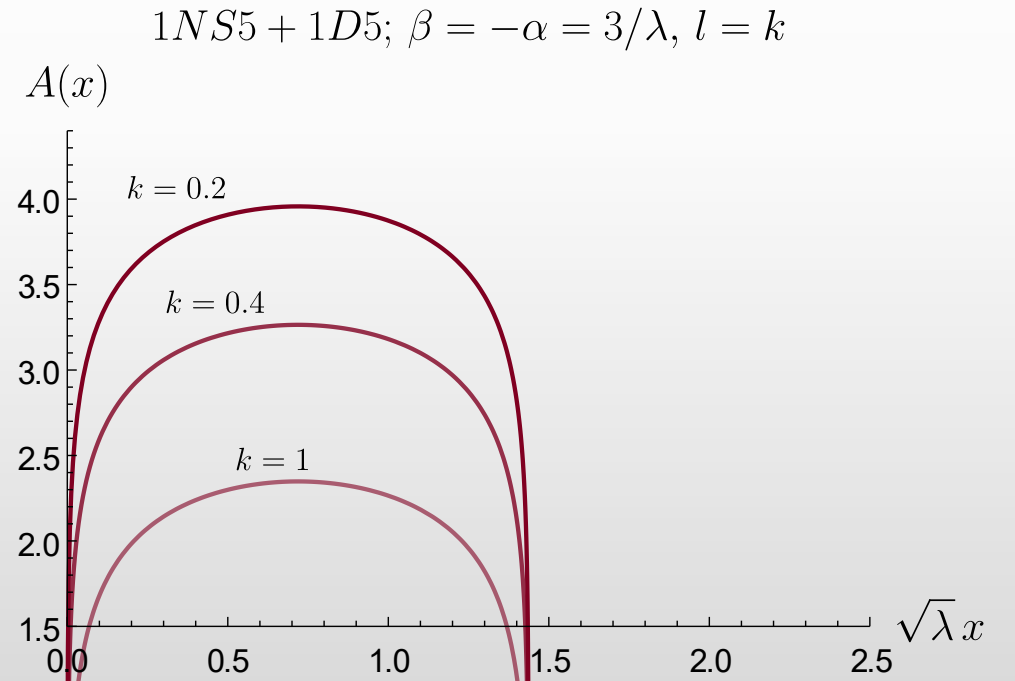
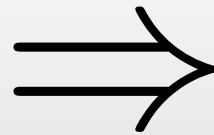
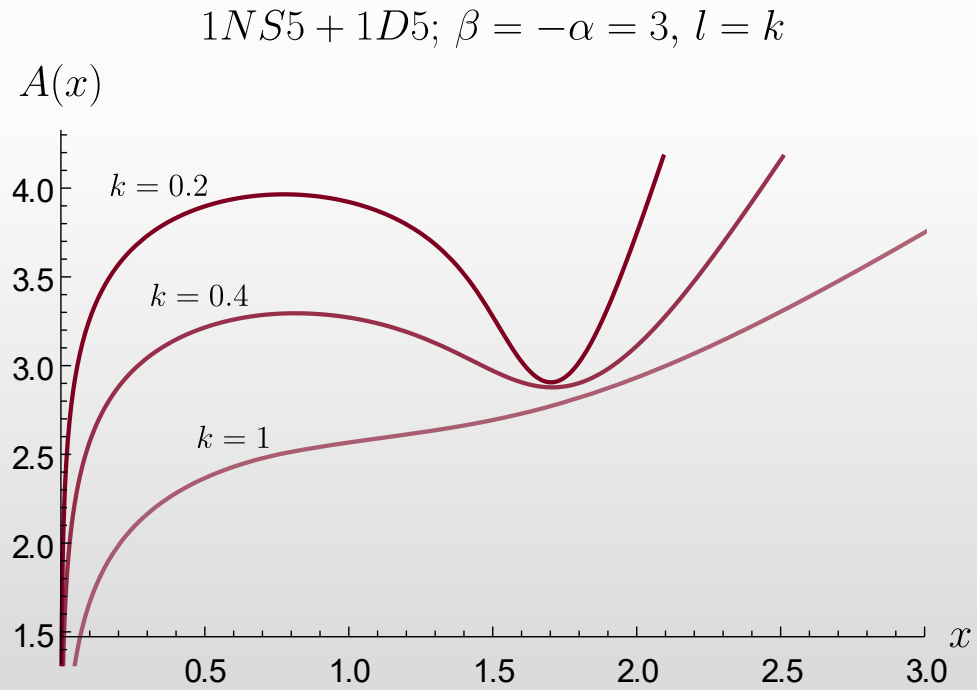
The double scaling limit makes the 5-form flux disappear and thus closes the AdS throat; so we get a compact geometry!

$$\text{AdS}_4 \times \mathbf{X}_6$$



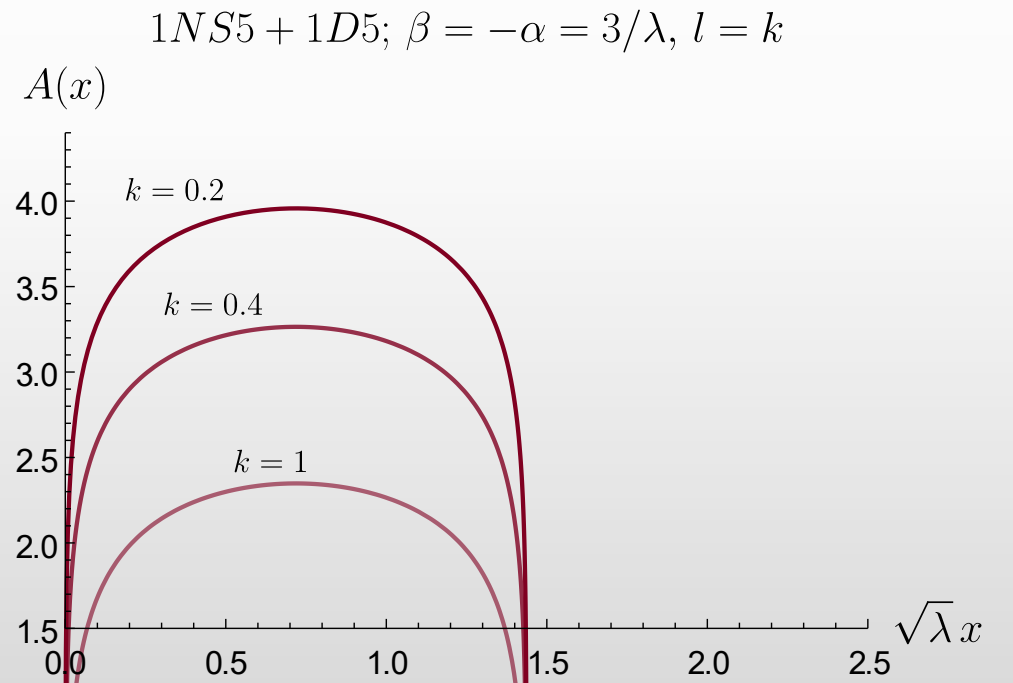
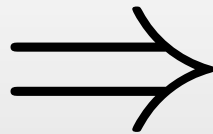
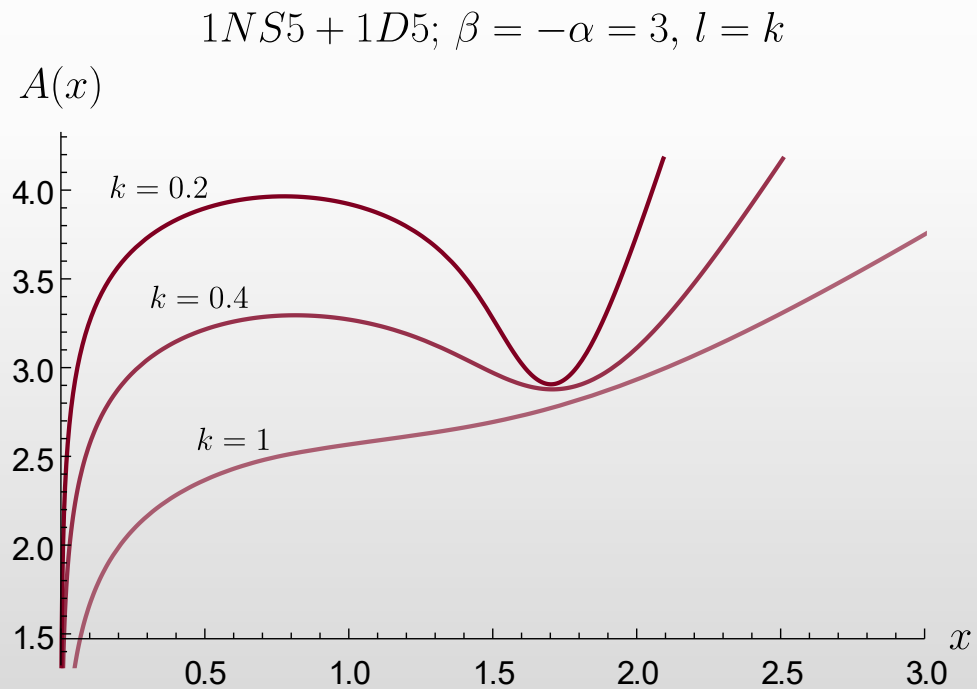
DOUBLE SCALING LIMIT

Only possible if you have both D5- and NS5-branes



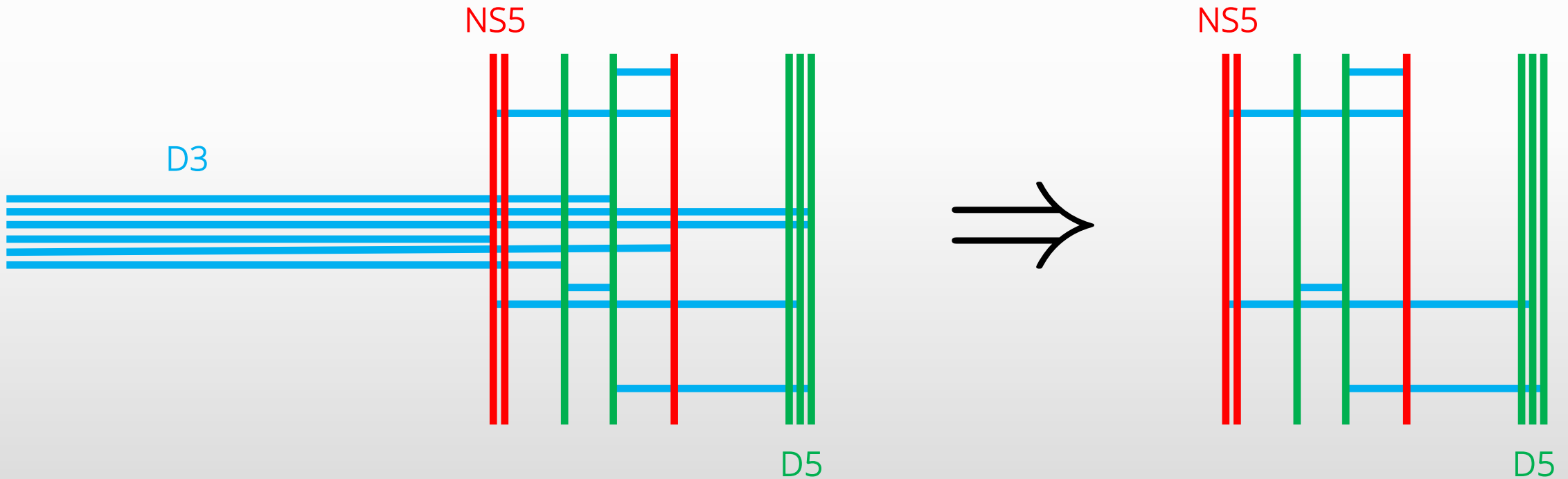
DOUBLE SCALING LIMIT

This double scaling limit isolates the dynamics of the ETW-brane!



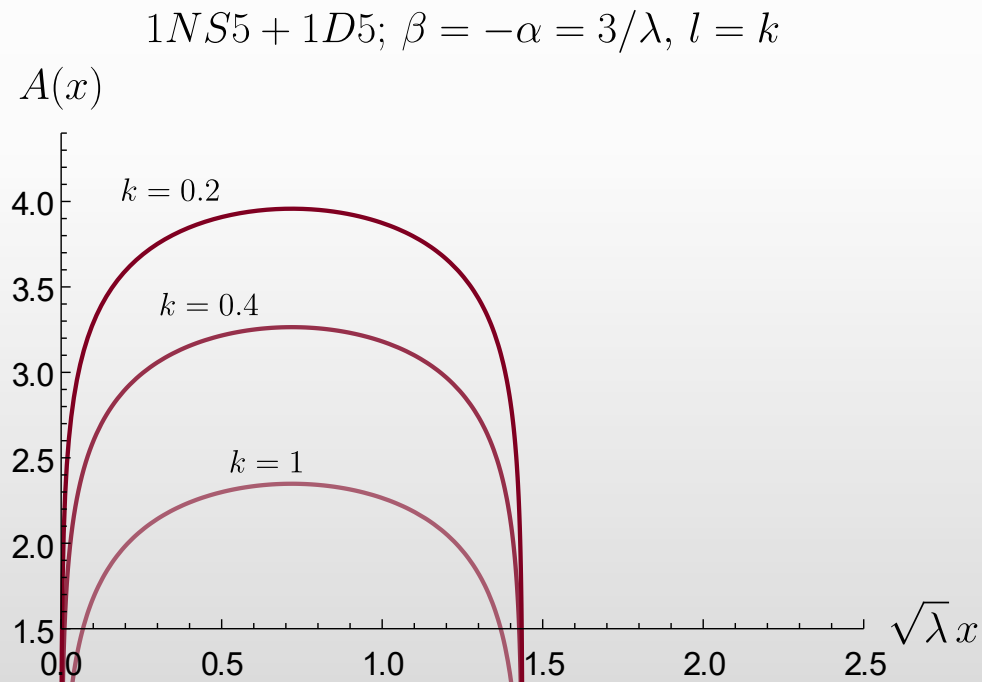
DOUBLE SCALING LIMIT

Which is that of a brane-web



DOUBLE SCALING LIMIT

In fact, provides the gravity dual of the 3d BCFT (wedge holography)



$$\text{AdS}_4 \times \mathbf{X}_6$$

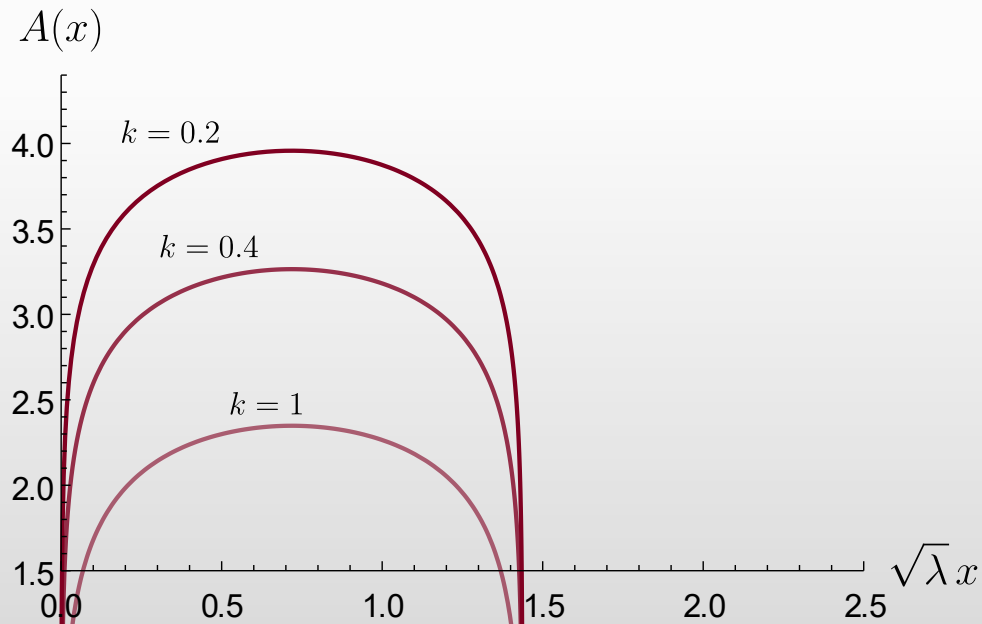
[Assel, Bachas, Estes, Gomis '11]

[Van Raamsdonk, Waddell, '21]

DOUBLE SCALING LIMIT

The fact that we can close the throat in a continuous fashion and make the space compact, is related to continuously making the 4d graviton massless

$$1NS5 + 1D5; \beta = -\alpha = 3/\lambda, l = k$$



*The mass of the graviton is proportional to the size of the throat

[Demulder, Gneccchi, Lavdas, Lust, '22]

CONCLUSIONS

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THANK YOU!