Cobordism and the half-spin group [C.K. 240X.XXXXX]

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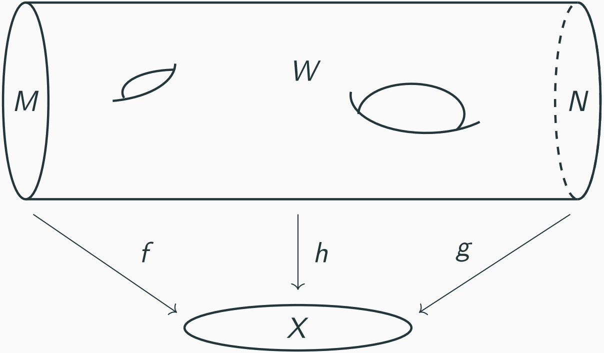
Setting the stage - Cobordism

- Mathematical model for topology-changing processes between n-dimensional manifolds M and N
- Cobordism groups denoted as $\Omega_n^{\xi}(X)$



$\Rightarrow$ here X a classifying space BG encoding a background gauge field

The Cobordism Conjecture [McNamara, Vafa '19]

- A non-vanishing cobordism group Ω_n^{ξ} can be understood as a higher-form global symmetry

  $\Rightarrow$ Absence of Global Symmetries in Quantum Gravity

  e.g. [Banks, Seiberg '10] implies $\Omega_n^{QG} = 0$

  $\Rightarrow$ Mechanism: Gauging or breaking

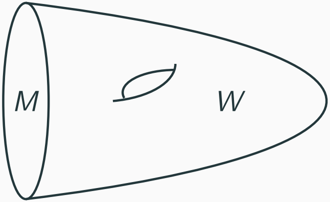


Figure 1: Vanishing cobordism group - nullbordant manifold

Spin(32)/$\mathbb{Z}_2$ or Spin(32)/$\mathbb{Z}_2$

- Aim: Study topological consistency conditions for type I (Spin(32)/$\mathbb{Z}_2$ heterotic string) arising from the Cobordism Conjecture
- Fixing tangential structure to be spin as required by type I string theory
- We have a background gauge field present due to background D9-branes $\Rightarrow$ SO(32)?
- Nonperturbatively has to be the same as the S-dual Spin(32)/$\mathbb{Z}_2$ heterotic string [Berkooz, Leigh, Polchinski, Schwarz, Seiberg, Witten '96]
- Spin(32) has center $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \Rightarrow$ two unique quotients: SO(32) and SemiSpin(32)
  $\Rightarrow$ Have to calculate $\Omega_n^{Spin}(BSs(32))$

Calculation of $\Omega^{Spin}(BSs(32))$

- For cobordism calculations involving classifying spaces usually Adams spectral sequence the preferred tool
- How do Spectral sequences work?
- Approximation tool, each step is called page

  $\Rightarrow$ Fill second page with a known commodity, e.g. for Adams spectral sequence we need to know $H^*(BG, \mathbb{Z}_2)$

  $\Rightarrow$ Calculate (finitely many) subsequent pages

  $\Rightarrow$ Retrieve cobordism groups from final page

Calculation of $\Omega^{Spin}(BSs(32))$ II

- However $H^*(BSs(32), \mathbb{Z}_2)$ is not known...

  $\Rightarrow$ More spectral sequences! techniques similar to [Kono, Mimura, Shimada '75, '76]

  $\Rightarrow$ The following Eilenberg-Moore-spectral sequence does the job

$$E_2 = \mathrm{Cotor}^{H^*(Ss(32),\mathbb{Z}_2)}(\mathbb{Z}_2, \mathbb{Z}_2) \Longrightarrow H^*(BSs(32), \mathbb{Z}_2)\,.$$

  $\Rightarrow$ Upshot: Calculate $H^*(BSs(32), \mathbb{Z}_2)$ from $H^*(Ss(32), \mathbb{Z}_2)$, feed it into Adams spectral sequence $\Rightarrow$ $\Omega_n^{Spin}(BSs(32))$

The ko-homology building blocks

- Compute $ko_n(BSs(32))$ use the ABP theorem [Anderson, Brown,Peterson '67]:

$$\Omega_n^{Spin}(X)_{\hat{2}} = ko_n(X)_{\hat{2}} \oplus ko_{n-8}(X)_{\hat{2}} \oplus ko_{n-10}\langle 2\rangle(X)_{\hat{2}} \oplus \ldots . \quad (1)$$

- $ko_n(BSs(32)) \cong ko_n(pt) \oplus \widetilde{ko}_n(BSs(32))$ informs us about type I open (fundamental) string sector
- What about S-dual heterotic strings?
  $\Rightarrow$ Indeed same background gauge group (more specifically Langlands dual group with $Ss(32) = {}^L Ss(32)$)
  $\Rightarrow ko_n(B^L Ss(32)) \cong ko_n(pt) \oplus \widetilde{ko}_n(B^L Ss(32))$ informs us about the heterotic open D-string sector! [Hull '97, '98] $\Rightarrow$ Jacob's talk yesterday

The final result

n	0	1	2	3	4	5	6	7
$\Omega_n^{Spin}(BSs(32))$	$\mathbb{Z}$	$\mathbb{Z}_2$	$2\mathbb{Z}_2$	0	$\mathbb{Z}_2 \oplus 2\mathbb{Z}$	0	$2\mathbb{Z}_2$	0

n	8	9	10	11	12
$\Omega_n^{Spin}(BSs(32))$	$5\mathbb{Z} \oplus \mathbb{Z}_8$	$5\mathbb{Z}_2$	$10\mathbb{Z}_2$	$3\mathbb{Z}_2$	$8\mathbb{Z} \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_8$

Some of the physics behind $\Omega^{Spin}(BSs(32))$

- Pick out one example $\Omega_4^{Spin}(BSs(32)) = \mathbb{Z}_2 \oplus 2\mathbb{Z}$
- Orientifold on orbifold limit of K3 [Gimon, Polchinski '96]

  $\Rightarrow$ 16 "hidden" instantons/D5-branes at fixed points and 8 dynamical D5-branes

  $\Rightarrow$ Fixed points can be smoothly blown up and retain smooth K3 spectrum $\Rightarrow$ only consistent if instantons are SemiSpin(32)-instantons [Berkooz, Leigh, Polchinski, Schwarz, Seiberg, Witten '96]

Some of the physics behind $\Omega^{Spin}(BSs(32))$ II

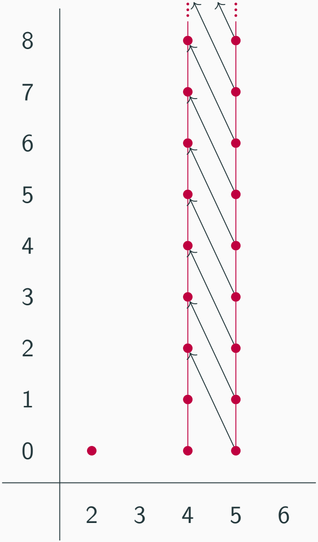
- There are three equivalence classes of K3-compactifications with SemiSpin(32)-bundle, enumerated by the homological invariants:

$$\widetilde{w}_2 = 0\,, \tag{2}$$
$$\widetilde{w}_2 \neq 0 \text{ and } \widetilde{w}_2^2 = 0 \text{ mod } 4\,, \tag{3}$$
$$\widetilde{w}_2 \neq 0 \text{ and } \widetilde{w}_2^2 = 2 \text{ mod } 4\,. \tag{4}$$

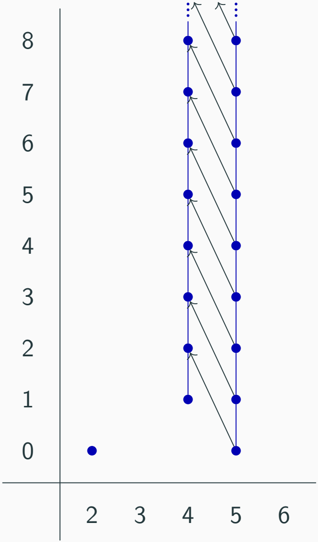
- Convenient general description as F-theory on elliptically fibered Calabi-Yau threefold with base $\mathbb{F}_\mathsf{n}$ where $\widetilde{w}_2^2 = 2(\mathsf{n}-4)$ mod 4 and $4 \geq n \geq -2$ [Aspinwall '96]

Some of the physics behind $\Omega^{Spin}(BSs(32))$ III

Second page E_2 for $H_4(B^2\mathbb{Z}_2;\mathbb{Z})_{\hat{2}}$



Second page E_2 for $ko_4(B^2\mathbb{Z}_2)_{\hat{2}}$



Some of the physics behind $\Omega^{Spin}(BSs(32))$ IV

- Our spin cobordism group is precisely detected by $\frac{1}{2}\mathcal{P}(x_2)$ mod 2 or equivalently $\widetilde{w}_2^2 = 0, 2$ mod 4

  $\Rightarrow$ So how is it trivialized?

- It turns out that the SemiSpin(32)-instantons always contribute a charge of $2\,\widetilde{w}_2^2 = 0$ mod 4 [Aspinwall '96] thereby we are always in the trivial class $[0] \in \Omega_4^{Spin}(BSs(32))$

Outlook

- Based on the constraint for $\frac{1}{2}\mathcal{P}(\widetilde{w}_2) \in \mathbb{Z}_2 \subset \Omega_4^{Spin}(BSs(32))$
  $\Rightarrow$ higher dimensional cobordism groups like
  $\widetilde{w}_2^4 \in \mathbb{Z}_8 \subset \Omega_8^{Spin}(BSs(32))$ gauged as well?
  $\Rightarrow$ Invariants related to frozen phase of F-theory/type I without vector structure compactifications [Witten '98]
- Map from Spin cobordism to KO-theory $\Rightarrow$ tadpole cancellation [Blumenhagen, Cribiori '21]
- Now that we have $\Omega_n^{Spin}(BSs(32))$ can uplift to twisted string structure (include Bianchi identity) for non-SUSY $(Ss(16) \times Ss(16)) \rtimes \mathbb{Z}_2$- and $Ss(32)$-heterotic string $\Rightarrow$ Are there new subtleties? [Basile, Debray, Delgado, Montero '23]