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STRING PHENO 2024

Università degli studi di Padova
Padova, Italy

The Scalar Weak Gravity Conjecture, moduli fields and gauged supergravity

Gabriele Casagrande

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GABRIELE CASAGRANDE

work in progress with E. Dudas and G. Dall'Agata

Weak Gravity Conjecture and scalar fields

WGC & scalar fields

- Weak Gravity Conjecture (WGC) in D spacetime dimensions: [ARKANI-HAMED, MOTL, NICOLIS, VAFA ’07]

$$\left(\frac{eq}{M}\right)^2 \geq \frac{D-3}{D-2}$$

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- WGC in presence of light scalar fields: [Palti '17]

$$\mathcal{Q}^2 \geq \left(\frac{D-3}{D-2}\right) m^2 + G_\phi^{ij} \partial_i m \partial_j m$$

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$$m = m(\phi)$$

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electromagnetic interactions

$$\mathcal{Q}^2 \geq \left(\frac{D-3}{D-2}\right) m^2 + G^{ij}_{\phi}\partial_i m \partial_j m$$

$$\mathcal{Q}^2 \equiv q_\Lambda \mathcal{I}^{\Lambda\Sigma} q_\Sigma \qquad \mathcal{L} \supset -\frac{1}{4}\mathcal{I}_{\Lambda\Sigma} F^\Lambda F^\Sigma$$

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scalar interaction

$$\mathcal{L} \supset -\frac{1}{2} G_{\phi ij} \partial_\mu \phi^i \partial^\mu \phi^j$$

WGC & scalar fields

- Natural scenario: the scalar fields are *compactification moduli*.

[Palti '17] [Lust, Palti '17]
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5D scalar QED coupled to gravity: [Lust, Palti '17]

$$S_5 = \int d^5x \sqrt{-G} \left\{ \frac{R_5}{2} - \frac{1}{4e^2} \mathcal{F}_{MN} \mathcal{F}^{MN} - \nabla_M H \nabla^M \bar{H} - M^2 \left| H \right|^2 \right\}$$

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5D Ricci scalar

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5D gauge field $\quad e \equiv$ gauge coupling

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5D charged scalar $\nabla_M H = \left(\partial_M - iq\mathscr{A}_M \right) H$

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$\Longrightarrow$ 5D WGC: $\left(\frac{eq}{M}\right)^2 \geq \frac{2}{3}$

WGC & scalar fields

⟶ Reduction to 4D on a circle: [LUST, PALTI '17]

$$S_4 = \int d^4x\sqrt{-g}\left\{\frac{R_4}{2} - \frac{1}{2}G_{ij}\partial_\mu z^i\partial^\mu z^j - \frac{1}{4}\mathcal{I}_{\Lambda\Sigma}F^\Lambda\cdot F^\Sigma - \nabla_\mu h\nabla^\mu \bar{h} - m^2\left|h\right|^2\right\}$$

WGC & scalar fields

⟶ Reduction to 4D on a circle: [LUST, PALTI '17]

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$z^i = (\varphi, a)$ moduli fields $\longrightarrow$ $G_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{e^2 r^2} \end{pmatrix}$ $r = e^{-\sqrt{\frac{2}{3}}\varphi}$

WGC & scalar fields

$\longrightarrow$ Reduction to 4D on a circle: [Lust, Palti '17]

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$F^\Lambda_{\mu\nu} = \left(G_{\mu\nu}, F_{\mu\nu}\right)$ gauge fields $\longrightarrow$ $\mathcal{I}_{\Lambda\Sigma} = \frac{r}{e^2}\begin{pmatrix}\frac{e^2}{2}r^2 + a^2 & a \\ a & 1\end{pmatrix}$

WGC & scalar fields

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charged scalar

$\longrightarrow$ field-dependent mass: $m^2 = \frac{M^2}{r}\left(1 + x^2\right)$ $x^2 \equiv \frac{q^2 a^2}{M^2 r^2}$

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⟶ 4D WGC with scalar fields: [Lust, Palti '17]

$$\mathcal{Q}^2 - \frac{1}{2}m^2 - G^{ij}\partial_i m \partial_j m$$

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$$\mathcal{Q}^2 - \frac{1}{2}m^2 - G^{ij}\partial_i m \partial_j m =$$

$$= \frac{M^2}{r\left(1+x^2\right)}\left\{\left[\left(\frac{eq}{M}\right)^2 + 2x^2\right]\left(1+x^2\right) - \frac{1}{2}\left(1+x^2\right)^2 - \frac{1}{6}\left(1+3x^2\right)^2 - \left(\frac{eq}{M}\right)^2 x^2\right\}$$

WGC & scalar fields

$\longrightarrow$ 4D WGC with scalar fields: [Lust, Palti '17]

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the moduli field
dependence factorizes

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5D WGC

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5D WGC

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$\longrightarrow$ the WGC is preserved by dimensional reduction

WGC & scalar fields

$\longrightarrow$ 4D WGC with scalar fields: [Lust, Palti '17]

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the moduli field dependence factorizes

$\longrightarrow$ the WGC is preserved by dimensional reduction

$\longrightarrow$ support for the WGC with scalar fields

The Scalar Weak Gravity Conjecture

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- <u>*Scalar Weak Gravity Conjecture*</u> *(SWGC)*: relation, analogous to the WGC, bounding scalar and gravitational interactions

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  - diagrammatic approach [Gonzalo, Ibanez '19] [Benakli, Branchina, LafforgueMarmet '20]

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$\longrightarrow$ see [Freivogel, Gasenzer, Hebecker, Leonhardt '20] for related discussions (and criticism)

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$$n m^2 + g^{ij}\partial_i m \partial_j m - \frac{1}{2} g^{ij} D_i \partial_j m^2 \leq 0$$

$n \equiv$ # scalars coupling to the SWGC state

The Scalar Weak Gravity Conjecture

- Algebraic identity in $\mathcal{N} = 2$ supergravity in 4D: [PALTI '17] [CERESOLE, D'AURIA, FERRARA '95]

$$K^{i\bar{j}} D_i \bar{D}_{\bar{j}} |Z|^2 = n_V |Z|^2 + K^{i\bar{j}} D_i Z \bar{D}_{\bar{j}} \bar{Z}$$

$Z \equiv$ central charge $K_{i\bar{j}} \equiv$ Kähler metric

$n_V \equiv$ # vector multiplets

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- Generalisation of the identity for $\mathcal{N} \geq 2$: [Dall'Agata, Morittu '20]

$$D_a \bar{D}^a \left(Z_{AB} Z^{AB}\right) = D_a Z_{AB} \bar{D}^a Z^{AB} + n Z_{AB} Z^{AB}$$

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$\longrightarrow$ Constraint on the ADM mass of BPS black holes:

$$D_a \bar{D}^a \left(M^2\right) = 4 D_a M \bar{D}^a M + n M^2$$

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support for the SWGC

The SWGC and dimensional reduction

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$\longrightarrow$ D-dimensional scalar QED:

$$S_D = \int d^D x \sqrt{-G} \left\{ \frac{R_D}{2} - \frac{1}{4e^2} \mathcal{F}_{MN} \mathcal{F}^{MN} - \nabla_M H \nabla^M \bar{H} - M^2 \left| H \right|^2 \right\}$$

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$\Longrightarrow$ *D-dimensional WGC*: $\quad \omega \equiv \dfrac{D-3}{D-2} - \left(\dfrac{eq}{M} \right)^2 \leq 0$

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$\longrightarrow$ # moduli $= \dfrac{N(N+3)}{2} = N + \dfrac{N(N+1)}{2}$ gauge moduli $\mathscr{A}_a \equiv a_a$ metric moduli $g_{ab} \equiv \phi_i$ $\longrightarrow z^I = (\phi_i, a_a)$

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$\longrightarrow$ moduli-dependent mass: $m^2 = \dfrac{M^2}{\Delta^{\frac{1}{d-2}}}\left(1 + x^2\right)$ $\quad x^2 = \left(\dfrac{q}{M}\right)^2 g^{ab} a_a a_b$ $\quad \Delta = \det\left(g_{ab}\right)$

The SWGC & dimensional reduction

- We study the SWGC in the form

$$m^2 + \alpha G^{IJ}\partial_I m \partial_J m - \frac{\beta}{2} G^{IJ} D_I \partial_J m^2 \leq 0$$

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$$m^2 + \alpha G^{IJ} \partial_I m \partial_J m - \frac{\beta}{2} G^{IJ} D_I \partial_J m^2 \leq 0$$

$$\longrightarrow \quad \left[\left(N\beta - \alpha \right) x^2 + N\beta \right] \omega + \left(1 + x^2 \right) P_N(x) \leq 0$$

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$$P_N(x) = 1 + \frac{N}{(d-2)(d+N-2)}\left\{\alpha - \left[d^2 + (N-4)d + 2(3-N)\right]\beta\right\} + \left\{1 + \frac{(d-1)\alpha - \left[(N+2)d - 2(N+1)\right]\beta}{d-2}\right\} x^2$$

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$$\longrightarrow \quad P_N(x) = 0 \quad \forall x, N \quad \Longleftrightarrow \quad \alpha = \frac{2}{N} \ \& \ \beta = \frac{1}{N}$$

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- We study the SWGC in the form

$$m^2 + \alpha G^{IJ}\partial_I m \partial_J m - \frac{\beta}{2} G^{IJ} D_I \partial_J m^2 \leq 0$$

$$\longrightarrow \quad \left[\left(N\beta - \alpha\right) x^2 + N\beta\right] \omega + \left(1 + x^2\right) P_N(x) \leq 0$$

$$\longrightarrow \quad P_N(x) = 0 \quad \forall x, N \quad \Longleftrightarrow \quad \boxed{\alpha = \frac{2}{N} \ \ \& \ \ \beta = \frac{1}{N}}$$

The SWGC & dimensional reduction

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Consistent with the WGC argument of

[Dall'Agata, Morittu '20]

The SWGC & dimensional reduction

$\longrightarrow$ with this choice we find:

$$Nm^2 + 2G^{IJ}\partial_I m \partial_J m - \frac{1}{2}G^{IJ}D_I\partial_J m^2 \leq 0 \qquad \Longleftrightarrow \qquad \left[1 + \left(1 - \frac{2}{N}\right)x^2\right]\omega \leq 0$$

The SWGC & dimensional reduction

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$\Longrightarrow$ $N \geq 2$: *the SWGC is exactly equivalent to the higher-dimensional WGC* $\omega \leq 0$

The SWGC & dimensional reduction

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$N \equiv$ # compact dimensions / gauge moduli

The SWGC & dimensional reduction

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The SWGC & dimensional reduction

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$\Longrightarrow$ $N \geq 2$: *the SWGC is exactly equivalent to the higher-dimensional WGC* $\omega \leq 0$

$N \equiv$ # compact dimensions / gauge moduli $\qquad N \neq$ # scalars coupling to the SWGC state

$\Longrightarrow$ $N = 1$: SWGC $\equiv \left(1 - x^2\right)\omega \leq 0 \longrightarrow$ *not* satisfied $\forall\, x$

The SWGC and gauged supergravity

The SWGC & gauged supergravity

- We want to study the SWGC in the presence of (local) supersymmetry.

$\longrightarrow$ 5D model: $\{E_M{}^A, \Psi^i_M, \mathscr{A}_M\} \quad + \quad \dfrac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times\mathrm{U}(1)} = \{\zeta^A, q^u\}$

The SWGC & gauged supergravity

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pure 5D SUGRA

The SWGC & gauged supergravity

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universal hypermultiplet

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universal hypermultiplet $\qquad$ hyperscalars

The SWGC & gauged supergravity

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$\longrightarrow$ we gauge a U(1) isometry with $\mathscr{A}_M$: [CERESOLE, DALL'AGATA, KALLOSH, VAN PROEYEN '04]

$$k^u = \frac{\sqrt{3}}{4}\begin{pmatrix} 2V\sigma \\ 1+\sigma^2-\left(V+\theta^2+\sigma^2\right)^2 \\ \sigma\theta-\tau\left(1+V+\theta^2+\tau^2\right) \\ \sigma\theta+\theta\left(1+V+\theta^2+\tau^2\right)\end{pmatrix} \quad \longleftrightarrow \quad P = \frac{\sqrt{3}}{4\sqrt{V}}\begin{pmatrix} \left(V-1-\theta^2-\tau^2\right)\theta-\sigma\tau \\ \left(V-1-\theta^2-\tau^2\right)\tau-\sigma\theta \\ -\frac{1}{4\sqrt{V}}\left[(V-1)^2+\sigma^2+\left(\theta^2+\tau^2\right)\left(\theta^2+\tau^2+2-6V\right)\right]\end{pmatrix}$$

Killing vector prepotential

The SWGC & gauged supergravity

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$\longrightarrow$ bosonic lagrangian: [Ceresole, Dall'Agata '00] [Ceresole, Dall'Agata, Kallosh, Van Proeyen '04]

$$S_5 = \int d^5xE\left\{\frac{R_5}{2} - \frac{1}{4}\mathcal{F}_{MN}\mathcal{F}^{MN} + \frac{E^{-1}}{6\sqrt{6}}\varepsilon^{MNRST}\mathcal{A}_M\mathcal{F}_{NR}\mathcal{F}_{ST} - \frac{1}{2}g_{uv}D_Mq^uD^Mq^v - \mathcal{V}(q)\right\}$$

The SWGC & gauged supergravity

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$D_Mq^u = \partial_Mq^u + \mathscr{A}_Mk^u$ $\qquad$ charged hyperscalars

The SWGC & gauged supergravity

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$\mathscr{V}(q)$ scalar potential

The SWGC & gauged supergravity

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$\longrightarrow$ 5D model: $\{E_M{}^A, \Psi^i_M, \mathscr{A}_M\} + \dfrac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times \mathrm{U}(1)} = \{\zeta^A, q^u\}$ $\quad q^u = (V, \sigma, \theta, \tau)$

$\longrightarrow$ bosonic lagrangian: [Ceresole, Dall'Agata '00] [Ceresole, Dall'Agata, Kallosh, Van Proeyen '04]

$$S_5 = \int d^5x E \left\{ \frac{R_5}{2} - \frac{1}{4}\mathscr{F}_{MN}\mathscr{F}^{MN} + \frac{E^{-1}}{6\sqrt{6}}\varepsilon^{MNRST}\mathscr{A}_M\mathscr{F}_{NR}\mathscr{F}_{ST} - \frac{1}{2}g_{uv}D_M q^u D^M q^v - \mathscr{V}(q) \right\}$$

$\mathscr{V}(q)$ scalar potential $\longrightarrow$ minimum at $q_\star = (1,0,0,0)$

The SWGC & gauged supergravity

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$\mathscr{V}(q)$ scalar potential $\longrightarrow$ minimum at $q_\star = (1,0,0,0)$ $\nearrow$ $\mathscr{V}(q_\star) = 0$ $\qquad$ Minkowski

The SWGC & gauged supergravity

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$\searrow$ $k^u(q_\star) = 0$ gauge symmetry preserving

The SWGC & gauged supergravity

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$\mathscr{V}(q)$ scalar potential $\longrightarrow$ minimum at $q_\star = (1,0,0,0)$

- $\mathscr{V}(q_\star) = 0$ — Minkowski
- $k^u(q_\star) = 0$ — gauge symmetry preserving
- $P(q_\star) = 0$ — supersymmetric

The SWGC & gauged supergravity

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$$q^u \;\leftrightarrow\; (z_1, z_2)$$

$$q_\star \;\leftrightarrow\; z_1^\star = z_2^\star = 0$$

The SWGC & gauged supergravity

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$$q^u \leftrightarrow (z_1, z_2)$$
$$q_\star \leftrightarrow z_1^\star = z_2^\star = 0 \qquad \longrightarrow \qquad q = \frac{\sqrt{3}}{2} \quad \& \quad M^2 = \frac{9}{8}$$

The SWGC & gauged supergravity

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$q^u \leftrightarrow (z_1, z_2)$

$q_\star \leftrightarrow z_1^\star = z_2^\star = 0$ $\quad \longrightarrow \quad q = \dfrac{\sqrt{3}}{2}$ & $M^2 = \dfrac{9}{8}$ $\quad \Longrightarrow \quad$ 5D WGC: $\omega = 0$ exactly saturated

The SWGC & gauged supergravity

- Reduction to 4D on the circle: $\left\{ e_\mu{}^a, \psi^i_\mu, A_\mu \right\} \; + \; \left\{ \zeta^A, q^u \right\} \; + \; \left\{ T, B_\mu, \lambda^i \right\}$

The SWGC & gauged supergravity

- Reduction to 4D on the circle: $\left\{ e_\mu{}^a, \psi^i_\mu, A_\mu \right\} + \left\{ \zeta^A, q^u \right\} + \left\{ T, B_\mu, \lambda^i \right\}$

$\mathcal{N} = 2$ SUGRA

The SWGC & gauged supergravity

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hypermultiplet

The SWGC & gauged supergravity

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vector multiplet modulus

The SWGC & gauged supergravity

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$\longrightarrow$ bosonic lagrangian:

$$e^{-1}\mathscr{L}_4 = \frac{R_4}{2} - \frac{1}{4}\mathscr{I}_{\Lambda\Sigma}F^\Lambda F^\Sigma + \frac{1}{4}\mathscr{R}_{\Lambda\Sigma}F^\Lambda \tilde{F}^\Sigma - \frac{3}{\left(T+\bar{T}\right)^3}\partial_\mu T\partial^\mu \bar{T} - K_{i\bar{j}}D_\mu z^i D^\mu \bar{z}^j - \mathscr{V}_{4D}$$

The SWGC & gauged supergravity

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$$\mathscr{I}_{\Lambda\Sigma} = \phi\begin{pmatrix} \frac{e^2}{2}\phi^2 + a^2 & a \\ a & 1 \end{pmatrix}$$

The SWGC & gauged supergravity

- Reduction to 4D on the circle: $\left\{e_\mu{}^a, \psi^i_\mu, A_\mu\right\} + \left\{\zeta^A, q^u\right\} + \left\{T, B_\mu, \lambda^i\right\}$ $\qquad T = \phi + i\sqrt{\frac{2}{3}}a$

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$$D_\mu z^i = \partial_\mu z^i - iqA_\mu z^i \qquad q = \frac{\sqrt{3}}{2}$$

The SWGC & gauged supergravity

- Reduction to 4D on the circle: $\left\{ e_\mu{}^a, \psi_\mu^i, A_\mu \right\} + \left\{ \zeta^A, q^u \right\} + \left\{ T, B_\mu, \lambda^i \right\}$ $\quad T = \phi + i\sqrt{\frac{2}{3}}a$

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$\mathscr{V}_{4D} \longrightarrow$ $z_1^\star = z_2^\star = 0$ minimum

The SWGC & gauged supergravity

- Reduction to 4D on the circle: $\left\{e_\mu{}^a, \psi^i_\mu, A_\mu\right\} \;+\; \left\{\zeta^A, q^u\right\} \;+\; \left\{T, B_\mu, \lambda^i\right\}$ $\qquad T = \phi + i\sqrt{\frac{2}{3}}a$

$\longrightarrow$ bosonic lagrangian:

$$e^{-1}\mathcal{L}_4 = \frac{R_4}{2} - \frac{1}{4}\mathcal{I}_{\Lambda\Sigma}F^\Lambda F^\Sigma + \frac{1}{4}\mathcal{R}_{\Lambda\Sigma}F^\Lambda \tilde{F}^\Sigma - \frac{3}{\left(T+\bar{T}\right)^3}\partial_\mu T\partial^\mu \bar{T} - K_{i\bar{j}}D_\mu z^i D^\mu \bar{z}^{\bar{j}} - \mathcal{V}_{4D}$$

$\mathcal{V}_{4D} \longrightarrow$ $z_1^\star = z_2^\star = 0$ minimum $\longrightarrow$ masses: $m^2 = \frac{M^2}{\phi}\left(1+x^2\right)\mathbb{1}_2$ with $x^2 = \frac{2}{3}\frac{a^2}{\phi^2}$

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathscr{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m$$

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathcal{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2 K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m$$

only the modulus defines a long-range interaction

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathcal{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m = 0$$

exactly satisfied $\longleftrightarrow$ WGC preserved by dimensional reduction

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathcal{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m = 0$$

exactly satisfied $\longleftrightarrow$ WGC preserved by dimensional reduction

- SWGC for complex scalars:

$$m^2 + 4K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m - K^{T\bar{T}} \partial_T \partial_{\bar{T}} m^2$$

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathcal{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m = 0$$

exactly satisfied $\longleftrightarrow$ WGC preserved by dimensional reduction

- SWGC for complex scalars:

$$n m^2 + 4K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m - K^{T\bar{T}} \partial_T \partial_{\bar{T}} m^2$$

$n = 1$

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathcal{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m = 0$$

exactly satisfied $\longleftrightarrow$ WGC preserved by dimensional reduction

- SWGC for complex scalars:

$$\boxed{m^2 + 4K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m - K^{T\bar{T}} \partial_T \partial_{\bar{T}} m^2 = 0}$$

SWGC exactly satisfied

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathcal{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m = 0$$

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- SWGC for complex scalars:

$$\boxed{m^2 + 4K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m - K^{T\bar{T}} \partial_T \partial_{\bar{T}} m^2 = 0}$$

SWGC exactly satisfied

$\longrightarrow$ agreement with [Dall'Agata, Morittu '20] $nm^2 + 4\,(Dm)^2 - D^2 m^2 \leq 0$

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathcal{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m = 0$$

exactly satisfied $\longleftrightarrow$ WGC preserved by dimensional reduction

- SWGC for complex scalars:

$$\boxed{m^2 + 4K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m - K^{T\bar{T}} \partial_T \partial_{\bar{T}} m^2 = 0}$$

SWGC exactly satisfied

$\longrightarrow$ agreement with [DALL'AGATA, MORITTU '20] $nm^2 + 4\,(Dm)^2 - D^2 m^2 \leq 0$

the SWGC identity uses physical inputs

The SWGC & gauged supergravity

- 4D WGC with scalar fields:

$$Q_\Lambda \mathcal{I}^{\Lambda\Sigma} Q_\Sigma - \frac{1}{2} m^2 - 2K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m = 0$$

exactly satisfied $\longleftrightarrow$ WGC preserved by dimensional reduction

- SWGC for complex scalars:

$$\boxed{m^2 + 4K^{T\bar{T}} \partial_T m \partial_{\bar{T}} m - K^{T\bar{T}} \partial_T \partial_{\bar{T}} m^2 = 0}$$

SWGC exactly satisfied

$\longrightarrow$ consistent with the previous formula $\left[1 + \left(1 - \frac{2}{N}\right) x^2\right] \omega \leq 0$ for $\omega = 0$

The SWGC & gauged supergravity

- Scalar potential of 4D gauged SUGRA:

$$\mathscr{V} = \bar{L}^{\Lambda} L^{\Sigma} \left(g_{uv} k^u_{\Lambda} k^v_{\Sigma} + \mathscr{K}_{i\bar{j}} k^i_{\Lambda} k^{\bar{j}}_{\Sigma} \right) + \left(U^{\Lambda\Sigma} - 3\bar{L}^{\Lambda} L^{\Sigma} \right) \mathscr{P}^r_{\Lambda} \mathscr{P}^r_{\Sigma}$$

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hypermultiplets sector

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vector multiplets sector

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prepotentials

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$$D_u \mathscr{P}^r_\Lambda = R^r_{uv} k^v_\Lambda \qquad R^r_{uv} \equiv \mathrm{SU(2)}\ \text{curvatures}$$

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Special geometry functions

The SWGC & gauged supergravity

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Special geometry functions

$$\nabla_i L^\Lambda \equiv \left(\partial_i + \frac{1}{2}\mathscr{K}_i \right) L^\Lambda \equiv f_i^\Lambda, \qquad \nabla_{\bar{i}} \bar{L}^\Lambda \equiv \left(\partial_{\bar{i}} + \frac{1}{2}\mathscr{K}_{\bar{i}} \right) \bar{L}^\Lambda \equiv f_{\bar{i}}^\Lambda,$$

$$\nabla_{\bar{i}} L^\Lambda \equiv \left(\partial_{\bar{i}} - \frac{1}{2}\mathscr{K}_{\bar{i}} \right) L^\Lambda = 0 \quad \longleftrightarrow \quad \partial_{\bar{i}} L^\Lambda = \frac{1}{2}\mathscr{K}_{\bar{i}} L^\Lambda$$

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$$U^{\Lambda\Sigma} \equiv \mathscr{K}^{i\bar{j}} f_i^\Lambda f_{\bar{j}}^\Sigma, \qquad \nabla_{\bar{j}} f_i^\Lambda = \mathscr{K}_{i\bar{j}} L^\Lambda .$$

The SWGC & gauged supergravity

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- We assume the existence of a vacuum $q^{\star}$ such that

$$\mathscr{P}_{\Lambda}^{r}(q^{\star}) = 0 \qquad \text{supersymmetric}$$

The SWGC & gauged supergravity

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$$\mathscr{P}^r_{\Lambda}(q^{\star}) = 0 \qquad \& \qquad k^u_{\Lambda}(q^{\star}) = 0$$

gauge-symmetry preserving

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$\longrightarrow$ hyperscalars mass matrix:

$$\left(m^2\right)^u{}_v = g^{uw} D_w D_v \mathscr{V}(q^{\star}) = \bar{L}^{\Lambda} L^{\Sigma} \left(\mathscr{H}_{\Lambda\Sigma}\right)^u{}_v$$

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$$\left(\mathscr{H}_{\Lambda\Sigma}\right)^u{}_v = g^{uw} g_{tz} \left(D_w k^t_{\Lambda} D_v k^z_{\Sigma} + D_v k^t_{\Lambda} D_w k^z_{\Sigma} \right)$$ hypermultiplets sector

The SWGC & gauged supergravity

- Derivatives with respect to the vector multiplets scalars:

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  - 3-point coupling $4\mathscr{K}^{i\bar{j}}\partial_i m \partial_{\bar{j}} m = \left(m^2\right)^{-1} U^{\Sigma\Lambda}\bar{L}^{\Gamma}L^{\Pi}\mathscr{H}_{\Gamma\Sigma}\mathscr{H}_{\Lambda\Pi}$

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analogy with [Dall'Agata, Morittu '20] but on physical vacua

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$\longrightarrow$ connection/extension to non-susy case by looking at non-susy vacua.

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- supersymmetric case $\nearrow$ exactly satisfied on physical vacua; $\searrow$ connection with structural identities

Summary & Conclusions

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    - more precise formulation of the identity on the scalar potential of gauged supergravity

Thank you!

The (S)WGC & Scherk-Schwarz compactification

- *Scherk—Schwarz* compactification:

$$\begin{pmatrix} \hat{z}_1 \\ \hat{z}_2 \end{pmatrix} = \begin{pmatrix} e^{im_{\mathrm{SS}}y} & 0 \\ 0 & e^{im_{\mathrm{SS}}y} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$\longrightarrow$ The scalar potential is modified to

$$\mathscr{V}_{\mathrm{SS}} = \frac{1}{\phi}\mathscr{V} + \frac{|z_1|^2 + |z_2|^2}{\left(1 - |z_1|^2 - |z_2|^2\right)^2}\frac{\left(qa + m_{\mathrm{SS}}\right)^2}{\phi^3}$$

$z_1 = z_2 = 0$ is now a vacuum with completely broken susy

The (S)WGC & Scherk-Schwarz compactification

- Masses of the scalar fields:

$$m^2 = \left[\frac{9}{8\phi} + \frac{1}{\phi^3} \left(\frac{\sqrt{3}a}{2} + m_{\rm ss} \right)^2 \right] \mathbb{1}_2$$

$\longrightarrow$ 4D WGC:

$$\mathcal{Q}^2 - \frac{1}{2}m^2 - 2K^{T\bar{T}}\partial_T m \partial_{\bar{T}} m = -\frac{2m_{\rm ss}\left(\sqrt{3}a + m_{\rm ss}\right)}{\phi^3}\mathbb{1}_2 \overset{?}{\geq} 0$$

the 4D WGC is *not automatically* satisfied anymore by the scalar fields

The (S)WGC & Scherk-Schwarz compactification

$\longrightarrow$ The 4D WGC is actually satisfied by the *hyperini* ζ^A:

- 5D: $q_\zeta = \dfrac{\sqrt{3}}{2}$ & $M_\zeta^2 = \dfrac{9}{8}$ $\implies$ 5D WGC: $\omega = \dfrac{2}{3} - \left(\dfrac{q}{M}\right)^2 = \dfrac{2}{3} = 0$

- 4D: $m_\zeta^2 = \dfrac{M_\zeta^2}{\phi}\left(1 + x^2\right)\mathbb{1}_2$ $\implies$ 4D WGC: $\mathcal{Q}^2 - \dfrac{1}{2}m^2 - 2K^{T\bar{T}}\partial_T m \partial_{\bar{T}} m = 0$

ζ^A unaffected by the SS reduction

exactly as the scalar fields in the KK reduction

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1-loop corrections?