Instituto de Física Teórica
UAM-CSIC

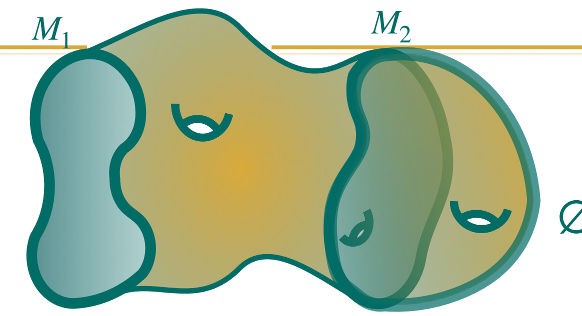
Dynamical Cobordism and Intersecting End-of-the-World Branes

Based on 2312.16286 with R. Angius, A. Uranga

Andriana Makridou

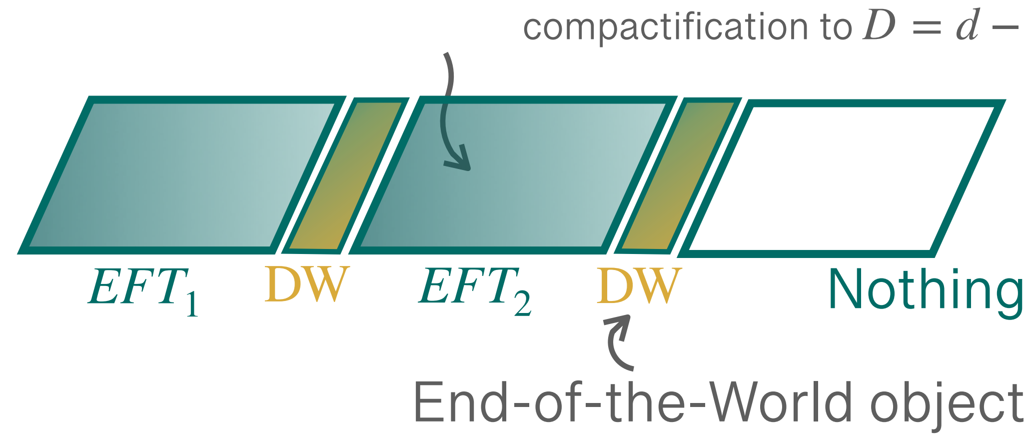
String Phenomenology 2024
Padova, June 25th 2024

Background



Cobordism Conjecture: [McNamara, Vafa '19]

All Cobordism Classes should be trivial: $\Omega_k^{QG} = 0$



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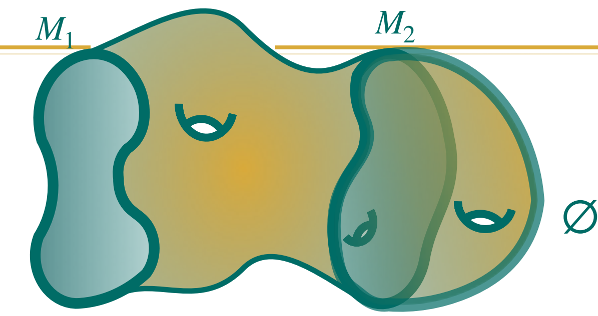

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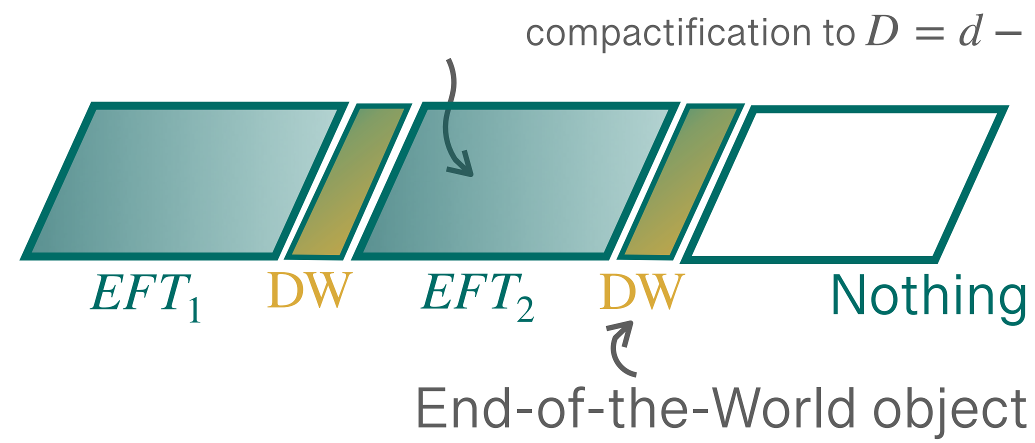

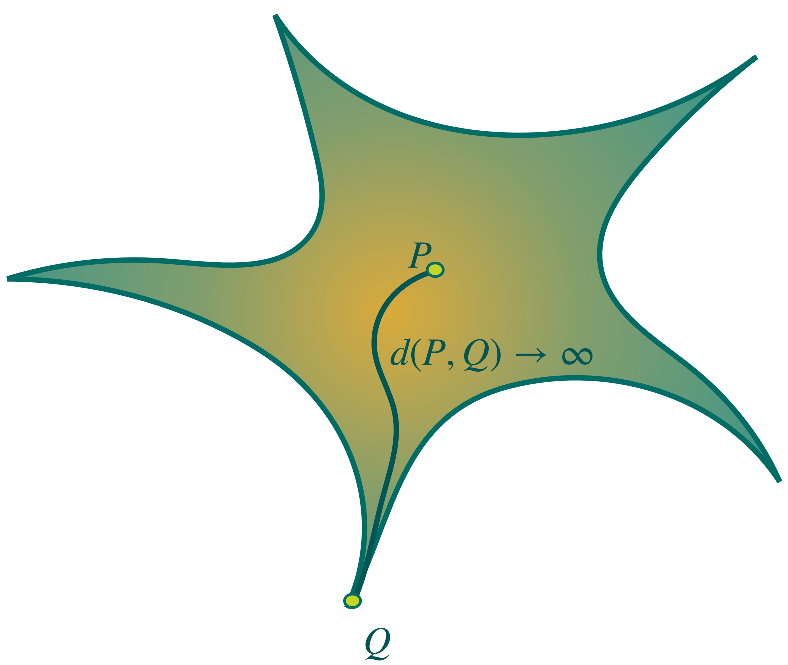

Infinite distance limits

How can we study them when we don't strictly have a moduli space?

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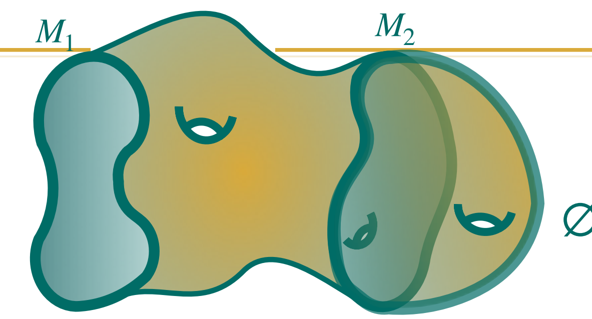

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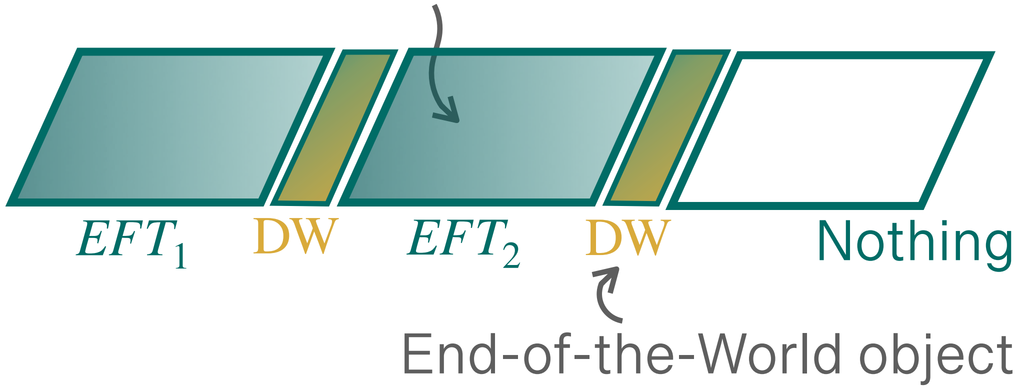
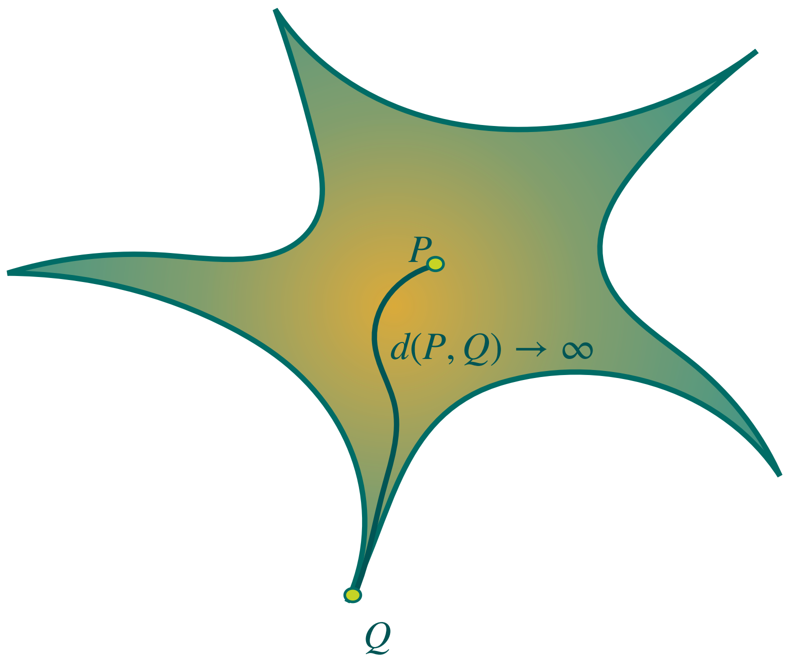
compactification to $D = d - k$





Infinite distance limits

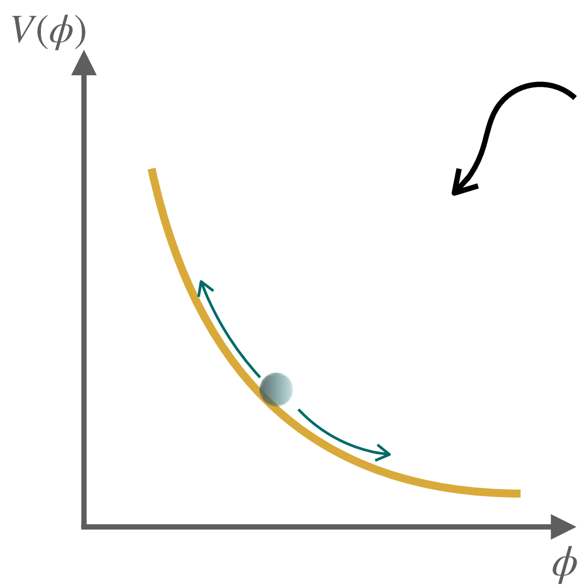
How can we study them when we don't strictly have a moduli space?

Idea: [Buratti, Calderon-Infante, Delgado, Uranga '21]

An infinite field distance limit can be realized as running into a cobordism wall of nothing.

When is this true?

How can this be realized?



Dynamical tadpoles (vs RR tadpoles)

...

How can this be realized?

[Sugimoto '99]
[Antoniadis, Dudas, Sagnotti '99]
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[Mininno, Uranga '20]
[Basile, Raucci, Thomée '22]
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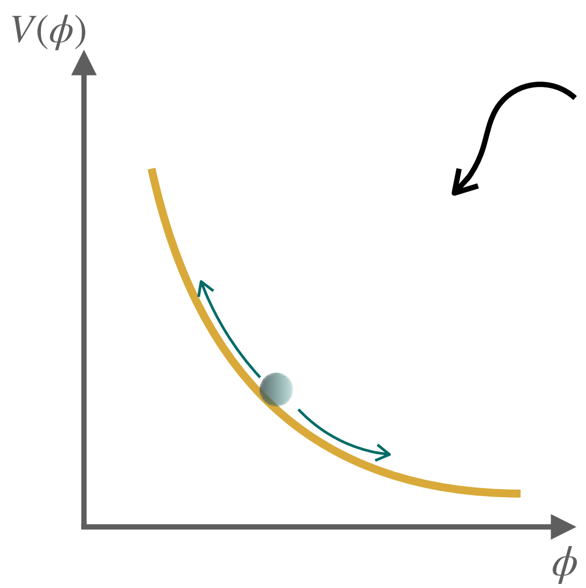

Dynamical tadpoles (vs RR tadpoles)

Spacetime-dependent solutions (instead of maximally-symmetric vacuum)

extending over **finite spacetime distance** Δ

featuring a Ricci curvature singularity at **infinite field distance** $D \to \infty$

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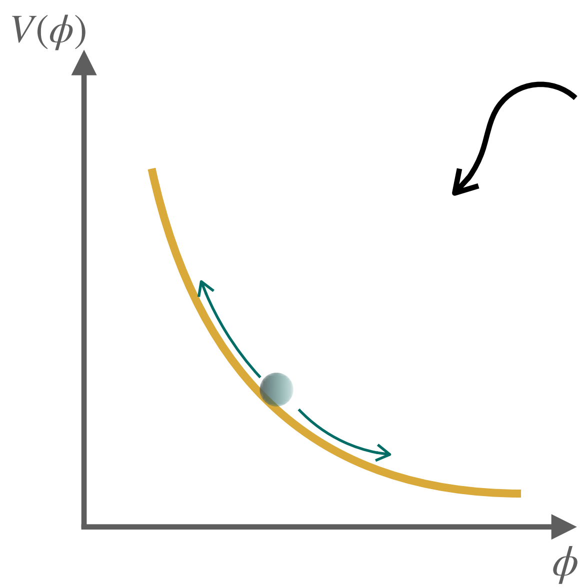

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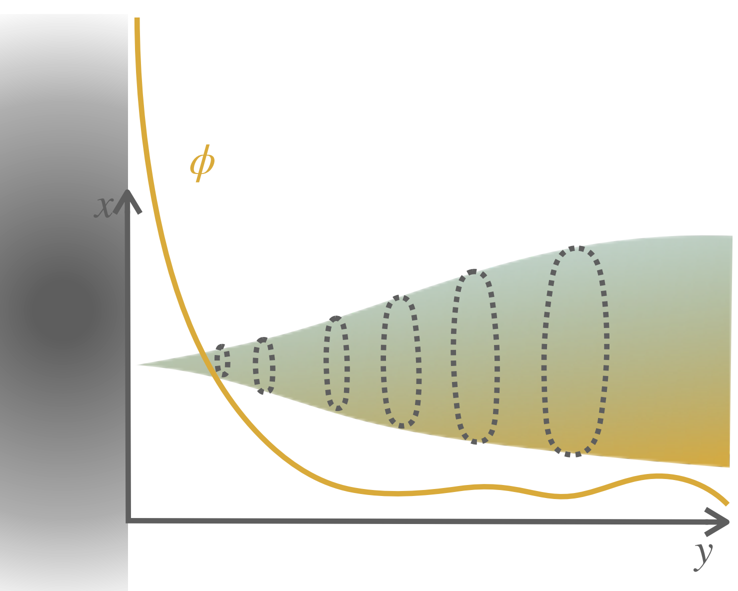

Interpretation: [Buratti, Delgado, Uranga '21]

Spacetime is cut-off
due to running onto a cobordism defect of the initial theory

Dynamical Cobordism

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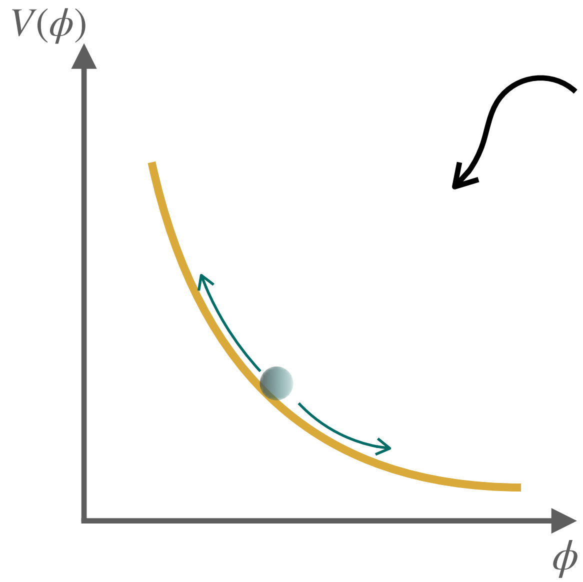

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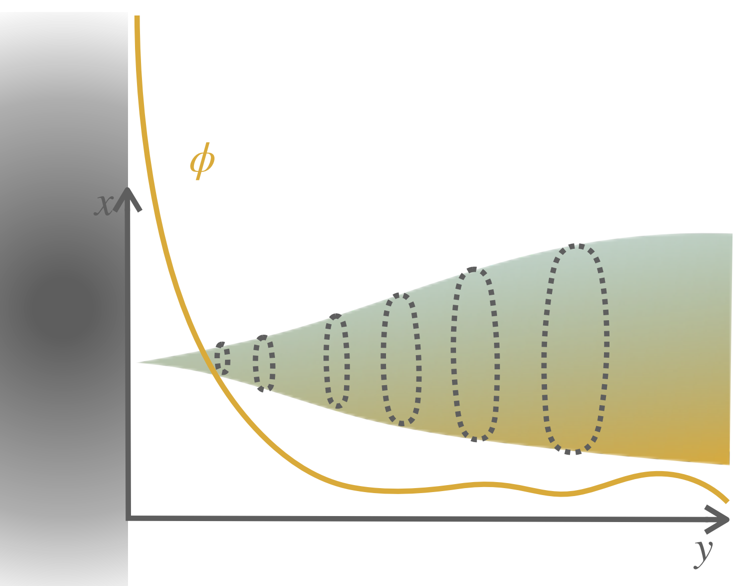

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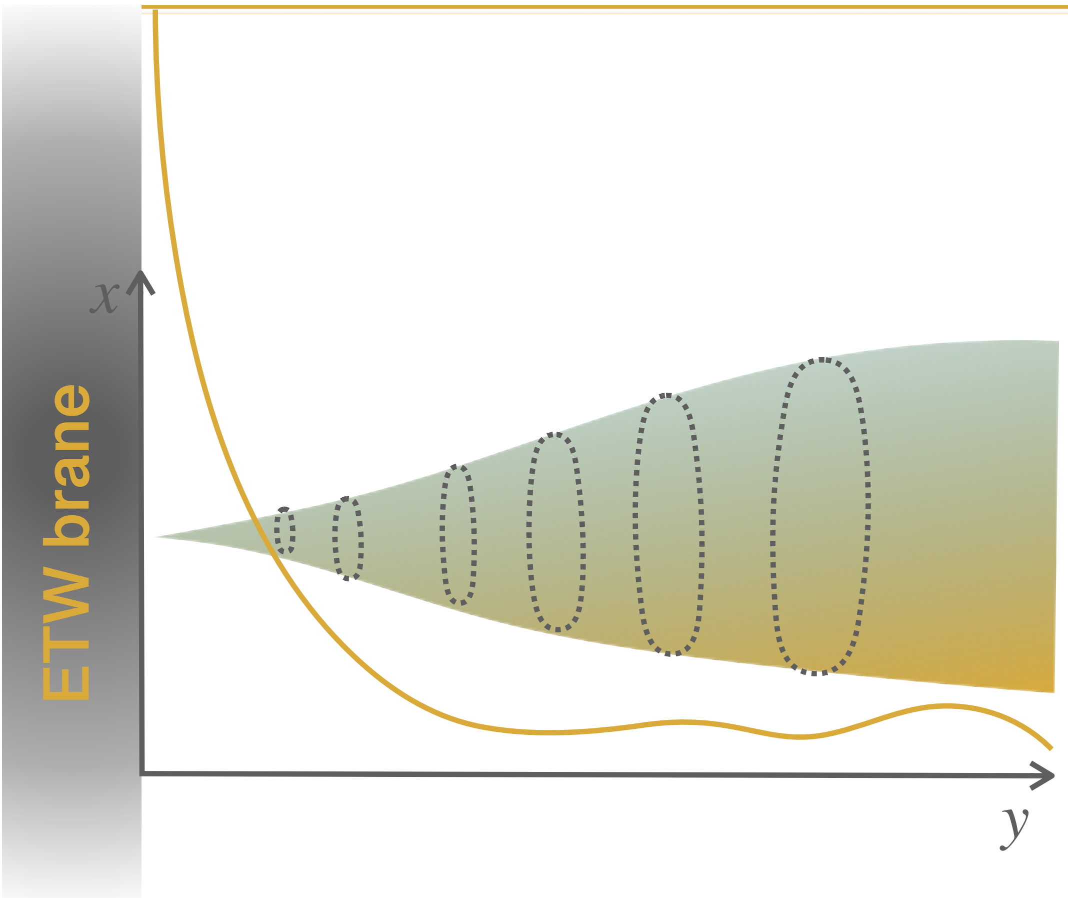
Dynamical Cobordism

Dynamical cobordisms obey the scaling relations $\Delta \sim e^{-\frac{1}{2}\delta D}, \quad |\mathscr{R}| \sim e^{\delta D}, \qquad \delta > 0.$

[Buratti, Calderon-Infante, Delgado, Uranga '21]

Universal description

[Angius, Calderon-Infante, Delgado, Huertas, Uranga '21]



critical exponent $\delta \leftrightarrow$ universal local description

$$S = \int d^d x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right)$$

$$ds_d^2 = e^{-2\sigma(y)} ds_{d-1}^2 + dy^2$$

Here ds_{d-1}^2 flat, see talk by Jesús for AdS case!

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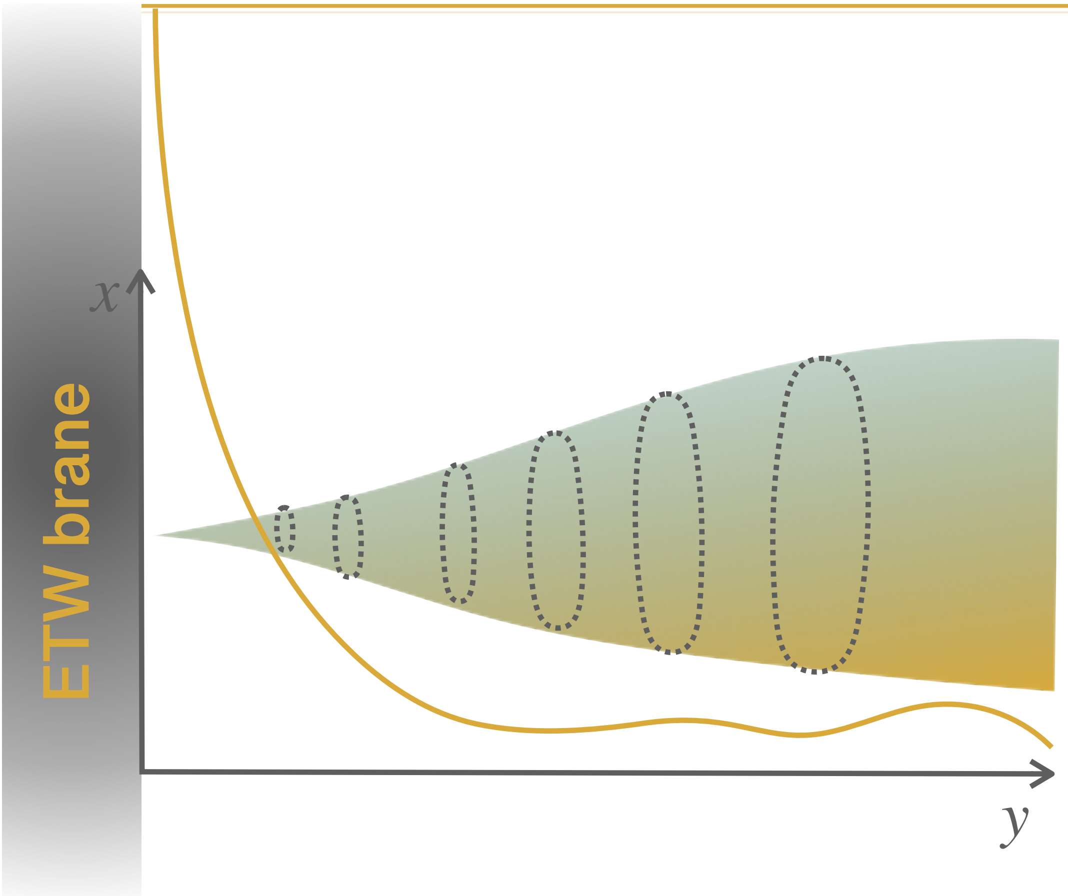

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Local description near ETW brane: $\phi(y) \sim -\frac{2}{\delta} \log y, \quad \sigma(y) \sim -\frac{4}{(d-2)\delta^2} \log y + \frac{1}{2} \log c$

Leading behaviour of potential: $V(\phi) \sim -ace^{\delta\phi}, \quad \delta = 2\sqrt{\frac{d-1}{d-2}(1-a)}$

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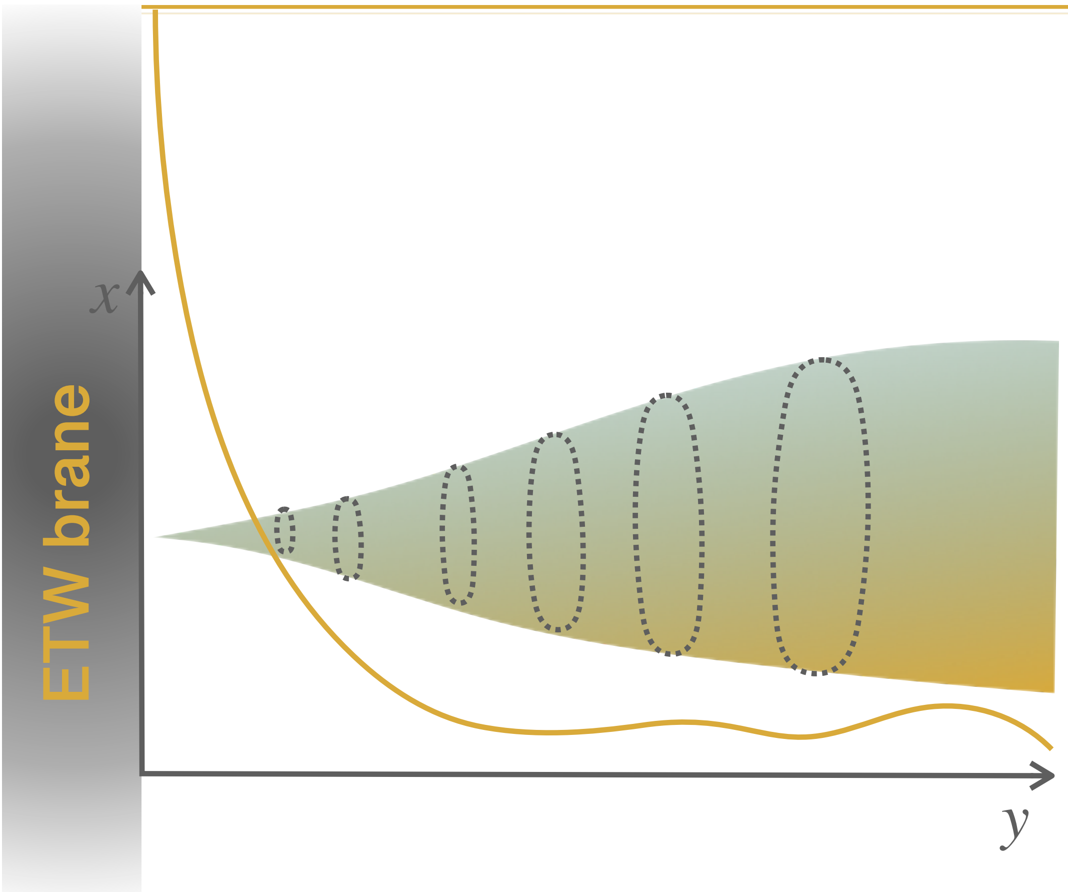

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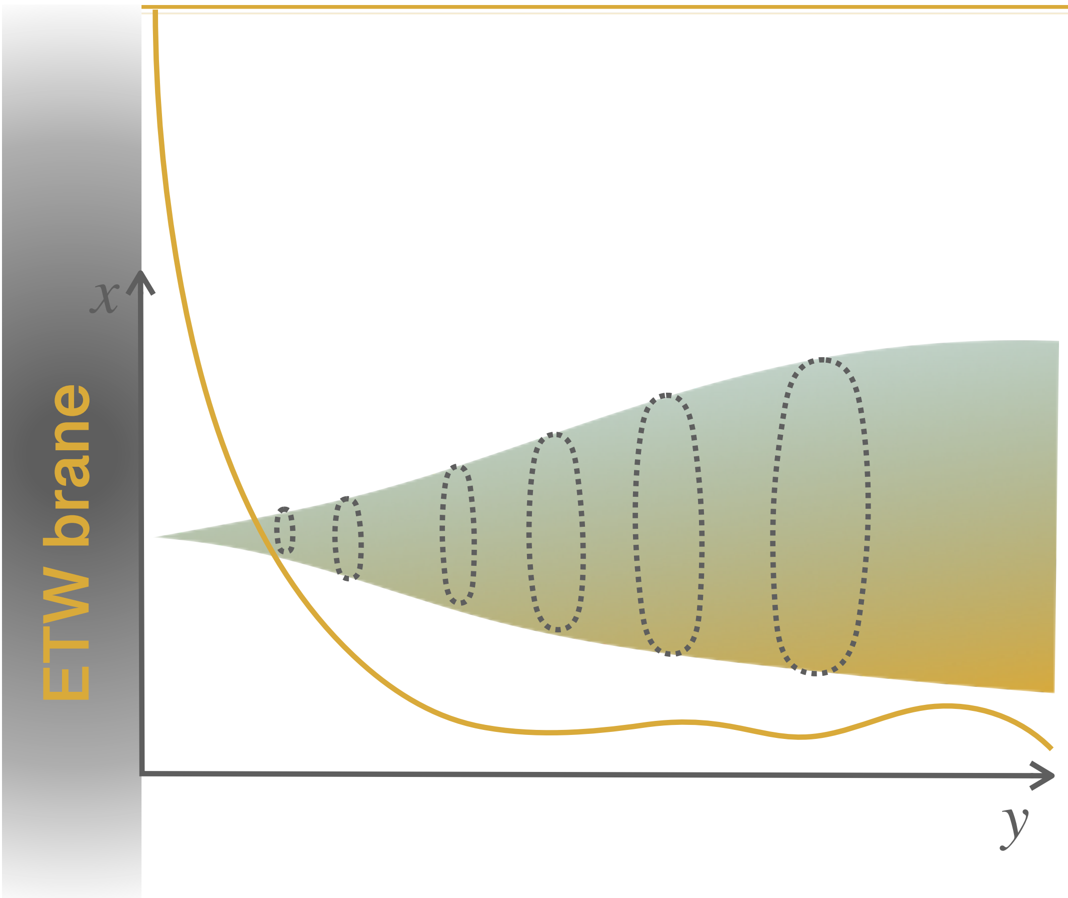
Many examples fall within this universal description [Angius, Calderon-Infante, Delgado, Huertas, Uranga '21]

[Witten '82]	Bubble of Nothing	$\delta = \sqrt{2/7}, \quad d = 4$	$a = 0$
	D2-brane	$\delta = \sqrt{6}, \quad d = 4$	$a = 20/21$
[Sugimoto '99]	USp(32) string	$\delta = \sqrt{6}, \quad d = 10$	$a = 0$

Why do we care?

[Angius, Calderon-Infante, Delgado, Huertas, Uranga '21]



Dynamical Cobordism solutions are quite constrained

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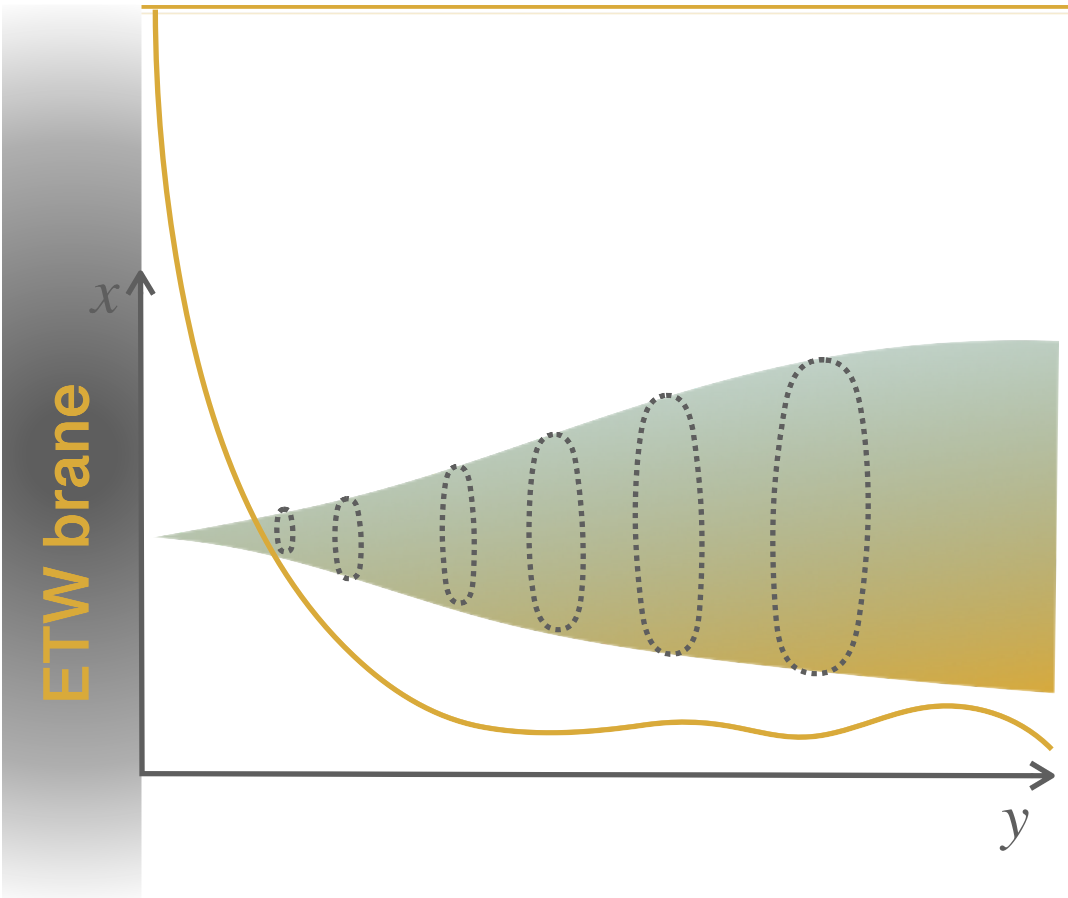

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Can Dynamical Cobordism be used as guiding principle for new solutions? Yes!

[e.g. Blumenhagen, Cribiori,Kneissl, AM '22, Blumenhagen, Kneissl, Wang '23]

see [Blumenhagen, Kneissl, Wang '23] for such a discussion

Parallels with (Sharpened) Distance Conjecture: $\lambda \geq \frac{1}{\sqrt{d-2}}$. Anything special at δ_{crit}?

[Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]

Can this be viewed as a criterion for "good" singularities? See e.g. [Gubser '00]

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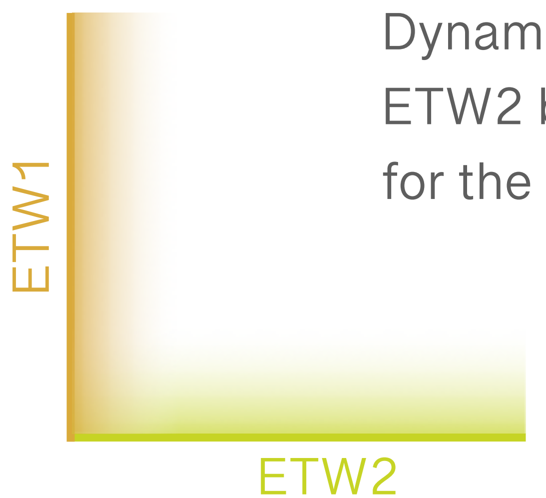

Dynamical cobordism with
ETW2 being the boundary
for the bulk theory ending on ETW1

Bulk theory ends on boundary which
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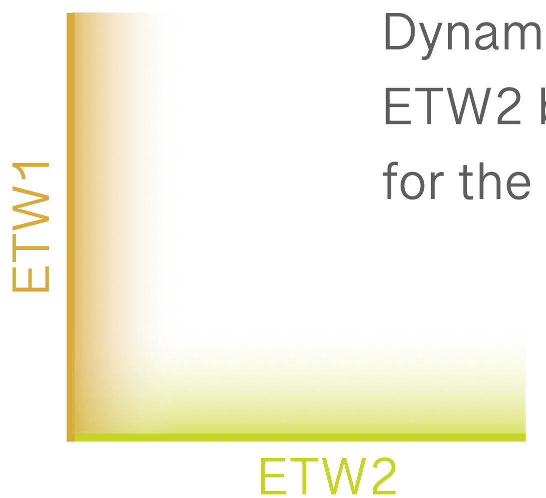

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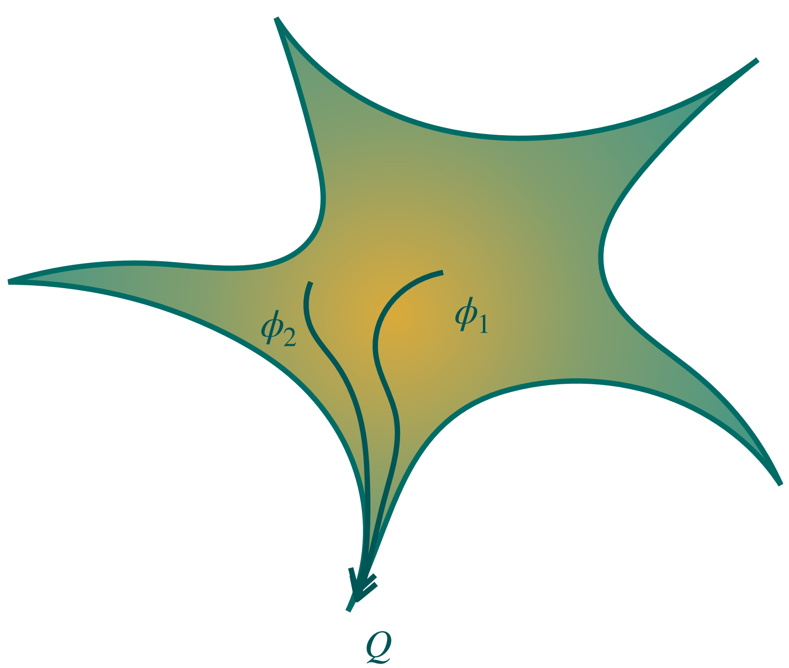
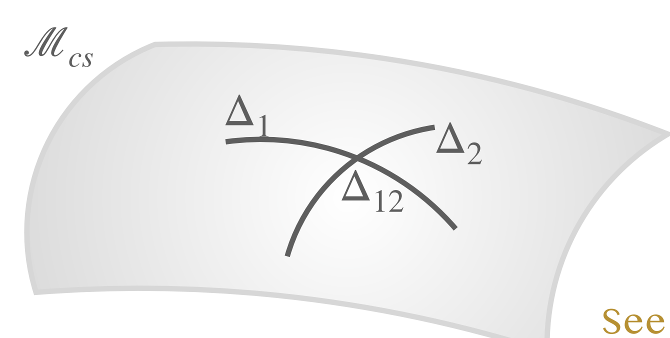


Exploration of network of infinite distance limits [Grimm, Li, Palti '18]

Intersecting divisors in Calabi Yau moduli space [Angius '24]



See Roberta's talk on Thursday

Possible cosmological applications? Collisions of cosmological bubbles?

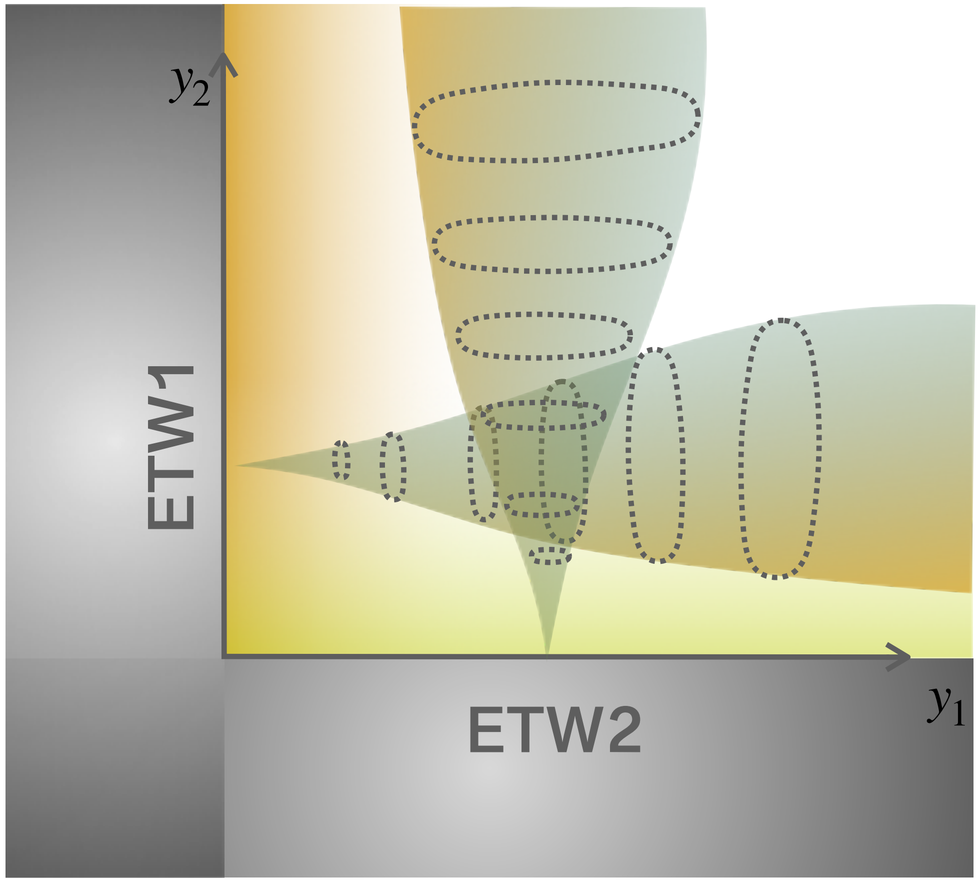
See talks by Jakob Moritz, Björn Friedrich

Codimension-2 case: Universal Description

$$S = \int d^{n+2}x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2 - \frac{\alpha}{2}\partial_\mu\phi_1\partial^\mu\phi_2 - V(\phi_1,\phi_2)\right)$$

$$ds^2_{n+2} = e^{2A(y_1,y_2)}ds^2_n + e^{2B(y_1,y_2)}dy_1^2 + e^{2C(y_1,y_2)}dy_2^2$$

$$\phi_1 = \phi_1(y_1),\quad \phi_2 = \phi_2(y_2)$$



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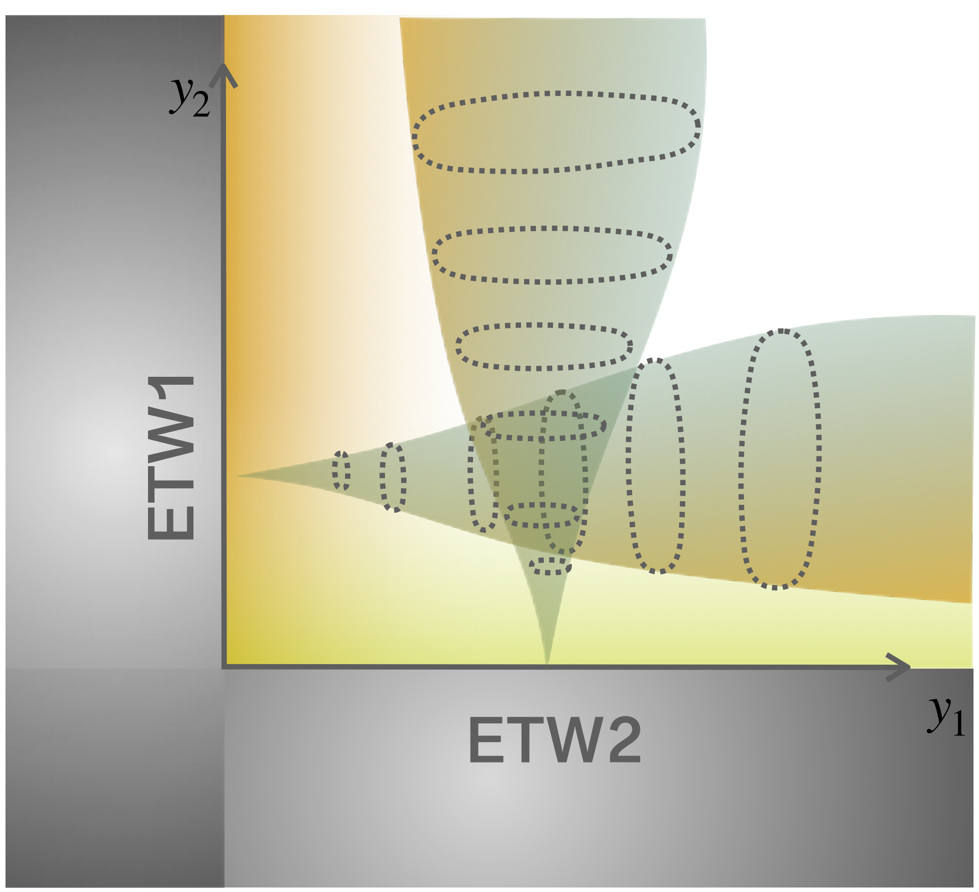

We want: ETW-1 solutions for constant y_i

Both scalars exploding around the origin

$$A(y_1,y_2) = -\sigma_1(y_1) - \sigma_2(y_2)$$

$$B(y_1,y_2) = -\sigma_2(y_2)$$

$$C(y_1,y_2) = -\sigma_1(y_1)$$

Intersecting solution conformally flat!

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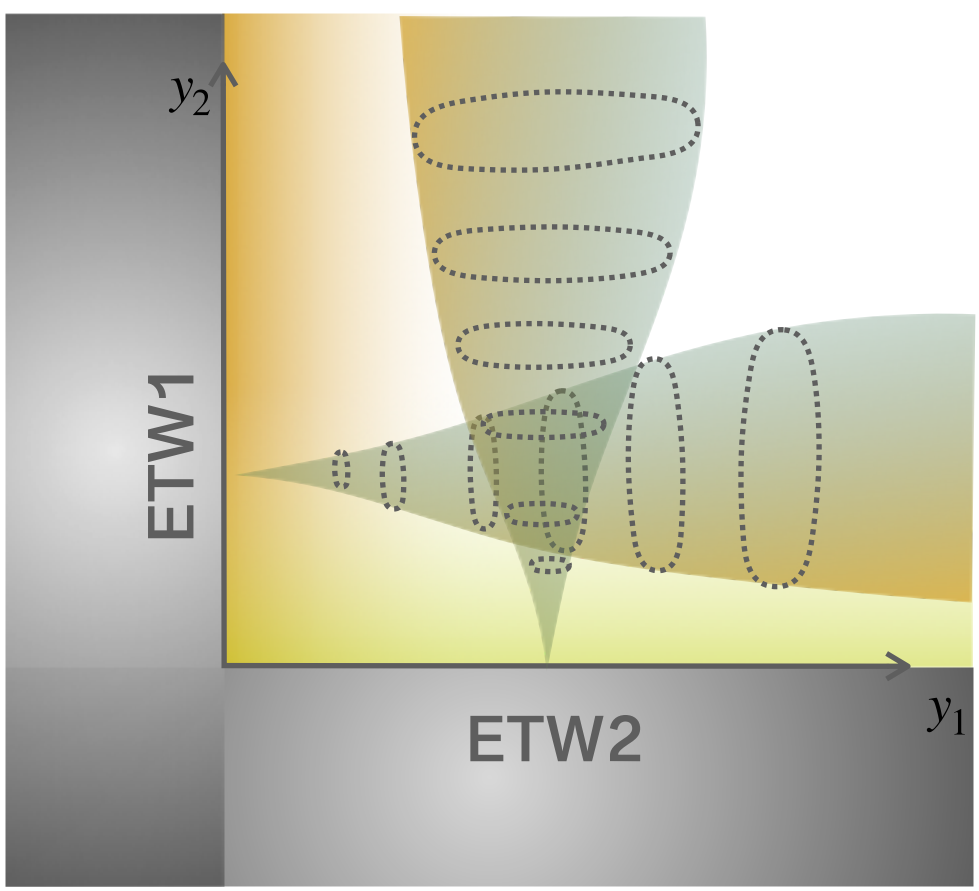


Local description ansatz:

$$\sigma_1(y_1) = -\frac{4}{n\delta_1^2}\log y_1 \qquad \sigma_2(y_2) = -\frac{4}{n\delta_2^2}\log y_2$$

$$\phi_1(y_1) = -\frac{2}{\delta_1}\log y_1 \qquad \phi_2(y_2) = -\frac{2}{\delta_2}\log y_2$$

$$V = -c_1 v_1 y_1^{-2} y_2^{-2a_2} - c_2 v_2 y_1^{-2a_1} y_2^{-2} \equiv V_1 + V_2$$

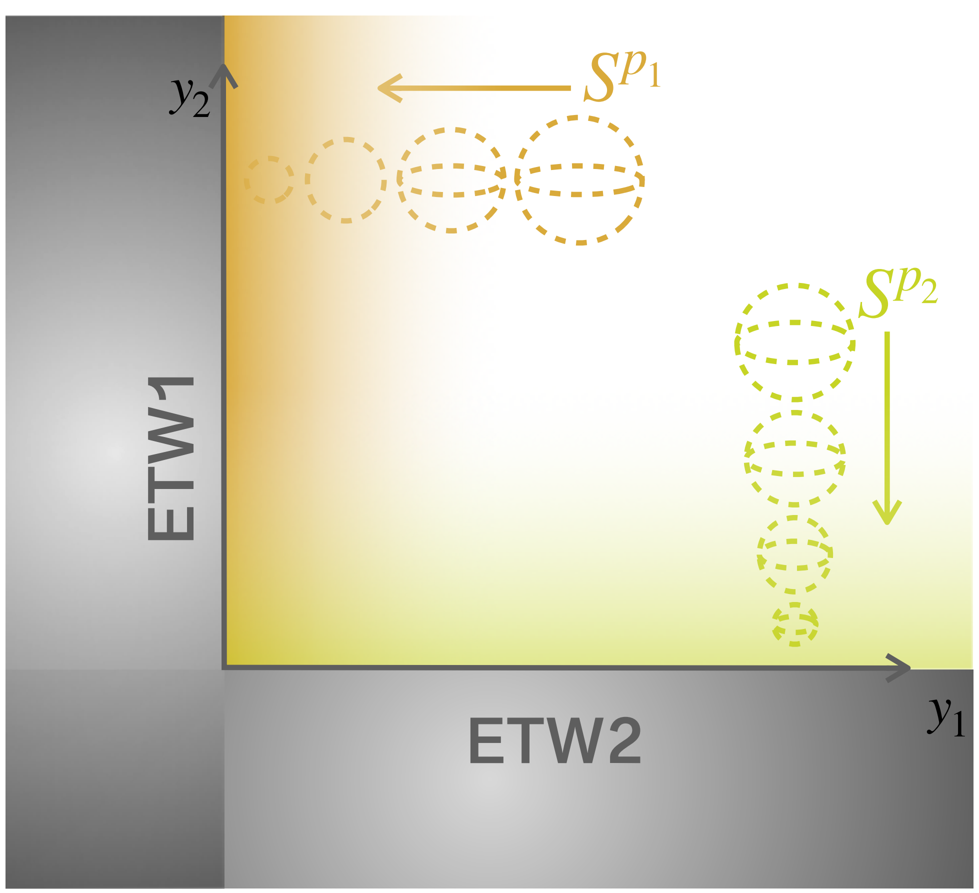
$$= -c_1 v_1 e^{\delta_1\phi_1}e^{a_2\delta_2\phi_2} + c_2 v_2 e^{a_1\delta_1\phi_1}e^{\delta_2\phi_2}$$

Example: $S^{p_1} \times S^{p_2}$ compactifications

Setup: Einstein gravity for $(n + p_1 + p_2 + 2)$-dimensional space, reduced over $S^{p_1} \times S^{p_2}$

$$S_{n+2} = \frac{1}{2}\int d^{n+2}x\sqrt{-g_{n+2}}\Big(R_{n+2} - |\partial\rho_1|^2 - |\partial\rho_2|^2 - \frac{2}{n+1}\partial_\mu\rho_1\partial^\mu\rho_2$$
$$+\frac{p_1(p_1-1)}{2}(\frac{n}{n+p_1})^2 e^{(\alpha_1+\beta_1)\rho_1+\alpha_2\rho_2} + \frac{p_2(p_2-1)}{2}(\frac{n}{n+p_2})^2 e^{(\alpha_2+\beta_2)\rho_2+\alpha_1\rho_1}\Big)$$

ETW-2 solution: $ds^2_{n+2} = y_1^{\frac{2p_1}{n+p_1}} y_2^{\frac{2p_2}{n+p+2}} ds^2_n + y_2^{\frac{2p_2}{n+p_2}} dy_1^2 + y_1^{\frac{2p_1}{p_1+1}} dy_2^2$



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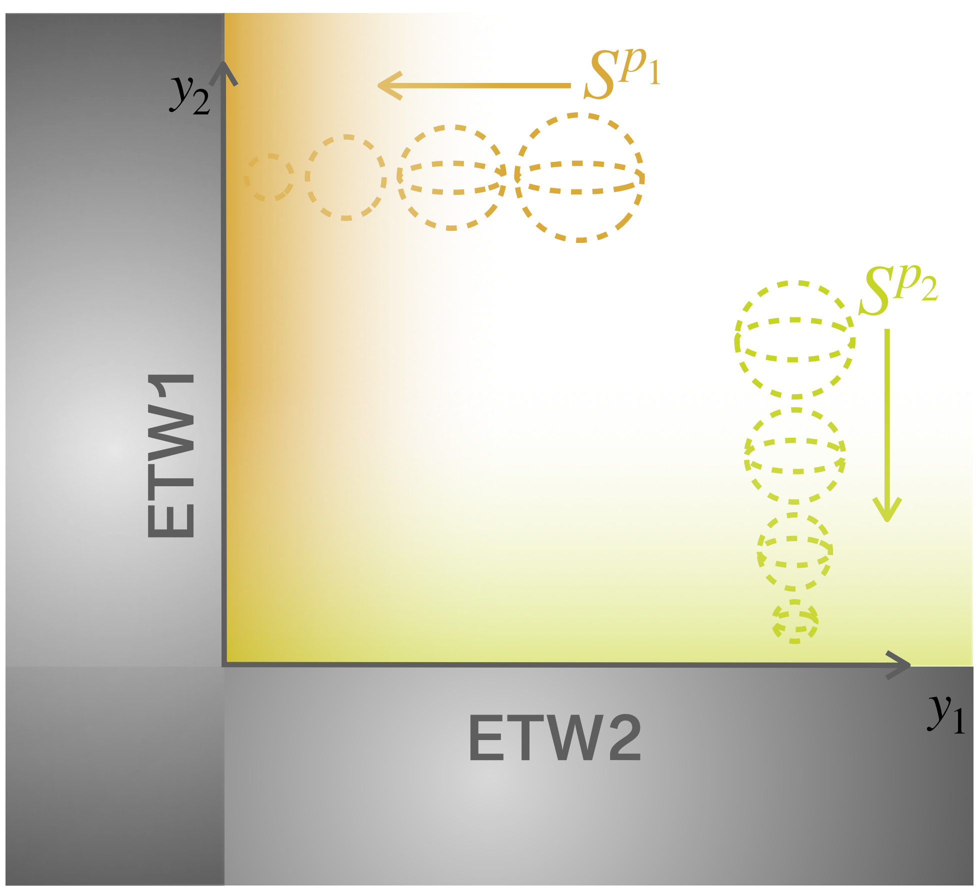

Can identify ρ_i with DC fields: $\phi_i = \rho_i = -\sqrt{\frac{np_i}{n+p_i}}\log y_i$

Read off: $\delta_1 = 2\sqrt{\frac{n+p_1}{np_1}}$ $\delta_2 = 2\sqrt{\frac{n+p_2}{np_2}}$

Codimension-2 case: Scaling relations

Q: Do the intersecting solutions obey the Dynamical Cobordism scaling relations?

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Spacetime distance:

$$\Delta = \int [e^{-2\sigma_1} dy_1^2 + e^{-2\sigma_2} dy_2^2]^{1/2}$$

Field space distance:

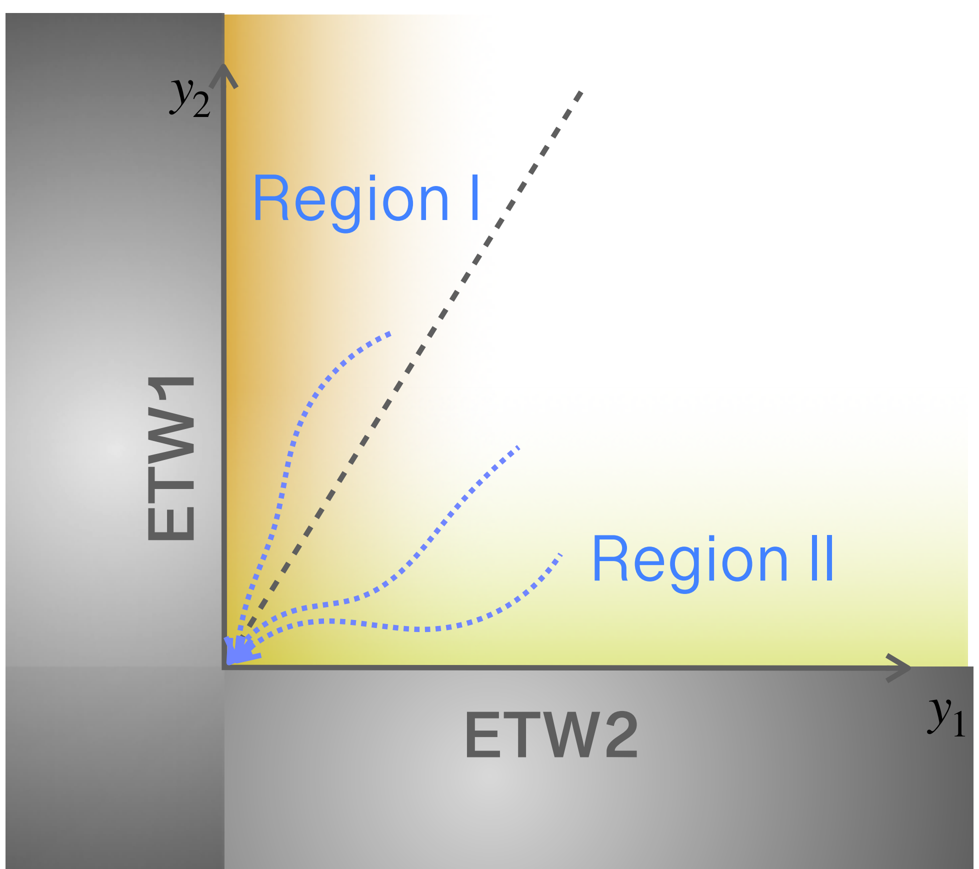
$$\mathscr{D} = \int [d\phi_1^2 + d\phi_2^2 + \alpha d\phi_1 d\phi_2]^{1/2}$$

Pick a path $y_i(t)$ →

$$y_1(t) = t^{\gamma_1}$$
$$y_2(t) = t^{\gamma_2}$$
$$r_1 = \frac{4\gamma_2}{n\delta_2^2} + \gamma_1 - 1$$

$$\Delta = \int [\gamma_1^2 t^{2r_1} + \gamma_2^2 t^{2r_2}]^{1/2} dt$$

$$\mathscr{D} = -2\left(\frac{\gamma_1^2}{\delta_1^2} + \frac{\gamma_2^2}{\delta_2^2} + \frac{\alpha\gamma_1\gamma_2}{\delta_1\delta_2}\right)$$



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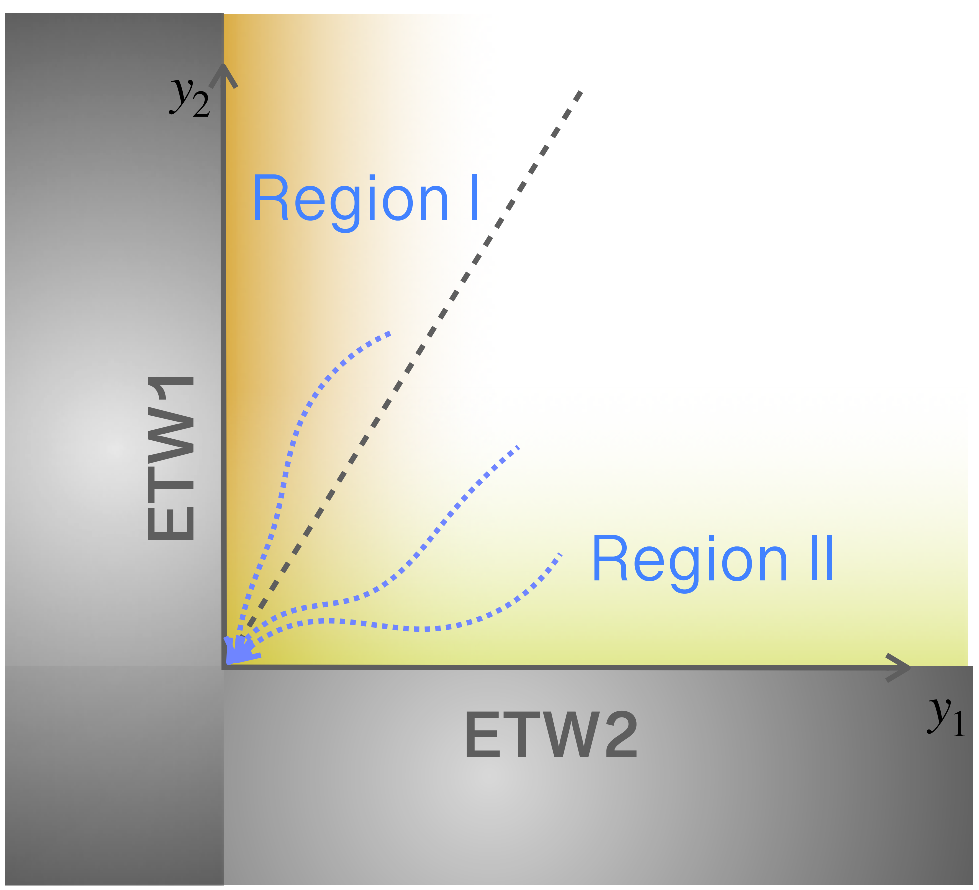
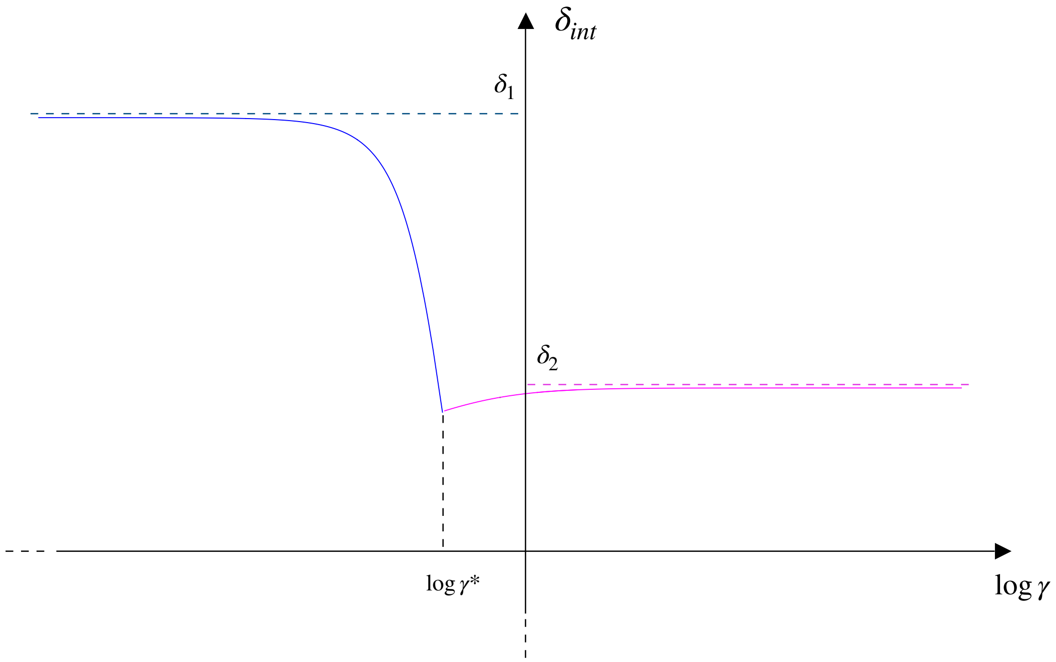


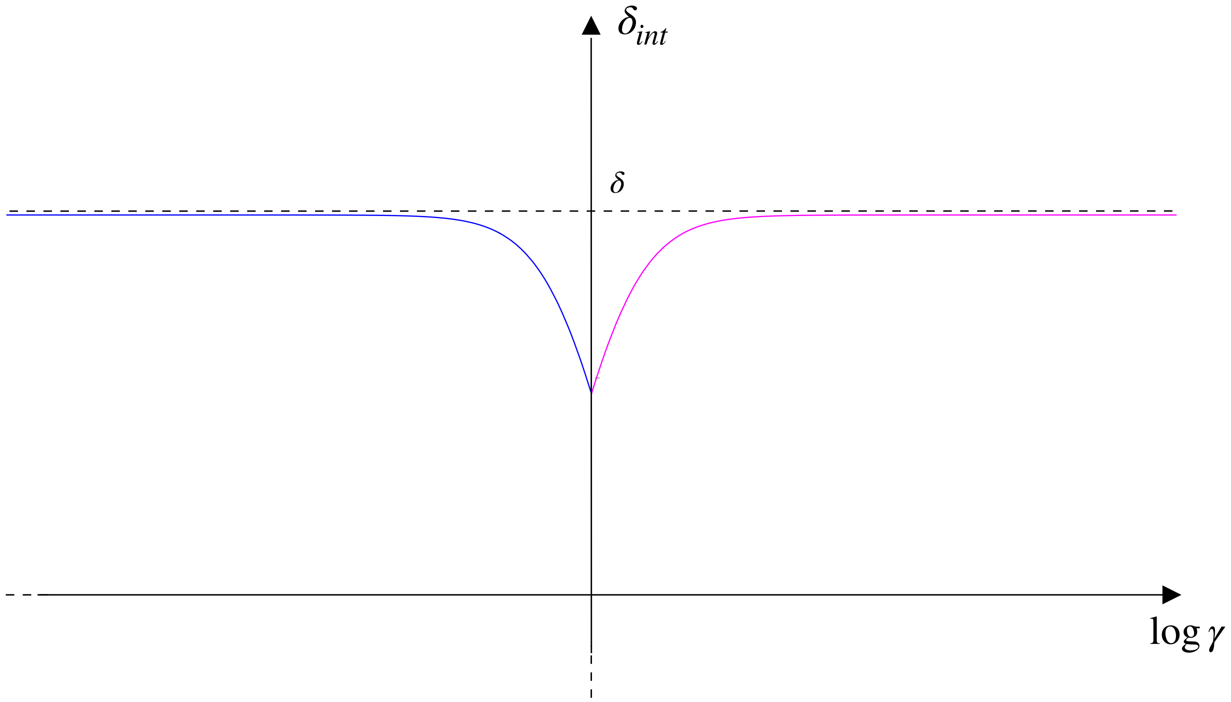
Scaling relations still hold

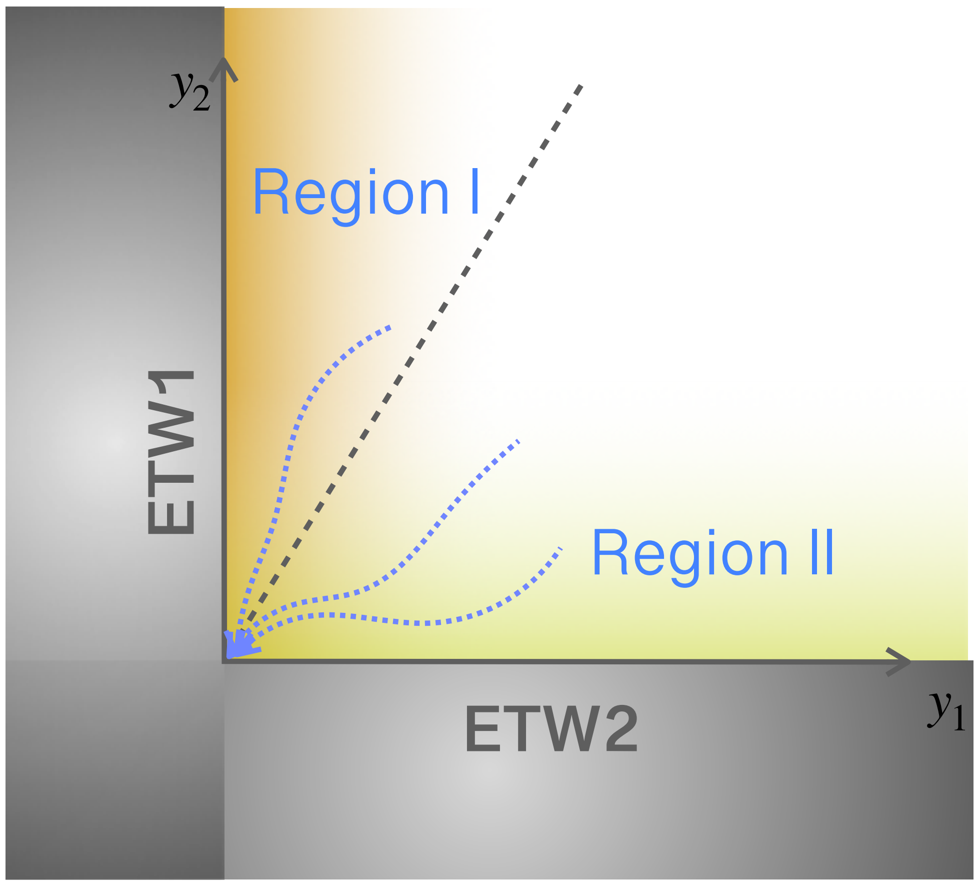
But are the critical exponent is **path-dependent**

$$\delta_{int} = \left(\frac{\gamma_1^2}{\delta_1^2} + \frac{\gamma_2^2}{\delta_2^2} + \frac{\alpha\gamma_1\gamma_2}{\delta_1\delta_2}\right)^{-1/2}(r_i + 1)$$

Codimension-2 case: Scaling relations







Scaling relations still hold

But are the critical exponent is **path-dependent**

$$\delta_{int} = \left(\frac{\gamma_1^2}{\delta_1^2} + \frac{\gamma_2^2}{\delta_2^2} + \frac{\alpha\gamma_1\gamma_2}{\delta_1\delta_2}\right)^{-1/2}(r_i + 1)$$

Is δ_{int} always path-dependent?

Consider the case of a single scalar: $$S = \int d^{n+2}x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right),$$

$$ds^2_{n+2} = e^{2A(y_1,y_2)}ds^2_n + e^{2B(y_1,y_2)}dy_1^2 + e^{2C(y_1,y_2)}dy_2^2$$

with the logarithmic dependence: $$A = a_1 \log y_1 + a_2 \log y_2, \quad B = b_2 \log y_2, \quad C = c_1 \log y_1$$

$$\phi = d_1 \log y_1 + d_2 \log y_2$$

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Eoms satisfied for: $d_1 = d_2 = -\sqrt{n}, a_1 = a_2 = b_2 = c_1 = 1$

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Explanation:

After a change of coordinates the scalar depends non-trivially on a single (new) coordinate

We have a codimension-1 ETW brane - recombined brane instead of intersection!

See [Angius, Delgado, Uranga '22] for an explicit such realization!

Summary and Outlook

Can construct intersecting Dynamical Cobordism solutions

A universal local description exists in terms of the critical exponents δ_1, δ_2

Scaling relations persist, but are now path-dependent

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see [Blumenhagen, Kneissl, Wang '23] for such a discussion

What is the interplay with relevant Swampland Conjectures?

e.g. Convex Hull Conjecture, Sharpened Distance Conjecture

[Calderón-Infante, Uranga, Valenzuela '20] [Etheredge, Heidenreich, Kaya, Kiu, Rudelius '22]

What is the interplay with other ETW/BON approaches?

e.g. Blanco-Pillado, Friedrich, Garcia-Etxebarria, Hebecker, Huertas, Montero, Moritz, Sousa, Sugimoto, Suzuki, Valenzuela, Walcher ….

Does δ have a physical meaning?

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Thank you!

For more Dynamical Cobordism see talks by Jesus, Roberta!

Bonus Slides

Possible generalizations

[Angius '24]
[Angius, AM, Uranga '23]

Up to now: Einstein gravity, 2 DC scalars ϕ_1, ϕ_2 with mixed kinetic term, potential

Solutions: $ds^2_{n+2} = e^{2A(y_1,y_2)} ds^2_n + e^{2B(y_1,y_2)} dy_1^2 + e^{2C(y_1,y_2)} dy_2^2$

$$\phi_1 = \phi_1(y_1), \quad \phi_2 = \phi_2(y_2)$$

$$A(y_1, y_2) = -\sigma_1(y_1) - \sigma_2(y_2), \quad B(y_1, y_2) = -\sigma_2(y_2), \quad C(y_1, y_2) = -\sigma_1(y_1)$$

Generalizations have a certain "cost":

I) $\phi_1 \to \psi_1 = \psi_1(y_1, y_2) = b_{11} \log y_1 + b_{12} \log y_2,$ $\quad \to \alpha = 0$: mixed kinetic term killed

$\phi_2 \to \psi_2 = \psi_2(y_2) = b_2 \log y_2$

but now ψ_1 explodes at both $y_1 = y_2 = 0$

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I) $\phi_1 \to \psi_1 = \psi_1(y_1, y_2) = b_{11} \log y_1 + b_{12} \log y_2,$

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$\to \alpha = 0$: mixed kinetic term killed but now ψ_1 explodes at both $y_1 = y_2 = 0$

II) $A(y_1, y_2) = -a_1 \log y_1 + a_2 \log y_2,$

$B(y_2) = a_2 \log y_2 - \frac{1}{2} \log c_2,$

$C(y_1) = (1 - a_1 n) \log y_1 - \frac{1}{2} \log c_1$

$\to V = -V_1 \cdot V_2 \sim -e^{\lambda_1 \phi_1} e^{\lambda_2 \phi_2}$: single contribution to the potential, but solution no longer conformally flat

Intersection at an angle

Metric Ansatz:

$$ds_{n+2}^2 = e^{-2\sigma_1 - 2\sigma_2}(ds_n^2 + dx_1^2 + dx_2^2 + f dx_1 dx_2)$$

$$= e^{-2\sigma_1 - 2\sigma_2} ds_n^2 + e^{-2\sigma_2} dy_1^2 + e^{-2\sigma_1} dy_2^2 + f e^{-\sigma_1 - \sigma_2} dy_1 dy_2$$

Using the logarithmic ansatz for the scalars and warp factors, the potential is now:

$$V = V_1 + V_2 + V_{12} = -c_1 v_1 y_1^{-2} y_2^{-2a_2} - c_2 v_2 y_1^{-2a_1} y_2^{-2} - c_1 c_2 \frac{2n^2 f a_1 a_2}{f^2 c_1 c_2 - 4} y_1^{-1-a_1} y_2^{-1-a_2}$$

↑

Subleading for $a_1, a_2 < 1$

$$V_1 = \frac{2c_1 n a_1}{f^2 c_1 c_2 - 4}[(n a_1 + a_1 - 1)] y_1^{-2} y_2^{-2a_2}$$

$$V_2 = \frac{2c_2 n a_2}{f^2 c_1 c_2 - 4}[(n a_2 + a_2 - 1)] y_1^{-2a_1} y_2^{-2}$$

with $a_i = \dfrac{n \pm \sqrt{n + 2(n+1) v_1 (f^2 c_1 c_2 - 4)}}{2n(n+1)}$ and $\delta_i^2 = \dfrac{4}{a_i n} = \dfrac{8(n+1) f^2}{n \pm \sqrt{n + 2(n+1) v_i (\; c_1 c_2 - 4)}}$

Codimension-2 case: Universal Description

$$S = \int d^{n+2}x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2 - \frac{\alpha}{2}\partial_\mu\phi_1\partial^\mu\phi_2 - V(\phi_1, \phi_2)\right)$$

$$ds^2_{n+2} = e^{2A(y_1,y_2)}ds^2_n + e^{2B(y_1,y_2)}dy_1^2 + e^{2C(y_1,y_2)}dy_2^2$$

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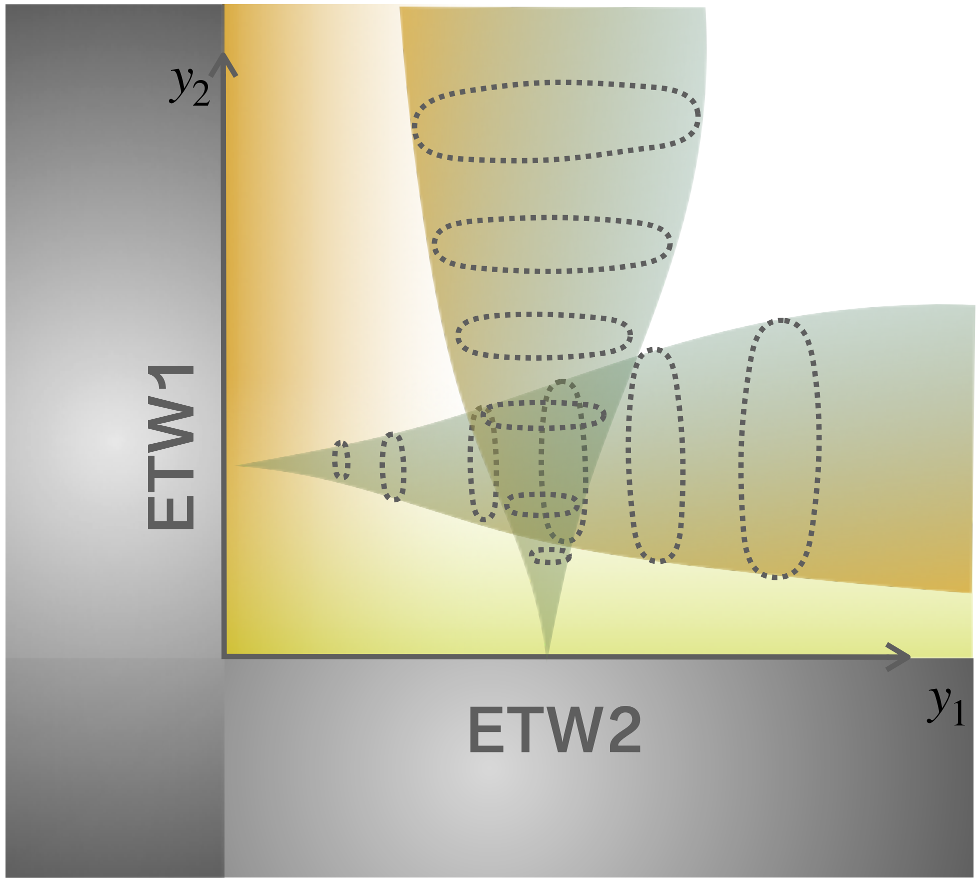

Local description ansatz:

$$\sigma(y_1) = -a_1 \log y_1 + \frac{1}{2}\log c_1$$

$$\sigma(y_2) = -a_2 \log y_2 + \frac{1}{2}\log c_2$$

(assuming $a_1, a_2 < 1$)

$$\phi_1(y_1) = -b_1 \log y_1 \qquad \phi_2(y_2) = -b_2 \log y_2$$

$$V = -c_1 v_1 y_1^{-2} y_2^{-2a_2} - c_2 v_2 y_1^{-2a_1} y_2^{-2} \equiv V_1 + V_2$$