Do black holes know about the emergent string conjecture?

Based on a series of works with Ivano Basile, Niccoló Cribiori and Dieter Lüst.
[2305.10489], *[2311.12113]*, [2401.06851]

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Carmine Montella, 27/06/2024

The emergent string conjecture

- From a bottom-up point of view, an infinite distance limit in a space of vacua is a factorisation limit, i.e. a $N-$point function can be reduced into $N-$one point functions. *[Stout '21]*

- However, gravity abhors factorisation (due to equivalence principle), thus it must couple to an infinite tower of species. *[Stout '22]* In string theory: tower is asymptotically massless. *(SDC!)*

  - The **Emergent String Conjecture** expresses the nature of the tower, stating that any infinite distance limit in the space of vacua is either a decompactification limit or a limit in which there is a weakly coupled (critical) string becoming tensionless. *[Lee, Lerche, Weigand '19]*

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The species scale

- The species scale is (an upper bound to) the cut-off of an effective theory of gravity.

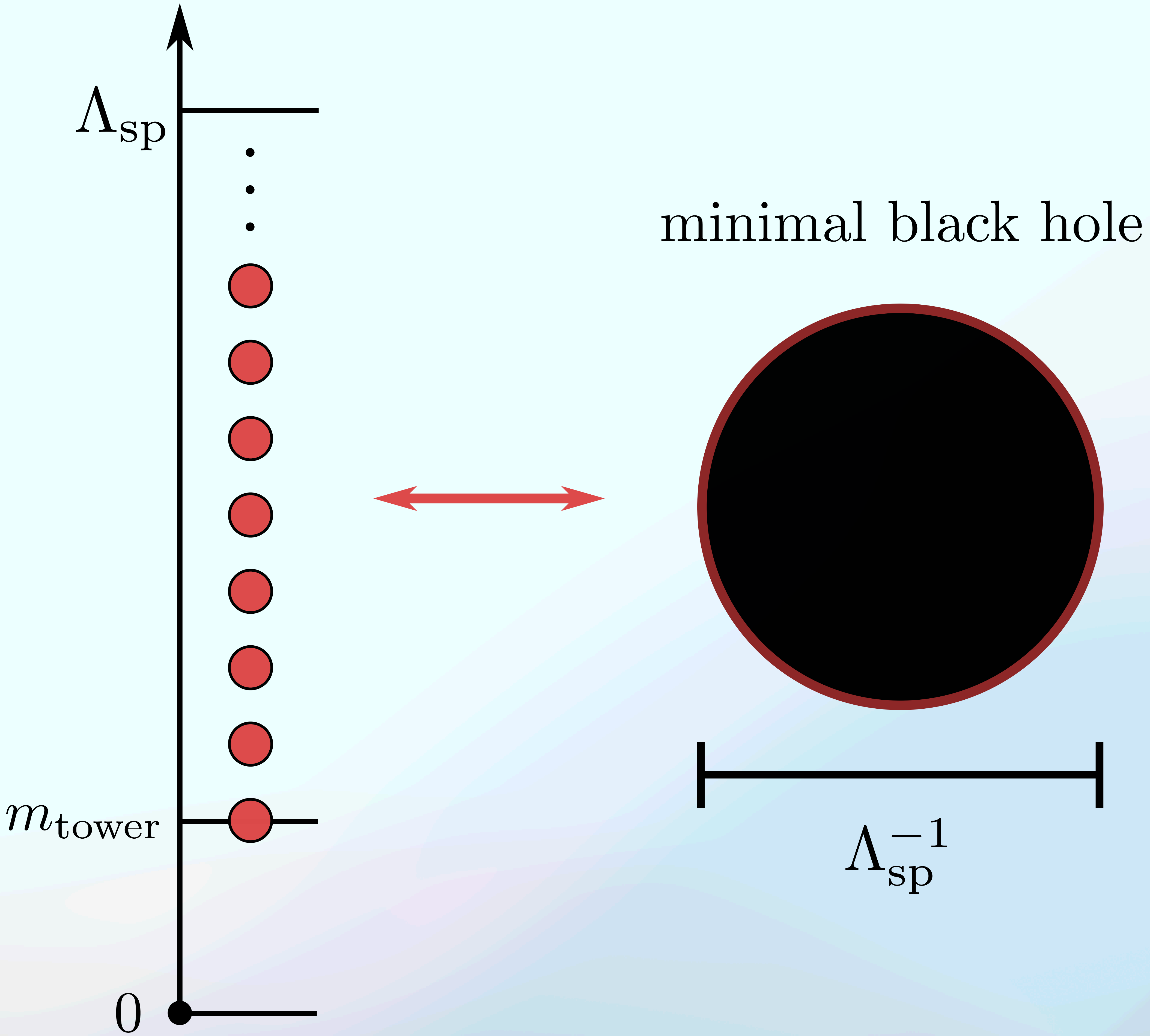
    1. The scale at which perturbative gravity breaks down due to the presence of $N_{sp} \gg 1$ species *[Dvali '09]*:

    $$\Lambda_{sp} = \frac{M_{pl,d}}{N_{sp}^{d-2}}$$

    2. The cut-off scale Λ_{UV} appears in the higher derivative terms of an effective gravitational action. A modern view in string theory context: *[van de Heisteeg, Vafa, Wiesner, Wu '22-'23]*:

    $$S_{\text{EFT}} \sim \frac{M_{\text{pl},d}^{d-2}}{2} \int d^d x \sqrt{-g} \left(R + \sum_n \frac{c_n}{\Lambda_{\text{UV}}^{2n-2}} O_n(g, \text{Riem}, \nabla) \right) .$$

Does the species scale define the smallest black hole?



A depiction of the correspondence or transition between (asymptotically) massless species and minimal black hole

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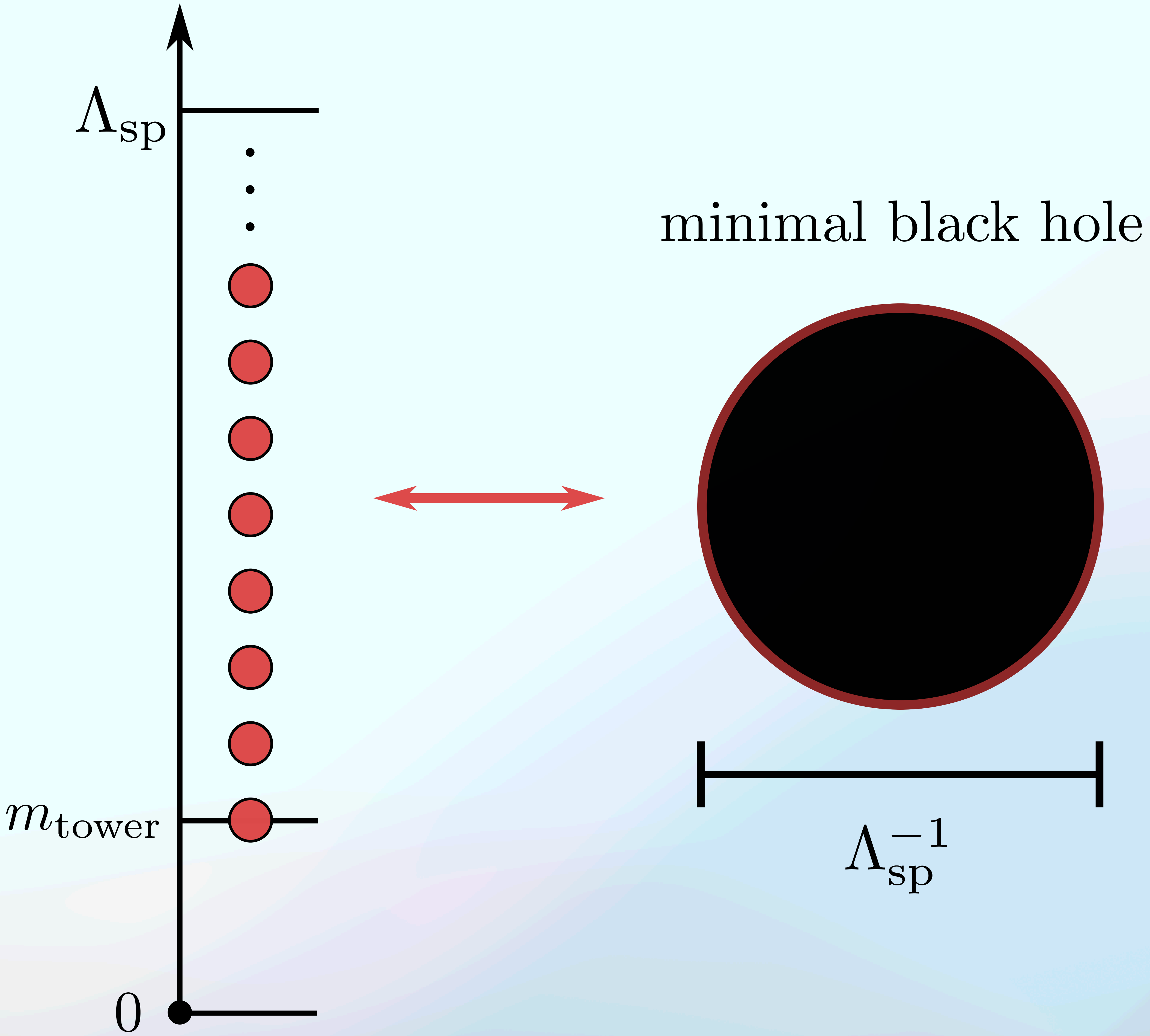

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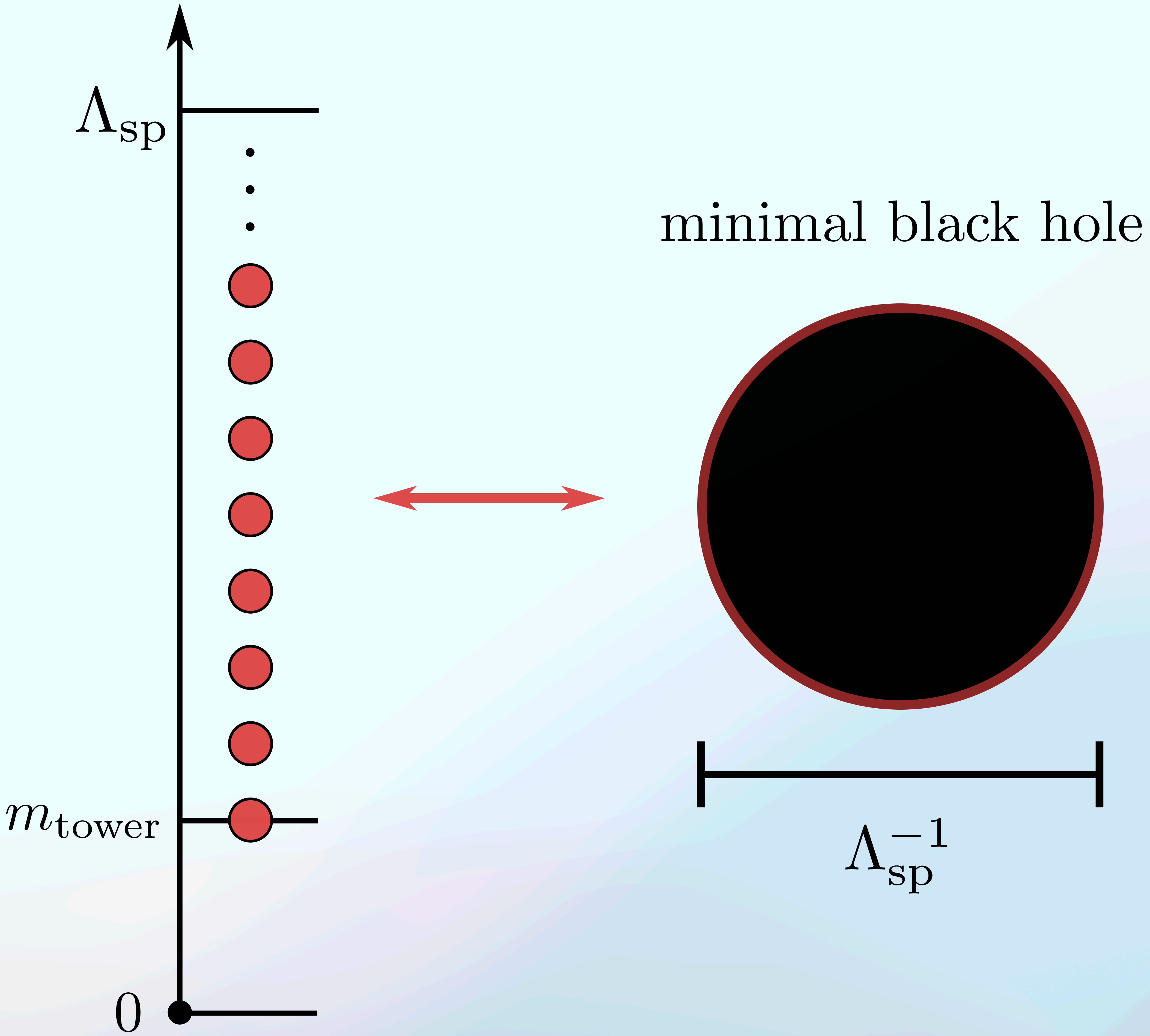

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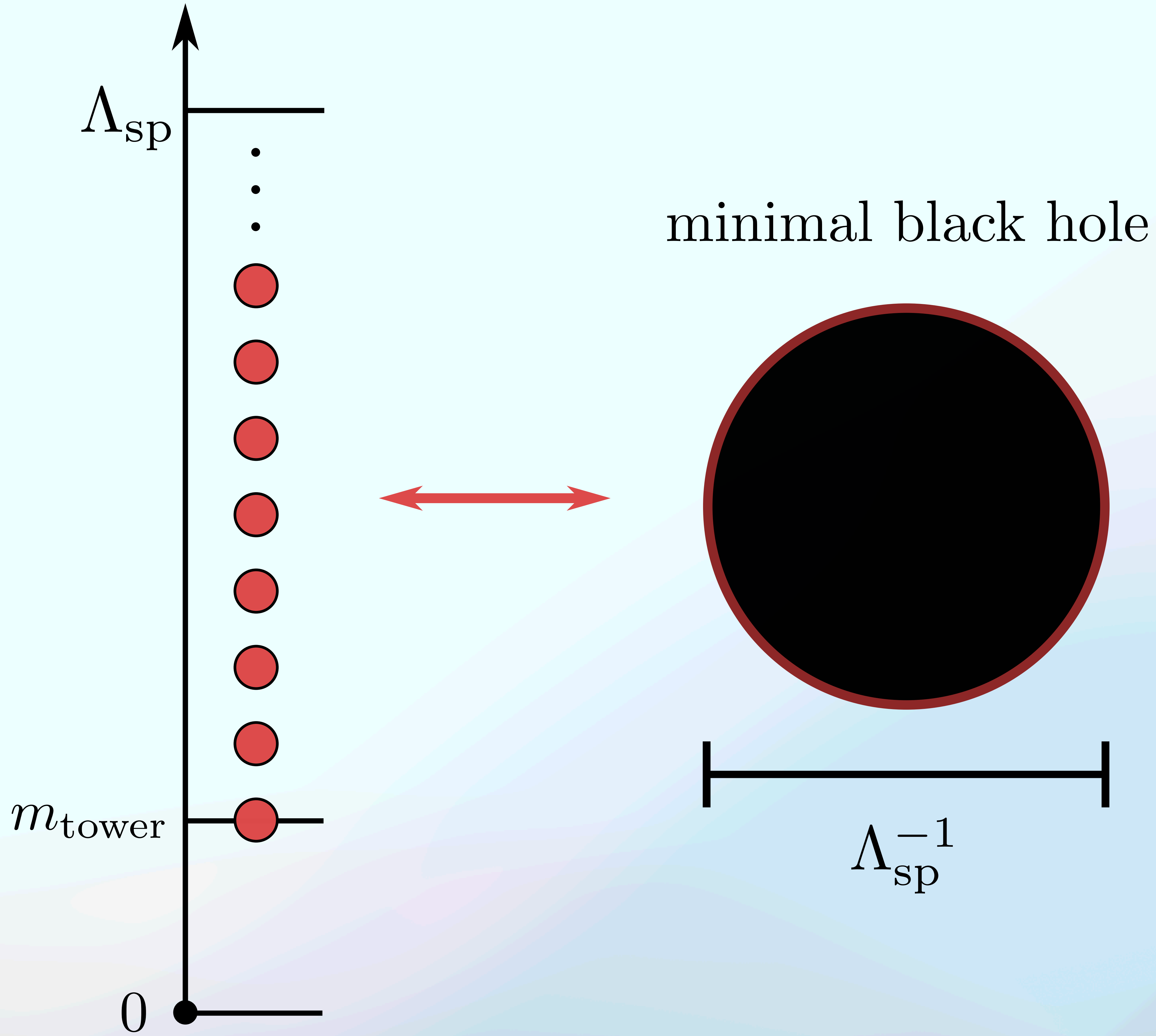

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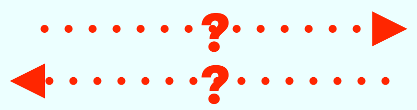
  Tower of species  Black Hole

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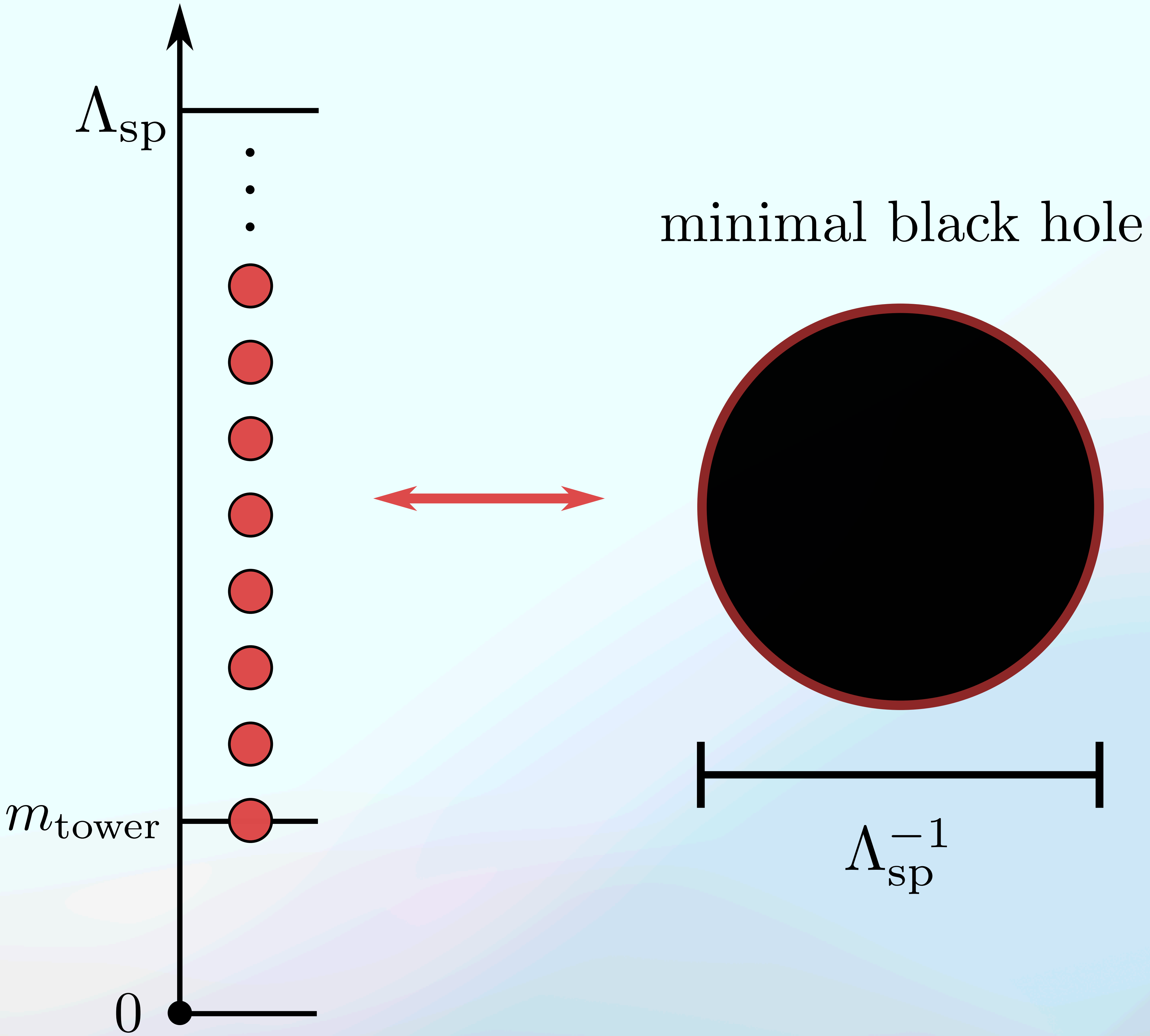

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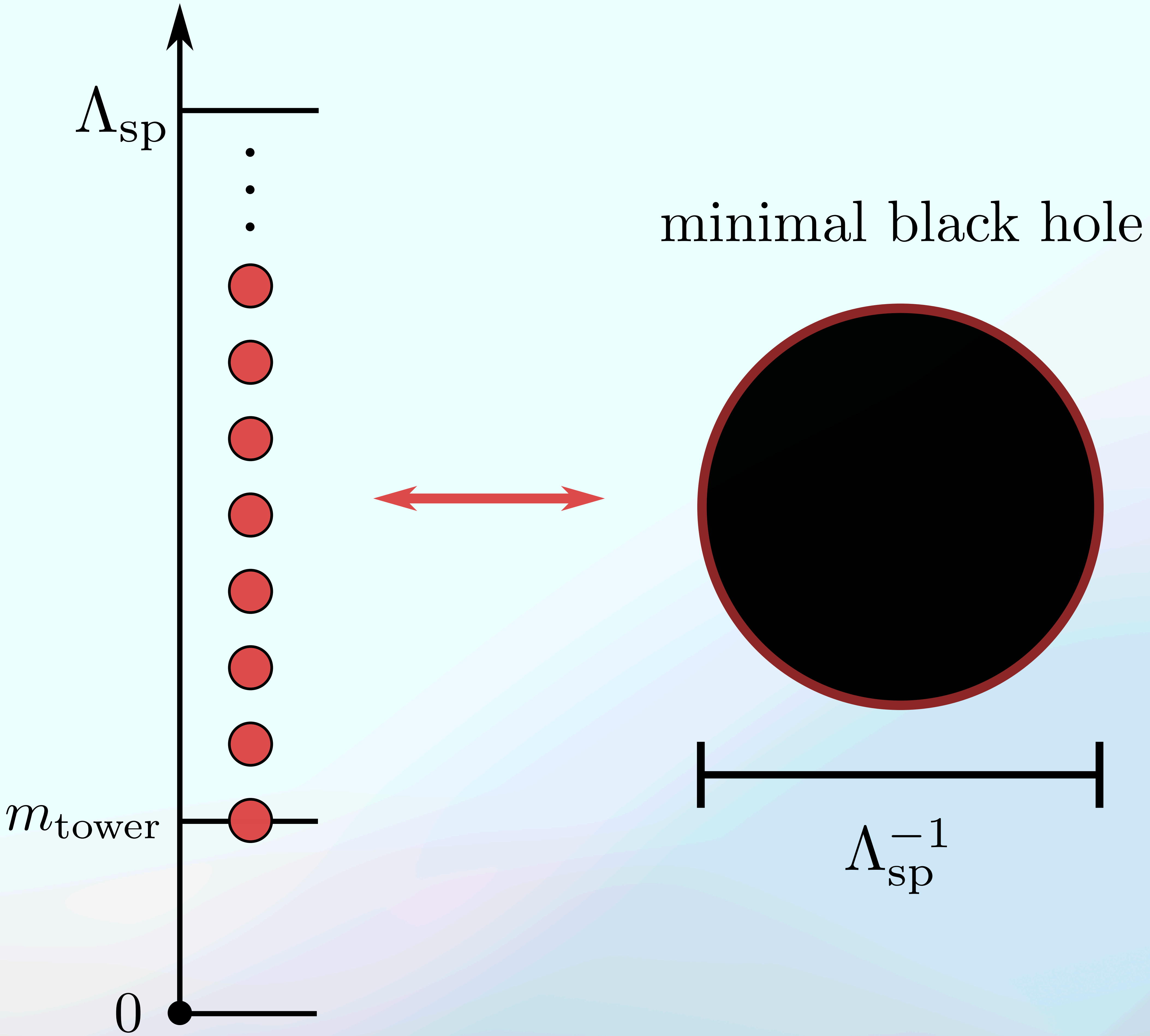

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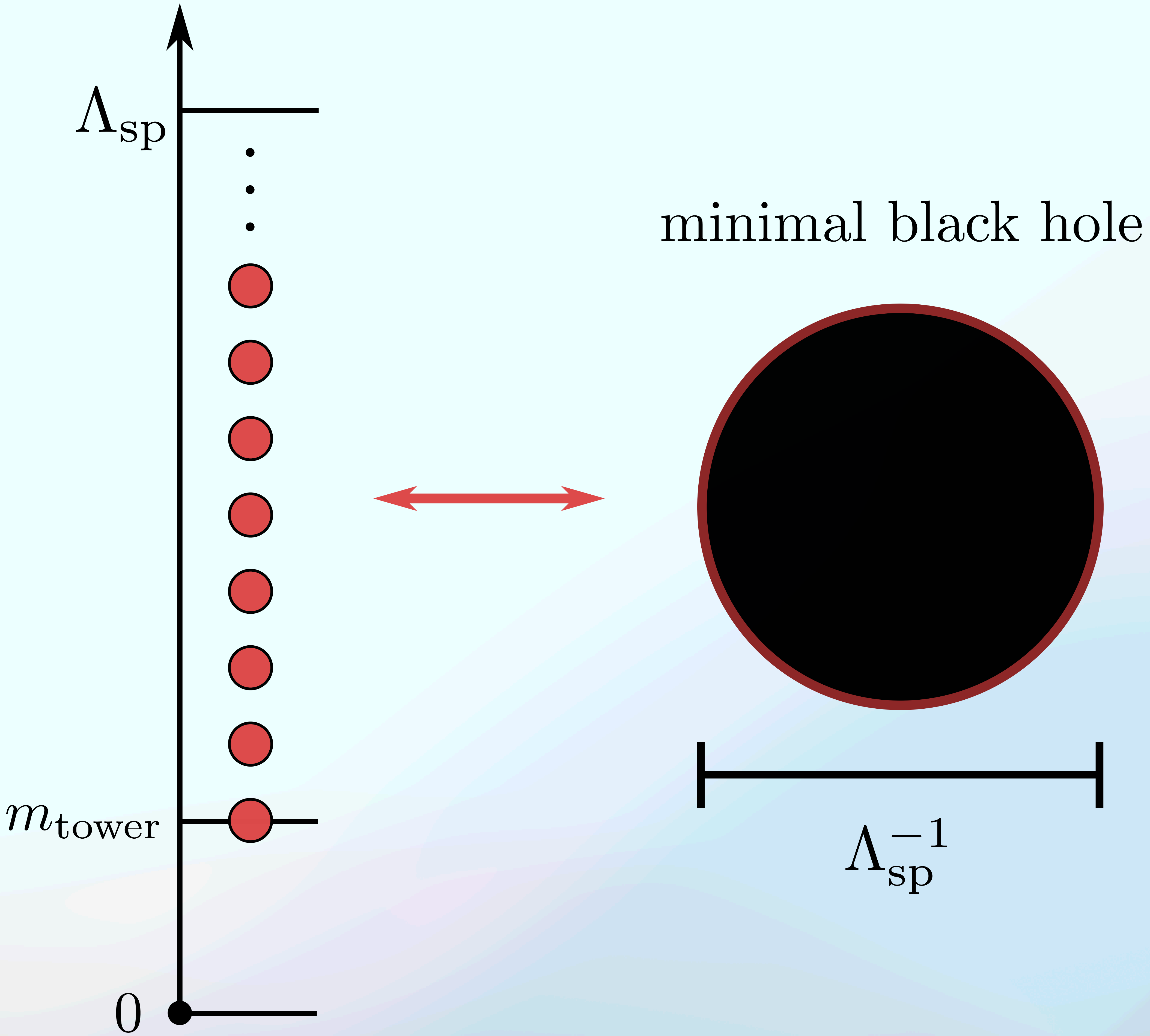

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See also coming talks!

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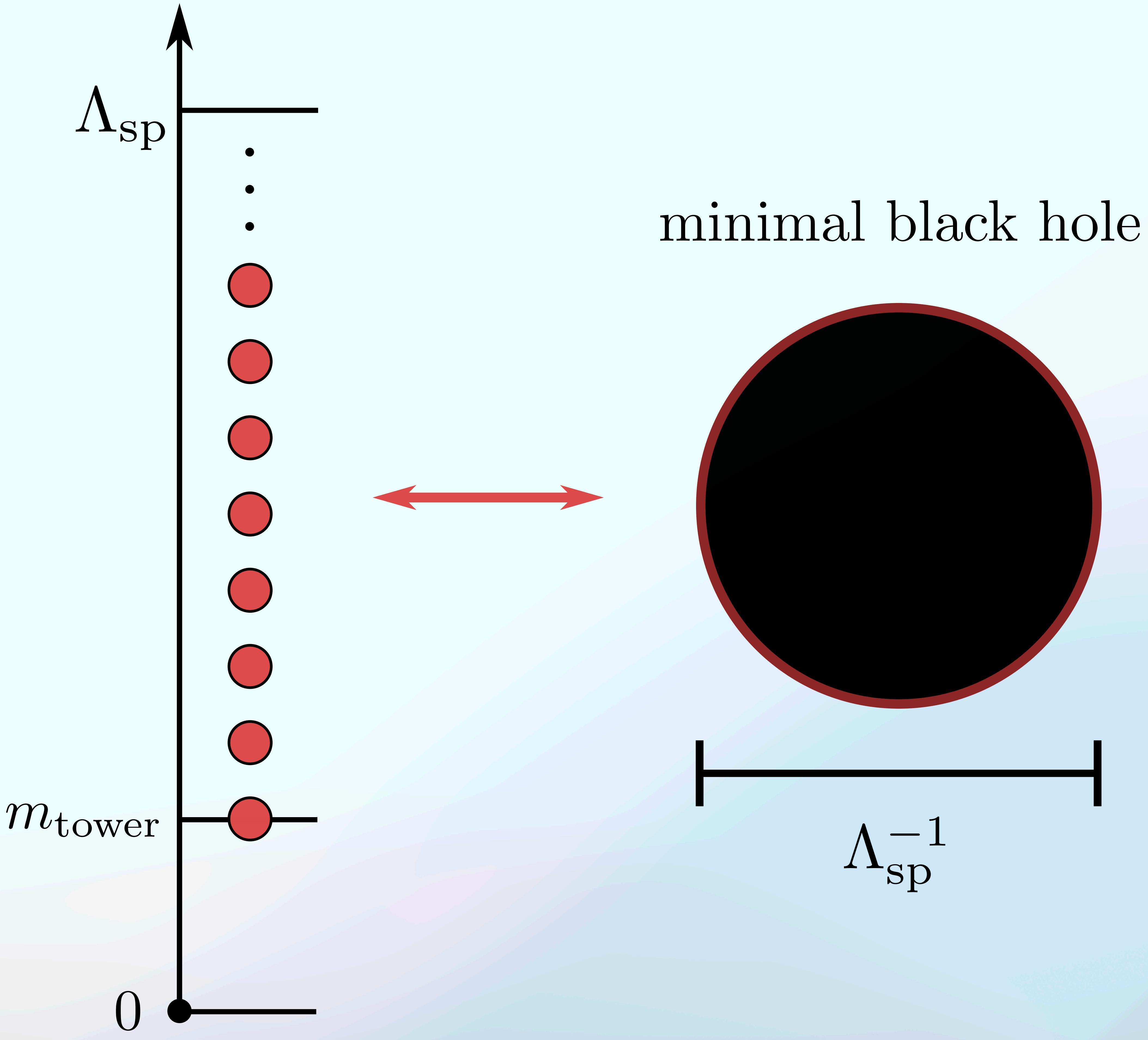

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Tower of species picture as black hole?

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- We parametrise a general tower as $m_n = m_{\text{tow}}\chi(n)$, with a degeneracy of d_n.
  - Fixing the energy E_{sp}, then the micro-canonical entropy can be calculated as $S_{sp} = \log D(E_{sp})$, i.e. $Z(q) = \sum_M q^M D(M) = \prod_{n \leq N} \left(1 - q^{\chi(n)}\right)^{-d_n}$

$$\longrightarrow \; S_{sp} \sim N_{\text{sp}} + \sum_{n \leq N} d_n \log \frac{E_{sp}}{N_{sp} m_n} + \text{corr.}$$

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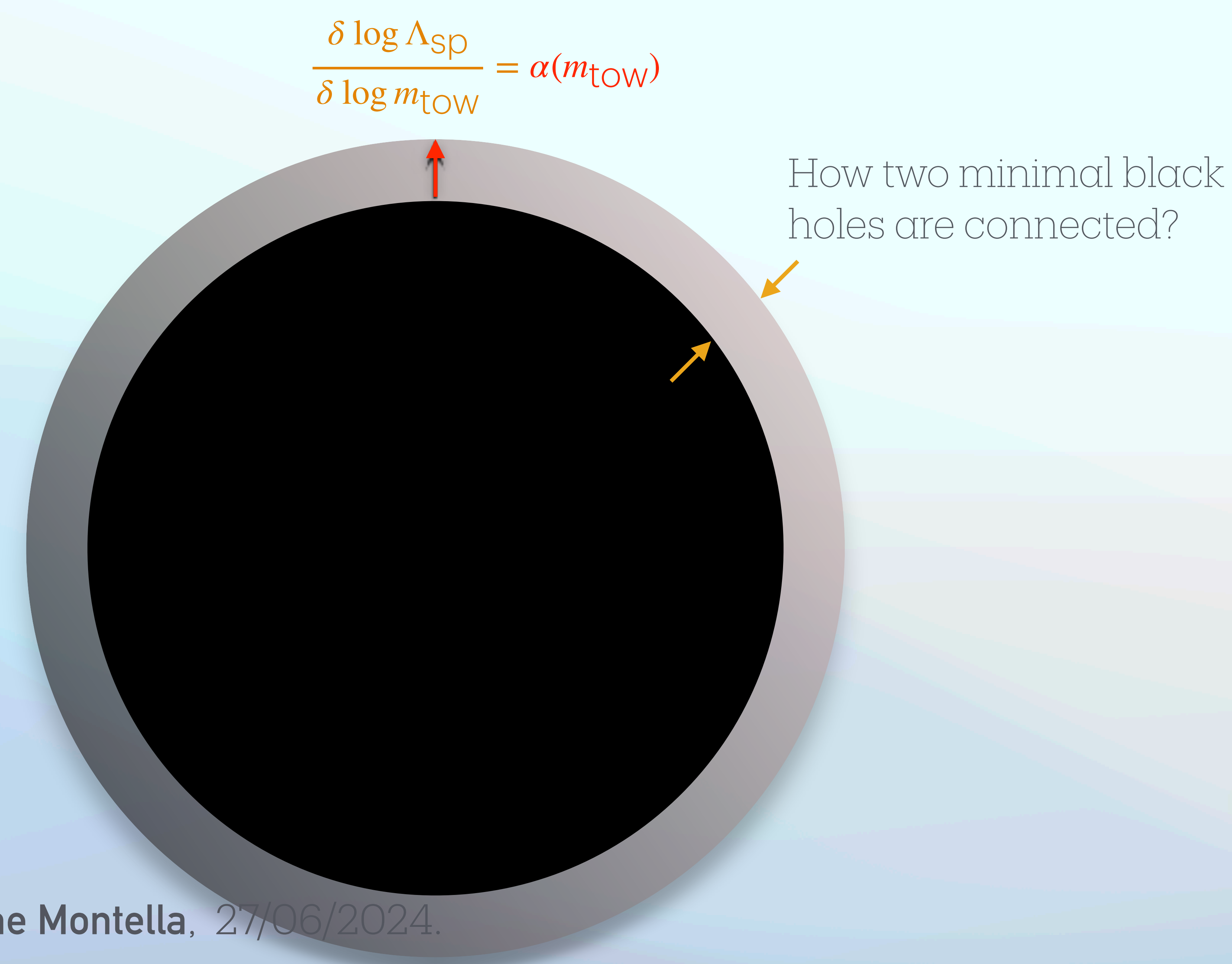
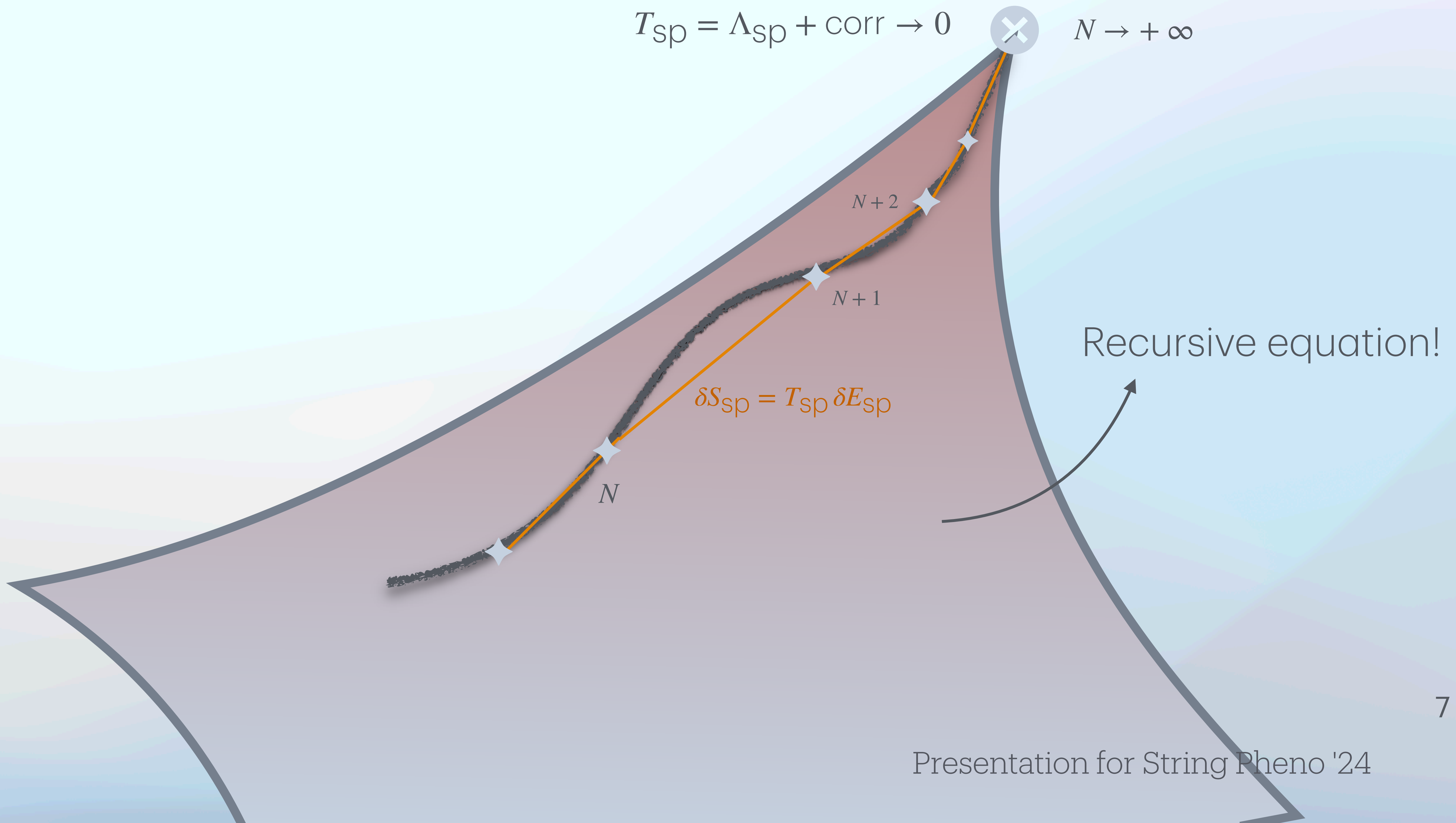
A. The question we asked ourselves is: given leading towers at infinite distance limit, when is it possible to have a correspondence/transition with the smallest black hole in the EFT?

A bottom-up approach

- **In a given EFT, any consistent tower at infinite distance limit must allow a tower-black hole transition (or correspondence) and viceversa [Basile, Lüst, Montella '23]**

- Due to $S_{sp} \sim \Lambda_{sp}^{2-d}$ + corr. for the leading towers, we impose $E_{sp} = \gamma \Lambda_{sp}^{3-d}$ + corr, for *any point* at infinite distance of the space of vacua.





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- In general, <u>every</u> different solution (m_n, d_n) always satisfies the <u>*same*</u> implicit pattern between Λ_{sp} and m_{tow}

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A. From a bottom-up approach, $\hat{p}\left(\hat{c}(m_n, d_n)\right) \geq 1$ represent a generic parameter defined by the black hole thermodynamics, and define different parametrizations of the "microstates".

B. From a top-down perspective it corresponds to the number of extra dimensions! $\longrightarrow m_{\text{tow}} \equiv m_{\text{KK}}$, $\Lambda_{\text{sp}} \sim M_{pl,d+\hat{p}}$

- The limit $\hat{p} \to \infty$ is not defined in terms of mass—degeneracy, but it is always well defined for every thermodynamics quantities. The tower-black hole transition returns (the already well known) string-BH transition $\longrightarrow m_{\text{tow}} \equiv M_{\text{S}}$, $\Lambda_{sp} \sim M_{\text{S}}$

$$T_{sp} \sim M_s \equiv T_{Hag} \qquad M_c \sim M_s^{3-d} = \frac{M_s}{g_s^2}$$

In Planck units.

Thanks!

Questions?