On the Origin of Species Thermodynamics

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String Phenomenology 2024, Padova

Based on [2406.xxxx, D. Lüst, A. Herráez, **JM,** M. Scalisi]

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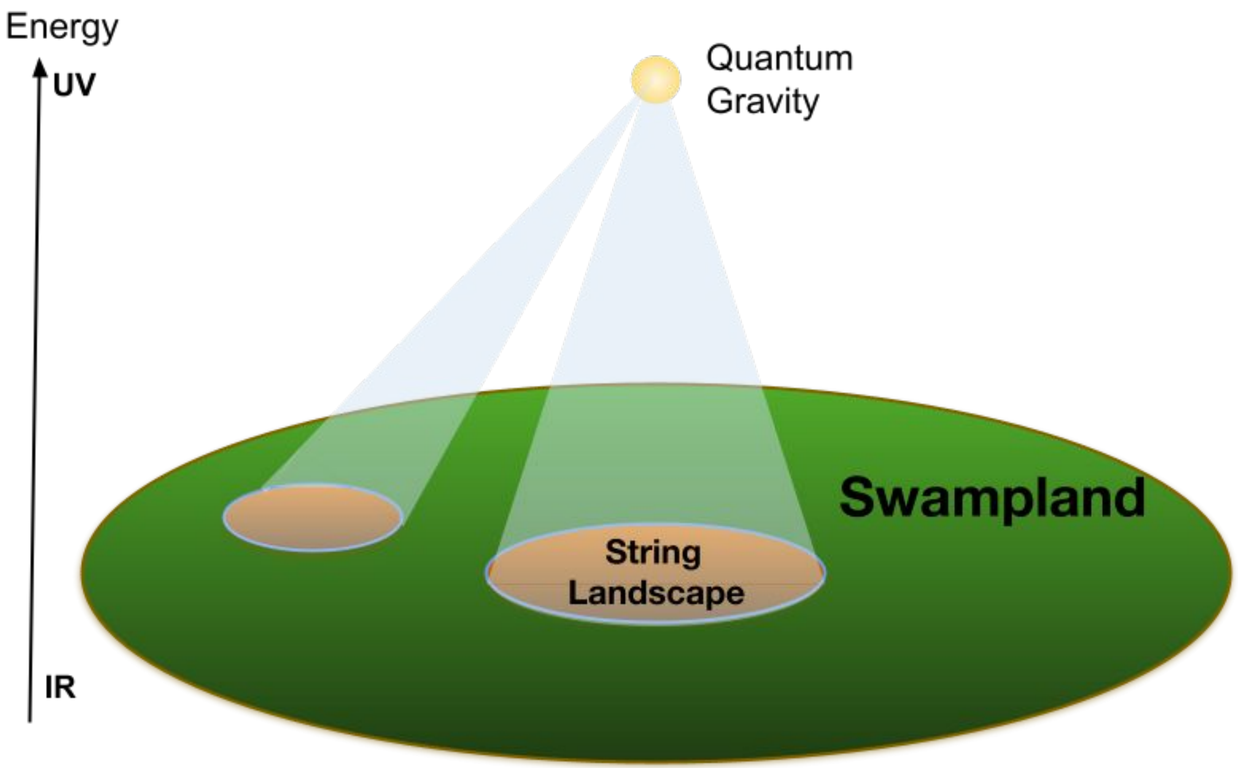
The Species Scale and the Emergent String Conjecture

- The Swampland: Set of seemingly consistent Effective Field Theories (EFT's) in the IR that cannot be consistently coupled to gravity in the UV. [Vafa '05]

- Swampland programme: Identify properties common to consistent theories in the IR.



The Species Scale

- Upper bound to the UV cut-off in QG in the presence of N_{sp} light species

$$\Lambda_{\rm sp} \simeq \frac{M_{\rm Pl,d}}{N_{\rm sp}^{\frac{1}{d-2}}}$$

[Dvali '07][Dvali, Redi '08]
[Dvali, Lüst '10] [Dvali, Gómez '10]

- For a spectrum of particles parameterized as $m_n = n^{1/p}\, m_t$

$$\Lambda_{\rm sp} \simeq m_t \left(\frac{m_t}{M_{{\rm Pl},d}}\right)^{\dfrac{d-2}{d+p-2}}$$

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- For an infinite tower parameterized as $m_n = n^{1/p}\, m_t$

$$\Lambda_{\rm sp} \simeq m_t \left(\frac{m_t}{M_{\text{Pl},d}}\right)^{\dfrac{d-2}{d+p-2}}$$

The Species Scale and the Emergent String Conjecture

- The emergent string conjecture (ESC) states

> **Any infinite distance limit is either a decompactification limit or a limit in which there is a weakly coupled string becoming tensionless.**
>
> [Lee, Lerche, Weigand ´19]

- For $m_n = n^{1/p}\, m_t$:

$p \simeq \mathcal{O}(1)$ — KK modes

$p \to \infty \quad (m_t \simeq M_s)$ — String excitations

- The species scale is then

$\Lambda_{\rm sp} \simeq M_{{\rm Pl},\, d+p}$ — d+p dim Planck mass

$\Lambda_{\rm sp} \simeq M_s$ — string scale

The Covariant Entropy bound and Gravitational Collapse

- For a configuration in a box of size L at temperature T, in order to avoid gravitational collapse we require:

$$L \geq R_{\rm BH}(E) = \left(\frac{E}{M_{{\rm Pl},d}}\right)^{\frac{1}{d-3}} M_{{\rm Pl},d}^{-1} \qquad E \lesssim T L^{d-2} \qquad S \lesssim \frac{L^{d-3}}{T}$$

- Similarly, in order to avoid violating the CEB:

[Bousso´99]

$$S \leq \frac{A}{4G_{N,d}} \sim (L M_{{\rm Pl},d})^{d-2} \qquad E \lesssim L^{d-3} \qquad S \lesssim L^{d-2}$$

- Both bounds coincide for $T \simeq 1/L$

[Castellano, AH, Ibáñez ´21] [AH, Lüst, Masías, Scalisi ´24]

$$E_{\rm max}(T) \simeq \frac{1}{T^{d-3}} \geq \frac{1}{\Lambda_{\rm sp}^{d-3}} \simeq \frac{N_{\rm sp}}{\Lambda_{\rm sp}} \qquad S_{\rm max}(T) \simeq \frac{1}{T^{d-2}} \geq \frac{1}{\Lambda_{\rm sp}^{d-2}} \simeq N_{\rm sp}$$

Field Theory and Species Entropy

- For N_T species in a box of size L at temperature T, such that $M_{\mathrm{Pl},d} \gg \Lambda_{\mathrm{sp}} \geq T$

[AH, Lüst, Masias, Scalisi '24]

- Partition Function

$$Z_{\mathrm{TOT}} = \frac{\prod_{n=1}^{N_T} (Z_{1,n})^{N_n}}{\prod_{n=1}^{N_T} N_n!}$$

$$N_n = Z_{1,n} \simeq (TL)^{d-1} \qquad T \gtrsim m_n$$

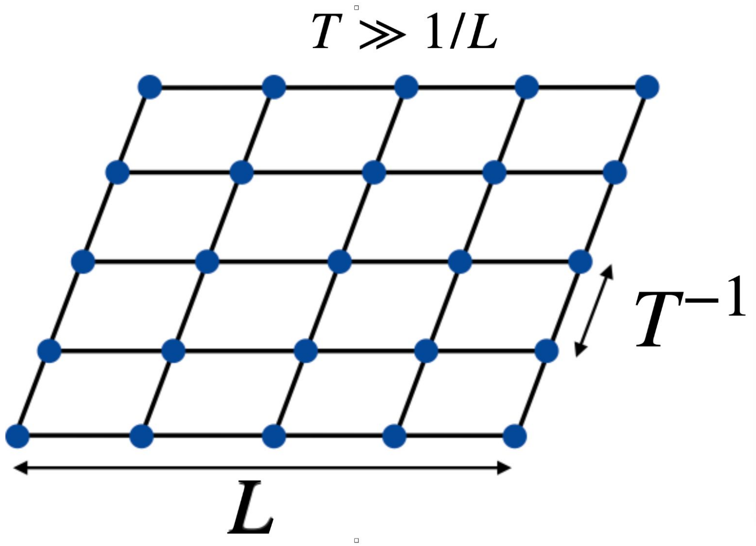
- Number of species for a tower of species $m_n, \quad d_n$

$$N_T \simeq \sum_{n=1}^{N} d_n$$

$$N : m_N = T$$

$$\log(Z_{\mathrm{TOT}}) \simeq (T\,L)^{d-1}\, N_T$$

$$E = (T\,L)^{d-1}\, N_T\, T \qquad S = (T\,L)^{d-1}\, N_T$$



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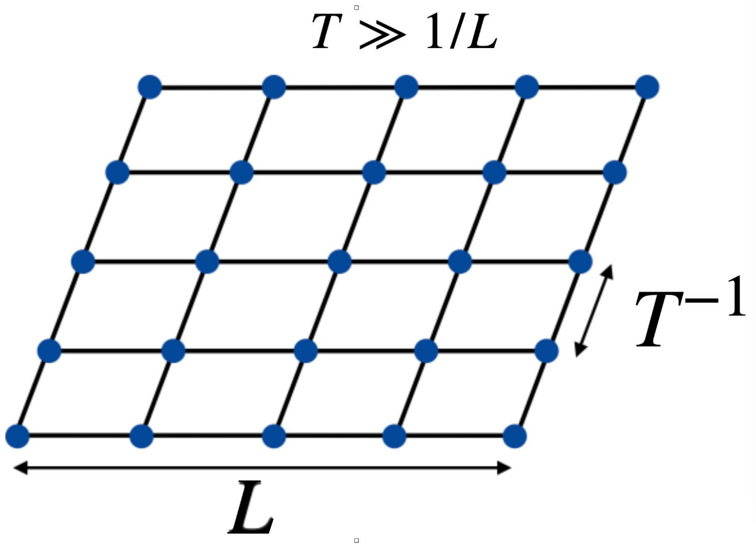


Field Theory and Species Entropy

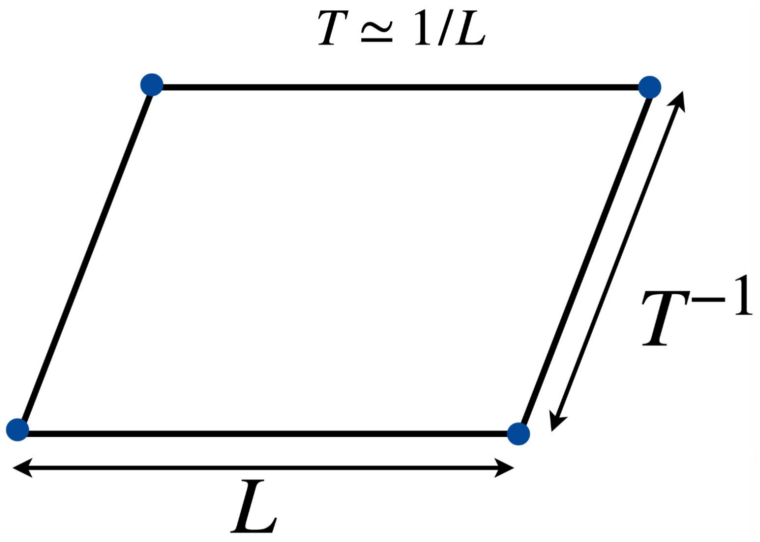
- We can arbitrary variate the temperature *T* along the trajectory $T = 1/L$

- Such that the Energy and entropy take the form

$$E = N_T T \qquad\qquad S = N_T$$

- In the limit $T \simeq \Lambda_{\rm sp}$, we reproduce the defining relations of Species Thermodynamics [Cribiori, Lüst, Montella, ´23]

$$E \simeq \Lambda_{\rm sp} N_{sp} \qquad S \simeq N_{sp}$$



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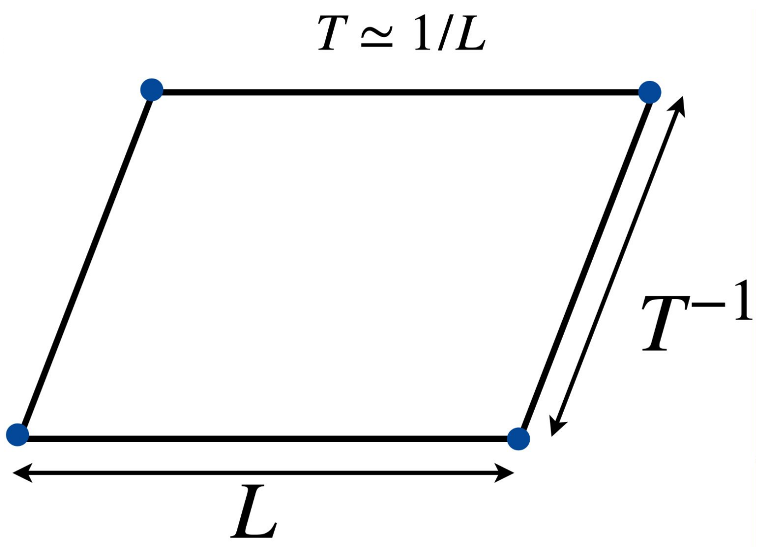

$$E \simeq \Lambda_{\text{sp}} N_{sp} \qquad S \simeq N_{sp}$$

The two Towers

- Can the defining relations of Species Thermodynamics be reproduced for any tower?

$$E = N_T\, T$$

$$E = T^2 \partial_T \left(\log Z\right) \qquad S = \partial_T \left(T \log Z\right)$$

$$S = N_T + \log Z \qquad \frac{\log Z}{N_T} \lesssim 1 \to S \simeq N_T$$

- Such that we recover the entropy of the minimal blackhole in the limit $T \to \Lambda_{\text{sp}}$, $N_T \to N_{\text{sp}} \gg 1$

$$S \simeq N_{\text{sp}}$$

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The two Towers

- Let us look at two very different spectra: $E = N_T T$

$$m_n = n^{1/p} m_t, \quad d_n = 1$$

$$S = \frac{p+1}{p} N_T$$

$$T \to \Lambda_{\rm sp}$$

$$N_T \to N_{\rm sp} \gg 1$$

$$S \simeq N_{\rm sp}$$

$$m_n = m_t, \quad d_n = N \delta_{1,n}$$

$$S = N \left(1 + \log \frac{T}{m_t}\right)$$

$$S = N \left(1 + \log \frac{\Lambda_{\rm sp}}{m_t}\right)$$

- Fixed mass "tower" isn't consistent with Species Thermodynamics

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- Fixed mass “tower” is an effective parameterization of a string tower!

$$\Lambda_{sp} = m_t \qquad\qquad S = N$$

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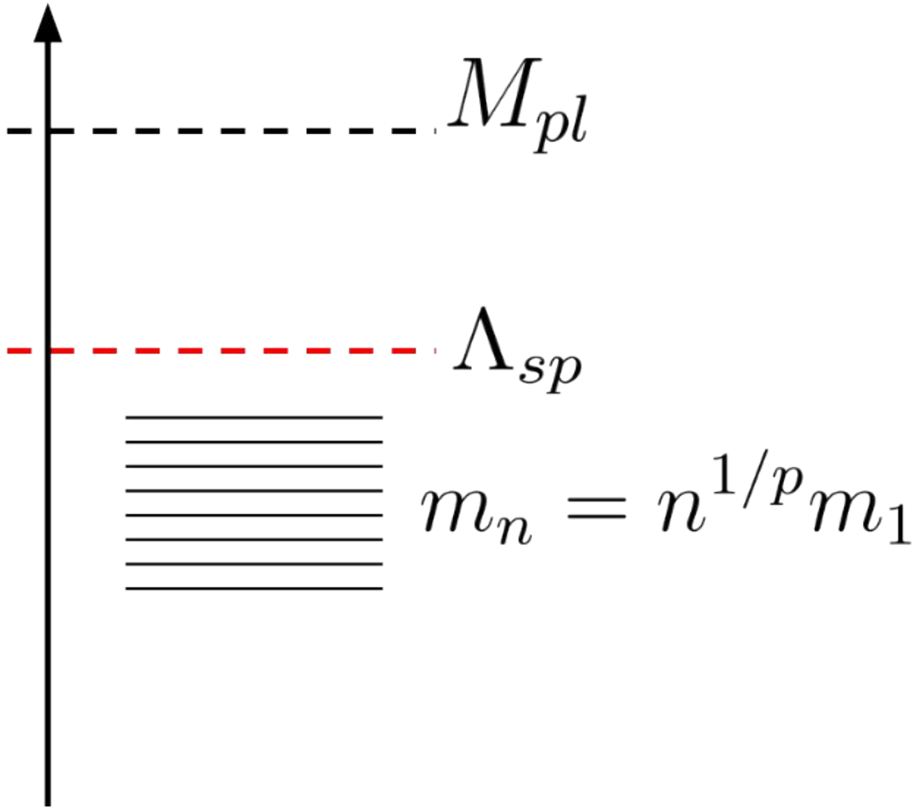
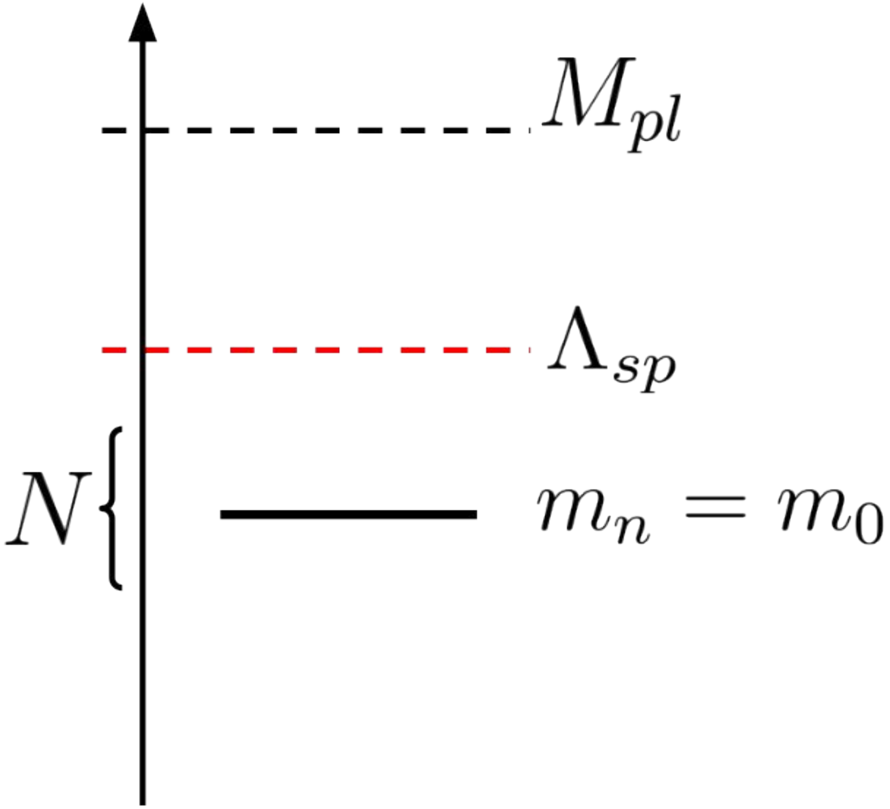
KK-like (+string) towers



Fixed mass “tower”



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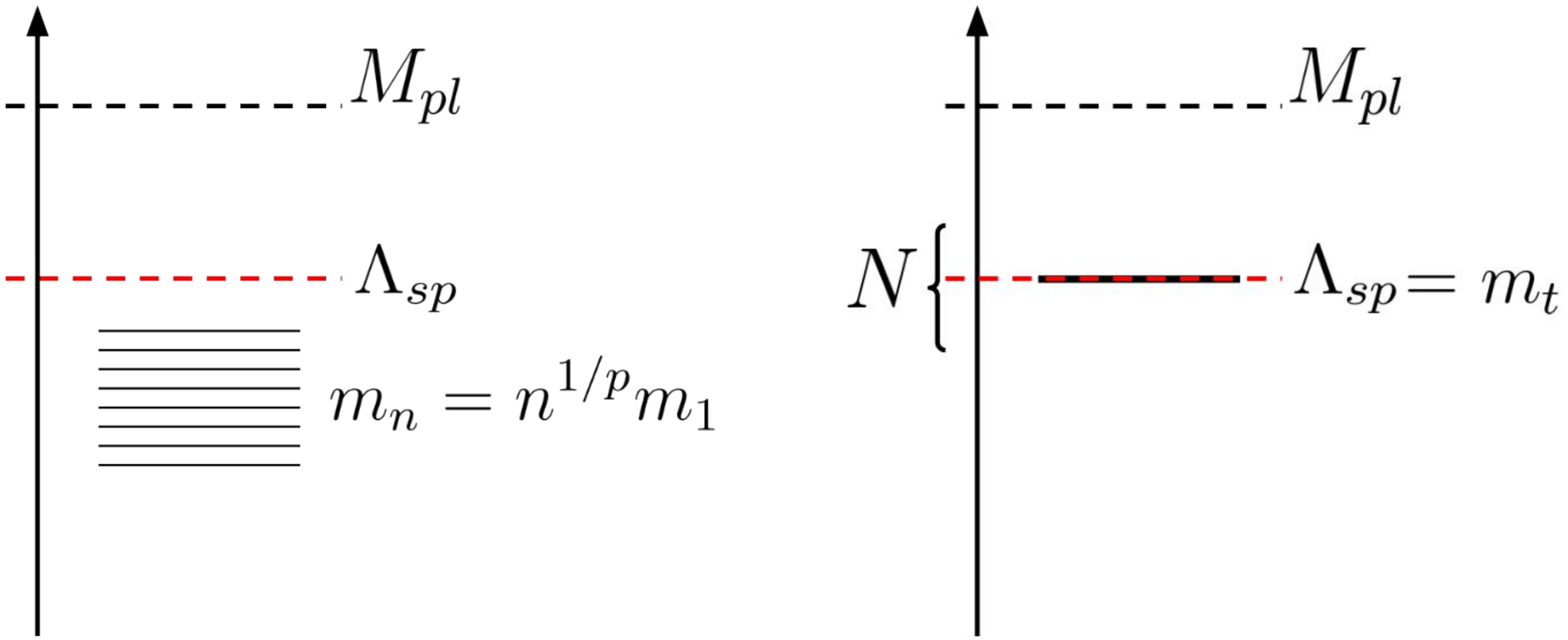
KK-like (+string) towers

String-like tower



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Super exponential towers: $d_{n+1} \gg d_n$ Divergent canonical partition function

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Super exponential towers with maximum level N_{UV}: $$m_n = N_{UV} m_t, \quad d_n = N_{\rm sp}\delta_{n,N_{UV}}$$

(Effective parameterization of) String towers

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Sub polynomial spectra: $d_n \gg d_{n+1}$ Effective fixed mass “tower”

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(Effective parameterization of) String towers

Sub polynomial spectra with $m_t = \Lambda_{\rm sp}$: $$m_n = m_t , \quad d_n = N_{\rm sp} \delta_{n,1}$$

Conclusions

- We provide evidence for the constitutive relations of species thermodynamics in the canonical ensemble $S_{\rm sp} \simeq E_{\rm sp}/T \simeq N_{\rm sp}$
- Extensive entropy scaling at low temperatures and species (intensive) scaling at high temperatures.
- We find polynomiallly (KK like) and exponentially degenerate spectra to give *appropriate* towers
- Super-exponential and sub-polynomial degeneracies are unsuitable, supporting the Emergent String Conjecture from a bottom up perspective.

- The partition has a contribution of the form

$$Z \sim \sum_{n}^{\infty} e^{-\frac{m_n}{T}} d_n$$

$$d_n < e^{\frac{m_n}{T_H}}$$

converges

$$d_n = e^{\frac{m_n}{T_H}}$$

converges for $T < T_H$

$$d_n > e^{\frac{m_n}{T_H}}$$

doesn't converge