The scale of many devices

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Effective theories (EFT) characterized at least by two scales

$$\Lambda_{IR} \qquad\qquad \Lambda_{UV}$$

This talk is about a third scale, the **species scale**, related to Λ_{UV} in *gravitational* EFTs.

It is a scale of many devices, intertwining different subjects: EFTs, black holes, thermodynamics, topological strings, . . . and more in general quantum gravity with mathematics.

Introduction

What is the shortest possible length?

Possible answer $L_p = 1/M_P$ and gravitational perturbation theory governed by E/M_p. However dimensionless parameters may produce scale $\Lambda < M_p$. How to understand this?

E.g. [Han, Willenbrock '04] used unitarity of scattering amplitudes. For 2→2 scattering, spin-2 partial wave breaks unitarity at

$$\Lambda \simeq M_p/\sqrt{N} < M_P \qquad \text{with} \quad N = \frac{2}{3}N_s + N_f + 4N_v$$

Standard Model has $N \sim \mathcal{O}(10 - 100)$, hence $\Lambda \simeq M_P$.
In quantum gravity $N \gg 1$ expected *universally* at boundary of moduli space [Ooguri, Vafa '06], hence $\Lambda \ll M_P$ therein.

The species scale

The species scale aims at making this concrete. It is proposed as **upper bound** on UV cutoff of d-dimensional gravitational EFTs

$$\Lambda_{UV} \lesssim \Lambda_{sp}$$

The point is that Λ_{sp} is *calculable* from properties of the EFT.

- Perturbatively
  - loop expansion [Veneziano '01; Dvali, Redi '07]
  - higher derivative expansion [Dvali, Redi '07; van de Heisteeg, Vafa, Wiesner, Wu '22, '23; NC, Lüst, Staudt '22]
- Non-perturbatively
  - black holes [Dvali, Redi '07; Dvali Lüst '09; Dvali, Gomez '10]

Interplay between these calculations under investigation
[Castellano, Herraez, Ibanez '22, '23; Calderon-Infante, Delgado, Uranga '24; Bedroya, Vafa, Wu '24; Aoufia, Basile, Leone '24; Bedroya, Mishra, Wiesner '24]

Species and uncertainty principle

Ideas in quantum gravity suggest modification of Heisenberg uncertainty principle at Planck scale (recent review [Bosso, Luciano, Petruzziello, Wagner '23])

$$\Delta x \Delta p \gtrsim 1 + N^2 L_P^2 (\Delta p)^2$$

implying minimal length $\Delta x \gtrsim N L_P$ with $N \sim \mathcal{O}(1)$.

We propose [NC, Lüst, Montella, in progress] that modification should occur at $1/\Lambda_{sp} > L_P$, i.e. $N \gg 1$. This makes concrete notion of species scale as shortest possible length

$$\Delta x \Delta p \gtrsim 1 + \Lambda_{sp}^{-2} (\Delta p)^2 \quad \Rightarrow \quad \Delta x \gtrsim 1/\Lambda_{sp}$$

Bound $N < 10^{30}$ [Bosso, Luciano, Petruzziello, Wagner '23] compatible with decompactification of one extra dimension *only*.

I will focus on the black hole picture of Λ_{sp}. The intuition is that in a theory with N_{sp} species, the minimal black hole should have entropy $\mathcal{S} \gtrsim N_{sp}$ [Dvali, Redi '07; Brustein, Dvali, Veneziano '09]. Hence, its inverse horizon radius should not be larger than

$$\Lambda_{sp} = \frac{M_P}{N_{sp}^{\frac{1}{d-2}}}$$

- It intertwines EFTs with: black holes, number and index theory, topological strings
- UV cutoff mapped to counting problem. Counting typically hard, especially at strong coupling, see e.g. [Long, Montero, Vafa, Valenzuela '21]

Black holes and species

In quantum gravity Λ_{sp} should be function of scalar fields, for there should not be free parameters. The goal is to find this function.

In [Bonnefoy, Ciambelli, Lüst, Lüst '19] general approach to extract info on EFT from entropy of small/large black holes. We can apply it to derive Λ_{sp} as function of scalar fields coupled to the black hole.

Controlled setup given by extremal black holes in 4d N=2 SUGRA. Described microscopically by [Maldacena, Strominger, Witten '97] via wrapped M5-branes in M-theory on $CY_3 \times S^1$ and macroscopically by [Cardoso, de Wit, Mohaput '98] via **higher derivative** supergravity.

Extremal black holes with R^2 corrections

Black hole coupled to scalars $X^\Lambda = (X^0, X^i)$, entering prepotential

$$F = \underbrace{\frac{1}{6} C_{ijk} \frac{X^i X^j X^k}{X^0}}_{\text{kin. term}} + \underbrace{c_{2i} \frac{X^i}{X^0} A}_{\text{higher der.}} \equiv F_0 + F_1 A$$

and with charges (attractor mechanism [Behrndt, Cardoso, de Wit, Kallosh, Lüst, Mohaupt '96])

$$p^\Lambda = -2\,\mathrm{Im} X^\Lambda, \qquad q_\Lambda = -2\,\mathrm{Im} \partial_\Lambda F$$

Composite N=2 chiral multiplet $A = (T_{\mu\nu})^2 = -64$ is (graviphoton) background. It contains R^2 as highest component [Antoniadis, Ferrara, Minasian, Narain '97]

$$S_{corr} = \int c_{2i} \mathrm{Im} \frac{X^i}{X^0} R \wedge *R + SUSY$$

Microscopically $c_{2i} = \int c_2(CY_3) \wedge \omega_i$ with ω Kähler 2-form.

Entropy and species scale

Black hole entropy given by [Cardoso, de Wit, Mohaupt '98]

$$\mathcal{S}_{BH} = \pi \left[Z\bar{Z} + 4\mathrm{Im}\left(A\,\partial_A F(X,A)\right)\right] = \pi\left[X^0\bar{X}^0 e^{-K} + \frac{1}{6}c_{2i}\mathrm{Im}\frac{\mathrm{X}^{\mathrm{i}}}{\mathrm{X}^0}\right]$$

For the specific model $\mathcal{S}_{BH} = \sqrt{\frac{q}{6}\left(C_{ijk}p^i p^j p^k + c_{2i}p^i\right)}$ and minimal entropy is for $\frac{1}{6}C_{ijk}p^i p^j p^k = 0$ but $c_{2i}p^i \neq 0$, giving

$$X^0\bar{X}^0 e^{-K} \sim \frac{1}{6}c_{2i}\mathrm{Im}\frac{X^i}{X^0}$$

Thus $\mathcal{S}_{BH} = \pi\left[X^0\bar{X}^0 e^{-K} + \frac{1}{6}c_{2i}\mathrm{Im}\frac{X^i}{X^0}\right] \gtrsim F_1 \simeq \mathcal{S}_{BH,min}$ and we find the species scale as function of scalars [NC, Lüst, Staudt '22]

$$N_{sp} \simeq S_{BH,min} \simeq F_1(X)$$

While the method is general and can be used elsewhere, the above setup exemplifies the interplay between various approaches and the richness of the notion of species scale:

- $N_{sp} \simeq F_1$ agrees with [Vafa, Van de Heisteeg, Wiesner, Wu '22] who argued via topological string. Indeed F_1 is genus-one topological string free energy; related to superstring by [Antoniadis, Gava, Narain, Taylor '93].
- $\mathcal{S}_{BH} \simeq F_1$ links directly perturbative (F_g) and non-perturbative ($\mathcal{S}_{BH}$) approach to species scale. Extending it to all orders would amount to "prove" the [Ooguri, Strominger, Vafa '04] conjecture $|\mathcal{Z}_{top}|^2 = \mathcal{Z}_{BH}$.
- $N_{sp} \simeq \mathcal{S}_{BH}$ hints at statistical interpretation of species scale along lines of black hole thermodynamics. This can be made more precise.

Species thermodynamics

[NC, Lüst, Montella '23; Basile, NC, Lüst, Montella '24]

$N_{sp} \gg 1$ natural in quantum gravity at boundary of moduli space. Here $N_{sp} \simeq (\Lambda_{sp}^{-1} M_P)^{d-2}$, which is intensive and can be understood as **entropy of species**

$$N_{sp} \leftrightarrow \mathcal{S}_{sp}$$

Rest of the dictionary can be built by exploiting **species/black hole correspondence** [Basile, NC, Lüst, Montella '23, '24]: *Set of species behaves collectively as (small) black hole.*
Talks by I. Basile, A. Herraez, D. Lüst, J. Masias, A. Mininno, C. Montella, T. Weigand. Poster by G. Staudt.

The framework allows for explicit calculations

$$T_{BH} \leftrightarrow T_{sp} \qquad M_{BH} \leftrightarrow M_{sp}$$

and it can provide bottom-up rationale to swampland conjectures.

Uncharged species

Consider set of species with

$$M_n = n^{\frac{1}{p}} \Delta m, \qquad p : \# \text{ of towers}$$

E.g.: $p = 1$ for single KK tower, $p \to \infty$ for string tower.
Dual to Schwarzschild BH with $\mathcal{S}_{sp}$, T_{sp}, M_{sp}. We find $(M_P = 1)$

- Mass: $$M_{sp} = \sum_{n=1}^{N_{sp}} M_n \simeq \frac{p}{p+1} \Lambda_{sp}^{3-d}$$
- Entropy: $$\mathcal{S}_{sp} = \log \frac{N_{sp}^{\frac{p+1}{p} N_{sp}}}{(N_{sp}!)^{\frac{p+1}{p}}} \simeq \frac{p+1}{p} N_{sp}$$
  (log of # of partitions of $N = M_{sp}/\Delta m$ into N_{sp} parts)
- Temperature: $T_{sp} \simeq \Lambda_{sp}$ (from $dM = T d\mathcal{S} + \dots$)

The Schwarzschild relation $\mathcal{S} T^{d-2} = 1$ becomes now $N_{sp} \Lambda_{sp}^{d-2} = 1$, namely the definition of the species scale we started from.

The laws of species thermodynamics

- **Zero-th law**: points in moduli space with the same $\Lambda_{sp}(\phi)$ have the same $T_{sp}(\phi)$.
- **First law**: $\delta E_{sp} = T_{sp}\delta\mathcal{S}_{sp} + \dots$
- **Second law**: species entropy does not decrease when moving adiabatically towards boundary of the moduli space

$$\delta\Lambda_{sp}(\phi) \leq 0, \qquad \delta\mathcal{S}_{sp}(\phi) \geq 0$$

- **Third law**: point $T_{sp} = 0$ impossible to reach with a finite sequence of steps.

Further developed and motivated in [Herraez, Lüst, Masias, Scalisi '24].

Charged species

If tower satisfies WGC, $Q_n > M_n$, then dual species black hole super-extremal, $Q_{sp} > M_{sp}$. Fixed by higher derivative corrections ($\propto \kappa$), leading to modified extremality parameter [Kats, Motl, Padi '06; Hamada, Noumi, Shiu '18]

$$c^2 = M^2 - Q^2 + 2\kappa M^{\frac{2d-8}{d-3}}$$

For simple tower $M_n = n\Delta m$, $Q_n = (n+\beta)\Delta m$, we get $c \simeq \kappa \mathcal{S}^{\frac{d-4}{d-2}}$ and then

$$T_{sp} \simeq \frac{c}{\mathcal{S}_{sp}} \simeq \sqrt{\kappa}\Lambda_{sp}^2$$

Suppressed by factor Λ_{sp} with respect to uncharged case.

Species in expanding universe

Consider d-dimensional expanding universe with Λ_{cc}.
Limit $\Lambda_{cc} \to 0$ conjectured (swampland [Lüst, Palti, Vafa '19] + unitarity) to be accompanied by tower of states with

$$m \sim \Lambda_{cc}^{\alpha}, \qquad \frac{1}{d} \leq \alpha \leq \frac{1}{2}$$

- Tower of species initially produced when temperature of universe reaches T_{sp}.
- For KK species with typical mass $m \simeq M_{KK}$ and corresponding to n extra dimensions, we have

$$\Lambda_{sp} \simeq M_{KK}^{\frac{n}{d+n-2}}$$

(valid also beyond flat space [Aoufia, Basile, Leone '24])

In scenario with $M_{KK} \simeq \Lambda_{cc}^{1/d}$ we can relate T_{sp} to Λ_{cc} via Λ_{sp}

$$\text{uncharged species} \qquad T_{sp} \simeq \Lambda_{cc}^{\frac{n}{d(d+n-2)}}$$

$$\text{charged species} \qquad T_{sp} \simeq \Lambda_{cc}^{\frac{2n}{d(d+n-2)}}$$

The Dark Dimension [Montero, Vafa, Valenzuela '22] is the particular case $d = 4 = 1/\alpha$, $n = 1$ and $m \equiv M_{KK} \sim \Lambda_{cc}^{1/4} \sim \Lambda_{sp}^{3}$ (up to prefactor $\sim \mathcal{O}(10^{-3})$) giving

$$\text{uncharged species} \qquad T_{sp} \simeq \Lambda_{cc}^{\frac{1}{12}}$$

$$\text{charged species} \qquad T_{sp} \simeq \Lambda_{cc}^{\frac{1}{6}}$$

The latter is initial temperature required for KK gravitons to be dark matter in Dark Dimension [Gonzalo, Montero, Obied, Vafa '22]. Here re-derived from species thermodynamics.

Species, emergence, ...

Species scale plays central role in the **emergence proposal**:
Dynamics for all fields are emergent in the infrared by integrating out towers of states down from ultraviolet scale Λ below the Planck scale. [Palti '19] [Heidenreich, Reece, Rudelius '17; Grimm, Palti, Valenzuela '18]

[Blumenhagen, NC, Gligovic, Paraskevopoulou '23] hinted at strong version of the proposal within M-theory (compactifications) by integrating out all infinite towers with typical mass $\lesssim \Lambda_{sp}$.
Talks by R. Blumenhagen, A. Gligovic, A. Paraskevopoulou

Similarly, R^4 term in 11D shown to be emergent in [Blumenhagen, NC, Gligovic, Paraskevopoulou '24]. It reduces to R^2 correction in 4D giving Λ_{sp} as function of type IIA Kähler moduli. In this sense, the species scale is emergent [Calderon-Infante, Delgado, Uranga '24].

. . . and more

The species scale has been also investigated in relation to

- 4d N=1 EFTs and wormholes [Martucci, Risso, Valenti, Vecchi '24]
- 4d N=2 EFTs [Marchesano, Melotti '22]
- Gravitino conjecture and SUSY breaking in Dark Dimension [NC, Lüst, Scalisi '22; Antoniadis, Anchordoqui, NC, Lüst, Scalisi '23]
- Inflationary cosmology [Scalisi, Valenzuela '18; Lüst, Masias, Muntz, Scalisi '23; Scalisi '24]
- . . .

Outlook

The species scale is rich notion of which much is to be understood.

- Unify various definitions of species scale?
- Improve on species thermodynamics. Does it imply swampland conjectures?
- Going away from asymptotic regions of moduli space?
- What the species scale can teach us about interplay between quantum gravity and mathematics?
- Can we relate it to observations?

Thank you!