Non-Invertible Symmetries on the Worldsheet

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String Pheno 2024

Based on [2402.00118], w/ J. Heckman, M. Montero, A. Sharon, C. Vafa, and I. Valenzuela

See also [2402.00105], by J. Kaidi, Y. Tachikawa, and H. Y. Zhang

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Introduction

Over the past few years, tremendous expansion in generalized notions of symmetry. (See talks by Clay, Mirjam, Jonathan, and many others)

In particular: **non-invertible symmetry**, replacing group law with fusion algebra:

$$\mathcal{N}_i \otimes \mathcal{N}_j = \sum_k \mathcal{T}_{ij}^k \mathcal{N}_k$$

Non-invertible symmetries arise in wide variety of QFTs in various dimensions.

Original home was 2D CFT, especially in the context of string theory. [Hamidi, Vafa '87; Verlinde '88; Dijkgraaf, Vafa, Verlinde, Verlinde, '89]

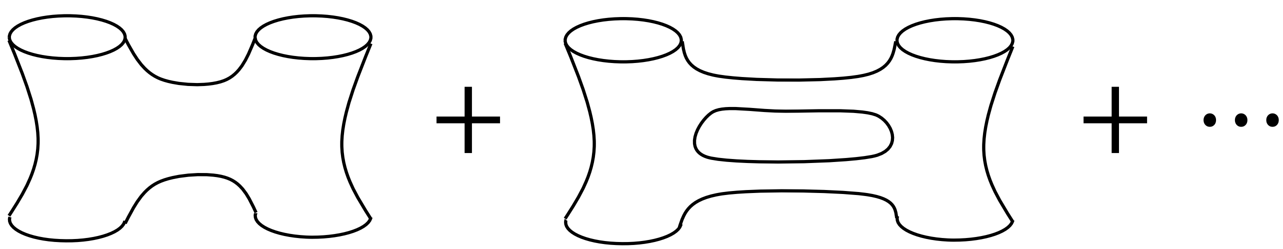
Goal: Revisit non-invertible worldsheet symmetries with modern understanding and determine implications for target space physics. [See also Cordova, Rizi '23; Angius, Giaccari, Volpato '24]

A Natural Guess

Following experience with ordinary invertible symmetries, natural to guess:

Non-Invertible Global Symmetry on the Worldsheet $\overset{?}{\Longrightarrow}$ **Non-Invertible Gauge Symmetry in Target Space**

Surprisingly, this is incorrect! **Reason:** string perturbation theory is not merely 2D CFT but 2D gravity:



Breaking by String Loops

Well-Known Fact: Selection rules for non-invertible symmetry violated on non-trivial topology.

Summing over topologies breaks symmetry. General feature of coupling QFT with non-invertible symmetry to gravity. [JM '21]

Upshot: Non-invertible worldsheet symmetries are **exact at string tree-level** but are **universally broken by string loops**.

Still useful for studying the Landscape at $g_s \ll 1$, where symmetry is approximately preserved!

Mechanism for deferring effect to given loop order. Pheno applications?

What is a Non-Invertible Symmetry?

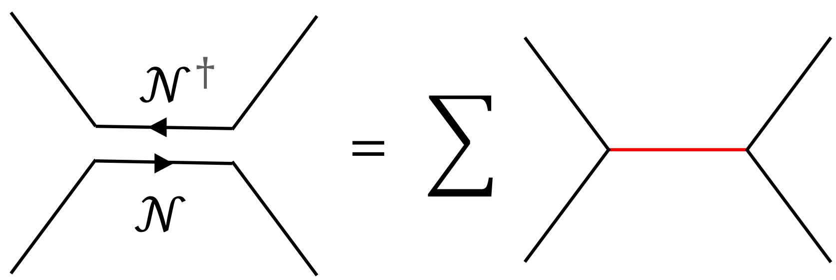
Non-Invertible Symmetries in 2D CFT

In a 2D CFT, a **non-invertible (0-form) symmetry** is a collection $\{\mathcal{N}_i\}$ of topological defect lines (TDLs) with fusion algebra:

$$\mathcal{N}_i \otimes \mathcal{N}_j = \sum_k \mathcal{T}_{ij}^k \mathcal{N}_k \,, \qquad \mathcal{T}_{ij}^k \in \mathbb{N}.$$

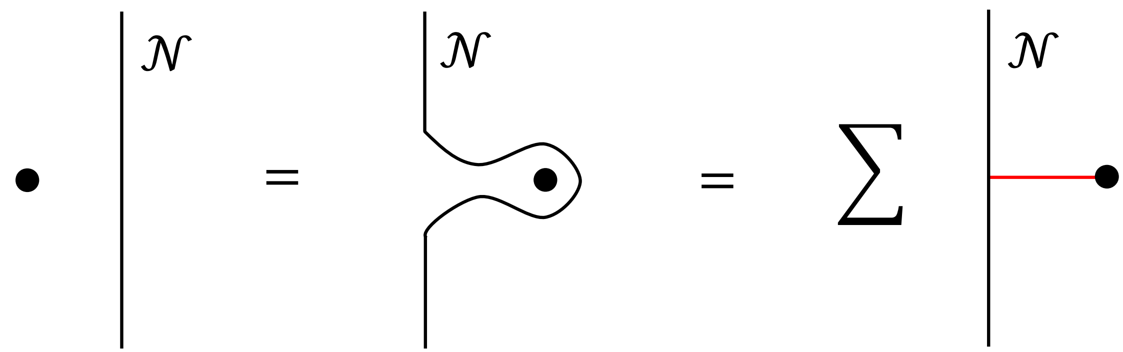
Fusion coefficient $\mathcal{T}_{ij}^k$ is number of topological junctions $\mathcal{N}_i \otimes \mathcal{N}_j \to \mathcal{N}_k$.

Non-invertible TDLs **cannot be freely reconnected**:



Action on Local Operators

Non-invertible symmetry acts by sweeping:



Maps **local operators** → **superpositions of local/non-local operators** (with branch cuts in correlation functions).

Which operators are local/non-local part of the data of the 2D CFT. Non-invertible symmetry does not preserve this choice.

Building Intuition

What does it mean when a local operator is mapped to a non-local operator?

Intuition: the symmetry action projects the local operator out of the spectrum of closed string states.

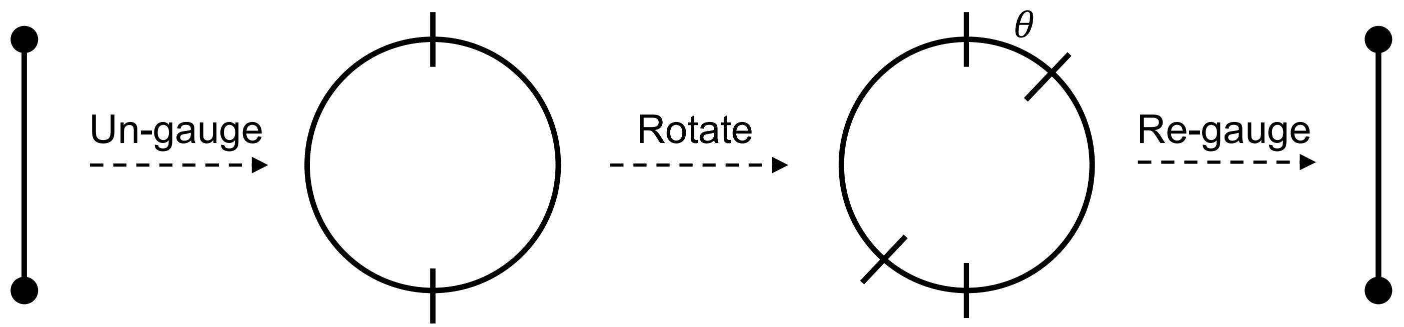
Conversely: when a non-local operator is mapped to a local one, it is added into the spectrum as a “twisted sector.”

In other words: a non-invertible symmetry acts on the 2D CFT by **performing an orbifold.** [Choi, Lu, Sun ’23]

Caveat: The orbifold being performed might, itself, involve gauging non-invertible symmetries.

Running Example: $S^1/\mathbb{Z}_2$

Consider the $c = 1$ orbifold $S^1/\mathbb{Z}_2$, with $\mathbb{Z}_2$ action $X \leftrightarrow -X$. Non-invertible symmetry:



Can be viewed as **non-invertible part of $U(1)_{\mathrm{KK}}$** that survives the orbifold projection. Denote the associated TDL by L_θ.

Exercise: Check that $L_\theta = e^{i\theta \frac{R}{2\pi} \int \star dX} + e^{-i \frac{R}{2\pi} \int \star dX}$. [Nguyen, Tanizaki, Ünsal '21; Heidenreich, JM, Montero, Reece, Rudelius, Valenzuela '21; Thorngren, Wang '21]

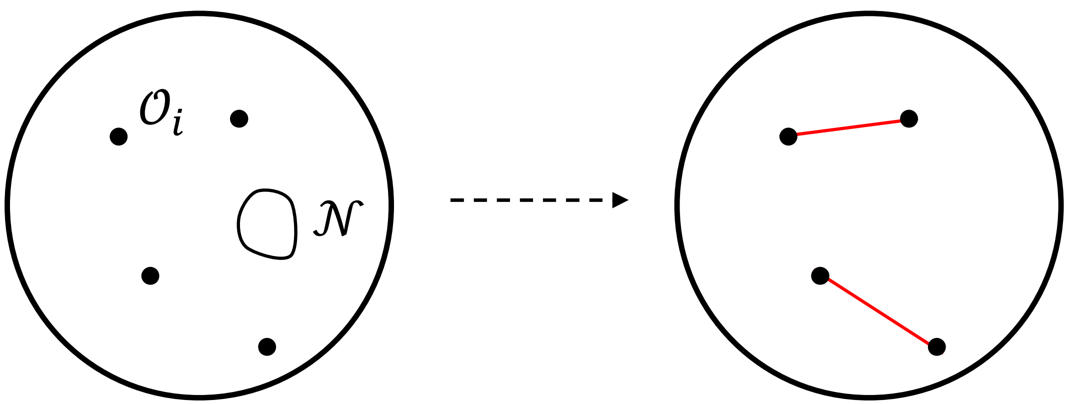
What are Symmetries Good For?

Representations and Selection Rules

Group operators into **representations** μ: collections of operators closed under the symmetry action. **Novelty:** irreps can contain both local and non-local operators.

Tree-Level Selection Rules: $\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle_{\mathbb{S}^2}$ can only be nonzero if the charges cancel, $\mu_1 \otimes \cdots \otimes \mu_n \supset 1$. [Lin, Okada, Seifnashri, Tachikawa '22]

Can be derived by sweeping TDLs across sphere and solving linear system of equations:



Selection Rules for $S^1/\mathbb{Z}_2$

Have operators: $\mathcal{O}_k^+ = \frac{1}{\sqrt{2}}(e^{\frac{ikX}{R}} + e^{-\frac{ikX}{R}})$, $\mathcal{O}_k^- = \frac{i}{\sqrt{2}}(e^{\frac{ikX}{R}} - e^{-\frac{ikX}{R}})$.

While $\mathcal{O}_k^+$ is a **local operator**, $\mathcal{O}_k^-$ is a **non-local operator** attached to a topological Wilson line. L_θ acts as a rotation matrix:

$$\mathcal{O}_k^+ \mapsto \cos(m\theta)\ \mathcal{O}_k^+ + \sin(m\theta)\ \mathcal{O}_k^-,$$
$$\mathcal{O}_k^- \mapsto -\sin(m\theta)\ \mathcal{O}_k^+ + \cos(m\theta)\ \mathcal{O}_k^-.$$

Pair $(\mathcal{O}_k^+, \mathcal{O}_k^-)$ forms an irrep under L_θ, labelled by $[k] = [-k]$.

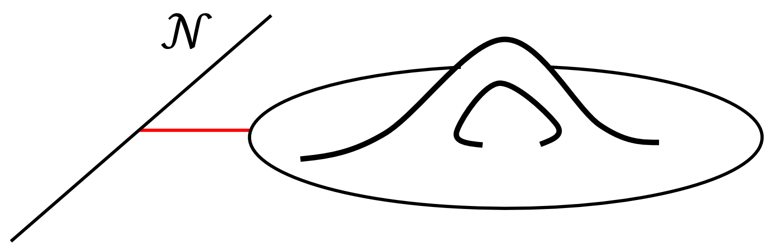
Fusion: $[k_1] \otimes [k_2] = [k_1 + k_2] \oplus [k_1 - k_2]$.

Selection rule: $\left\langle \mathcal{O}_{k_1}^\pm \ldots \mathcal{O}_{k_n}^\pm \right\rangle_{\mathbb{S}^2}$ nonzero implies $k_1 \pm \cdots \pm k_n = 0$, for some choice of signs. Conservation of **KK momentum up to sign**.

Breaking by String Loops

Selection Rules at Higher Genus?

Deriving selection rules at higher genus, non-invertible TDLs get caught on handles:



Handle **sources non-invertible symmetry charge**. View as mixed ‘t Hooft anomaly: nonzero source charge in presence of background topology.

Intuition: Only local operators run in loop, breaking symmetry between local and non-local operators.

General pattern: charges sourced at one loop break non-invertible symmetry to the **maximal invertible subgroup**.

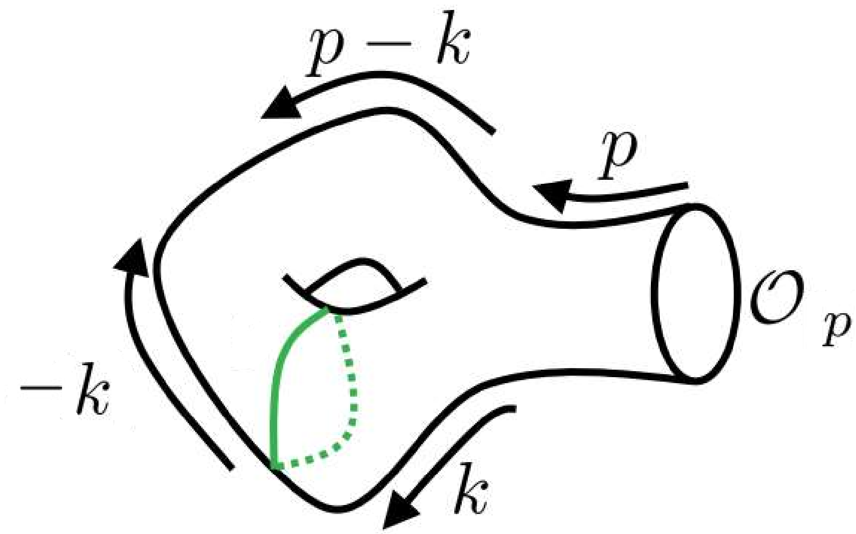
Selection Rules for $S^1/\mathbb{Z}_2$ at Loop Level

Consider torus one-point function $\langle \mathcal{O}_p^+ \rangle_{\mathbb{T}^2}$.

Orbifold $S^1/\mathbb{Z}_2$ involves sum over $\mathbb{Z}_2$ lines on both cycles.

Track flow of KK momentum through $\mathbb{T}^2$.

Learn: $\langle \mathcal{O}_p \rangle_{\mathbb{T}^2}$ can be nonzero if there exists p such that $p - k = k$, i.e., if $p = 2k$.



Remaining Selection Rule: Conservation of **KK momentum mod 2**. Non-invertible symmetry broken to $(\mathbb{Z}_2)_{\mathrm{KK}}$, maximal invertible subgroup.

Applications at $g_s \ll 1$

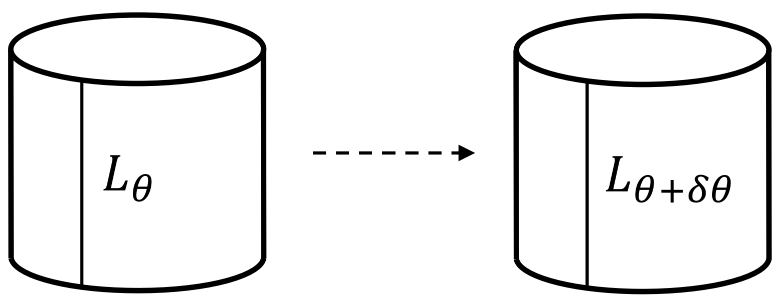
Tower of States for $S^1/\mathbb{Z}_2$ Decompactification

In decompactification limit of the S^1 σ-model, tower of light states is WGC tower for $U(1)_{\mathrm{KK}}$. What about $S^1/\mathbb{Z}_2$?

Non-invertible variant of spectral flow argument for Sublattice WGC. [Arkani-Hamed, Motl, Nicolis, Vafa '06; Heidenreich, Reece, Rudelius '16; Montero, G. Shiu, Soler '16; Heidenreich, Lotito '24]

Implies infinite tower of states (at string tree-level), charged under the non-invertible symmetry.

Intuition: Berry connection for continuous family of defect Hilbert spaces:



(Sub-)Lattice WGC and Non-Invertible Symmetry

Counterexamples to Lattice WGC take the form of orbifolds where one or more $U(1)$ symmetries are projected out. [Arkani-Hamed, Motl, Nicolis, Vafa '06; Heidenreich, Reece, Rudelius '16]

Ex: $\frac{T^3}{\mathbb{Z}_2\times\mathbb{Z}_2'}$, where orbifold action is

$$\mathbb{Z}_2:\ \mathrm{X} \to X+\pi, \qquad Y \to Y+\pi,$$
$$\mathbb{Z}_2':\ X \to -X, \qquad Z \to Z+\pi.$$

Always have **continuous non-invertible symmetries**, generalizing $S^1/\mathbb{Z}_2$.

Lattice sites without WGC states charged under non-invertible symmetry, spectrum generated via spectral flow.

Is there a **Non-Invertible Lattice WGC** (at least in asymptotic limit $g_s \ll 1$ where symmetry is restored)?

Conclusion

Summary

In this talk, I discussed the implications of **non-invertible worldsheet symmetry** for target space physics.

Punchline: these symmetries are **exact at tree-level** but are **broken by string loops to the maximal invertible subgroup**.

Still a useful tool in the perturbative regime $g_s \ll 1$. Connections to Distance Conjecture and WGC in cases where $U(1)$ symmetries are projected out.

Tip of the iceberg of applications of non-invertible worldsheet symmetries to the string Landscape. (See Thomas's talk, possibly?)

Thank you for listening!