Conifold Transitions and New N=1 Dualities

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String Phenomenology 2024

Based on work with James Gray and Callum Brodie
arXiv:2211.05804
And with JG, Sunit Patil, and Caoimhin Scanlon, 24xx.xxxx
and JG and Xingyang Yu, 24xx.xxxxx

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Motivating Questions

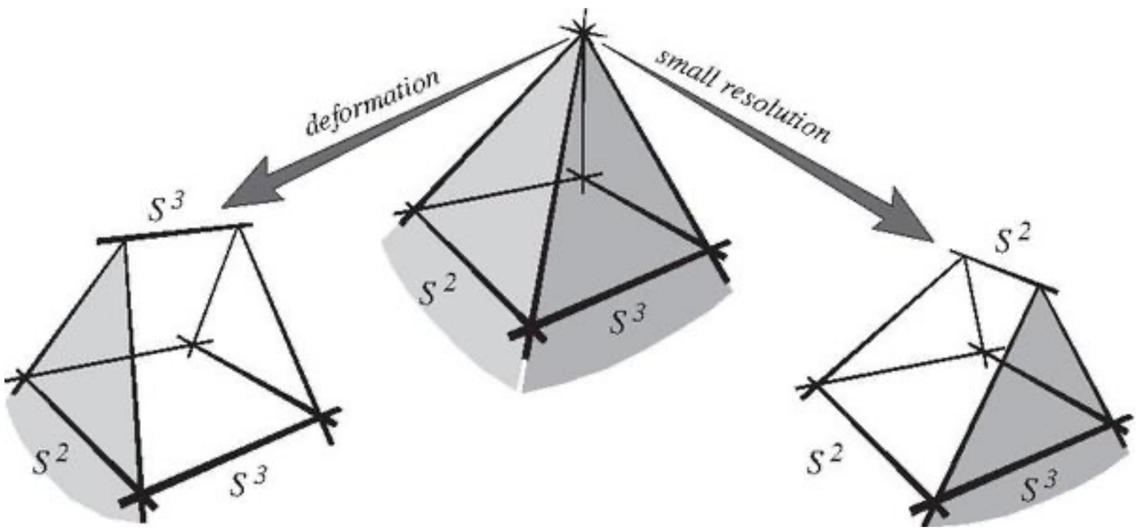
- In string compactifications:

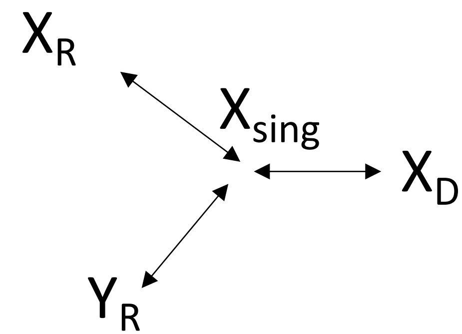
  which field theories <-> which geometries?

- We understand how to build QFTs that model nature (e.g. Standard Model of particle physics)
- What string compactifications (if any) produce vacua like our universe? What "geometry" matters?
- String theory contains topology changing transitions. Can we understand these in relevant contexts? (Dynamics?)

Geometric Transitions

- Topology changing transitions in CY 3-folds: Conifolds and Flops

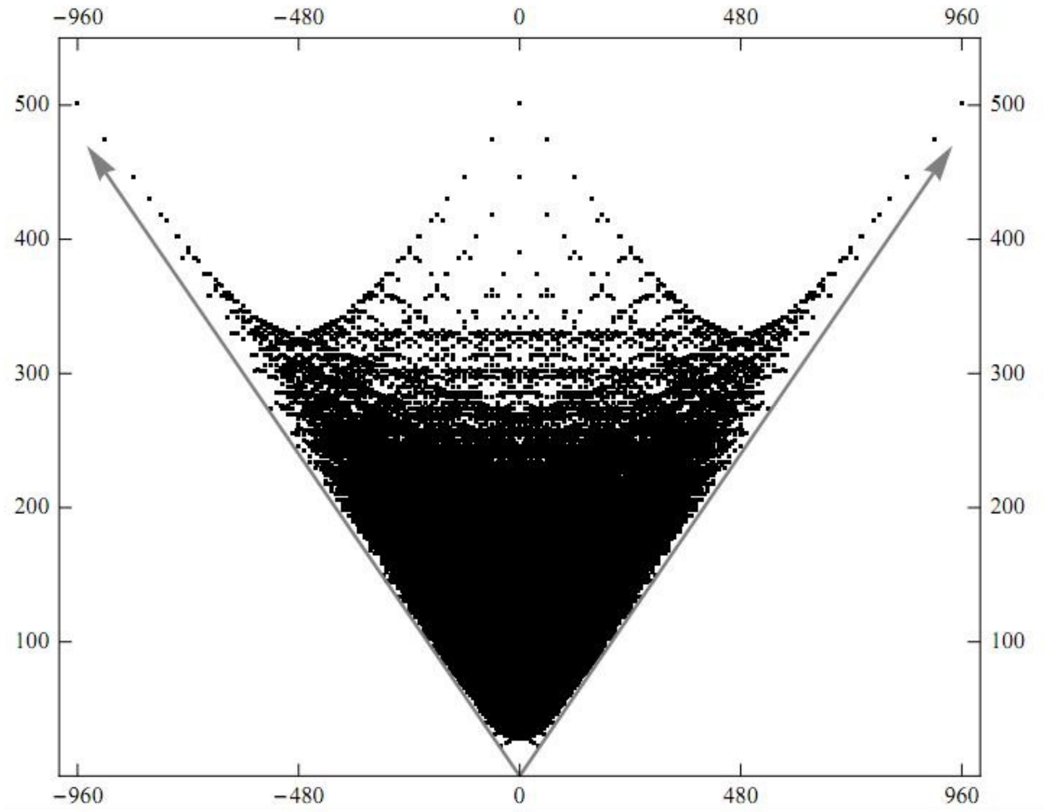




- At the level of CY geometry, these transitions are well understood.
- They are also well understood field-theoretically in some contexts (i.e. Type IIA/B) (Strominger, Greene+Morrison, etc)

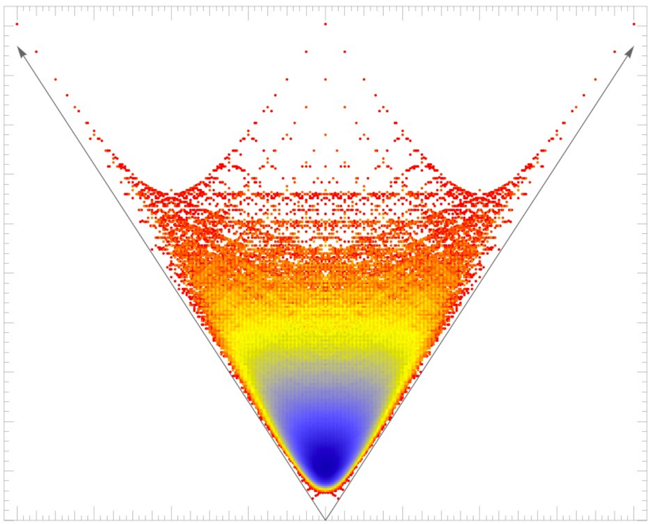
Geometric Transitions

- Connect (most) known CY manifolds
- All CY manifolds? (Reid)
- All SU(3) Structure manifolds??
- Key for attempts at bounding all CY or SU(3) structure manifolds

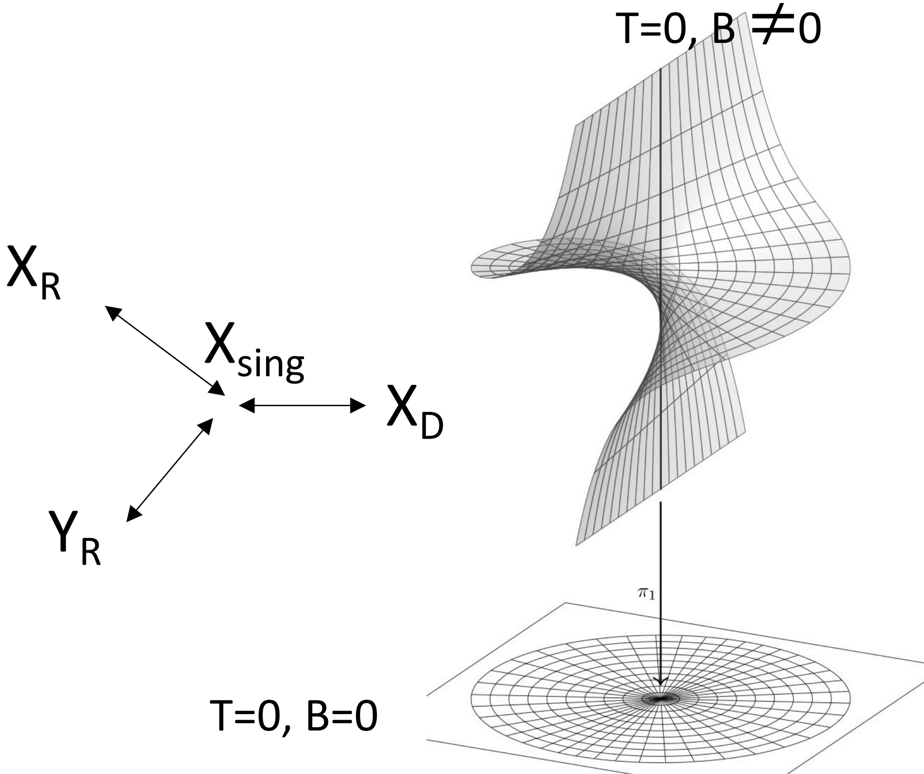


Geometric Transitions

String Duality <-> Geometric “Redundancy”



- We know in many contexts that more than one geometry can lead to the same EFT in string theory
- E.g. N=2 Theories
- Mirror Symmetry (Here Type IIB <-> IIA)
- Type IIB Flops



B-field allows for field space to stay smooth, despite

the CY singularity

Key questions for this talk

- Are there phenomena like mirror symmetry or "smooth" geometric transitions (i.e. flops) in the context of N=1 compactifications?
- Can we fully characterize what "geometry" matters for the N=1 compactifications (i.e. manifolds+ bundles/branes/fluxes)? Redundancy?
- Can we understand/characterize/bound topology changing transitions in the N=1 context?
- Difficulties:
    1. Notions of "moduli space" (For N=1: non-trivial superpotentials, etc).
    2. Intrinsically coupled problem between manifolds and other background geometry (bundles, branes,etc)

- Conifolds in the N=1 context?
- In heterotic string theory cannot ignore the gauge fields/bundle. The theory has a Bianchi Identity:

$$dH = -\alpha' \mathrm{tr} F \wedge F + \alpha' \mathrm{tr} R \wedge R$$

Three-form

Negative sources from gauge and positive sources from gravitational sectors

- Chern class condition:

$$[\mathrm{tr} R \wedge R] = [\mathrm{tr} F \wedge F] \Rightarrow c_2(\Omega_X) = c_2(V)$$

- With 5-branes: $c_2(\Omega_X) = c_2(V) + [C]$

- This means that we cannot set the gauge fields to zero.

  Need to understand the change in the bundle during a geometric transition in addition to the CY geometry itself. This has been a stumbling block for decades.

  Motivation:

  Try and understand what happens to the gauge fields in heterotic string theory during a certain type of geometric transition (a conifold).

See also Collins, Gukov, Picard, Yau math.DG/2102.11170 and Candelas, de la Ossa, He, Szendroi hep-th/0706.3134

Examples observed of N=1 “redundancy” (not descended from N=2)

- Known examples of heterotic compactifications with distinct geometry (i.e. pairs $(X, \pi: X \to V)$) that lead to the same massless spectrum
- E.g. (0,2) GLSM realizations (0,2) Target Space Duality (Distler/Kachru, Blumenhagen). (0,2) GLSMs that share a non-geometric vacuum

Calabi-Yau Defining Relations ⟷ Monad Maps

- Observation: base CY manifolds (X, X’) in these examples generically related by conifold transitions.

Distler and Kachru hep-th/9707198, Blumenhagen hep-th/9707198 and hep-th/9710021, Blumenhagen and Rahn 1106.4998, Anderson and Feng 1607.04628

- In an example:

Smooth quintic

$$X_D = \left[\begin{array}{c|c} \mathbb{P}^4 & 5 \end{array} \right]$$

Nodal quintic

Deform complex structure

$$X_0 = \left[\begin{array}{c|c} \mathbb{P}^4 & 5 \end{array} \right]$$

Blow up singular points of nodal quintic with (16) $\mathbb{P}^1 s$

$$X_R = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array} \right]$$

- These manifolds come equipped with bundles:

Deformation side bundle:

$$0 \to \mathcal{O}(-5) \to \mathcal{O}(-1)^{\oplus 5} \to V_D \to 0$$

Resolution side bundle:

$$0 \to \begin{array}{c} \mathcal{O}(-1,-5) \\ \oplus \\ \mathcal{O}(0,-4) \end{array} \to \begin{array}{c} \mathcal{O}(-1,0) \oplus \mathcal{O}(0,-5) \\ \oplus \\ \mathcal{O}(0,-1)^{\oplus 4} \end{array} \to V_R \to 0$$

What happens to the degrees of freedom of the theory?
E.g. Singlets:

Deformation side:

Kahler moduli: $h^{1,1} = 1$

Complex Structure: $h^{2,1} = 101$

Bundle Moduli: $h^1(\text{End}_0(V)) = 324$

Total = 426

Resolution side:

Kahler moduli: $h^{1,1} = 2$

Complex Structure: $h^{2,1} = 86$

Bundle Moduli: $h^1(\text{End}_0(V)) = 338$

Total = 426

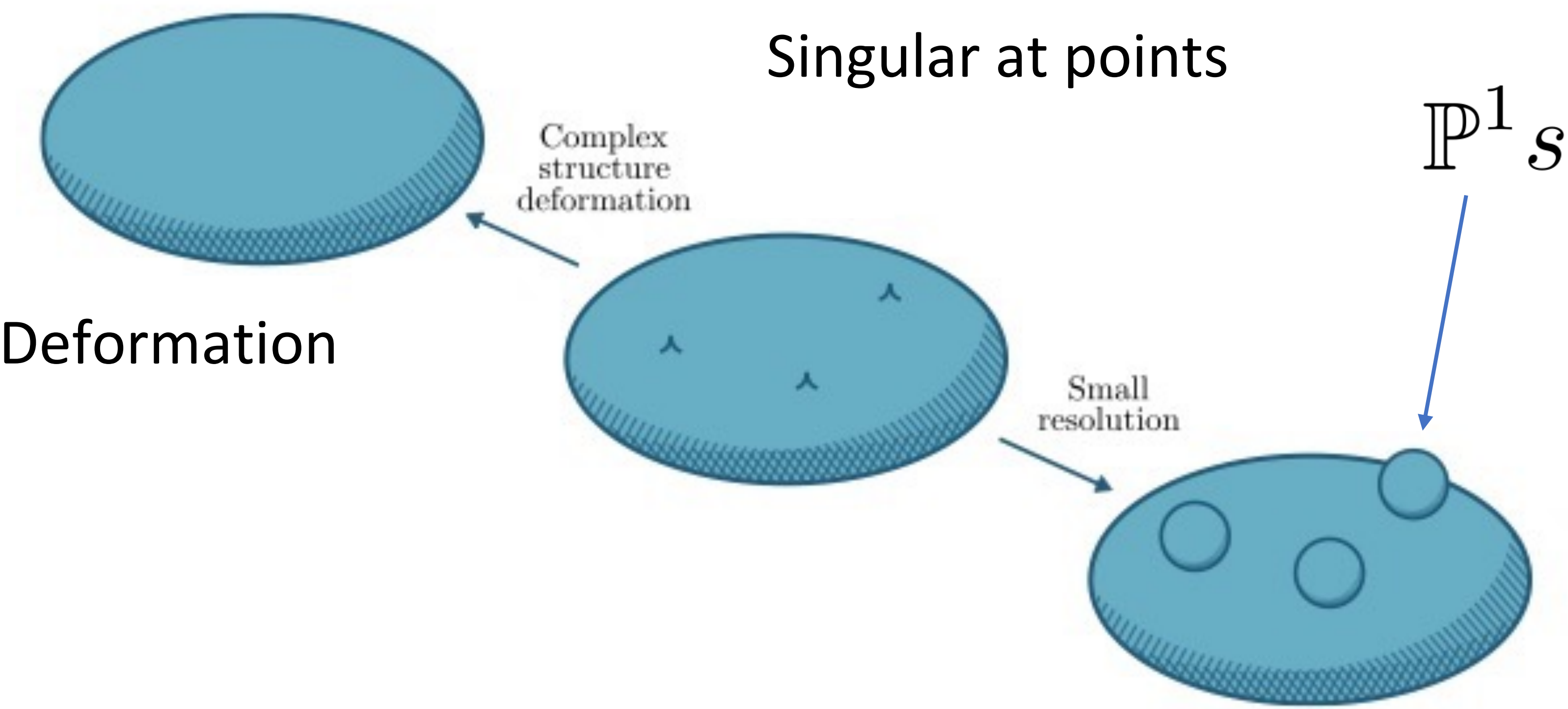
(SO(10) Gauge symmetry and charged matter also agrees)

Heterotic Conifold Transitions

- With Gray + Brodie, we pulled apart the geometry of examples like these and developed more general rules for how a bundle must adjust across a conifold to maintain a consistent heterotic theory.
- Contains (0,2) TSD, but more general.
- New features of CY conifold transitions
- Can be applied to other classes of N=1 compactifications (i.e. Type I, Type IIB orientifolds, F-theory, etc).
- Will give a heterotic overview and then talk about the more general effect.

Conifold transitions

- At the level of geometry we have schematically:



How do the tangent bundles change?

- Topology change:

$$ch_2(X_R) = ch_2(X_D) + [\mathbb{P}^1]$$

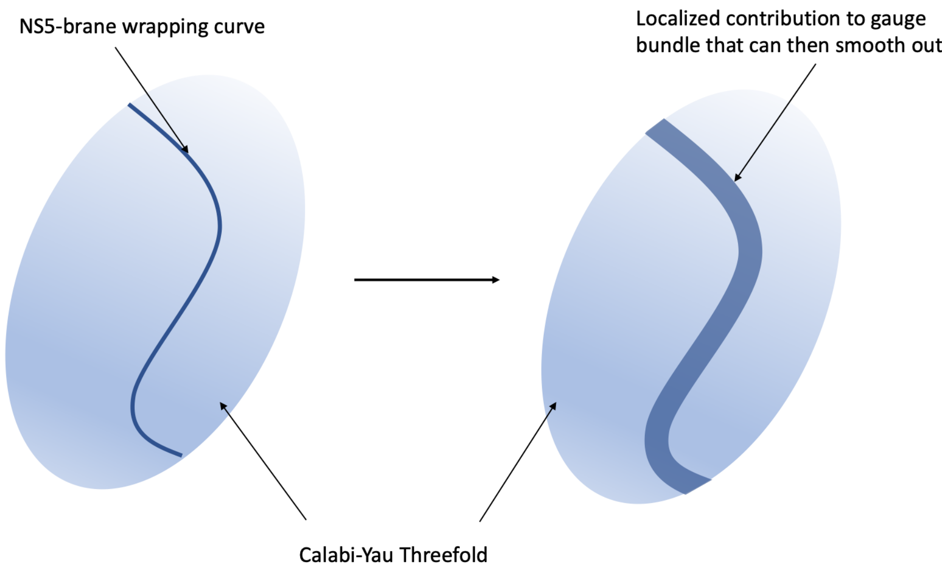
$$h^{1,1}(X_R) = h^{1,1}(X_D) + 1$$

$$h^{2,1}(X_R) = h^{2,1}(X_D) - \Delta$$

- As bundles on the resolution manifold

$$0 \to f^*\Omega_{X_0} \to \Omega_{X_R} \to \mathcal{O}_{\mathbb{P}^1 s}(-2) \to 0$$

Small contraction: $f : X_R \to X_0$



- This looks familiar: Small Instanton Transition (Hecke Transform)

Including a gauge bundle in the transition

- Recall the anomaly cancelation condition in heterotic string-theory:

$$c_2(\Omega_{X_R}) = c_2(V_R)$$

- How does this change during the transition?

$$c_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = c_2(V_R) + [\mathbb{P}^1 s]$$

This is how the gravitational sector changes given the transition we have seen in the cotangent bundle.

The gauge sector must change in the same way.

The full structure of the transition

- On the next slide is the map of the transition, presented at the level of classes for clarity.

- All of the sequences of sheaves required for this process to occur exist and can be written down explicitly.

- In what follows V_s is a “spectator bundle” which goes through the transition in a trivial manner.

$$\mathrm{c}_2(\Omega_{X_R}) = \mathrm{c}_2(V_R)$$

“Pair create” curve supported sheaves ↓ “Pair create” curve supported sheaves

$$\mathrm{c}_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = \mathrm{c}_2(V_R) + [\mathbb{P}^1 s]$$

SIT in cotangent bundle ↓ SIT in gauge bundle

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_R] + [\mathbb{P}^1 s]$$

↓ “Brane” recombination

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_D]$$

↓ SIT in gauge bundle

$$\mathrm{c}_2(f^*\Omega_{X_0}) = \mathrm{c}_2(V_D)$$

$$c_2(\Omega_{X_R}) = c_2(V_R)$$

Pair create curve supported sheaves | Pair create curve supported sheaves

$$c_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = c_2(V_R) + [\mathbb{P}^1 s]$$

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“Brane” recombination

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_D]$$

SIT in gauge bundle

$$c_2(f^*\Omega_{X_0}) = c_2(V_D)$$

Bridging Branes

- The spectra matching we find in the heterotic case is based on the properties of special NS 5-branes (C_D, C_R) that are linked to the conifold pair. We call these Bridging Branes.
- Geometric property: Curves in X_R which do not collapse in conifold, but intersect collapsing cycles at a point.
- Moduli of pure 5-brane theory: $h^{1,1}(X) + h^{1,1}(X) + h^0(C, N_C)$

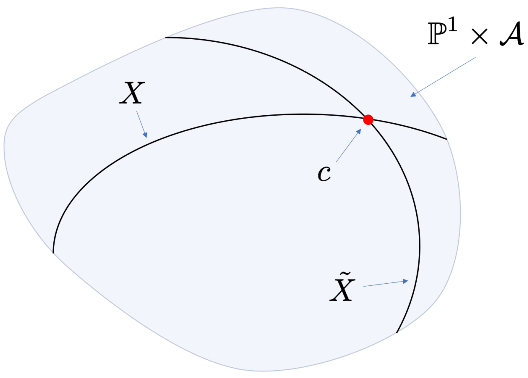
For Bridging Curves:

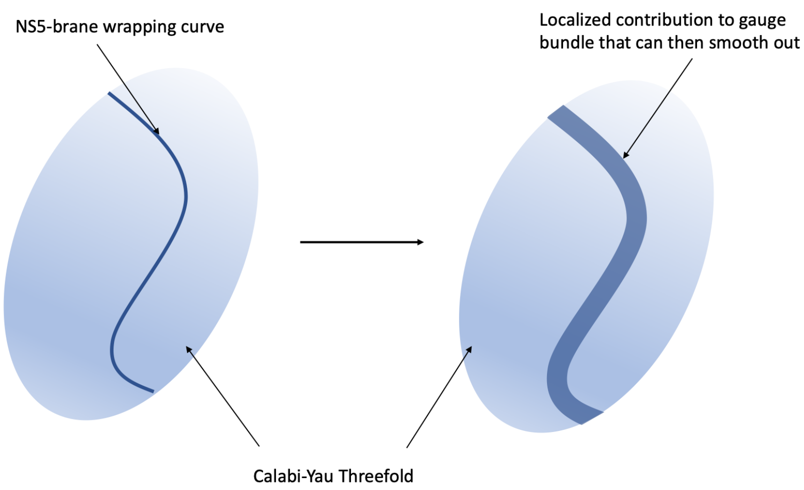
$h^0(C, N_C)$ adjusts across the conifold to exactly compensate for the change in hodge numbers! Every CY conifold determines a pair (C_D, C_R)

5-brane duality

$$\left[\ \mathcal{A}\ \middle|\ \mathbf{v}_0+\mathbf{v}_1\quad \mathbf{R}\ \right] \longleftrightarrow \left[\begin{array}{c|ccc} \mathbb{P}^1 & 1 & 1 & 0 \\ \mathcal{A} & \mathbf{v}_0 & \mathbf{v}_1 & \mathbf{R} \end{array}\right]$$

- Pure 5-brane heterotic theories identical
- For CICY 3-folds bridging curves for 5-brane correspond to the intersection of the two CY 3-folds in an ambient space.
- General heterotic conifold duality induced from this phenomenon by small instanton transitions
- Every CY 3-fold conifold pair contains bridging curves





Questions:

- We observe correlated geometry and matching massless spectra.
- Suggestion of a smooth moduli space?
- Is this a true duality? Or distinct theories with intersecting vacuum spaces?
- Can we use the geometry of these bridging branes in other string compactifications?

Testing a duality: Heterotic Superpotenials

- Explore whether the theories (not just their spectra) are identical? N=1 theories -> superpotential.
- Need a full mapping of fields to do this
- Perturbative Yukawa couplings
  - E.g. E6 theory, $\mathbf{27}^3$ coupling: $H^1(V) \times H^1(V) \times H^1(V) \to H^3(\wedge^3 V)$
  - Complex functions of bundle/complex structure moduli
- Non-perturbative terms $W \sim \sum Pfaff(c,b)_i e^{-T}$
  - Could obstruct the transition?
  - Agreement?

An E6 example:

Deformation side:

$$\left[\mathbb{P}^4 | 5\right]$$

$$0 \to V_D \to \mathcal{O}(1)^3 \oplus \mathcal{O}(2) \to \mathcal{O}(5) \to 0$$

Resolution side:

$$\left[\begin{array}{ccccccc|cc} y_0 & y_1 & y_2 & y_3 & y_4 & x_0 & x_1 & p_1 & p_2 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 & 4 & 4 \end{array}\right]$$

$$0 \to V_R \to \mathcal{O}(0,2)^2 \oplus \mathcal{O}(0,1) \oplus \mathcal{O}(1,0) \to \mathcal{O}(1,5) \to 0$$

Family structure and Yukawa Couplings

- Deformation side:

Equivalence class describing families ($H^1(V_D)$):

$$P_5 \sim P_5 + \alpha p_{(5)} + \sum_{a=1}^{3} \beta_a q^a_{(4)} + \gamma c_{(3)}$$

Equivalence class describing Yukawa coupling ($H^3(\wedge^3 V_D)$):

$$P_{15} \sim P_{15} + A p_{(5)} + \sum_{a=1}^{3} B_a q^a_{(4)} + D c_{(3)}$$

There is one equivalence class of degree 15 polynomials of this type. Multiply degree 5 polynomials together and compare to a degree 15 representative to obtain coupling.

- Resolution side:

Equivalence class describing families ($H^1(V_R)$):

$$P_{1,5} \sim P_{1,5} + \tilde{\alpha} p_{(0,5)} + \sum_{a=1}^{3} \beta_a q^a_{(1,4)} + \sum_{i=1}^{2} \gamma^i c^i_{(1,3)}$$

without loss of generality take one of the $c_{(1,3)}$ maps to be x_0 . Then this becomes equivalent to:

$$x_1 P_{0,5} \sim x_1 P_{0,5} + x_1 \alpha p_{(0,5)} + x_1 \sum_{a=1}^{3} \beta_a q^a_{(0,4)} + x_1 \gamma c_{(0,3)}$$

There is then an obvious mapping between these family equivalence classes and those for the quintic.

Equivalence class describing Yukawa coupling ($H^3(\wedge^3 V_R)$):

$$P_{3,15} \sim P_{3,15} + \tilde{A} p_{(0,5)} + \sum_{a=1}^{3} \tilde{B}_a q^a_{(1,4)} + \sum_{i=1}^{2} \tilde{D}_i c^i_{(1,3)}$$

Maps are same as for families so one of the $c_{(1,3)}$ maps has the same effect

$$x_1^3 P_{0,15} \sim x_1^3 P_{0,15} + x_1^3 A p_{(0,5)} + x_1^3 \sum_{a=1}^{3} B_a q^a_{(0,4)} + x_1^3 D c_{(0,3)}$$

At this stage we can see that the entire resolution side Yukawa computation is identical to the deformation side one, multiplied by spectator factors of x_1

Matching superpotentials

In this example we find that all 147,440 independent, non-vanishing, Yukawa couplings correctly match as holomorphic functions of the moduli on either side of the duality (95 families in this case).

- More generally, for instanton contributions, we can prove
- Potentially obstructing terms

$$W \sim \sum \psi_i e^{-T_{new}}$$

vanish

- Remaining non-perturbative terms have non-trivial matching of Pfaffians, moduli dependence (work in progress)

Type IIB orientifolds (D5/O5)

- CY3 Conifold pair

$$\left[\begin{array}{c|ccccc} \mathbb{P}^4_y & 1 & 1 & 1 & 1 & 1 \\ \mathbb{P}^4_z & 1 & 1 & 1 & 1 & 1 \end{array}\right]_{2,52} \leftrightarrow \left[\begin{array}{c|cccccc} \mathbb{P}^1_x & 1 & 1 & 0 & 0 & 0 & 0 \\ \mathbb{P}^4_z & 1 & 0 & 1 & 1 & 1 & 1 \\ \mathbb{P}^4_z & 0 & 1 & 1 & 1 & 1 & 1 \end{array}\right]_{3,47}$$

- Orientifold actions

$$(y_0 \to -y_0 \quad , \quad z_1 \to -z_1) \leftrightarrow (x_0 \to -x_0 \ , \ y_0 \to -y_0 \ , \ z_1 \to -z_1)$$

- D5s (bridging curves) and O5s (by action above) satisfy tadpoles
- Spectrum agrees

closed string d.o.f.	
chiral multiplets	$h^{1,1}_+ + h^{1,1}_- + h^{2,1}_+ + 1$
vector multiplets	$h^{2,1}_-$
open string d.o.f.	
chiral multiplets	$h^0_+(C, N_C) + h^{1,0}(C)$
vector multiplets	1

Further Topics

- We are developing new and intrinsically N=1 dualities(?)
- Not dual “pairs” of theories, but long chains.
- Of clear relevance for model building/scans. Large redundancy of theories
- Embedding of theories (low $h^{1,1}$—> high $h^{1,1}$) is potentially powerful.
- What geometry matters? N=1 “Wall’s Data”?

- Work to appear (ask me if curious):
- What about N=1 F-theory/CY 4-folds?
- What is the effective physics of such apparently “smooth” N=1 conifolds?