Towards string perturbation theory in Ramond backgrounds

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Based on Minjae Cho, MK 2311.04959

String Phenomenology 2024

Summary of the talk

- Many interesting backgrounds in string theory involve Ramond-Ramond fluxes

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- We found background solutions in SFT that corresponds to GKP type flux compactifications
- With this "worldsheet" description, we can now compute stringy amplitudes in flux backgrounds
- e.g., direct computations of α' and g_s corrections to the effective action in flux backgrounds

Plan of the talk

- Why string field theory?
- What is string field theory?
- Review of GKP
- SFT for GKP with small flux superpotential
- Conclusions

Chapter 0: Why string field theory?

String perturbation theory in RR backgrounds

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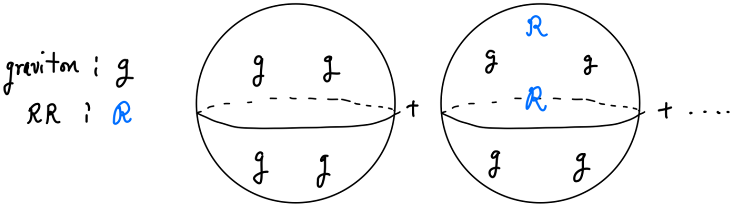
- Therefore, it is hard to make sense of the deformed CFT
- But, this does not yet imply that we cannot compute amplitudes in background field method

String perturbation theory in RR backgrounds

- Because scattering amplitudes involving RR fields are well defined, one can still attempt to compute scattering amplitudes in RR backgrounds with the background field method

- Let's try to formulate four-graviton amplitude in RR backgrounds at string tree-level



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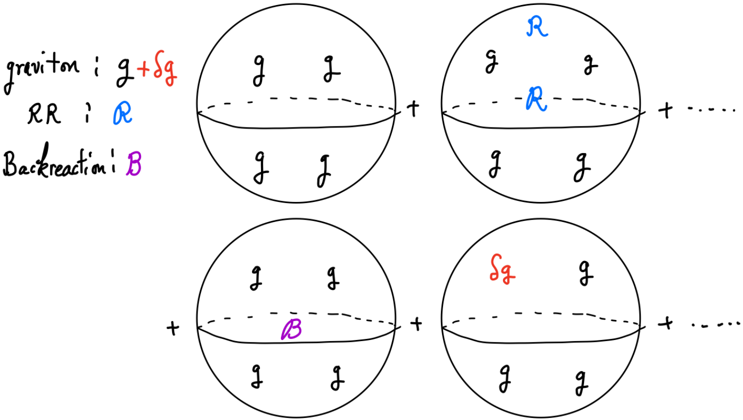

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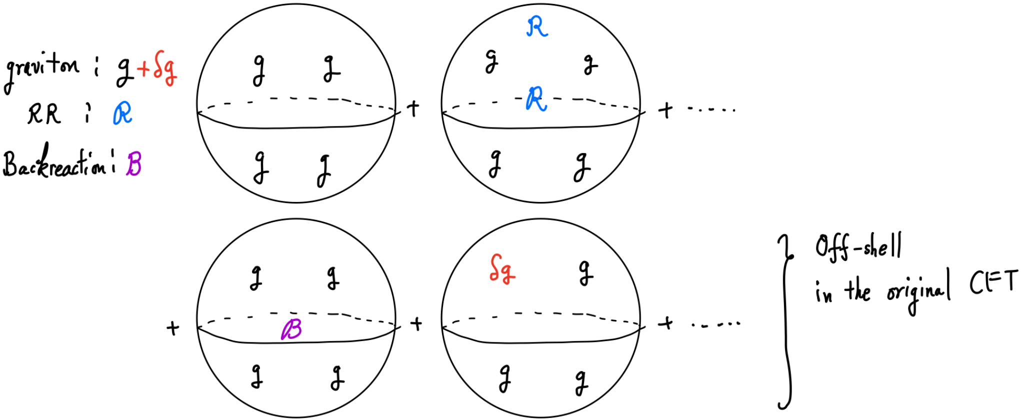

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- If we can make sense of off-shell amplitudes in string theory, we can understand RR backgrounds.
- We should use string field theory!

Chapter 1: What is string field theory?

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- It has not yet been shown SFT is the right approach to formulate non-perturbative string theory

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- On the other hand, SFT gives a systematic prescription to handle the above problems Cho, Collier, Yin 18, Sen 20, 21, Alexandrov, Sen, Stefanski 21, Agmon, Balthazar, Cho, Rodriguez, Yin 22, Eniceicu, Mahajan, Murdia, Sen 22, Alexandrov, Hilmi Fırat, MK, Sen, Stefanski 22, Alexandrov, Mahajan, Sen 23, Cho, Mazel, Yin 23, Mazel, Sandor, Wang, Yin 24..

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- Importantly, to do perturbative calculations with SFT, one does not need much more than Polchinski

  “Anyone who’s taken a string theory class with Polchinski can do it”

  \- Minjae Cho (paraphrased)

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- In usual string perturbation theory, on shell states are constructed as

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- One can construct string field Ψ, by

$$\Psi = Tc\bar{c}e^{ik\cdot X} + \epsilon_{\mu\nu}c\bar{c}\partial X^\mu \bar{\partial} X^\nu e^{ik\cdot X} + \ldots\,,$$

  where polarizations are now taken as string fields.

- Crucially, in SFT, on-shell condition is not imposed and k can take an arbitrary value.

What is string field theory?

- With the string field, the goal is to construct an off-shell action

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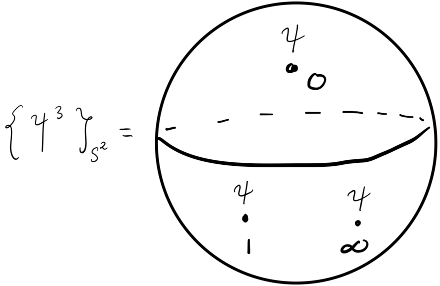
- What about interaction vertices?

- The idea is to read off Feynmann vertices from off-shell scattering amplitudes

Three-point vertex

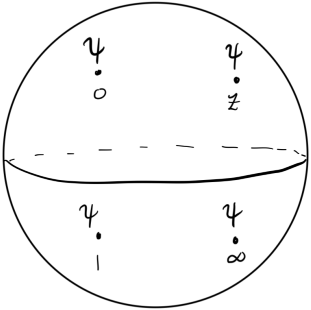
- The three point vertex is determined by the following off-shell amplitude



- $\{\Psi^3\}$ is a complicated function of polarization/string fields.

Four-point vertex

- To compute the four-point vertex, we need to do a little more work.
- Let's first compute four-point amplitude



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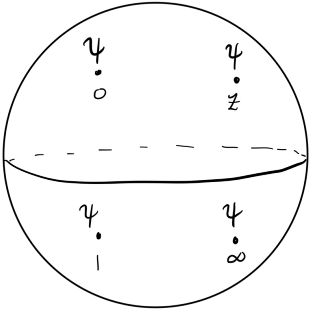

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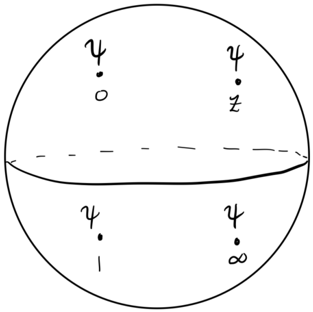
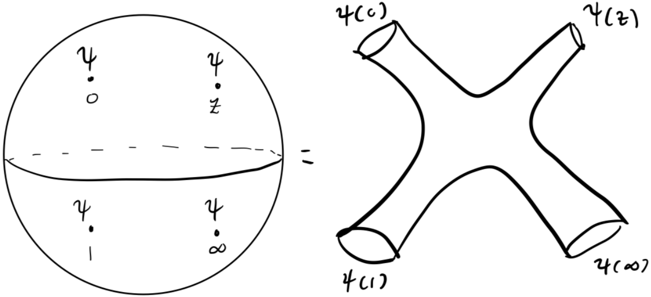


- We expect that some contributions to the four-point amplitude come from joining three-point vertices
- The goal is to isolate the contribution that comes purely from the four-point vertex

Four-point vertex

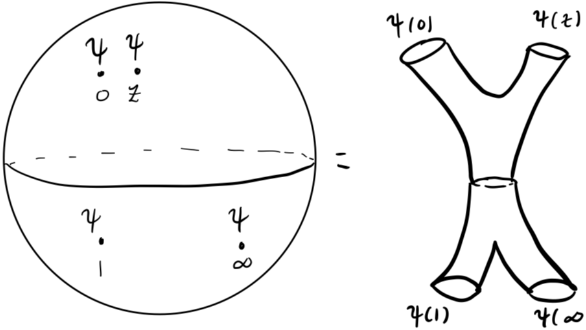
- We can put z at a generic point



- For generic z, we have a four-point vertex contribution

Four-point vertex

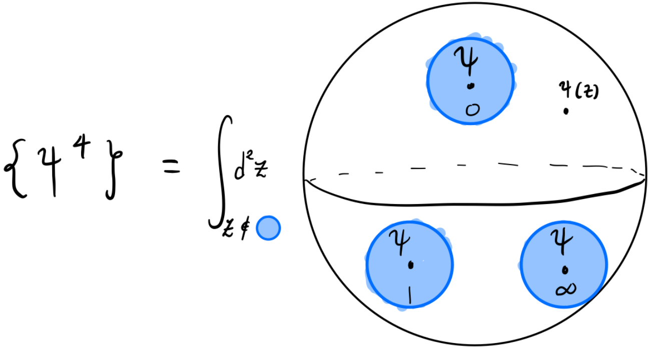
- We can bring z to 0



- When z is close to 0, we have t-channel

Four-point vertex

- To find the four-point vertex contribution, we can excise local coordinate charts around $0,\ 1,\ \infty$
- and integrate over z away from the blue regions



- Different choices of local coordinates correspond to field redefinitions

What is string field theory?

- Finally, we have constructed string field action

$$S(\Psi) = -\frac{1}{2g_s^2}\langle\Psi|c_0^- Q_B|\Psi\rangle + \sum_{N,g}\frac{g_s^{2-2g+N}}{N!}\{\Psi^N\}_{\Sigma_g}\,.$$

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- In essence, SFT as we know is a self-consistent set of rules that allows off-shell computations in string perturbation theory
- The SFT action involves infinitely many terms for infinitely many field. So, we should carefully choose a problem

Chapter 2: Review of GKP background.

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- The low-energy action contains the following terms

$$S_{bulk} \supset -\frac{1}{4\kappa_{10}^2}\int_{\mathbb{R}^{1,3}\times X/\mathcal{I}} d^{10}X\sqrt{-G}\left(\frac{|H_3|^2}{g_s^2}+|F_3|^2\right)\,,\ S_{D3/O3} \supset \sum_i -\mu_3 Q_i \int_{\mathbb{R}^{1,3}} d^4x\sqrt{-G}\frac{1}{g_s}$$

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- One can massage the above equations to obtain

$$S \supset -\frac{1}{2\kappa_{10}^2}\int_{\mathbb{R}^{1,3}} d^4X\left[\int d^6X\sqrt{-G}\frac{G_-\cdot\bar{G}_-}{\mathrm{Im}\tau}\right]\,,$$

$$\int_{X/\mathcal{I}} H\wedge F + N_{D3} = Q_{D3}\,,\ G_3 := F_3 - \frac{i}{g_s}H_3\,,\ G_- := G_3 + i\star_6 G_3\,.$$

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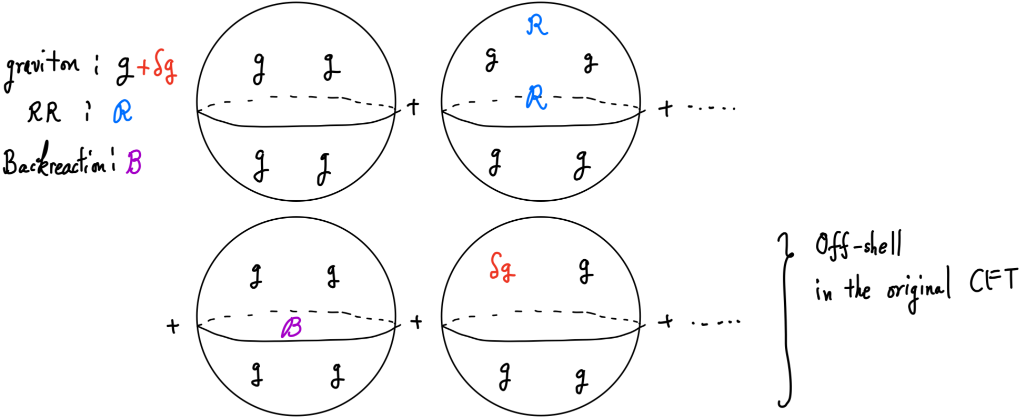
- G_- vanishes if G_3 is a linear combination of complex $(2,1)\oplus(0,3)$ forms.
- Therefore, quantized fluxes H_3 and F_3 induce potential for z and $1/g_s$.
- At the minimum of the potential, one finds

$$-\star_6\frac{H_3}{g_s} = F_3\,.$$

Chapter 3: SFT for GKP.

Goal

- Today we will find the background solution $\equiv B$ in string field theory for GKP backgrounds
- and show that vacua with small flux superpotential admit double scaling expansion



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- We want to turn on *quantized* fluxes $F_3,\ H_3$ in SFT to find a nearby vacuum

Puzzle

- To find GKP solution in SFT we need to ensure that we can treat quantized fluxes as a small perturbation

$$\delta\Psi = c\bar{c}H_{ijk}Y^i e^{-\phi}\psi^j e^{-\bar{\phi}}\bar{\psi}^k + g_s c\bar{c}e^{-\phi/2}\Sigma_\alpha F^{\alpha\beta}e^{-\bar{\phi}/2}\overline{\Sigma}_\beta\,.$$

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- H_{ijk} and $F^{\alpha\beta}$ are quantized fluxes. So, we cannot treat them as small numbers.
- Naively, this seems to suggest that we cannot treat quantized fluxes as a perturbation.
- Then, string field theory is practically useless in the context of flux compactifications.

Resolution

- Let's look at OPEs of the worldsheet fields

$$Y^i(x)Y^j(0) \sim -\frac{\alpha'}{2}G^{ij}(z)\log|x|^2\,,\ \psi^i(x)\psi^j(0) \sim \frac{G^{ij}(z)}{x}$$

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- Following Demirtas, MK, McAllister, Moritz 19 (PFV), one can choose H and F such that

$$\mathcal{O}\left(H_{ijk}Y^i e^{-\phi}\psi^j e^{-\bar{\phi}}\bar{\psi}^k\right) = \mathcal{O}(z^{-1/2})\,,\;\; \mathcal{O}\left(g_s e^{-\phi/2}\Sigma_\alpha F^{\alpha\beta}e^{-\bar{\phi}/2}\overline{\Sigma}_\beta\right) = \mathcal{O}(g_s z^{1/2})$$

c.f., Cicoli, Licheri, Mahanta, Maharana 22

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- By taking the following double scaling expansion

$$g_s \to 0\,,\ z^{-1} \to 0\,,\ zg_s = fixed$$

we can treat $\delta\Psi$ as a small perturbation

Solving EOM perturbatively

- We call the following double scaling expansion the ϵ expansion

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as we treat $\mathcal{O}(g_s) = \mathcal{O}(z^{-1}) = \mathcal{O}(\epsilon)$.

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- In this talk, we will study eom up to the second order

$$Q_B|\Psi_1\rangle = 0\,,$$

$$Q_B|\Psi_2\rangle = \frac{1}{2}\left[\Psi_1^2\right]_{S^2} + []_{D^2+\mathbb{RP}^2}\,,$$

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- Let's now study the second-order eom

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- Let's define a projection operator $\mathbb{P}$ that projects states to $L_0^+ := L_0 + \bar{L}_0$ nilpotent (massless) states
- Then we can find two independent equations

$$Q_B\mathbb{P}|\Psi_2\rangle = \frac{1}{2}\mathbb{P}\left[\Psi_1^2\right]_{S^2} + \mathbb{P}[]_{D^2+\mathbb{RP}^2}$$

$$Q_B(1-\mathbb{P})|\Psi_2\rangle = \frac{1}{2}(1-\mathbb{P})\left[\Psi_1^2\right]_{S^2} + (1-\mathbb{P})[]_{D^2+\mathbb{RP}^2}$$

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- Let's study the massive part of the second-order eom

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$$(1-\mathbb{P})|\Psi_2\rangle = \frac{b_0^+}{L_0^+}\left[\frac{1}{2}(1-\mathbb{P})\left[\Psi_1^2\right]_{S^2} + (1-\mathbb{P})[]_{D^2+\mathbb{RP}^2}\right]$$

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- Note that b_0^+/L_0^+ corresponds to the Green's function in target space.

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- Let's study the L_0^+ nilpotent part of the second-order eom

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- Because Q_B is not an invertible operator for L_0^+ nilpotent states, one needs to do an actual work here.
- After CFT gymnastics, at the F-term minimum, one arrives at

$$\begin{aligned}\frac{4\alpha'}{g_c^2}\mathbb{P}(\Psi_2)_{NSNS} = &-\frac{\pi}{18\kappa_{10}^2 g_s^2 \epsilon} c\bar{c}\Big(B_{ab}B^{ab}(\eta\bar{\partial}\bar{\xi}e^{-2\bar{\phi}} - \partial\xi\bar{\eta}e^{-2\phi}) - 2B_{ac}B^{cb}e^{-\phi}\psi^a e^{-\bar{\phi}}\bar{\psi}_b \\ &- 2i\sqrt{\frac{\alpha'}{2}}B_{ab}H^{abc}(\partial c + \bar{\partial}\bar{c})\left(e^{-\phi}\psi_c e^{-2\bar{\phi}}\bar{\partial}\bar{\xi} + e^{-\bar{\phi}}\bar{\psi}_c e^{-2\phi}\partial\xi\right)\Big).\end{aligned}$$

- Existence of the solution to low-energy SUGRA is not a sufficient condition for the existence of the SFT background.

Conclusions

Take home messages

- String field theory provides a systematic framework to study generic backgrounds.
- Provided that sugra solutions are well controlled, finding SFT counterpart isn't very difficult.
- Using the background solution in SFT, one can now compute string amplitudes in RR backgrounds
- e.g., α' and g_s corrections in the flux backgrounds, or more ecenonomic choice is to extend SFT solutions to higher orders. (c.f., talk by Liam McAllister and Andreas Schachner)
- The rules of the computations are not completely known. Opportunities for investigations.

Future directions

- We are computing killing spinor equations to extend the solutions to higher orders Minjae Cho, MK 24xx.xxxxx
- One-loop graviton amplitudes in orientifold compactifications W. I. P.
- One can also study flux compactifications in type IIA, (non-supersymmetric) heterotic string theories.
- Probably there are many more exciting directions! If you are interested, let's chat!