Heterotic asymmetric orbifolds revisited

Anamaría Font V.

Facultad de Ciencias, Universidad Central de Venezuela

In collaboration with:

G. Aldazabal, E. Andrés, K. Narain, I. Zadeh. To appear.

Aims and motivation

- go beyond geometric compactifications

  * orbifold action can include elements of duality group

  * moduli stabilization (action exists at special values of moduli)

  * new ground to look for universal properties and test swampland conjectures

- provide explicit world-sheet description, using methods developed for heterotic non-supersymmetric $\mathbb{T}^3/\mathbb{Z}_2$ asymmetric orbifolds Acharya, Aldazabal, AF, Narain, Zadeh '22

  e.g. for heterotic on $\mathbb{T}^4/\mathbb{Z}_M$, with 8 and 16 supercharges

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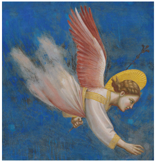
* Some recent work on asymmetric heterotic orbifolds:

Groot-Nibbelink, Vaudrevange '17	Faraggi, Groot-Nibbelink, Percival '23
Harvey, Moore '18	Baykara, Hamada, Tarazi, Vafa '23
Groot-Nibbelink '21	Baykara, Tarazi, Vafa '24
Baykara, Harvey '21	Baykara, Tarazi, Vafa '24



Outline

- Aims and Motivation ✓
- Basics of heterotic asymmetric orbifolds
- Heterotic on $\mathbb{T}^4/\mathbb{Z}_2$ with 8 supercharges
- Heterotic on $\mathbb{T}^4/\mathbb{Z}_K$ with 16 supercharges and rank reduction
- Summary and Outlook

Basics of heterotic asymmetric orbifolds

Heterotic in 10 dim

Gross, Harvey, Martinec, Rohm '85

- R-movers (superstring) $+$ L-movers (bosonic)

  $\psi_R^M,\ X_R^M,\ \ M = 0, \ldots, 9$ $\qquad$ $X_L^M,\ Y_L^{\mathrm{I}},\ \ \mathrm{I} = 1, \ldots, 16$

- modular invariance $\Rightarrow$ Y_L^{I} must live on a 16-dim torus with even selfdual lattice

- only 2 such lattices

  $\Gamma_8 \oplus \Gamma_8$, for the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic

  Γ_{16}, for the $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic

Γ_8 : root lattice of E_8

$$\Gamma_{8q} = \left\{ (m_1, \ldots, m_{8q}), (m_1 + \tfrac{1}{2}, \ldots, m_{8q} + \tfrac{1}{2}) | m_k \in \mathbb{Z}, \sum_{k=1}^{8q} m_k = \text{even} \right\}$$

Compactification on $\mathbb{T}^d$ and the lattice $\Gamma(16+d,d)$ Narain '86

▷ R-movers (superstring) $\quad + \quad$ L-movers (bosonic)

ψ_R^μ, X_R^μ, $\mu = 0, \ldots, 9\text{-}d$	X_L^μ
ψ_R^j, X_R^j, $j = 1, \ldots, d$	X_L^j, Y_L^{I}, $\mathrm{I} = 1, \ldots, 16$

▷ modular invariance $\Rightarrow$ $(Y_L^{\mathrm{I}}, X_L^j; X_R^j)$ must have momenta in an even selfdual lattice $\Gamma(16+d,d)$, signature $(16+d,d)$

▷ infinite such lattices, parametrized by $d(16+d)$ continuous moduli

background values of metric g_{ij}, Kalb-Ramond field b_{ij}, Wilson lines A_i^{I}

Narain, Sarmadi, Witten '86

▷ $\Gamma(16+d,d)$ lattice vectors = momenta of $(Y_L^{\mathrm{I}}, X_L^j; X_R^j) = (P_L; P_R)$

▷ toroidal partition function

see e.g. Blumenhagen, Lüst, Theisen

$$\mathcal{Z} = \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^{8-d}} \times \frac{1}{2\,\bar{\eta}^4}\left[\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4 - \bar{\vartheta}_2^4 + \bar{\vartheta}_1^4\right] \times \frac{1}{\eta^{16+d}\bar{\eta}^d} \sum_{(P_L,P_R)\in\Gamma(16+d,d)} q^{\frac{1}{2}P_L^2}\,\bar{q}^{\frac{1}{2}P_R^2}$$

▷ $(10-d)$-dimensional theory with 16 supercharges

massless gravity multiplet (with d graviphotons)

massless gauge multiplets of $\mathrm{U}(1)^{16+d}$ at generic moduli

supersymmetry breaking, rank reduction $\longrightarrow$ asymmetric orbifolds

Asymmetric orbifolds 1

Narain, Sarmadi, Vafa '87

▷ in closed strings orbifold action can be different on L and R

▷ need to specify orbifold action on $\Gamma(16+d,d) \equiv \Gamma$

go to pt in moduli space where Γ admits authomorphism Θ not mixing L and R

Invariant lattice: $\mathrm{I} = \{ P \in \Gamma \,||\, \Theta P = P \}$

Normal lattice: $\mathrm{N} = \{ P \in \Gamma \,||\, P \cdot X = 0, \forall X \in \mathrm{I} \}$ = orthogonal complement of I in Γ
a.k.a. coinvariant lattice

$$\mathrm{N}^*/\mathrm{N} = \mathrm{I}^*/\mathrm{I}$$

$$\forall P \in \Gamma, \quad P = (P_\mathrm{N}, P_\mathrm{I}), \; P_\mathrm{N} \in \mathrm{N}^*, \; P_\mathrm{I} \in \mathrm{I}^*$$

$$\Gamma = (\mathrm{N}, \mathrm{I}) + \coprod_{w \in \mathrm{N}^*/\mathrm{N}} (w, \varsigma(w)) \qquad \varsigma : \mathrm{N}^*/\mathrm{N} \to \mathrm{I}^*/\mathrm{I}$$

glue vectors $(w, \varsigma(w))$ have even norm and integer scalar product with each other

Recall orbifold partition function:

- in Abelian $\mathbb{Z}_K$ orbifolds with generator g

$$\mathcal{Z}(\tau,\bar\tau) = \sum_{\ell=0}^{K-1}\left[\frac{1}{K}\sum_{m=0}^{K-1}\mathcal{Z}(g^\ell, g^m)\right] \; ; \qquad \mathcal{Z}(g^\ell, g^m) = \mathrm{Tr}_{\mathcal{H}_\ell}\Big(g^m\, q^{L_0}\, \bar q^{\bar L_0}\Big)$$

$\mathcal{H}_\ell$: g^ℓ-twisted Hilbert space

$$X(t,\sigma+2\pi) = g^\ell X(t,\sigma)$$

- sum over ℓ is a sum over twisted sectors
- sum over m enforces the orbifold projection: $\displaystyle\frac{1}{K}\sum_{m=0}^{K-1} g^m$ inserted in trace
- double sum $\Leftrightarrow$ modular invariance

Asymmetric orbifolds 2

Narain, Sarmadi, Vafa '87

- orbifold generator g

  on Γ, $\quad g|P_{\rm N},P_{\rm I}\rangle = e^{2i\pi P_{\rm I}\cdot v}|\Theta P_{\rm N},P_{\rm I}\rangle$, $\qquad v$: constant shift along $\rm I$ directions

  g on $(Y_L,X_L;X_R)$ along $\rm N$ given by $\Theta=(\Theta_L,\Theta_R)$

  g on ψ_R given by Θ_R to preserve world-sheet supersymmetry

  defines action in untwisted sector $\longrightarrow$ $\mathcal{Z}(\mathbb{1},g)=\mathrm{Tr}_{\mathcal{H}_0}\Big(g\,q^{L_0}\,\bar q^{\bar L_0}\Big)\supset \sum_{P\in{\rm I}} q^{\frac12 P_L^2}\,\bar q^{\frac12 P_R^2}\,e^{2i\pi P\cdot v}$

- modular transformations $\to$ action on twisted sectors, e.g.

$$\mathcal{Z}(\mathbb{1},g)\quad\xrightarrow{\tau\to-\frac1\tau}\quad\mathcal{Z}(g,\mathbb{1})\supset\sum_{P\in{\rm I}^*} q^{\frac12(P+v)_L^2}\,\bar q^{\frac12(P+v)_R^2}$$

- modular invariance puts conditions on g

$$\mathcal{Z}(g,\mathbb{1})\quad\xrightarrow{\tau\to\tau+K}\quad\mathcal{Z}(g,g^K)\equiv\mathcal{Z}(g,\mathbb{1})\quad\text{for } g^K=\mathbb{1}$$

Heterotic on $\mathbb{T}^4/\mathbb{Z}_2$ with 8 supercharges

Setup

- $\mathbb{Z}_2$: reflection of s from 20 L-movers and 4 R-movers (superstring sector)
  $r = (20 - s)$ L-movers are invariant

- on (bosonized) world-sheet fermions, with r an $\mathrm{SO}(8)$ weight

  $$\mathbb{Z}_2 : \ |r\rangle \to e^{-2\pi i r\cdot v_f}|r\rangle, \qquad v_f = (0, 0, \tfrac{1}{2}, -\tfrac{1}{2})$$

  breaks half supersymmetries

- on $\Gamma(20, 4)$, $\mathbb{Z}_2 : \ \Theta(P_{\mathrm{N}}, P_{\mathrm{I}}) = (-P_{\mathrm{N}}, P_{\mathrm{I}})$, $\qquad \det'(1 - \Theta) = 2^s 2^4$

  invariant lattice I, signature $(r, 0)$,

  normal lattice $\mathrm{N} = \mathrm{I}^{\perp}$, signature $(s, 4)$

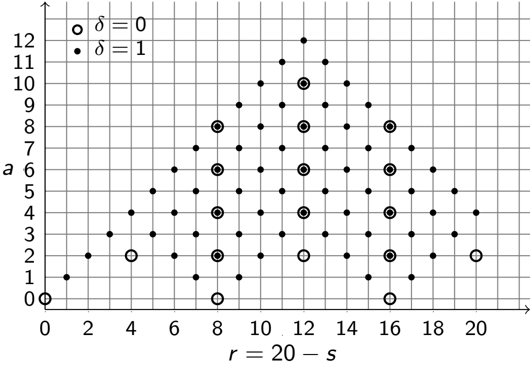
  $$\mathrm{I}^*/\mathrm{I} = \mathrm{N}^*/\mathrm{N} = \mathbb{Z}_2^a$$

  there exist 84 such involutions

  classified using results of Nikulin '80

Classification of half-supersymmetric involutions of $\Gamma(20,4)$

▷ characterized by (r, a, δ), $r = \mathrm{rk}(\mathrm{I})$, $\mathrm{I}^*/\mathrm{I} = \mathbb{Z}_2^a$, $\delta = \begin{cases} 0 & \text{if } P_\mathrm{I}^2 \in \mathbb{Z} \quad \forall P_\mathrm{I} \in \mathrm{I}^* \\ 1 & \text{otherwise} \end{cases}$



▷ e.g. $(r, a, \delta) = (12, 10, 0) \Rightarrow \mathrm{I} = \mathrm{E}_8(2) \oplus \mathrm{D}_4,\ \mathrm{N} \sim \mathrm{E}_8(2) \oplus \mathrm{D}_4(-1)$

$(r, a, \delta) = (7, 1, 1) \Rightarrow \mathrm{I} = \mathrm{E}_7,\ \mathrm{N} \sim \mathrm{E}_8 \oplus \mathrm{A}_1 \oplus 4\mathrm{U}$

$\mathrm{L}(n)$: lattice with Gram matrix rescaled by n, U: even, selfdual, signature (1,1)

- $\triangleright$ N unique up to $\mathrm{SO}(s,4)$

- $\triangleright$ I has no moduli   no Coulomb branch in 6d with (0,1) supersymmetry

- $\triangleright$ symmetric orbifolds: $(r,a,\delta)=(16,0,0)$, $\mathrm{N}=4\mathrm{U}$, $\mathrm{I}=2\mathrm{E}_8$ or Γ_{16}

- $\triangleright$ degeneracy factor in twisted sector: $\sqrt{\dfrac{\det'(1-\Theta)}{|\mathrm{I}^*/\mathrm{I}|}}=4\times 2^{\frac{s-a}{2}}\in\mathbb{Z}$

- $\triangleright$ $\delta=0,\quad e^{2i\pi P^2}=1,\ \forall\, P\in\mathrm{I}^*$

- $\triangleright$ $\delta=1,\quad \exists\, w\in\mathrm{I}^*,\ w^2+\dfrac{s}{2}\in 2\mathbb{Z}\ ||\ e^{2i\pi P^2}=e^{2i\pi P\cdot w},\ \forall\, P\in\mathrm{I}^*$

Modular invariance

▷ $\mathcal{Z}(g,g^2) \stackrel{?}{=} \mathcal{Z}(g,\mathbb{1})$

$$e^{2i\pi(v^2+\frac{s}{8})} \sum_{P\in \mathrm{I}^*} q^{\frac{1}{2}(P+v)_L^2}\, \bar{q}^{\frac{1}{2}(P+v)_R^2}\, e^{2i\pi P\cdot 2v} e^{2i\pi P^2} \stackrel{?}{=} \sum_{P\in \mathrm{I}^*} q^{\frac{1}{2}(P+v)_L^2}\, \bar{q}^{\frac{1}{2}(P+v)_R^2}$$

▷ $\delta=0,\ e^{2i\pi P^2}=1,\ \forall\, P\in \mathrm{I}^*,\quad 2v\in \mathrm{I},\quad v^2+\frac{s}{8}\in\mathbb{Z}\ \Rightarrow\ \mathcal{Z}(g,g^2)=\mathcal{Z}(g,\mathbb{1})$

$$g|P_{\rm N},P_{\rm I}\rangle = e^{2i\pi P_{\rm I}\cdot v}|-P_{\rm N},P_{\rm I}\rangle \quad\Rightarrow\quad g^2|P_{\rm N},P_{\rm I}\rangle = |P_{\rm N},P_{\rm I}\rangle$$

$$e^{2i\pi P^2}=1,\ \forall\, P\in \mathrm{I}^*,\quad\Longleftrightarrow\quad x\cdot\Theta x = \text{even},\ \forall\, x\in\Gamma$$

Narain, Sarmadi, Vafa '86

can be relaxed without doubling the order !

▷ $\delta=1,\ e^{2i\pi P^2}=e^{2i\pi P\cdot w},\ \forall\, P\in \mathrm{I}^*,\quad 2v+w\in \mathrm{I},\quad v^2+\frac{s}{8}\in\mathbb{Z}\ \Rightarrow\ \mathcal{Z}(g,g^2)=\mathcal{Z}(g,\mathbb{1})$

$$g|P_{\rm N},P_{\rm I}\rangle = f(P_{\rm N})e^{2i\pi P_{\rm I}\cdot v}|-P_{\rm N},P_{\rm I}\rangle$$

Acharya, Aldazabal, AF, Narain, Zadeh '22

$$f(0)=1,\quad f(P_{\rm N})f(-P_{\rm N}) = e^{2i\pi P_{\rm N}^2} = e^{2i\pi P_{\rm I}^2} = e^{2i\pi P_{\rm I}\cdot w} \quad\Rightarrow\quad g^2|P_{\rm N},P_{\rm I}\rangle = |P_{\rm N},P_{\rm I}\rangle$$

▷ $2v + w \in \mathrm{I}, \quad v^2 + \frac{s}{8} \in \mathbb{Z} + \frac{1}{2} \;\Rightarrow\; \mathcal{Z}(g, g^4) = \mathcal{Z}(g, \mathbb{1})$

gives $\mathcal{Z} = \mathcal{Z}(\mathbb{1}, \mathbb{1})$, i.e. $\mathbb{T}^4$ compactification with 16 supercharges

▷ $2v + w \notin \mathrm{I}, \quad 4v \in I, \quad 2v^2 + \frac{s}{4} \in \mathbb{Z} \;\Rightarrow\; \mathcal{Z}(g, g^4) = \mathcal{Z}(g, \mathbb{1})$

twisted sectors can be rearranged into a $\mathbb{Z}_2$ with I', N'

▷ end result: full modular invariant partition function $\longrightarrow$ spectrum of states

- ∗ in cases with 8 supercharges, massless matter in 1 tensor multiplet, n_V vector multiplets of some G, n_H hypermultiplets transforming under G
- ∗ can check cancellation of tr R^4 anomalies, $n_H - n_V = 244$ and cancellation of tr F^4 anomalies

Examples

▷ $(r,a,\delta)=(20,2,0)$, $\mathrm{I}=2\mathrm{E}_8+\mathrm{D}_4$, $\mathrm{N}=\mathrm{D}_4(-1)$, $v=(1,0^7)\times(1,0^7)\times(0^4)$

$2v\in\mathrm{I}$

$\mathrm{G}=\mathrm{SO}(16)\times\mathrm{SO}(16)\times\mathrm{SO}(8)$

hypers: $(\mathbf{128},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{128},\mathbf{1})+(\mathbf{16},\mathbf{16},\mathbf{1})$

no neutral hypermultiplets

Baykara, Hamada, Tarazi, Vafa '23

▷ $(r,a,\delta)=(20,4,1)$, $\mathrm{I}=2\mathrm{E}_8+4\mathrm{A}_1$, $\mathrm{N}=4\mathrm{A}_1(-1)$, $v=(0^8)\times(\frac{1}{8}^7,\frac{5}{8})\times(0^4)$

$w=(0^8)\times(0^8)\times(\frac{1}{\sqrt{2}}^4)$, $2v+w\notin\mathrm{I}$

$\mathrm{G}=\mathrm{E}_8\times\mathrm{U}(1)\times\mathrm{SU}(8)\times\mathrm{SU}(2)^4$

hypers: no neutral

$(\mathbf{28},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})+(\overline{\mathbf{28}},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{56},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{28},\underline{\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1}})+(\mathbf{8},\underline{\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1}})+(\mathbf{1},\underline{\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{1}})+(\mathbf{8},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$

▷ $(r,a,\delta)=(6,2,1)$, $\mathrm{I}=\mathrm{D}_6$, $\mathrm{N}=2\mathrm{A}_1+\Gamma(12,4)$, $v=(\frac{1}{2},0^5), w=(1,0^5)$

$2v+w\in\mathrm{I}$, $v^2+\frac{s}{8}\in\mathbb{Z}$

$\mathrm{G}=\mathrm{E}_7\times\mathrm{SU}(2)$

hypers: $12(\mathbf{1},\mathbf{1})+(\mathbf{56},\mathbf{2})+128(\mathbf{1},\mathbf{2})$

Heterotic on $\mathbb{T}^4/\mathbb{Z}_K$ with 16 supercharges and rank reduction

Setup

▷ $\mathbb{Z}_K$: acts on s (even) of 20 L-movers, eigenvalues $e^{\pm 2\pi i t_m}$, $m = 1, \dots, \frac{s}{2}$

leaves invariant $(20 - s)$ L-movers and all 4 right movers

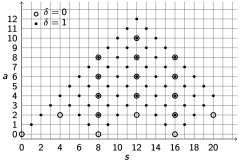
no action on world-sheet fermions, all 16 supersymmetries unbroken

▷ on $\Gamma(20,4)$, $\mathbb{Z}_K$ automorphism Θ, $\quad \det{}'(1-\Theta) = \prod_m 4\sin^2 \pi t_m, \quad t_m = \frac{\ell}{K}$

invariant lattice I, signature $(20 - s, 4)$

normal lattice $\mathrm{N} = \mathrm{I}^{\perp}$, signature $(s, 0)$

▷ full classification for $\mathbb{Z}_2$ (exchange I with N in $\mathbb{T}^4/\mathbb{Z}_2$ with 8 supercharges)



Rank reduction

▷ typical example: CHL $\Gamma(20,4) = \mathrm{E}_8 + \mathrm{E}_8 + 4\mathrm{U}$ (Θ exchanges the two E_8) Chaudhuri, Polchinski '95

$\mathrm{I} = \mathrm{E}_8(2) + 4\mathrm{U}, \quad \mathrm{N} = \mathrm{E}_8(2)$

N does not have roots, i.e. vectors of squared norm 2

Θ kills massless states from oscillators along N directions

invariant combinations $|P_\mathrm{N}\rangle + \Theta|P_\mathrm{N}\rangle + \cdots + \Theta^{K-1}|P_\mathrm{N}\rangle$ allowed, but $\nexists\, P_\mathrm{N} \,||\, P_\mathrm{N}^2 = 2$

rank reduction requires absence of roots in N

▷ other examples: 7d asymmetric $\mathbb{T}^3/\mathbb{Z}_K$, $K = 2, 3, 4, 5, 6$ de Boer et al '01

∗ explicitly constructed as $(\mathbb{T}^2 \times S^1)/\mathbb{Z}_K$ Fraiman, Parra De Freitas '21

$\mathbb{Z}_K$ automorphism of $\Gamma(18,2)$ + translation by $\frac{2\pi\mathcal{R}}{K}$ in S^1

all have N without roots e.g. $\mathbb{Z}_3$, $\mathrm{I} = 2\mathrm{A}_2 + 2\mathrm{U}(3) + \mathrm{U}$, $\mathrm{N} = \mathrm{K}_{12}$ (Coxeter-Todd lattice)

∗ equivalently $\mathbb{T}^3/\mathbb{Z}_K$ with $\mathbb{Z}_K$ automorphism of $\Gamma(19,3)$

- ▷ $\mathbb{Z}_K$ automorphisms of $\Gamma(19,3)$ with I of signature $(19-s,3)$, N of signature $(s,0)$, and N without roots, have been classified — Nikulin '80

  a.k.a. symplectic automorphisms of $\mathrm{K3}$ surfaces

- ▷ also $K=7,8$ allowed, but no rank reduction in 7d asymmetric $\mathbb{T}^3/\mathbb{Z}_7$ and $\mathbb{T}^3/\mathbb{Z}_8$

  extra massless vectors in twisted sectors $\Leftarrow$ $v \in \mathrm{I}^*$, $\forall v$ satisfying modular invariance

  still can construct $(\mathbb{T}^3 \times S^1)/\mathbb{Z}_7$ and $(\mathbb{T}^3 \times S^1)/\mathbb{Z}_8$ with rank reduction in 6d

  including a translation by $\frac{2\pi\mathcal{R}}{K}$ in S^1

- ▷ for rank reduction in 6d asymmetric $\mathbb{T}^4/\mathbb{Z}_K$ want automorphisms of $\Gamma(20,4)$ with I of signature $(20-s,4)$, N of signature $(s,0)$, and N without roots

  automorphisms of this type are symmetries of $\mathrm{K3}$ sigma models

  Gaberdiel, Hohenegger, Volpato '11

Rank reduction in $\mathbb{T}^4/\mathbb{Z}_K$ via Leech lattice Λ

- $\triangleright$ N without roots $\rightsquigarrow$ look at automorphisms of Λ — Gaberdiel, Hohenegger, Volpato '11

  Λ: even self-dual $(24, 0)$ without roots, automorphism group = Conway group Co_0

- $\triangleright$ sublattices of Λ fixed by elements of Co_0 classified up to conjugacy — Höhn, Mason '16

  $\exists$ 290 distinct invariant lattices $\tilde{\mathrm{I}}$ $(r, 0)$, with normal N $(s, 0)$, $r + s = 24$

  * if $r \geq 4$ look for I of signature $(r-4, 4)$ || $\mathrm{N}^*/\mathrm{N} = \mathrm{I}^*/\mathrm{I}$ — Baykara, Harvey '21

    and there exist glue vectors to construct $\Gamma(20, 4)$

  * $\Theta : \mathbb{Z}_K$ automorphism of N that preserves correlated classes of N^*/N

- $\triangleright$ $\mathbb{T}^4/\mathbb{Z}_K$, $K = 2(\mathrm{CHL}), 3, 4, 5, 6, 7, 8$, with rank reduction $\longleftrightarrow$ HM# 2,4,9,20,18,52,55

- $\triangleright$ other components of the moduli space of 6d theories with 16 supercharges ?

  Fraiman, Parra De Freitas '22

$\mathbb{T}^4/\mathbb{Z}_2$ with rank 8

▷ HM5, $s = 12$, $\mathrm{N} = \mathrm{D}_{12}^{+}(2)$, $\mathrm{N}^{*}/\mathrm{N} = \mathbb{Z}_2^{12}$, $\tilde{\mathrm{I}} = \mathrm{D}_{12}^{+}(2)$ $\mathrm{D}_{12}^{+} = \mathrm{D}_{12} + (\mathsf{Sp})$

$\mathrm{I} = 8\mathrm{A}_1 + 4\mathrm{A}_1(-1)$ or $\mathrm{I} = \mathrm{E}_8(2) + 4\mathrm{A}_1(-1)$ or $\mathrm{I} \sim 7\mathrm{A}_1 + \mathrm{U}(2) + 3\mathrm{A}_1(-1)$

$\Theta = -\mathbb{1}$ acting on N

▷ $e^{2i\pi P^2} \neq 1,\ \forall\, P \in \mathrm{I}^*$, but $\exists\ w \in \mathrm{I}^*,\ w^2 + \dfrac{s}{2} \in 2\mathbb{Z}\ ||\ e^{2i\pi P^2} = e^{2i\pi P\cdot w},\ \forall\, P \in \mathrm{I}^*$

▷ $\exists\, v\ ||\ 2v + w \in I,\ 2v^2 + \dfrac{s}{4} \in 2\mathbb{Z}$

▷ can construct modular invariant partition function with $\mathcal{Z}(g, g^2) = \mathcal{Z}(g, \mathbb{1})$

$$g|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = f(P_{\mathrm{N}})e^{2i\pi P_{\mathrm{I}}\cdot v}|-P_{\mathrm{N}}, P_{\mathrm{I}}\rangle, \qquad f(0) = 1,\ f(P_{\mathrm{N}})f(-P_{\mathrm{N}}) = e^{2i\pi P_{\mathrm{I}}\cdot w}$$

$$g^2|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = e^{2i\pi P_{\mathrm{I}}\cdot(2v+w)}|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = |P_{\mathrm{N}}, P_{\mathrm{I}}\rangle$$

▷ partition function

$$\mathcal{Z} = \frac{1}{\left(\sqrt{\tau_2}\eta\bar{\eta}\right)^4} \times \frac{1}{2\,\bar{\eta}^4}\left[\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4 - \bar{\vartheta}_2^4 + \bar{\vartheta}_1^4\right] \times \frac{1}{2}\sum_{\ell=0}^{1}\sum_{m=0}^{1}\mathcal{Z}_\Gamma(g^\ell, g^m)$$

untwisted sector

$$\mathcal{Z}_\Gamma(\mathbb{1},\mathbb{1}) = \frac{1}{\eta^{20}\bar{\eta}^4}\sum_{P\in\Gamma} q^{\frac{1}{2}P_L^2}\bar{q}^{\frac{1}{2}P_R^2}$$

$$\mathcal{Z}_\Gamma(\mathbb{1},g) = \left(\frac{2\eta}{\vartheta_2}\right)^{s/2}\frac{1}{\eta^8\bar{\eta}^4}\sum_{P\in\mathrm{I}} q^{\frac{1}{2}P_L^2}\bar{q}^{\frac{1}{2}P_R^2}\, e^{2\pi i P\cdot v} \qquad s = 12$$

twisted sector

$$\mathcal{Z}_\Gamma(g,\mathbb{1}) = \left(\frac{\eta}{\vartheta_4}\right)^{s/2}\frac{1}{\eta^8\bar{\eta}^4}\sum_{P\in\mathrm{I}^*} q^{\frac{1}{2}(P+v)_L^2}\bar{q}^{\frac{1}{2}(P+v)_R^2}$$

$$\mathcal{Z}_\Gamma(g,g) = e^{i\pi(v^2+\frac{s}{8})}\left(\frac{\eta}{\vartheta_3}\right)^{s/2}\frac{1}{\eta^8\bar{\eta}^4}\sum_{P\in\mathrm{I}^*} q^{\frac{1}{2}(P+v)_L^2}\bar{q}^{\frac{1}{2}(P+v)_R^2}\, e^{2\pi i P\cdot v}\, e^{i\pi P^2}$$

▷ partition function encodes spectrum

∗ can choose v satisfying modular invariance || $(P+v)_R \neq 0, \forall P \in \mathrm{I}^*$

$\Rightarrow$ no massless states in twisted sector

∗ massless states only in untwisted sector

from R-movers: $\mathrm{NS}\ \mathbf{8}_v,\ \mathrm{R}\ \mathbf{8}_s,\quad P_R = 0$

from L-movers: $\frac{1}{2}P_L^2 + N_L - 1 = 0$

vector multiplets from oscillator number $N_L = 1$ along 8 left I directions

at generic points of I, group $\mathrm{U}(1)^8$

enhancement at special points, e.g. $\mathrm{SO}(9) \times \mathrm{SO}(9),\ \mathrm{SU}(9)$

Fraiman, Parra De Freitas '22

Candidate heterotic island in 6d

▷ HM149, $s = 20$, $\mathrm{N}^*/\mathrm{N} = \mathbb{Z}_2^2 \times \mathbb{Z}_{10}^2$,

$$\mathrm{N} = \begin{pmatrix}
4 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 2 & 1 & 2 & 2 & 0 & -2 & 1 & -2 & -2 & -1 & -2 & -1 \\
1 & 4 & -2 & -1 & 1 & -2 & 1 & -2 & 2 & -1 & 2 & 0 & 1 & -1 & 2 & -1 & -2 & -2 & 0 & -2 \\
-1 & -2 & 4 & -1 & -1 & 2 & 1 & 0 & -2 & -1 & -1 & 0 & 1 & 0 & -1 & 2 & 1 & 0 & -1 & 2 \\
1 & -1 & -1 & 4 & 0 & 0 & 0 & 2 & 1 & 2 & -1 & 1 & -2 & 0 & -1 & 0 & -1 & 0 & -1 & -1 \\
-1 & 1 & -1 & 0 & 4 & -2 & 0 & 1 & 1 & 1 & -1 & -2 & 0 & 2 & 0 & 0 & 1 & 1 & 2 & -1 \\
-1 & -2 & 2 & 0 & -2 & 4 & 0 & 1 & -1 & 0 & -1 & 1 & -1 & 0 & 0 & 2 & 1 & 1 & 0 & 2 \\
-1 & 1 & 1 & 0 & 0 & 0 & 4 & 0 & -1 & -2 & -1 & -1 & 0 & 0 & -1 & 2 & 0 & -2 & 0 & 0 \\
-1 & -2 & 0 & 2 & 1 & 1 & 0 & 4 & 0 & 2 & -2 & -1 & -2 & 1 & -2 & 1 & 1 & 2 & 1 & 0 \\
2 & 2 & -2 & 1 & 1 & -1 & -1 & 0 & 4 & 1 & 2 & 1 & 0 & -1 & 2 & -1 & -2 & 0 & 0 & -2 \\
1 & -1 & -1 & 2 & 1 & 0 & -2 & 2 & 1 & 4 & -1 & 1 & -2 & 1 & 0 & -1 & 0 & 2 & 0 & -1 \\
2 & 2 & -1 & -1 & -1 & -1 & -1 & -2 & 2 & -1 & 4 & 1 & 2 & -2 & 2 & -2 & -2 & -1 & -1 & -1 \\
2 & 0 & 0 & 1 & -2 & 1 & -1 & -1 & 1 & 1 & 1 & 4 & 0 & -1 & 1 & -1 & -2 & -1 & -2 & 0 \\
0 & 1 & 1 & -2 & 0 & -1 & 0 & -2 & 0 & -2 & 2 & 0 & 4 & -1 & 1 & 0 & -1 & -1 & 0 & 0 \\
-2 & -1 & 0 & 0 & 2 & 0 & 0 & 1 & -1 & 1 & -2 & -1 & -1 & 4 & 0 & 0 & 2 & 2 & 2 & 0 \\
1 & 2 & -1 & -1 & 0 & 0 & -1 & -2 & 2 & 0 & 2 & 1 & 1 & 0 & 4 & -1 & -1 & 0 & 0 & -1 \\
-2 & -1 & 2 & 0 & 0 & 2 & 2 & 1 & -1 & -1 & -2 & -1 & 0 & 0 & -1 & 4 & 1 & 0 & 1 & 1 \\
-2 & -2 & 1 & -1 & 1 & 1 & 0 & 1 & -2 & 0 & -2 & -2 & -1 & 2 & -1 & 1 & 4 & 2 & 2 & 2 \\
-1 & -2 & 0 & 0 & 1 & 1 & -2 & 2 & 0 & 2 & -1 & -1 & -1 & 2 & 0 & 0 & 2 & 4 & 2 & 0 \\
-2 & 0 & -1 & -1 & 2 & 0 & 0 & 1 & 0 & 0 & -1 & -2 & 0 & 2 & 0 & 1 & 2 & 2 & 4 & 0 \\
-1 & -2 & 2 & -1 & -1 & 2 & 0 & 0 & -2 & -1 & -1 & 0 & 0 & 0 & -1 & 1 & 2 & 0 & 0 & 4
\end{pmatrix}$$

$$\mathrm{I} = 2\mathrm{A}_1(-1) + 2\mathrm{A}_1(-5) \quad \text{or} \quad \mathrm{I} = \begin{pmatrix} -6 & 4 \\ 4 & -6 \end{pmatrix}^{\oplus 2}$$

Θ acting on N: $\mathbb{Z}_{10}$ with eigenvalues $e^{\pm 2\pi i t_i}$, $t = \frac{1}{10}(1^2, 2^2, 3^2, 4^2, 5^2)$

▷ $e^{10 i\pi P^2} \neq 1,\ \forall\, P \in \mathrm{I}^*$, but $\exists\, w \in \mathrm{I}^* \mid\mid e^{10 i \pi P^2} = e^{2 i \pi P\cdot w},\ \forall\, P \in \mathrm{I}^*$

▷ $\exists\, v \mid\mid 10v + w \in \mathrm{I},\ 10(v^2 + t^2) \in 2\mathbb{Z} \quad \Rightarrow \quad \mathcal{Z}(g, g^{10}) = \mathcal{Z}(g, \mathbb{1})$

▷ action of g^ℓ on momenta determined by $\mathcal{Z}(g^\ell, g^{-1})$ derived from $\mathcal{Z}(\mathbb{1}, g)$, e.g.

$$\mathcal{Z}(\mathbb{1}, g) \xrightarrow{\tau \to -1/\tau} \mathcal{Z}(g, \mathbb{1}) \xrightarrow{\tau \to \tau + 2} \mathcal{Z}(g, g^2) \xrightarrow{\tau \to -1/\tau} \mathcal{Z}(g^2, g^{-1})$$

$\Big($

▷ $\mathcal{Z}(g, g^2) \supset \sum_{P\in \mathrm{I}^*} \bar{q}^{\frac{1}{2}(P+v)^2}\, e^{2i\pi P\cdot 2v} e^{2i\pi P^2}$

$\mathrm{I}^*/\mathrm{I} = \mathbb{Z}_2^4 \times \mathbb{Z}_5^2, \quad P = P_1 + P_2, \quad P_1 \in \mathrm{I}, \mathbb{Z}_2$ classes, $\quad P_2 \in \mathbb{Z}_5$ classes

$e^{2\pi i P_1^2} = e^{2\pi i P_1\cdot w} \quad \rightsquigarrow \quad$ can do Poisson resummation in P_1

▷ $\mathcal{Z}(g^2, g^{-1}) \supset \sum_{P\,\in\,\mathrm{I},\mathbb{Z}_5 \text{ classes}} \bar{q}^{\frac{1}{2}(P+2v+w)^2}\, e^{-2i\pi(P+2v+w)\cdot v}$

▷ lattice sum in $\mathcal{Z}(\mathbb{1}, g^2)$ is over I_2 (invariant under Θ^2)

▷ lattice sum in $\mathcal{Z}(g^2, \mathbb{1})$ is over I_2^* shifted by $2v + w$

g projects onto $\mathrm{I}, \mathbb{Z}_5$ classes $\subset \mathrm{I}_2^*$

▷ g^2 action on $P \in \mathrm{I}_2$ must include $e^{2\pi i P\cdot(2v+w)}$

$(P', P_\mathrm{I}) \in \mathrm{I}_2, \quad \Theta P' = -P'$

$g|P', P_\mathrm{I}\rangle = f(P')e^{2i\pi P_\mathrm{I}\cdot v}|-P', P_\mathrm{I}\rangle \quad \Rightarrow \quad g^2|P', P_\mathrm{I}\rangle = e^{2i\pi P_\mathrm{I}\cdot(2v+w)}|P', P_\mathrm{I}\rangle$

as before $f(P')f(-P') = e^{2i\pi P_\mathrm{I}\cdot w}$ $\Big)$

- ▷ can construct full modular invariant partition function

- ▷ normal lattice N 20 dim & $\nexists\, P \in \mathrm{N} \,||\, P^2 = 2$

  $\Rightarrow$ no massless vector multiplets in untwisted sector

- ▷ shifts v and w are such that there are no massless states in twisted sectors

- ▷ only massless gravity multiplet

  no moduli except dilaton $\longrightarrow$ island Dabholkar, Harvey '98

- ▷ in progress: check operator interpretation, in particular integer multiplicities of massive states

Summary and Outlook

- Studied heterotic asymmetric $\mathbb{T}^4/\mathbb{Z}_2$ with 8 supercharges
  - * classified the relevant automorphisms of the $\Gamma(20,4)$ lattice
  - * constructed the partition function, with novel ways to define the $\mathbb{Z}_2$ action
  - * worked out several examples, rediscovered some without neutral hypermultiplets
  - ◇ extensions: $\mathbb{T}^5/\mathbb{Z}_2, \mathbb{T}^6/\mathbb{Z}_2, \mathbb{Z}_2 \to \mathbb{Z}_p$ Gkountoumis et al '23, Baykara et al '23
    also type II Gkountoumis, Hull, Vandoren '24, Baykara, Tarazi, Vafa '24, Nian@SPheno24, Tarazi@SPheno24
  - ◇ to do: examine dualities

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∗ noticed that rank reduction requires N lattice of $\mathbb{Z}_K$ in $\Gamma(20,4)$ with no roots

∗ constructed examples via Leech lattice, including candidate $\mathbb{T}^4/\mathbb{Z}_{10}$ island

∗ developed formalism for $\mathbb{T}^4/\mathbb{Z}_{2\mathfrak{m}}$, $\mathfrak{m}$ prime

◇ extensions: $(\mathbb{T}^4 \times \tilde{S}^1 \times S^1)/\mathbb{Z}_K$ or $\mathbb{T}^6/\mathbb{Z}_K$ e.g. $\mathbb{T}^6/\mathbb{Z}_{22}$ island AAFNZ '24
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- Methods can be applied to **non-supersymmetric** asymmetric orbifolds

Fraiman et al '23, Parra De Freitas '24, Baykara, Tarazi, Vafa '24, Tarazi@SPheno24, Fraiman@SPheno24
Raucci@SPheno24, Leone@SPheno24, Larotonda@SPheno24, Abel, Dudas, Heckman, Montero, Sagnotti@SPheno24


Fin