Surprises from Non-Supersymmetric Strings

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I. 10D Tachyon-Free Models

The (SUSY) 10D-11D Hexagon

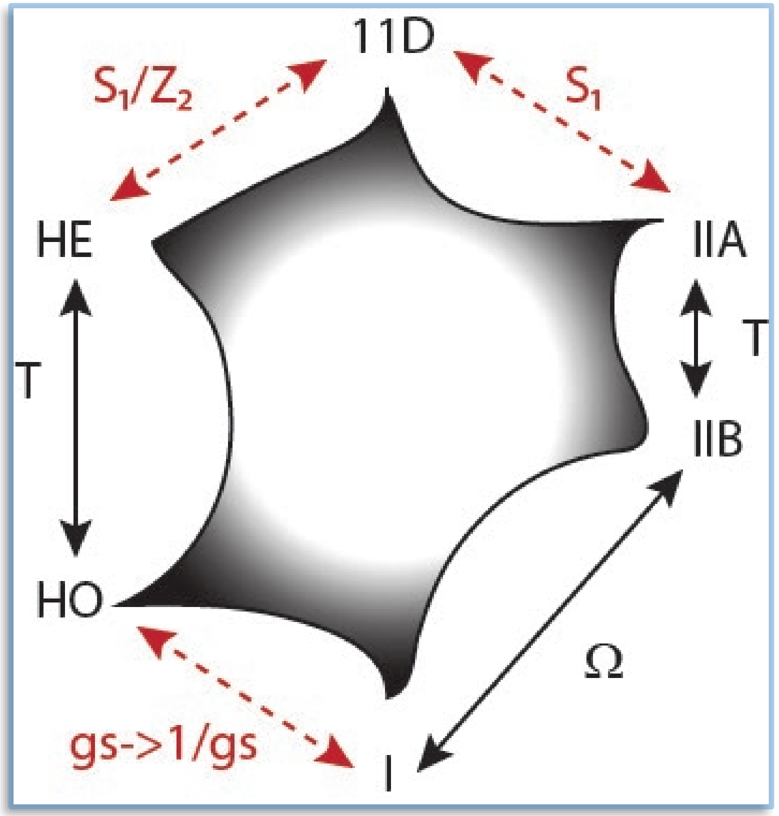
- Perturbative → **Solid arrows**
- **[** 10&11D supergravity → **Dashed arrows]**
- **Highest point** of (SUSY) String Theory

BUT:

- Exhibits **dramatically our limitations**

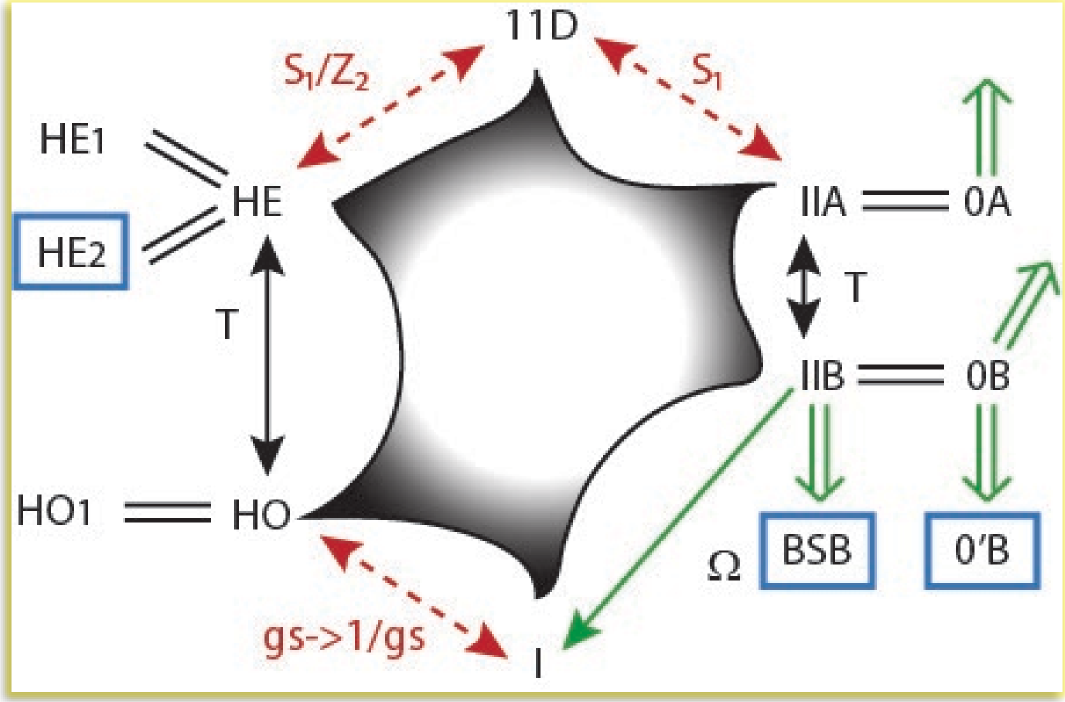
(Witten, 1995)

- **SUSY**: **stabilizes** the 10D Minkowski vacua



BROKEN SUSY ?

The 10D-11D Zoo



- Non-SUSY closed & orientifolds

(Seiberg, Witten, 1986)
(Dixon, Harvey, 1986)
(Bianchi, AS, 1990)

∃ 3 non-SUSY non-tachyonic strings

- SO(16)xSO(16) — *(Dixon, Harvey, 1986)* *(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)*
- 0'B U(32) — *(AS, 1995)*
- [BSB: Usp(32)] — *(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)*

- **NO SUPERSYMMETRY → (typically) TACHYONS**
- Fairly enough: we are still UNABLE to cope with them
- ∃ three 10D theories without supersymmetry BUT NO TACHYONS:

1) Heterotic variant 2) Exotic descendant of "tachyonic 0B" 3) Brane SUSY breaking

The Non-Tachyonic 10D String Models

$$O_{2n} = \frac{\theta^n\left[{0 \atop 0}\right](0|\tau) + \theta^n\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,, \qquad S_{2n} = \frac{\theta^n\left[{1/2 \atop 0}\right](0|\tau) + i^{-n}\,\theta^n\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}$$

$$V_{2n} = \frac{\theta^n\left[{0 \atop 0}\right](0|\tau) - \theta^n\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,, \qquad C_{2n} = \frac{\theta^n\left[{1/2 \atop 0}\right](0|\tau) - i^{-n}\,\theta^n\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}$$

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1 - q^n)\,, \qquad q = e^{2\pi i \tau}\,,$$

$$\theta\left[{\alpha \atop \beta}\right](z|\tau) = \sum_{n\in Z} q^{\frac{1}{2}(n+\alpha)^2}\, e^{i2\pi(n+\alpha)(z-\beta)}$$

SO(16)xSO(16):

$$\mathcal{T}_{SO(16)\times SO(16)} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{1}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8}\,\left[O_8(\bar{V}_{16}\,\bar{C}_{16} + \bar{C}_{16}\,\bar{V}_{16}) + V_8(\bar{O}_{16}\,\bar{O}_{16} + \bar{S}_{16}\,\bar{S}_{16}) - S_8(\bar{O}_{16}\,\bar{S}_{16} + \bar{S}_{16}\,\bar{O}_{16}) - C_8(\bar{V}_{16}\,\bar{V}_{16} + \bar{C}_{16}\,\bar{C}_{16})\right]$$

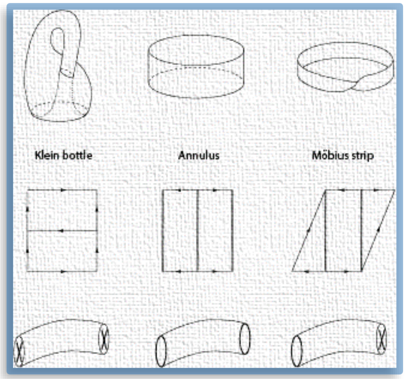
(Dixon, Harvey, 1987)
(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)

U(32):

$$\frac{1}{2}(\mathcal{T} + \mathcal{K}) = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8} + \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\,\frac{(-)\,O_8 + V_8 + S_8 - C_8}{(\tau_2)^4\,\eta^8}\,[2i\tau_2]$$

$$\frac{1}{2}(\mathcal{A} + \mathcal{M}) = \int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\,\frac{\mathcal{N}\overline{\mathcal{N}}\,V_8 - \frac{1}{2}(\mathcal{N}^2 + \overline{\mathcal{N}}^2)\,C_8}{(\tau_2)^4\,\eta^8}\,[i\tau_2/2] - \frac{\mathcal{N} + \overline{\mathcal{N}}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\,\frac{\hat{C}_8}{(\tau_2)^4\,\hat{\eta}^8}\,[i\tau_2/2 + 1/2]$$

(AS, 1995)



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(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)

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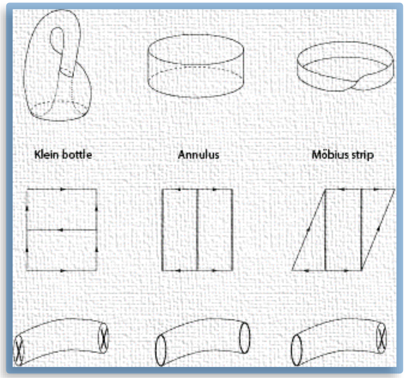

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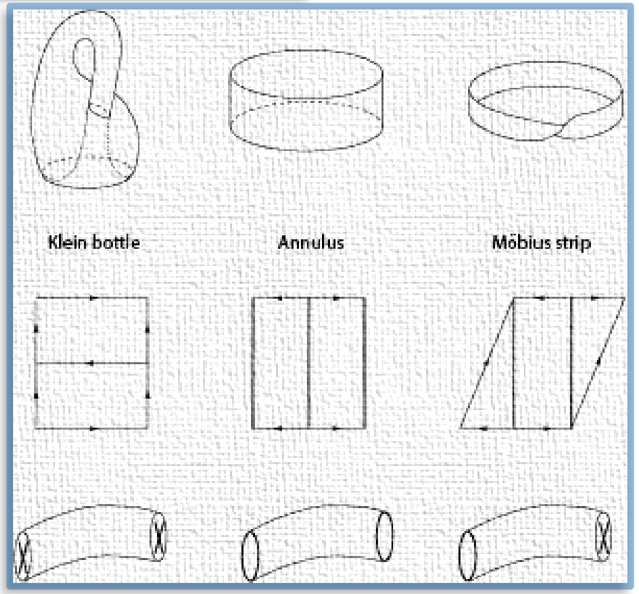
$$V_8 = \frac{\theta^4\left[{0 \atop 0}\right](0|\tau) - \theta^4\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^4(\tau)}\,, \qquad C_8 = \frac{\theta^4\left[{1/2 \atop 0}\right](0|\tau) - \theta^4\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^4(\tau)}$$

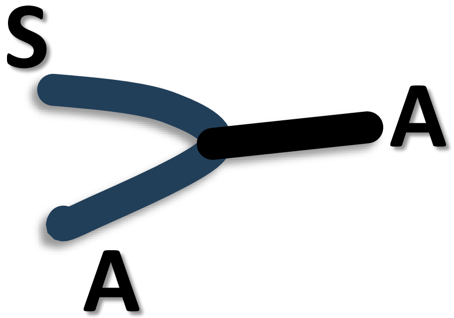
$$\begin{pmatrix} O_8 \\ V_8 \\ -S_8 \\ -C_8 \end{pmatrix} \xrightarrow{s} \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} O_8 \\ V_8 \\ -S_8 \\ -C_8 \end{pmatrix}$$



"- signs": (Schellekens and Warner, 1987)

$$\frac{1}{2}\,\mathcal{K} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\;\frac{\epsilon_o\, O_8 + \epsilon_v\, V_8 + \epsilon_s\,(-S_8) + \epsilon_c\,(-C_8)}{(\tau_2)^4\;\eta^8}\,[2i\tau_2]$$

Standard choice: $\epsilon_i = (1,1,1,1)$



Can change it compatibly with the fusion rules:
(as in 2D WZW models of ADE series)

$$\epsilon_i = (1,1,-1,-1)$$

$$\epsilon_i = (-1,1,1,-1)$$

(Pradisi, AS, Stanev, 1994)

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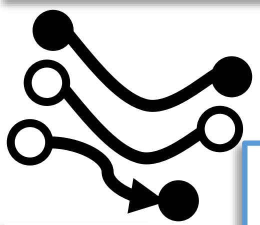

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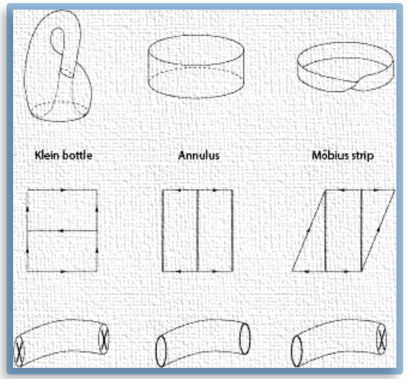

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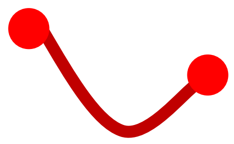

(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)

Back-Reaction on the Vacuum

- **Dual Role** of **Vacuum amplitudes** in **String Theory:**
  - a. **Consistency conditions**
  - b. **Backreaction on vacuum**
- **AT BEST: Double expansion** in powers of $\alpha' R$ and $g_s = e^{\phi}$
- **VERY DIFFICULT: one can at least EXPLORE the dominant terms … AND YET …**

$$\mathcal{S} = \frac{1}{2\,k_{10}^2} \int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-[0,1]\phi} + \ldots\right\}$$

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VACUUM ENERGY → "TADPOLE POTENTIAL"

II. Brane Supersymmetry Breaking

(non-linear supersymmetry in D=10)

Brane SUSY Breaking (BSB)

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

- ❖ NO TACHYONS
- ❖ Non-linear SUSY: ∃ goldstino!

(Dudas, Mourad, 2000)
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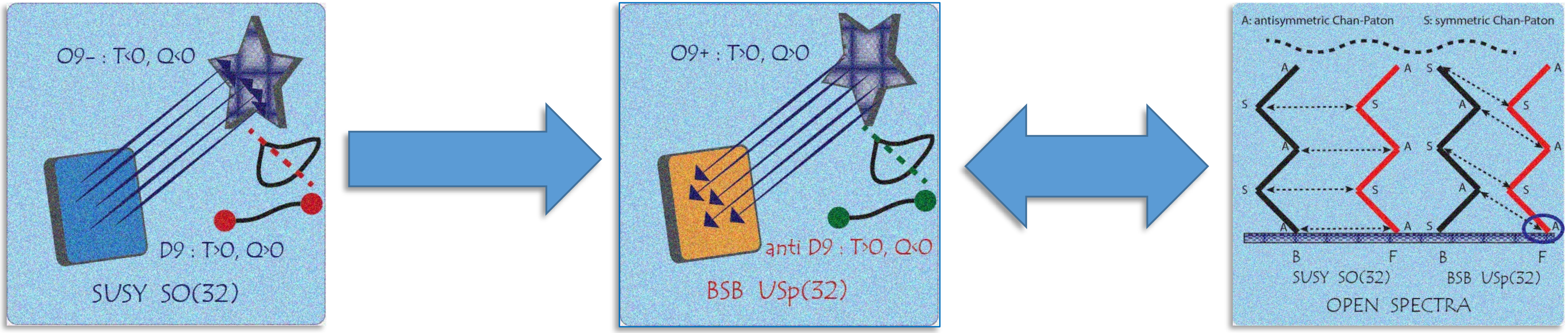
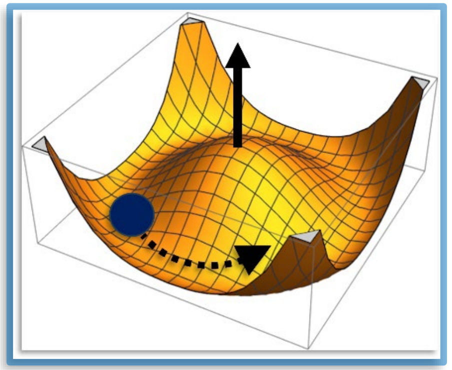


NON-LINEAR REALIZATIONS: USUALLY limits of linear ones. WHAT ARE THE "HIGGS" MODES HERE?

In D=10 BSB an OPTION, in lower dimensions **INEVITABLE WITH** special Klein-bottle projections

SUSY IN CLOSED SPECTRUM, NOT IN OPEN: a puzzle noted in Rome in the early '90's (see hep-th/9302099),

Work with **M. Bianchi and G. Pradisi** [See also 2403.02392 for recent developments]



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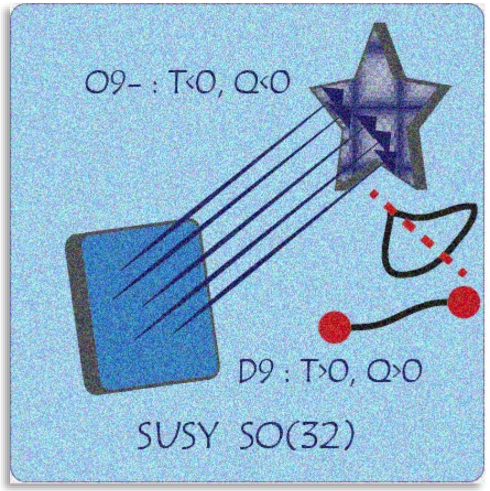

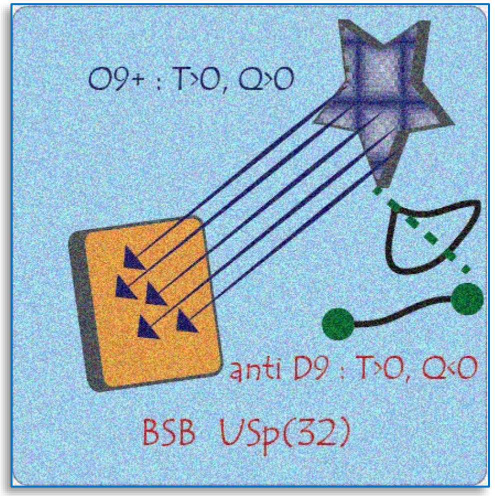

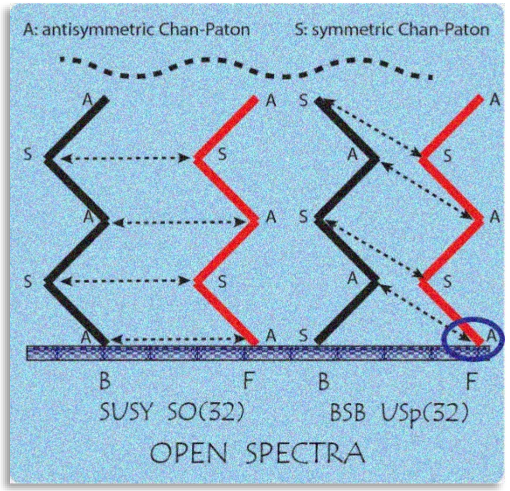

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III. The Climbing Scalar

(Different Cosmological dynamics with $V = e^{\gamma\phi}$ for $\gamma < \gamma_c$ & $\gamma \geq \gamma_c$)

Cosmology: “Critical”Potential & Climbing Scalar

WHAT POTENTIALS LEAD TO SLOW-ROLL, AND WHERE ? *(Dudas, Kitazawa, AS, 2010)*

$$ds^2 = -dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x} \;\longrightarrow\; \ddot{\phi} + 3\dot{\phi}\sqrt{\frac{1}{3}\dot{\phi}^2 + \frac{2}{3}V(\phi)} + V' = 0$$

Driving force from V’ *vs* friction from V

- **IF V does not vanish : a convenient gauge “makes the damping term neater”** *(Dudas and Mourad, 2000)*

$$ds^2 = e^{2\mathcal{B}(t)}\, dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\, d\mathbf{x}\cdot d\mathbf{x}$$

$$V e^{2\mathcal{B}} = V_0$$

$$\tau = t\sqrt{\frac{d-1}{d-2}}\,, \qquad \varphi = \phi\sqrt{\frac{d-1}{d-2}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

- **NOW: driving from logV** *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ **Quadratic potential?** **Far away from origin** *(Linde, 1983)*

❖ **Exponential potential?** **YES or NO**

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

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$$V e^{2\mathcal{B}} = V_0$$

$$\tau = t\sqrt{\frac{d-1}{d-2}}\,, \qquad \varphi = \phi\sqrt{\frac{d-1}{d-2}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

$$m\ddot{x} + b\dot{x} = f$$

- **NOW: driving from logV** *vs* O(1) damping

$$V = \varphi^n \;\longrightarrow\; \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ **Quadratic potential?** **Far away from origin** *(Linde, 1983)*

❖ **Exponential potential?** **YES or NO**

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \;\longrightarrow\; \frac{V'}{2V} = \gamma$$

$V = e^{2\gamma\varphi}$: *Climbing & Descending Scalars*

(HERE we work with $\gamma_c = 1$)

(Halliwell, 1987;..., Dudas and Mourad, 2000; Russo, 2004)
(Dudas, Kitazawa, AS, 2010)

Follow solutions back to the initial singularity:

- $\gamma < 1$? Both signs of speed allowed

a. **"Climbing" solution** (φ climbs, then descends):

b. **"Descending" solution** (φ only descends):

Limiting τ- speed (LM attractor): $$\mathbf{v}_{\mathbf{lim}} = - \frac{\gamma}{\sqrt{\mathbf{1} - \gamma^{\mathbf{2}}}}$$

(Lucchin and Matarrese, 1985)

$\gamma = 1$ is "critical": **LM attractor & descending solution disappear for $\gamma \geq 1$**

$$V_S \sim e^{-\phi}$$
$$V_E = e^{\frac{3}{2}\phi}$$
$$(V_E = e^{2\varphi})$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1 + \dot{\varphi}^2\right) = 0$$

$$\ddot{\varphi} + \dot{\varphi}\,|\dot{\varphi}| + \gamma\,\dot{\varphi}^2 \simeq 0 \longrightarrow \dot{\varphi} = \frac{C}{t}$$

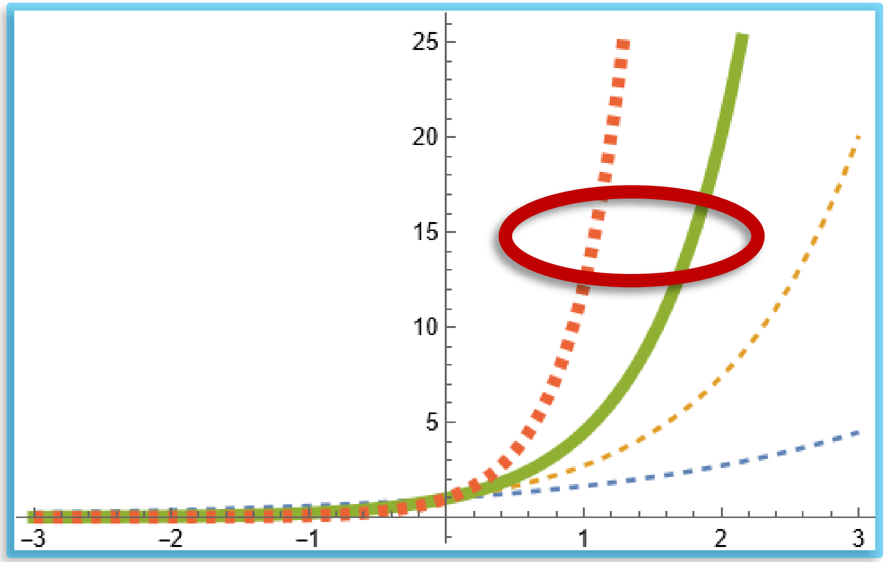
$$|\mathbf{C}| = \frac{\mathbf{1}}{\mathbf{1} + \epsilon\gamma}, \quad \epsilon = \pm 1$$

$$V = T e^{2\gamma\varphi}$$

$V = e^{2\gamma\varphi}$: Climbing & Descending Scalars

(HERE we work with $\gamma_c = 1$)

(Halliwell, 1987;…, Dudas and Mourad, 2000; Russo, 2004)
(Dudas, Kitazawa, AS, 2010)



Follow solutions back to the initial singularity:

- $\gamma < 1$? Both signs of speed allowed

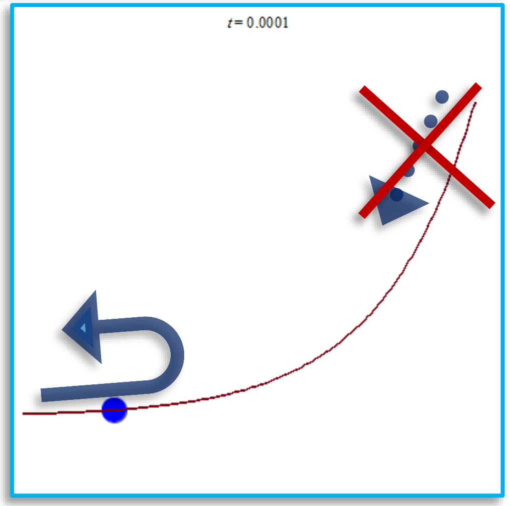
a. **"Climbing" solution** (φ climbs, then descends):

b. **"Descending" solution** (φ only descends):

Limiting τ- speed (LM attractor): $v_{\lim} = - \frac{\gamma}{\sqrt{1-\gamma^2}}$

(Lucchin and Matarrese, 1985)

$\gamma = 1$ is "critical": **LM attractor & descending solution disappear for $\gamma \geq 1$**



$$V_S \sim e^{-\phi}$$

$$V_E = e^{\frac{3}{2}\phi}$$

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$$|\mathbf{C}| = \frac{1}{1 + \epsilon\gamma}, \quad \epsilon = \pm 1$$

$$V = T e^{2\gamma\varphi}$$

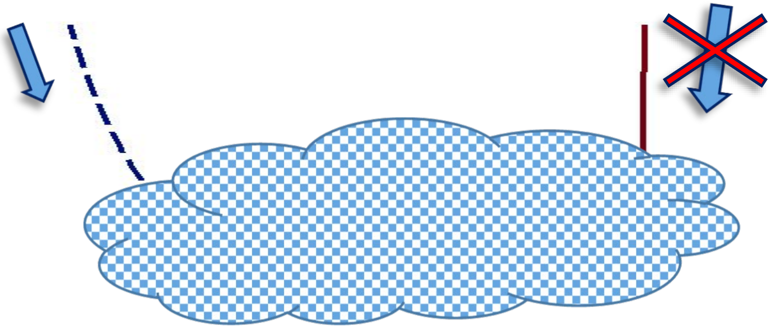
Cosmology: a Climbing Scalar as Trigger of Inflation?

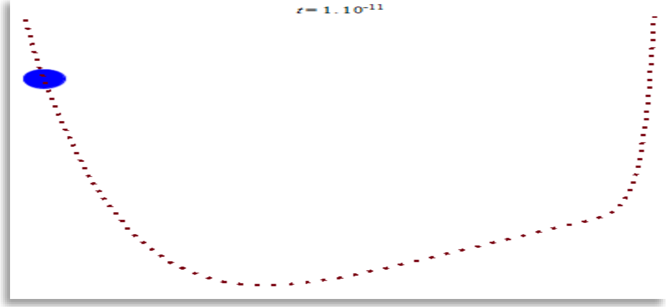
CLIMBING & SLOW-ROLL ? With (super)critical Exponential (e.g. + Starobinsky) → **FIXED INITIAL CONDITIONS**

$$V(\phi) = T\, e^{2\varphi} + v(\phi)$$

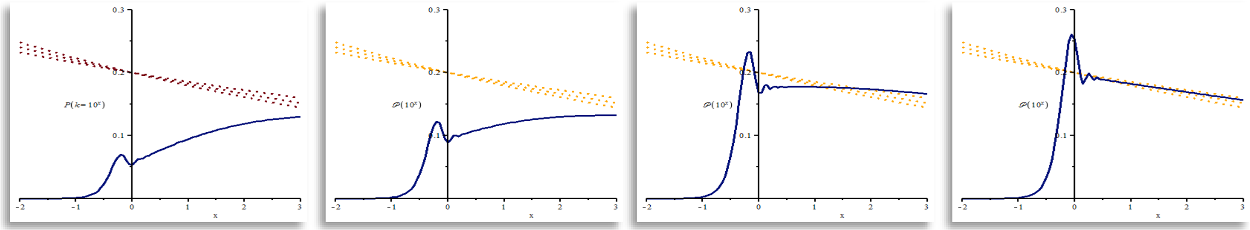
$$e.g.\ \ v(\phi) = v_0 \left(1 - e^{-\frac{2}{3}\varphi}\right)^2$$

(Dudas, Kitazawa, Patil, AS, 2013)
(Kitazawa, AS, 2014)



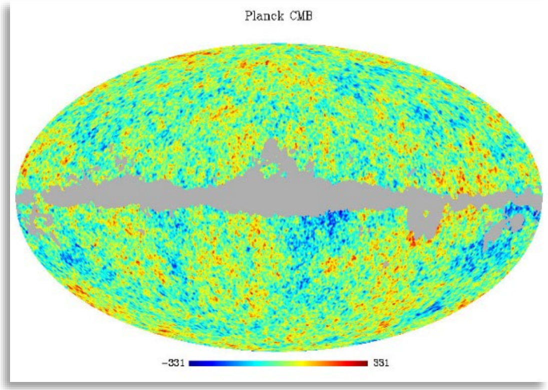


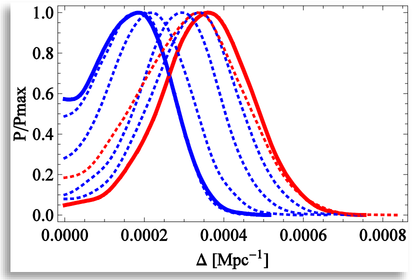
DAMPED LOW END of primordial power spectrum → POSSIBLY: damping of first CMB multipoles (cfr. lack-of-power)
[+ enhanced tensor-to-scalar ratio at the transition]



$$P(k) \sim k^{3-3\nu} \longrightarrow P(k) \sim \frac{k^3}{\left[k^2 + \Delta^2\right]^{\nu}}$$

[Corrects Chibisov-Mukhanov tilt by Δ]





$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\mathrm{Mpc}^{-1}$$

$$\Delta_{infl} \sim 10^{12} - -10^{14}\ GeV\ \text{for}\ N \sim 60$$

$[\ \mathrm{RED} : +30\text{−degree extended mask}\ ]$

(Gruppuso, Mandolesi, Natoli, Kitazawa, AS, 2015)
(+ Lattanzi, 2017)

Climbing Scalar : Instability of Isotropy

(Basile, Mourad, AS, 2018)

- **COSMOLOGY : the issue is the time evolution of perturbations**
- **INITIALLY (large η) V is negligible: tensor perturbations evolve as**

$$
\begin{aligned}
h''_{ij} \quad &+ \quad \frac{1}{\eta}\, h'_{ij} \; + \; \mathbf{k}^2\, h_{ij} \;=\; 0 \\
h_{ij} \quad &\sim \quad A_{ij}\, J_0(k\eta) \; + \; B_{ij}\, Y_0(k\eta) \qquad (\mathbf{k} \neq 0) \\
h_{ij} \quad &\sim \quad A_{ij} \; + \; B_{ij}\, \log\left(\frac{\eta}{\eta_0}\right) \qquad (\mathbf{k} = 0)
\end{aligned}
$$

- **NOTE: logarithmic growth for k=0 (instability of isotropy) !!**
- **RESONATES with** *(Kim, Nishimura, Tsuchiya, 2018)* *(Anagnostopoulos, Auma, Ito, Nishimura, Papadoulis, 2018)*

(HINT of) Dynamical origin of compactification ?

IV. The Dudas-Mourad Vacua

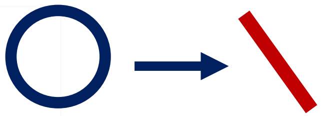
(Tadpole-driven compactifications on intervals,
with strongly coupled portions and yet perturbatively stable)

9D Dudas-Mourad Vacua (for orientifolds)

(Dudas, and Mourad, 2000, 2001)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \,..\right\}$$

9D solutions → T DRIVES compactification & KK CIRCLE→INTERVAL



[For Usp(32) and U(32), & similar for SO(16) x SO(16)]

❖ SPONTANEOUS COMPACTIFICATIONS: INTERVALS of FINITE length ~ $\frac{1}{\sqrt{T}}$

- ❖ **FINITE 9D Planck mass & gauge coupling**
- **At ends: g_s→ (∞,0) & curvature diverges**
- **ASYMPTOTICS: Kasner-like (FREE!)**

$$e^{\phi} = e^{u+\phi_0}\,u^{\frac{1}{3}}$$
$$ds^2 = e^{-\frac{u}{6}}\,u^{\frac{1}{18}}\,dx^2 + \frac{2}{3\,T\,u^{\frac{3}{2}}}\,e^{-\frac{3}{2}(u+\phi_0)}\,du^2$$

$$u\to 0\ :\ ds^2 \sim (\mu_0\,\xi)^{\frac{2}{9}}\,dx^2 + d\xi^2\,,\quad e^{\phi}\sim(\mu_0\,\xi)^{\frac{4}{3}}$$
$$u\to\infty\ :\ ds^2 \sim [\mu_0\,(\xi_m-\xi)]^{\frac{2}{9}}\,dx^2 + d\xi^2\,,\quad e^{\phi}\sim[\mu_0\,(\xi_m-\xi)]^{-\frac{4}{3}}$$

- **EXTENSIONS:** $V_E = T\,e^{\frac{3}{2}\phi} \longrightarrow V_E = T\,e^{\gamma\,\phi}$ **Orientifold γ : "CRITICAL" !**

- **ARE large values of curvature & g_s INEVITABLE in these non-SUSY compactifications?**
- **STABILITY ?**

Dudas-Mourad Vacua : Stability , I

(Basile Mourad, AS, 2018)

❖ **Dudas-Mourad: ∃STRONG COUPLING END but STABLE VACUUM !**

- **SETUP : Scalar perturbations:**

$$ds^2 \;=\; e^{2\Omega(z)}\left[(1+A)\,dx^\mu\,dx_\mu \;+\; (1-7A)\,dz^2\right] \;,$$

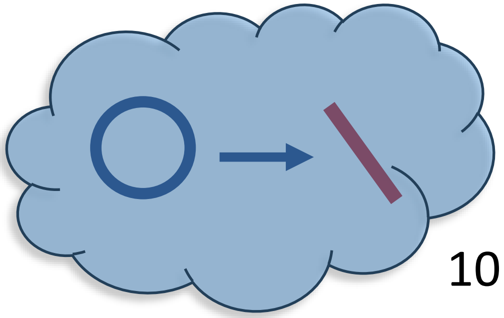
$$A'' + A'\left(24\,\Omega' \;-\; \frac{2}{\phi'}\;e^{2\Omega}\,V_\phi\right) \;+\; A\left(m^2 \;-\; \frac{7}{4}\;e^{2\Omega}\,V \;-\; 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) \;=\; 0$$

❖ **Schrödinger-like form:**

$$m^2\,\Psi \;=\; \left(b \;+\; \mathcal{A}^\dagger\,\mathcal{A}\right)\Psi$$

$$\mathcal{A} \;=\; \frac{d}{dr} \;-\; \alpha(r)\;, \qquad \mathcal{A}^\dagger \;=\; -\,\frac{d}{dr} \;-\; \alpha(r)\;, \qquad b \;=\; \frac{7}{2}\,e^{2\Omega}\,V\,\frac{1}{1 \;+\; \frac{9}{4}\,\alpha_O\,y^2} \;>\; 0$$

BUT: Boundary Conditions !



Self-Adjoint Extensions

(Mourad, AS, 2023)

- SELF-ADOINT EXTENSIONS (boundary conditions) → COMPLETE SETS of modes
- In conformal coordinate along the $[0,z_m]$ interval → Schrödinger form

$$-\frac{d^2\psi(z)}{dz^2} + V(z)\,\psi(z) = m^2\,\psi(z)$$

& ∀ two solutions

$$\Big[\psi^\star\,\partial_z\,\chi - \partial_z\,\psi^\star\,\chi\Big]_0^{z_m} = 0$$

$$\underline{\psi} = \begin{pmatrix} \psi \\ z_m\,\partial_z\,\psi \end{pmatrix}, \quad \underline{\chi} = \begin{pmatrix} \chi \\ z_m\,\partial_z\,\chi \end{pmatrix} \longrightarrow \underline{\psi}^\dagger(z_m)\;\sigma_2\;\underline{\chi}(z_m) = \underline{\psi}^\dagger(0)\;\sigma_2\;\underline{\chi}(0)$$

$$e^{i\beta}\,\underline{\psi}(z_m) = U\,\underline{\psi}(0)\,, \qquad e^{i\beta}\,\underline{\chi}(z_m) = U\,\underline{\chi}(0) \qquad U^\dagger\,\sigma_2\,U = \sigma_2$$

- Generic self-adjoint boundary conditions: points in SL(2,R) x U(1)

$$U(\rho,\theta_1,\theta_2) = \cosh\rho\,(\cos\theta_1\,\underline{1} - i\,\sigma_2\,\sin\theta_1) + \sinh\rho\,(\sigma_3\,\cos\theta_2 + \sigma_1\,\sin\theta_2)$$

- Link between ends ALSO characterized by a matrix **V** of SL(2,R) (Wronskian is CONSTANT)
- Eigenvalue equation: $$\mathrm{Tr}\left[U^{-1}\,V\right] = 2\,\cos\beta$$
- Boundary conditions given independently at the ends: ρ → ∞ (Boundary of SL(2,R))

Singular Potentials & Self-Adjoint Extensions

(Mourad, AS, 2023)

❖ SELF-ADOINT EXTENSIONS (boundary conditions) → COMPLETE SETS of NORMALIZABLE modes

$$V(z) \sim \frac{\mu^2 - \frac{1}{4}}{z^2}\,, \qquad V(z) \sim \frac{\tilde{\mu}^2 - \frac{1}{4}}{(z_m - z)^2} \longrightarrow \psi \sim z^{\frac{1}{2}\pm\mu}\,, \qquad \psi \sim (z_m - z)^{\frac{1}{2}\pm\tilde{\mu}}$$

- Two choices at z=0 ONLY IF $\mu < 1$ (and similarly at right end)
- In the cases of interest for $\gamma \leq \gamma_c$: $\mu = \tilde{\mu}$

❖ The possible self—adjoint extensions depend on μ

a) $\mu \geq 1$: UNIQUE b.c. → SCALAR MODES (MASSIVE)

b) $\mu < 1$: b.c. $\in SL(2,R) \times U(1)$ → [indep.: AdS_3 boundary (θ_1,θ_2)] → TENSOR & VECTOR MODES

STABILITY ANALYSIS ($m^2 > 0$) → EXACT LEGENDRE EIGENVALUE EQUATION

$$H = \mathcal{A}^\dagger \mathcal{A} \qquad V_\pm = \left(\frac{\pi}{z_m}\right)^2 \left[\frac{\left(\mu^2 - \frac{1}{4}\right)}{\sin^2\left(\frac{\pi z}{z_m}\right)} - \left(\frac{1}{2} \pm \mu\right)^2\right]$$

Legendre functions
(& Exact zero modes)

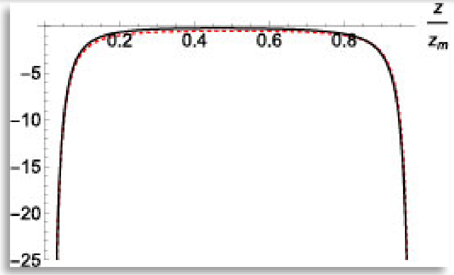
Dudas-Mourad Vacua : Stability , II

(Mourad, AS, 2023)

- (Singular) potentials closely approximated by Legendre ones
- Exact eigenvalue equations
- Vertical adjustments: compare with the exact zero modes

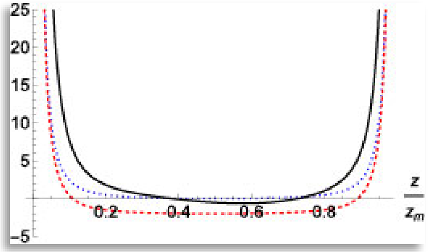
$$V_{\pm} = \left(\frac{\pi}{z_m}\right)^2 \left[\frac{\left(\mu^2 - \frac{1}{4}\right)}{\sin^2\left(\frac{\pi z}{z_m}\right)} - \left(\frac{1}{2} \pm \mu\right)^2\right]$$

Tensor Modes (μ=0) :

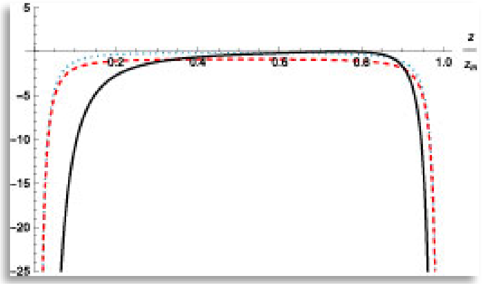


Contour lines of fixed tachyon mass

Scalar Modes (μ=1)



Vector Modes (μ=3/8) :



Dudas-Mourad Vacua : Stability , II

(Mourad, AS, 2023)

- (Singular) potentials closely approximated by Legendre ones
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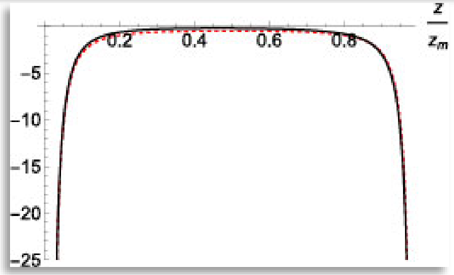
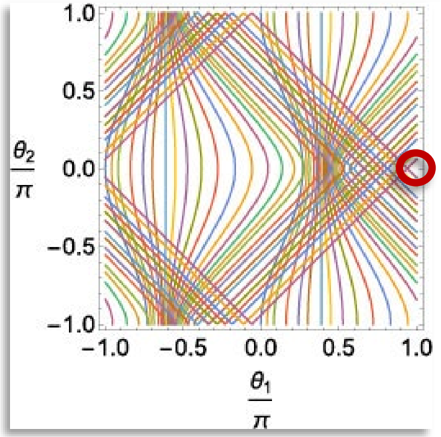
$$V_{\pm} = \left(\frac{\pi}{z_m}\right)^2 \left[\frac{\left(\mu^2 - \frac{1}{4}\right)}{\sin^2\left(\frac{\pi z}{z_m}\right)} - \left(\frac{1}{2} \pm \mu\right)^2\right]$$

Contour lines of fixed tachyon mass

Tensor Modes (μ=0) :

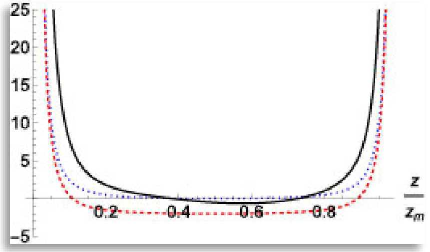




$$m^2 \simeq \left(\frac{\pi}{z_m}\right)^2 n\,(n+1)\ ,\ n = 0, 1, 2, \ldots$$

UNIQUE stable b.c. (π,0)
(massless 9D graviton !)

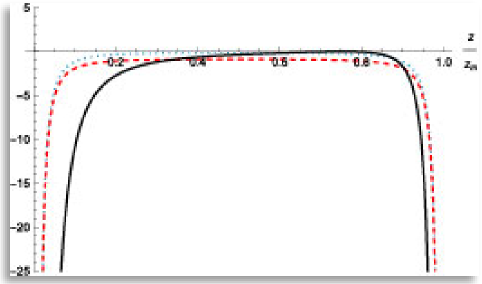
Scalar Modes (μ=1)

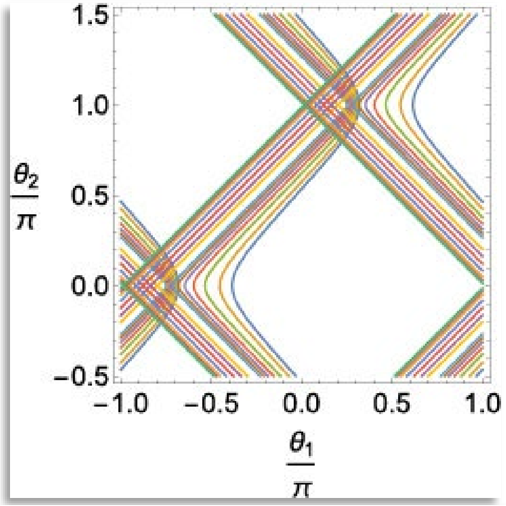


UNIQUE b.c.
[up to vertical adjustment]
(massive scalar)

$$m^2 \simeq \left(\frac{\pi}{z_m}\right)^2 \left[n(n+1) - \frac{7}{8}\right]\ ,\ n = 1, 2, \ldots$$

Vector Modes (μ=3/8) :





WIDE stability regions
(generic massive/massless vector)

V. Some Further Developments

(Insights on the nature of boundaries)

M_d x I x T^{9-d} Vacua with TADPOLE Potential

(Mourad, AS, 2021)

Solution	Left(L)	Left(T)	Left(g_s)	Right(L)	Right(T)	Right(g_s)
$\gamma = \gamma_c,\ \beta = 0$	F	is. (0)	∞	F	is. (0)	∞
$\gamma = \gamma_c,\ \beta \neq 0,\ p = 8$	F	is. (0)	0	F	is. (0)	∞
$\gamma = \gamma_c,\ \beta \neq 0,\ p < 8$	F	anis. (0)	A	F	is. (0)	∞
$\gamma < \gamma_c,\ p = 8$	F	is. (0)	0	F	is. (0)	∞
$\gamma < \gamma_c,\ \cos\eta < -\frac{\gamma}{\gamma_c}$	F	anis. (0)	0	F	anis. (0)	∞
$\gamma < \gamma_c,\ \cos\eta > \frac{\gamma}{\gamma_c}$	F	anis. (0)	∞	F	anis. (0)	0
$\gamma < \gamma_c,\ \lvert\cos\eta\rvert < \frac{\gamma}{\gamma_c}$	F	anis. (0)	∞	F	anis. (0)	∞
$\gamma > \gamma_c,\ p = 8,\ E = 0$	F	is. ($\neq 0$)	∞	∞	is. ($\neq 0$)	0
$\gamma > \gamma_c,\ E = 0,\ \phi_1 > 0$	F	is. ($\neq 0$)	∞	F	anis. (0)	∞
$\gamma > \gamma_c,\ E = 0,\ \phi_1 = 0$	F	is. ($\neq 0$)	∞	∞	is. ($\neq 0$)	0
$\gamma > \gamma_c,\ E = 0,\ \phi_1 < 0$	F	is. ($\neq 0$)	∞	∞	anis. (0)	0
$\gamma > \gamma_c,\ p = 8,\ E > 0,\ (u)$	F	is. ($\neq 0$)	∞	F	is. (0)	0
$\gamma > \gamma_c,\ p = 8,\ E > 0,\ (l)$	F	is. ($\neq 0$)	∞	∞	is. (0)	0
$\gamma > \gamma_c,\ E > 0,\ (u)$	F	is. ($\neq 0$)	∞	F	anis. (0)	A
$\gamma > \gamma_c,\ E > 0,\ (l)$	F	is. ($\neq 0$)	∞	∞	anis. (0)	0
$\gamma > \gamma_c,\ E < 0,$	F	is. ($\neq 0$)	∞	F	is. ($\neq 0$)	∞

INTERNAL INTERVAL again: the TWO GROUPS OF COLUMNS refer to its LEFT and RIGHT ends

- **F and ∞ : finite or infinite distances from r=0**
- **(0, A, ∞): g_s vanishes, can be anything (zero, finite or infinite, depending on parameters)**
- **(0): the vacuum approaches asymptotically the (an)isotropic T=0 solutions**

  [links with Blumenhagen et al (2021-23) & Uranga et al (2022-23)]
- **(≠0): the tension T is NOT sub-dominant in the limiting region**

NOTE: SHARP CHANGE OF BEHAVIOR across $\gamma=\gamma_c$

M_d x I x T $^{9-d}$ Vacua with TADPOLE Potential

(Mourad, AS, 2021)

Solution	Left(L)	Left(T)	Left(g_s)	Right(L)	Right(T)	Right(g_s)
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$\gamma < \gamma_c,\ \cos\eta > \frac{\gamma}{\gamma_c}$	F	anis. (0)	∞	F	anis. (0)	0
$\gamma < \gamma_c,\ \lvert\cos\eta\rvert < \frac{\gamma}{\gamma_c}$	F	anis. (0)	∞	F	anis. (0)	∞
$\gamma > \gamma_c,\ p = 8,\ E = 0$	F	is. ($\neq 0$)	∞	∞	is. ($\neq 0$)	0
$\gamma > \gamma_c,\ E = 0,\ \phi_1 > 0$	F	is. ($\neq 0$)	∞	F	anis. (0)	∞
$\gamma > \gamma_c,\ E = 0,\ \phi_1 = 0$	F	is. ($\neq 0$)	∞	∞	is. ($\neq 0$)	0
$\gamma > \gamma_c,\ E = 0,\ \phi_1 < 0$	F	is. ($\neq 0$)	∞	∞	anis. (0)	0
$\gamma > \gamma_c,\ p = 8,\ E > 0,\ (u)$	F	is. ($\neq 0$)	∞	F	is. (0)	0
$\gamma > \gamma_c,\ p = 8,\ E > 0,\ (l)$	F	is. ($\neq 0$)	∞	∞	is. (0)	0
$\gamma > \gamma_c,\ E > 0,\ (u)$	F	is. ($\neq 0$)	∞	F	anis. (0)	A
$\gamma > \gamma_c,\ E > 0,\ (l)$	F	is. ($\neq 0$)	∞	∞	anis. (0)	0
$\gamma > \gamma_c,\ E < 0,$	F	is. ($\neq 0$)	∞	F	is. ($\neq 0$)	∞

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- **(≠0): the tension T is NOT sub-dominant in the limiting region**

NOTE: SHARP CHANGE OF BEHAVIOR across $\gamma=\gamma_c$

D=4 with Fluxes on T^5 x I

(Mourad, AS, 2023)

❖ **Five-form flux in IIB → ϕ CONSTANT, SPATIAL INTERVAL of length l**

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu}{[h\, \sinh(\tilde{r})]^{\frac{1}{2}}} + [\sinh(\tilde{r})]^{\frac{1}{2}} \left[\ell^2\, e^{-\frac{\sqrt{10}}{2}\tilde{r}}\, d\tilde{r}^2 + (2\,\Phi\,\ell)^{\frac{2}{5}}\; e^{-\frac{\sqrt{10}}{10}\tilde{r}}\, (d\,\tilde{y}^i)^2\right]$$

$$\mathcal{H}_5^{(0)} = \frac{1}{2\,h}\,\frac{dx^0 \wedge \ldots \wedge dx^3 \wedge d\tilde{r}}{[\sinh(\tilde{r})]^2} + \Phi\, d\tilde{y}^1 \wedge \ldots \wedge d\tilde{y}^5$$

❖ **FINITE gs , BUT STILL CURVATURE SINGULARITY]**

USED EXTENSIVELY: (Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen, 2001)

- **SUSY BREAKING scale ~ 1/ℓ**
- **SUSY recovered asymptotically at one end**
- **Less familiar tensor eqs: (+ Einstein eqs.)**
- **Interval of FINITE length :** $\ell \sim H^{\frac{1}{4}}\ \rho^{\frac{5}{4}}$
- **PERTURBATIONS: → Schrödinger-like systems ∃ STABLE BOUNDARY CONDITONS! (Hypergeometric setup)**

A Closer Look at the Interval, I

(Mourad, AS, 2022, 2023)

One can "explore" the interval with a probe brane :

$$\frac{S}{V_3} = -T_3 \int dt\, e^{4A(r(t))} \sqrt{1 - e^{2(B-A)(r(t))}\, \dot r(t)^2} + q_3 \int b[r(t)]\, dt$$

$$b(r) = -\frac{1}{4\,\rho\, H}\left[\coth\left(\frac{r}{\rho}\right) - 1\right]$$

$$E = \frac{T_3\; e^{4A(r(t))}}{\sqrt{1 - e^{2(3A+5C)(r(t))}\, \dot r(t)^2}} - q_3\, b$$

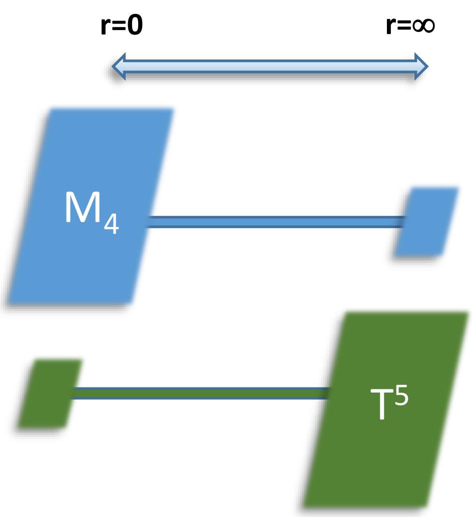
The probe brane feels the potential below:

$$V(r) = T_3 e^{4A} - q_3 b = \frac{1}{2\,|H|\,\rho}\left[\frac{T_3}{\sinh\left(\frac{r}{\rho}\right)} + \frac{q_3\,\text{sign}(H)}{2}\left(\coth\left(\frac{r}{\rho}\right) - 1\right)\right]$$

$$\mathbf{V} \sim \boxed{\frac{\mathbf{1}}{\mathbf{r}}\left[\mathbf{T_3} + \frac{\mathbf{q_3}}{\mathbf{2}}\,\text{sign}\,(\mathbf{H})\right]}$$

BPS r=0 endpoint ! (consistently w. Killing spinor emerging as ρ→∞)

NO FORCE: if T3 and q3 are TUNED (the factor ½ depends on conventions)



A Closer Look at the Interval, II

(Mourad, AS, 2022, 2023)

Einstein action with York-Gibbons-Hawking term & its variation :

$$
\begin{aligned}
\mathcal{S}_{grav} &= \frac{1}{2\,k_{10}^2}\int_{\mathcal{M}} d^9x\,dr\,\sqrt{-\tilde{\mathbf{g}}}\,\mathcal{N}\left[\tilde{\mathbf{R}} + \mathcal{K}_{\mathbf{mn}}\,\mathcal{K}_{\mathbf{pq}}\left(\tilde{\mathbf{g}}^{\mathbf{mn}}\,\tilde{\mathbf{g}}^{\mathbf{pq}} - \tilde{\mathbf{g}}^{\mathbf{mp}}\,\tilde{\mathbf{g}}^{\mathbf{nq}}\right)\right] \\
\tilde{g}_{mn} &= g_{mn}\,, \qquad \mathcal{N}_m = g_{mr}\,, \qquad \mathcal{N}^2 + \mathcal{N}^m\,\mathcal{N}_m = g_{rr}\,, \qquad \mathcal{K}_{mn} = \frac{1}{2\,\mathcal{N}}\left(\partial_r\,\tilde{g}_{mn} - \tilde{D}_{(m}\,\mathcal{N}_{n)}\right) \\
\mathbf{G}_{\mathbf{mn}} &- \mathbf{T}_{\mathbf{mn}} + \frac{\delta(\mathbf{r}-\mathbf{R}^\star)}{\mathcal{N}}\left[\mathcal{K}_{\mathbf{mn}} - \tilde{\mathbf{g}}_{\mathbf{mn}}\,\mathcal{K}\right] - \frac{\delta(\mathbf{r}-\mathbf{r}^\star)}{\mathcal{N}}\left[\mathcal{K}_{\mathbf{mn}} - \tilde{\mathbf{g}}_{\mathbf{mn}}\,\mathcal{K}\right] = \mathbf{0}\,,
\end{aligned}
$$

This reveals TENSION (and CHARGE) of an EFFECTIVE BPS O_3 orientifold at r=0
Neat realization of "dynamical cobordism" (HERE protected by SUSY)

(McNamara, Vafa, 2019)
(Uranga et al, 2021)
(Blumehagen et al, 2021)
(Raucci, 2022)

$$
\begin{aligned}
G_{\mu\nu} &- T_{\mu\nu} + H\,\tilde{g}_{\mu\nu}\,\sqrt{-\det\,\tilde{g}_{\mu\nu}}\;\delta(z-z^\star) = 0 \\
\mathcal{S}_T &= \frac{H}{k_{10}^2}\int d^9x\,\sqrt{-\det\,\tilde{g}_{\mu\nu}}\,\Big|_{z^\star} \longrightarrow \boxed{\mathbf{T} = -\,\frac{\mathbf{\Phi}}{\mathbf{k}_{10}^2}}
\end{aligned}
$$

- **SITUATION LESS CLEAR at other NON-SUSY end (BUT opposite charge)**

Summarizing

❖ **Tadpoles → Dudas-Mourad vacua: BOUNDARIES play a key role !**

✓ **STABILITY:** NO tachyon modes emerge [cfr UNSTABLE AdS x S !]

(Basile, Mourad, AS, 2018)
(Raucci, 2023)
(Mourad, AS, 2023)

- (Proportional) Tension & charge of EFFECTIVE (SUSY) ORIENTIFOLD at one end

- ∃ (explicit) correspondence with work on "Dynamical Cobordism"

  [See: (Bergshoeff, Riccioni et al, 2006 –) for a wide zoo of lower-dimensional branes built via SUGRA U-dualities]

(McNamara, Vafa, 2019)
(Uranga et al, 2021)
(Blumehagen et al, 2021)
(Raucci, 2022)

✓ **COSMOLOGY:** climbing & inflation → (lack-of-power [enhanced tens.-to-scal. ratio]
[& non-Gaussianities?]

- INTRIGUING **INSTABILITY OF ISOTROPY** (k=0) in "climbing scalar" Cosmology : **4D by accident?**

➢ **BRANES & TADPOLES** → (un)charged branes in Dudas-Mourad vacua (Salvatore Raucci's talk)

(2406.14296, 2406.16327)

Thank You
for
Your Attention