The Tadpole Conjecture beyond geometry

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Work in collaboration with

Katrin Becker, Nathan Brady, Miguel Morros, Anindya Sengupta and Qin You
arXiv: 2407.xxxxx

Iosif Bena, Johan Blåbäck and Severin Lüst
arXiv: 2010.10519

Bena, Braun, Brodie, Fraiman, Grimm, van de Heisteeg, Herraez, S. Lust, Parra de Freitas, Plauschinn
20-23

String Phenomenology, June 2024

Introduction

- Flux compactifications: building block in string pheno because of moduli stabilization

Dasgupta, Rajesh, Sethi 99

Giddings, Kachru, Polchinski 01

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(1) fluxes back-react on geometry

(2) fluxes induce positive charges that needs to be cancelled globally Maldacena, Nuñez 00

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(1) IIB/F-theory most studied setup: flux solutions $M_{\text{ink}4} \times_w \text{CY}$

→ Drawback: odd fluxes $(H_3, F_3) \Rightarrow$ only complex structure mod stabilized

Kähler moduli not stabilized

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- Tadpole conjecture: common lore not true!

The tadpole conjecture

Bena, Blåbäck, M.G., Lüst 20

For a large number of moduli stabilized at a generic point in moduli space, the induced charge N_{flux} satisfies

$$N_{\text{flux}} > \alpha\, n_{\text{stab}}$$

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(2) with $\alpha > \ldots$ stay awake 😆

Tadpole cancelation condition

- Fluxes induce D3-charge. In a compact space total charge should be zero

- In type IIB with 3-form fluxes

$$N_{\text{flux}} = \int F_3 \wedge H_3 \leq |Q_{O3}|$$

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- D7-branes wrapped on 4 cycles also have negative charge (and D7-moduli)

- Unified description in F-theory

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$$N_{\text{flux}} = \frac{1}{2} \int G_4 \wedge G_4 \quad \leq \frac{\chi(CY_4)}{24}$$

H_3, F_3 and flux on D7

all the negative 3-charge from D7/O7

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c.s., dilaton and D7 moduli
(can be stabilized by G_4)

$$N_{\text{flux}} = \frac{1}{2}\int G_4 \wedge G_4 \leq \frac{\chi(CY_4)}{24}$$

$$\frac{\chi}{24} = \frac{1}{4}(h^{3,1} + h^{1,1} - h^{2,1} + 8)$$

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Tadpole conjecture

$$N_{\text{flux}} > \alpha n_{\text{stab}}$$

If $\alpha > \frac{1}{4}$, cannot stabilize
all moduli in F-theory
(if number is large)!

Supporting examples for $\frac{N_{\text{flux}}}{n_{\text{stab}}} > \frac{1}{3}$ in CY, with $n_{\text{stab}} = n_{\text{moduli}}$

Description	n_{stab}	N_{flux}	$\alpha = \frac{N_{\text{flux}}}{n_{\text{stab}}}$	Ref
IIB at symm pt in mod space	$h^{2,1} = 128$	48	0.38	Giryavets, Kachru, Tripathy, Trivedi 03
	$h^{2,1} = 272$	124	0.46	Demirtas, Kim, Mc Allister, Morritz 19
F-theory on sextic CY at symm point	$h^{3,1} = 426$	775/4 587/4	0.45 0.34	Braun, Valandro 20 Braun, Fortin, Lopez Garcia, Villaflor Loyola 24 See Braun's talk
F-theory on $\mathbb{CP}^3$ base	$n_7 = 3728$	1638	0.44	Collinucci, Denef Esole 08
F-theory on K3xK3	$n_{\text{mod}} = 57$	25	0.44	Bena, Blåbäck, M.G., Lust 20
IIB on (3,51) CY$_3$ at large complex structure	$h^{2,1} = 51$	36	0.35	Coudarchet, Marchesano, Prieto, Urkiola '23

Supporting examples for linear behavior $N_{\text{flux}} > \alpha\, n_{\text{stab}}$

Description	n_{stab}	N_{flux}	$\alpha = \frac{N_{\text{flux}}}{n_{\text{stab}}}$	Ref
F-theory on any weak-Fano base	$n_7 = 58c_1^3(B) + 16$	$\frac{7}{16}(58c_1^3(B) + 15)$	0.44	Bena, Brodie, M.G. 21
F-theory on CY at LARGE complex structure	$n_{\text{stab}} \leq n_{\text{mod}}$	$\alpha\, n$	In all exemples out of a large set $\alpha > 0.7$	M.G., Grimm, van de Heisteeg, Herraez, Plauschinn 22

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HERE!				

Tadpole conjecture beyond geometry:
mirror duals of rigid Calabi-Yau manifolds

Tadpole conjecture beyond geometry: mirror duals of rigid Calabi-Yau manifolds

Hodge diamond of a Calabi-Yau

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{1,2} & & 1 \\ & 0 & & h^{2,2} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

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On the two-dimensional (2,2) SCFT on the world-sheet of strings in CY:

$h^{2,1}$: marginal deformations in the (c,c) ring

$h^{1,1}$: marginal deformations in the (a,c) ring

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↕ symmetry

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Lecher, Vafa, Warner '89

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$h^{1,1} = 0\,!$

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No volume form

$$\begin{tikzcd} J \wedge J \wedge J = \mathrm{vol} \\ \in h^{1,1} \arrow[u, dotted] \end{tikzcd}$$

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$\in h^{1,1}$

Not a manifold

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But perfectly fine from the world-sheet point of view

Description in terms of Landau-Ginzburg models Vafa ‘89

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Standard notions in geometric flux compactifications (flux superpotential, tadpole) still apply

Becker, Becker, Vafa, Walcher '06

IIB (geometric) flux Compactifications on Calabi-Yau orientifolds

$$M_{10} = M_4 \times \mathrm{CY}_3$$

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- $h^{2,1}$ complex structure moduli (volumes of 3-cycles) ~ $\mathcal{O}(100)$

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- $h^{2,1}$ complex structure moduli (volumes of 3-cycles) $\sim \mathcal{O}(100)$

- Add 3-form fluxes

$$\int_{\Gamma_n} F_3 = \overset{\in \mathbb{Z}}{M^n} \qquad \int_{\Gamma_n} H_3 = K^n \qquad n = 1, \ldots, 2h^{2,1} + 2$$

basis of 3-cycles

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basis of 3-cycles

- In the 4d EFT: potential for complex structure moduli (and dilaton)

$$G_3 = F_3 - \tau H_3$$

$$V = e^K \left(|D_I W|^2 - 3|W|^2 \right) \qquad \text{with} \qquad W = \int_{CY} G_3 \wedge \Omega \quad \sim (M - \tau K) f(z)$$

Gukov, Vafa, Witten 99

IIB geometric flux Compactifications on Calabi-Yau orientifolds

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- SUSY minima at

- $D_I W = 0 \rightarrow$ equation for complex structure moduli: get a vev depending on M^n, K^n

$$D_I W = \int_{CY} G_3 \wedge \chi_I \qquad \Rightarrow G^{(1,2)} = 0$$

(2,1) forms

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(2,1) forms

- $D_a W = \partial_a K\, W = 0 \rightarrow W_0 = 0$. No equation for Kähler moduli. Unfixed by fluxes

$$\Rightarrow G^{(0,3)} = 0$$

IIB geometric flux Compactifications on Calabi-Yau orientifolds

- In the 4d $\mathcal{N} = 1$ EFT

$$V = e^K \left(|DW|^2 - 3|W|^2 \right) \qquad \text{with} \qquad W = \int_{CY} G_3 \wedge \Omega \sim (M - \tau K) f(z^I)$$

- SUSY minima at

- $D_I W = 0 \rightarrow$ equation for complex structure moduli: get a vev depending on M^n, K^n

$$D_I W = \int_{CY} G_3 \wedge \chi_I \qquad \Rightarrow G^{(1,2)} = 0$$

(2,1) forms

- $D_a W = \partial_a K\, W = 0 \rightarrow W_0 = 0$. No equation for Kähler moduli. Unfixed by fluxes

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- Tadpole cancelation condition

$$N_{\text{flux}} = \int F_3 \wedge H_3 = \underbrace{M^n K_n}_{> 0} \leq |Q_{O3}|$$

at minimum
$H_3 = \star F_3$
(dilaton eq says $G^{(3,0)} = 0$)

IIB Landau Ginzburg models with flux

- h^{2,1} complex structure moduli ((c,c) marginal deformations or RR ground states in CFT)

- Add 3-form fluxes

$$\int_{\Gamma_n} F_3 = \overset{\in \mathbb{Z}}{M^n} \qquad \int_{\Gamma_n} H_3 = K^n \qquad n = 1, \dots, 2h^{2,1} + 2$$

basis of 3-cycles (susy cycles wrapped by A-branes ↔ bdy cond in the CFT)

- 4d $\mathcal{N} = 1$ EFT

$$G_3 = F_3 - \tau H_3$$

$$V = e^K \left(|D_I W|^2 - 3|W|^2 \right)$$

with

Becker, Becker, Vafa, Walcher 06

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Landau Ginzburg models

Vafa '89

Lerche, Vafa, Warner '89

- 2d $\mathcal{N} = (2{,}2)$ theories of r chiral fields $\Phi_i,\ i = 1,\ldots,r$

$$S_{2d} = \underbrace{\int d^2z\, d^4\theta\, \mathscr{K}(\Phi_i, \bar{\Phi}_i)}_{\text{world-sheet Kähler potential}} + \underbrace{\int d^2z\, d^2\theta\, \mathscr{W}(\Phi_i)}_{\text{world-sheet superpotential}}$$

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E.g. $k_1 = k_2 = \ldots k_r = k$ $\qquad g(\Phi_i) = e^{i\omega}\Phi_i \qquad \omega = 2\pi/(k+2)$

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- Lead to 4-dimensional $\mathcal{N} = 2$ string vacua (as CY)

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- Can orientifold; quotient by $\Omega\,\sigma$. $\mathscr{W}(\sigma(\Phi)) = -\,\mathscr{W}(\Phi)$

  E.g. in k^r model $(\mathscr{W} = \sum_{i=1}^{r} \Phi_i^{k+2})$ can take $\sigma(\Phi_i) = e^{i\pi/(k+2)}\Phi_i$

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Two particularly interesting k^r models with $c = 9$

1^9 2^6

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1^9

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 0 & & 0 & \\
1 & & 63 & & 63 & & 1 \\
 & 0 & & 0 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}$$

2^6

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 0 & & 0 & \\
1 & & 90 & & 90 & & 1 \\
 & 0 & & 0 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}$$

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With $\sigma(\Phi_1, \ldots, \Phi_9) = -\,(\Phi_2, \Phi_1, \Phi_3, \ldots, \Phi_9)$ $\sigma(\Phi_1, \ldots, \Phi_6) = ie^{i\pi/4}(\Phi_1, \ldots, \Phi_6)$

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$|Q_{O3}| = 12$

mirror of $\dfrac{T^6}{\mathbb{Z}_3\times\mathbb{Z}_3}$

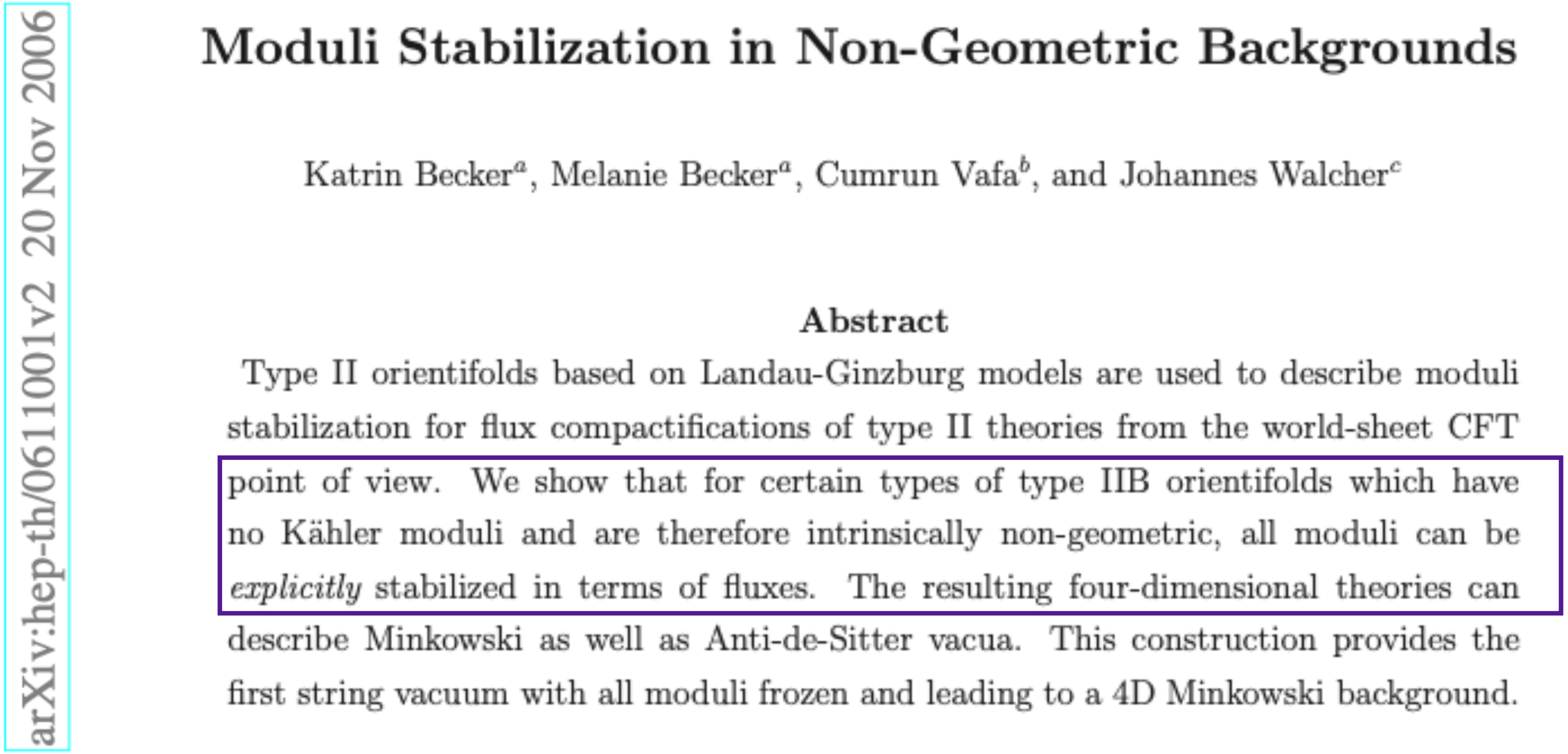
$\sigma(\Phi_1,\ldots,\Phi_6) = ie^{i\pi/4}(\Phi_1,\ldots,\Phi_6)$

$|Q_{O3}| = 40$

mirror of $\dfrac{T^6}{\mathbb{Z}_4\times\mathbb{Z}_4}$

Moduli stabilisation in these Landau Ginzburg models

Becker, Becker, Vafa, Walcher 06



arXiv:hep-th/0611001v2 20 Nov 2006

Moduli Stabilization in Non-Geometric Backgrounds

Katrin Becker[a], Melanie Becker[a], Cumrun Vafa[b], and Johannes Walcher[c]

Abstract

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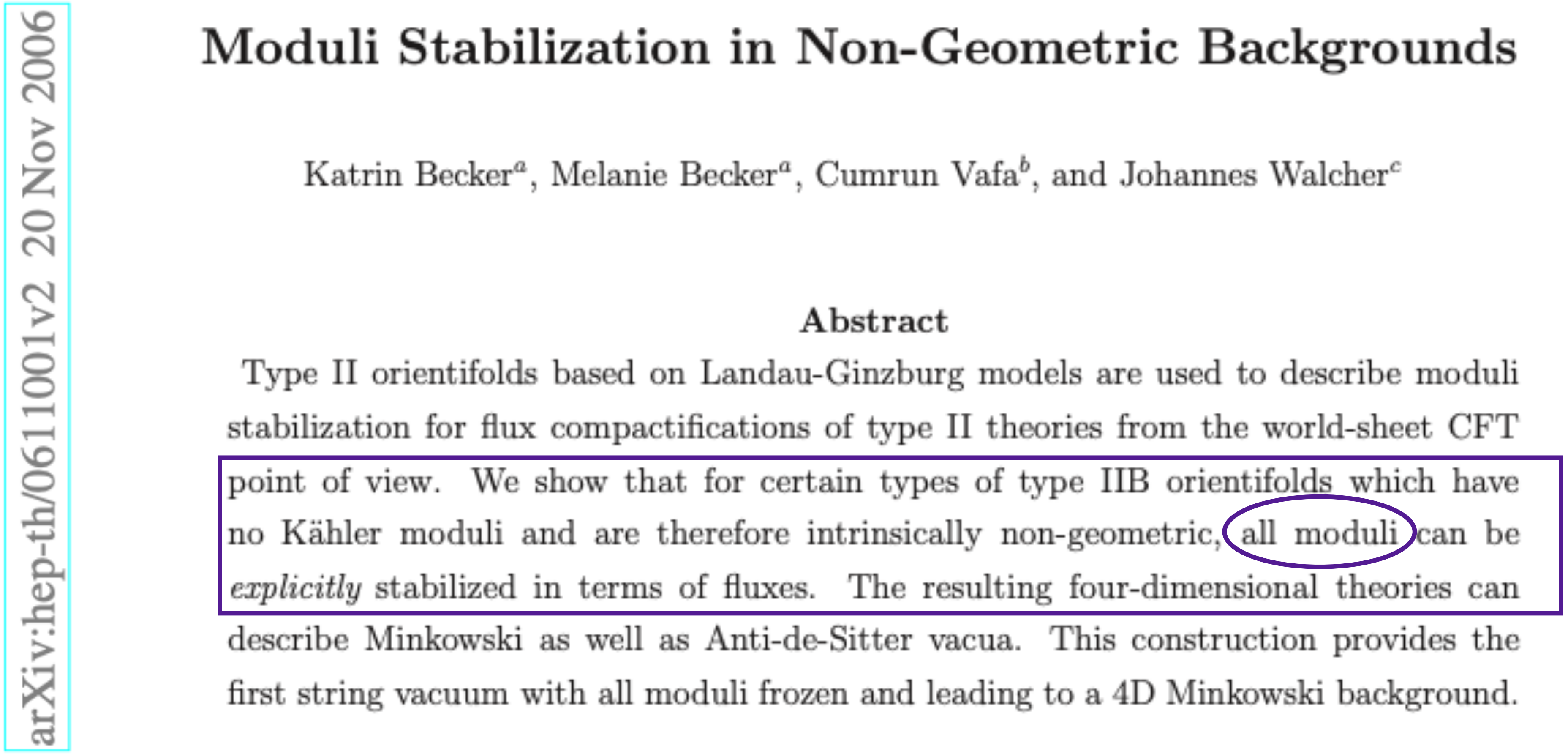

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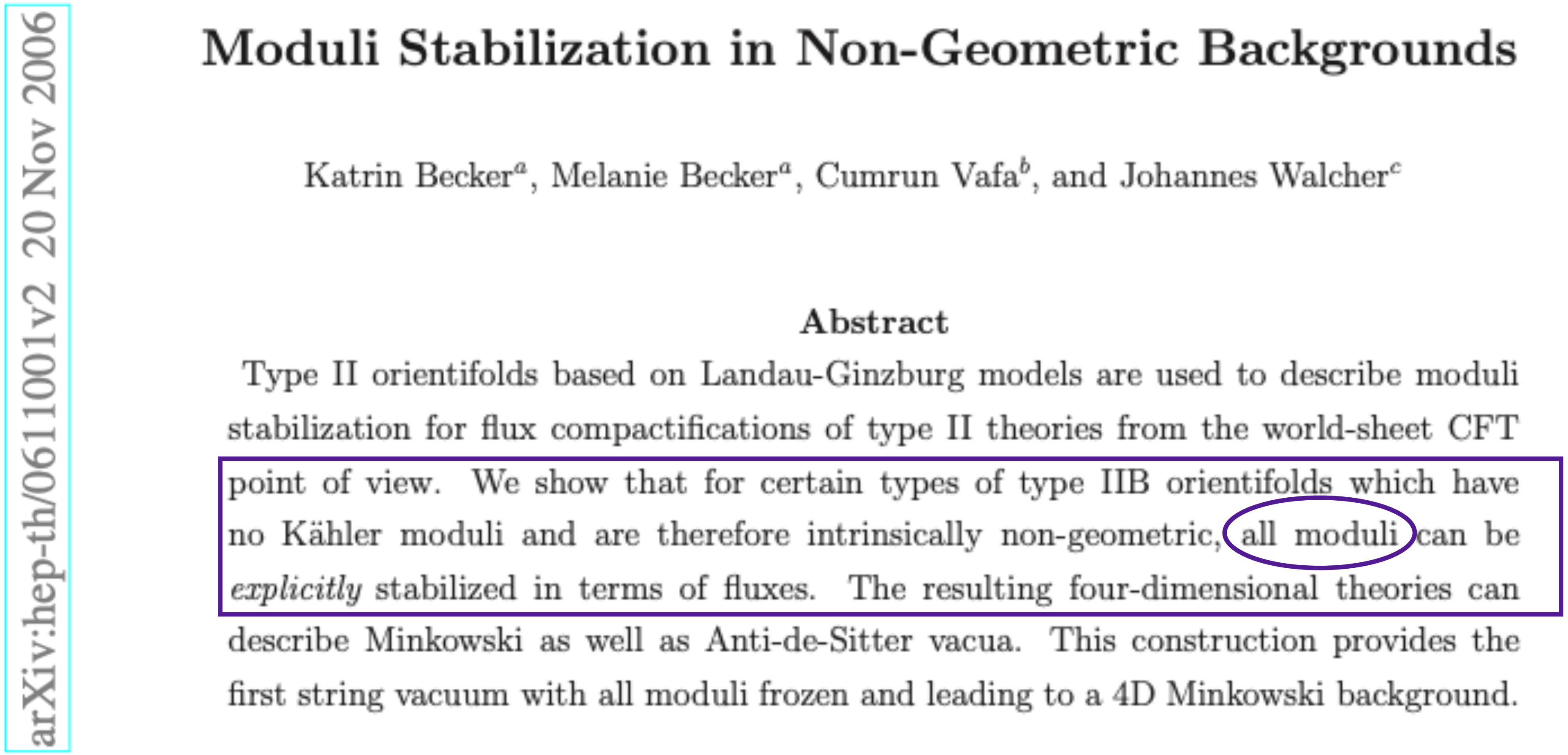

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All moduli?

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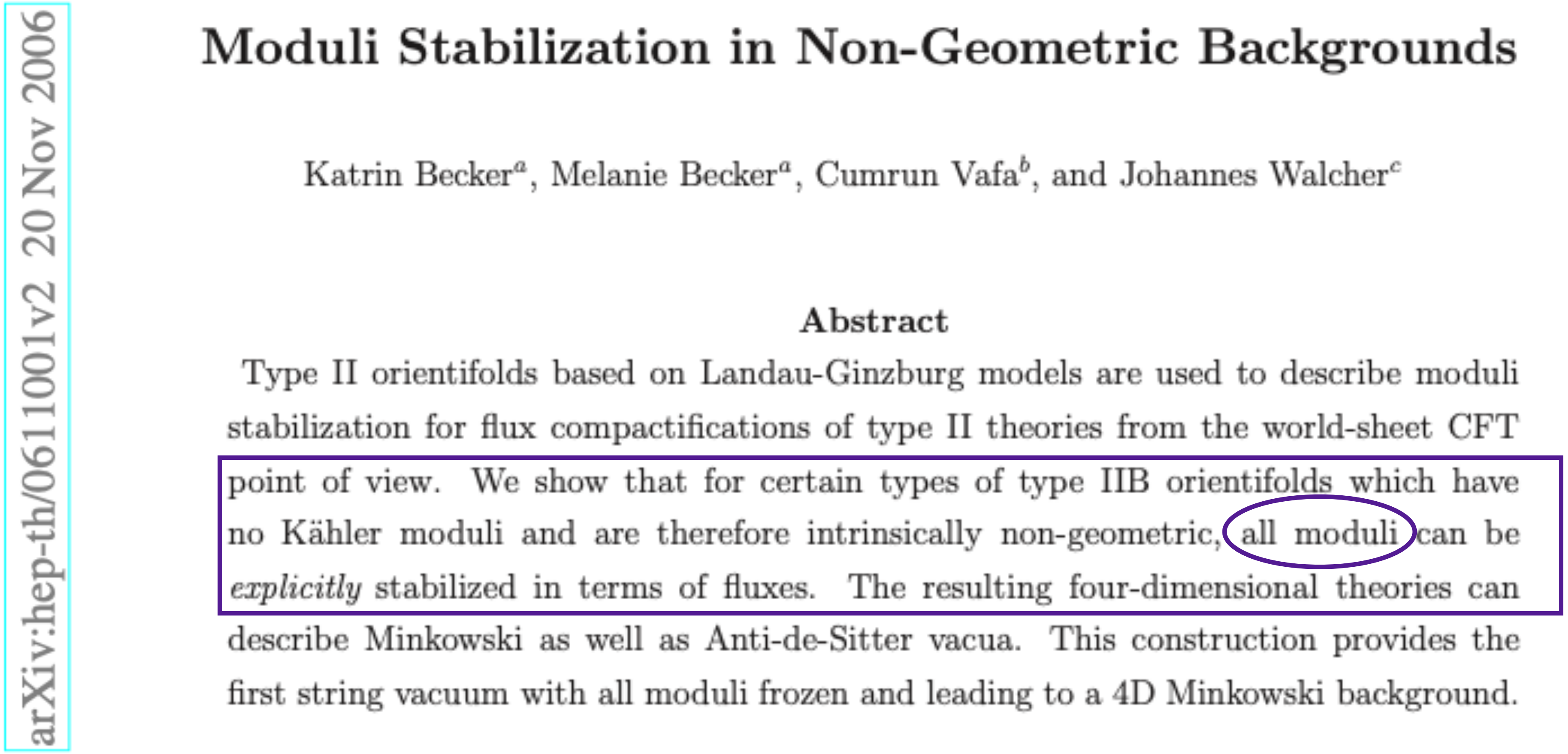

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If $N_{\text{flux}} > \frac{1}{3} n_{\text{stab}}$ $\Rightarrow$ to fix all moduli need $\longrightarrow 1^9 : N_{\text{flux}} > \frac{1}{3} 63 = 21$

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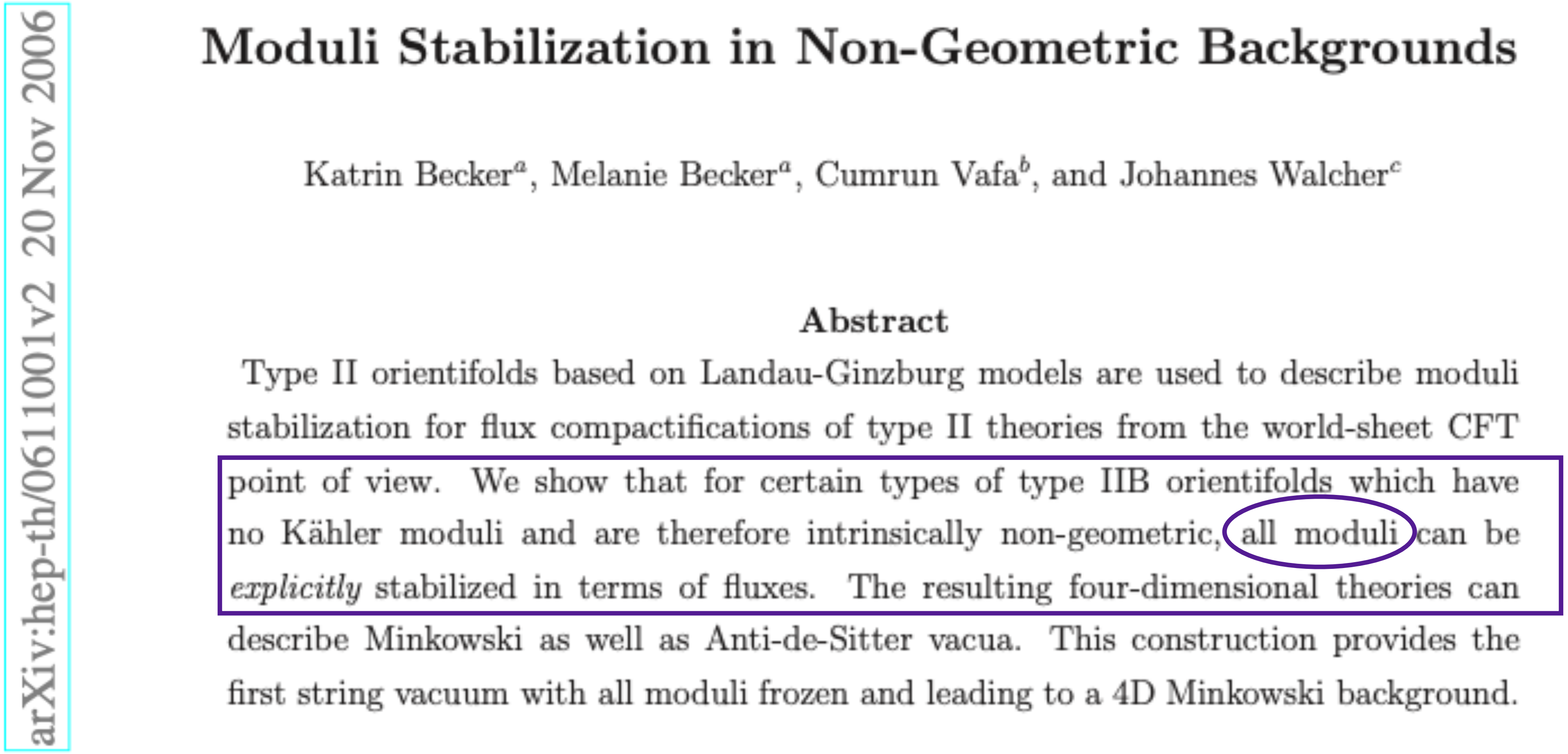

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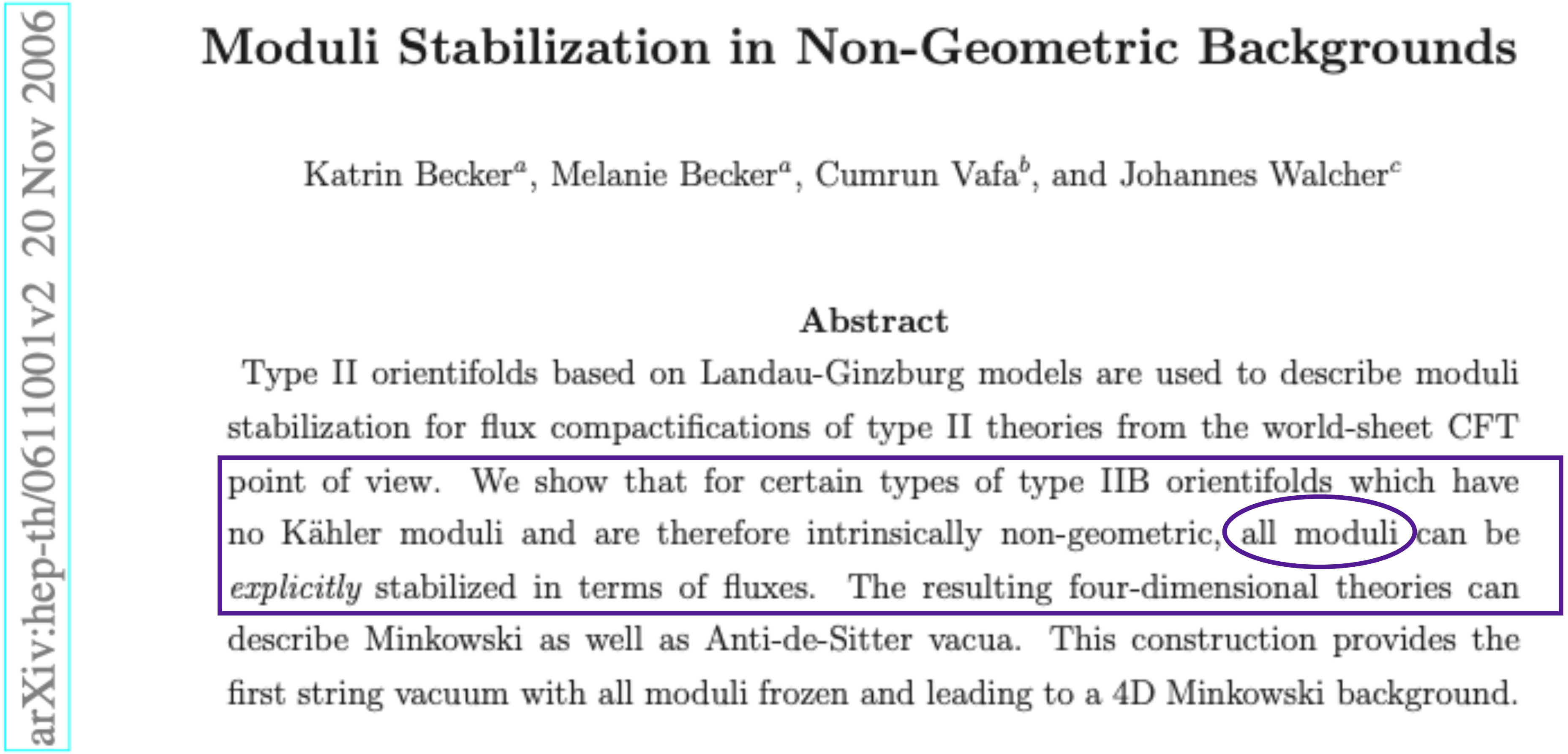

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Type II orientifolds based on Landau-Ginzburg models are used to describe moduli stabilization for flux compactifications of type II theories from the world-sheet CFT point of view. We show that for certain types of type IIB orientifolds which have no Kähler moduli and are therefore intrinsically non-geometric, all moduli can be *explicitly* stabilized in terms of fluxes. The resulting four-dimensional theories can describe Minkowski as well as Anti-de-Sitter vacua. This construction provides the first string vacuum with all moduli frozen and leading to a 4D Minkowski background.

All moduli?

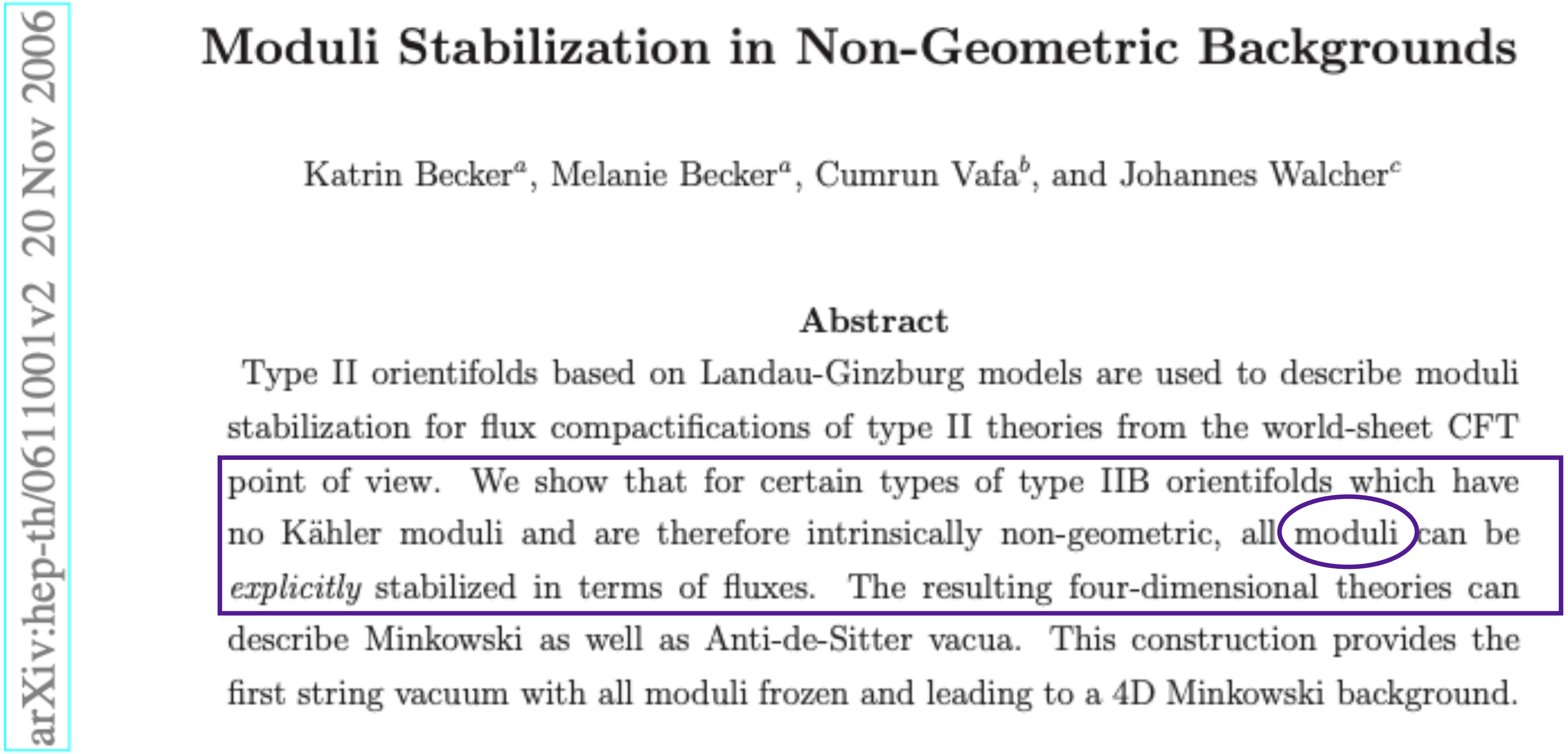
If $N_{\text{flux}} > \frac{1}{3} n_{\text{stab}}$ $\Rightarrow$ to fix all moduli need

$\longrightarrow 1^9 \; : \; N_{\text{flux}} > \frac{1}{3} 63 = 21$ but $|Q_{O3}| = 12\,!$

$\longrightarrow 2^6 \; : \; N_{\text{flux}} > \frac{1}{3} 90 = 30$ $|Q_{O3}| = 40$

Moduli stabilisation in these Landau Ginzburg models

Becker, Becker, Vafa, Walcher 06



arXiv:hep-th/0611001v2 20 Nov 2006

Moduli Stabilization in Non-Geometric Backgrounds

Katrin Becker[a], Melanie Becker[a], Cumrun Vafa[b], and Johannes Walcher[c]

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Type II orientifolds based on Landau-Ginzburg models are used to describe moduli stabilization for flux compactifications of type II theories from the world-sheet CFT point of view. We show that for certain types of type IIB orientifolds which have no Kähler moduli and are therefore intrinsically non-geometric, all moduli can be *explicitly* stabilized in terms of fluxes. The resulting four-dimensional theories can describe Minkowski as well as Anti-de-Sitter vacua. This construction provides the first string vacuum with all moduli frozen and leading to a 4D Minkowski background.

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Moduli

- Moduli: Deformations of $\mathscr{W}$ (for concreteness all that follows for 2^6)

$$\mathscr{W} = \sum_{i=1}^{6} \Phi_i^4 + \sum_{L=1} t^L \, \Phi^{\boldsymbol{L-1}}$$

$$\Phi^{\boldsymbol{L-1}} \equiv \Phi_1^{l_1-1} \, \Phi_2^{l_2-1} \ldots \Phi_6^{l_r-1}$$

$$L = (l_1, \ldots, l_6)$$

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$\Sigma\, l_i$	6	10	14	18
(p, q)	(3,0)	(2,1)	(1,2)	(0,3)

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Fluxes & Moduli stabilisation

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$$\int_{\Gamma_N} F_3 = \overset{\in \mathbb{Z}}{M^N} \qquad \int_{\Gamma_N} H_3 = K^N$$

$$L = (l_1, \ldots, l_6) \quad l_i = 1,2,3$$

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(not all independent)

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$$W = \int G_3 \wedge \Omega = \sum_N (M^N - \tau K^N)\, \Omega_N\,, \qquad \Omega_N = \int_{\Gamma_N} e^{-\mathscr{W}(\Phi,t)}\, d^4\Phi \sim \sum_p t_1 \ldots t_p\, i^{(L_1+\ldots+L_p)\cdot N}$$

Fluxes & Moduli stabilisation

- Massive moduli

tadpole conjecture

Fluxes & Moduli stabilisation

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$$\Sigma\, l_i = 10 \;\Rightarrow I$$

$$n_{\text{mass}} = \text{rank}\, M \qquad \Sigma\, l_i = 14 \;\Rightarrow \bar{I}$$

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- Here testing a weaker form of tadpole conjecture

Two alternative procedures

- Turn on G_3 on one, two, three,… L^I component ($\Sigma\, l_i = 10$)

  $\longrightarrow \in H^{(2,1)}$ automatic

  $\longrightarrow M^N, K^N \in \mathbb{Z}$ to be imposed

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- Beyond: use algorithms for smart search (start from a set of minimal length vectors)
- Compute $N_{\text{flux}}\,, n_{\text{mass}}$

$$N_{\text{flux}} = \int F_3 \wedge H_3 = M^N K_N$$

$$N_{\text{flux}} = \frac{i}{\tau - \bar{\tau}} \int G_3 \wedge \bar{G}_3 = \frac{1}{2\tau_2} \,|\, G_I|^2$$

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Results: 1^9

Becker, Gonazlo, Walcher, Wrase '22

Becker, Brady, Sengupta '23

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$$|Q_{O3}| = 12$$

$$h^{2,1} = 63$$

$$\tau = e^{2\pi i/3}$$

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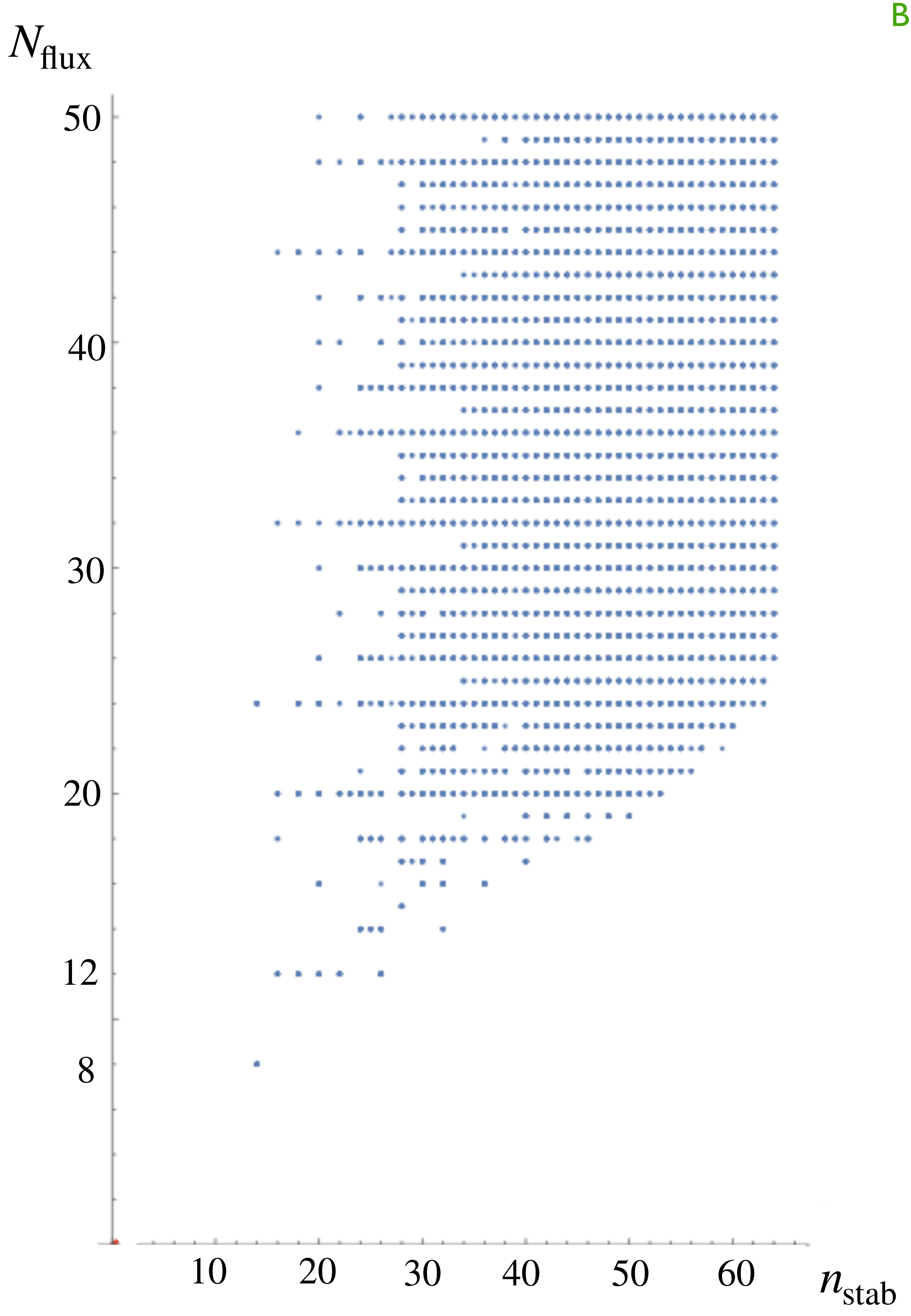

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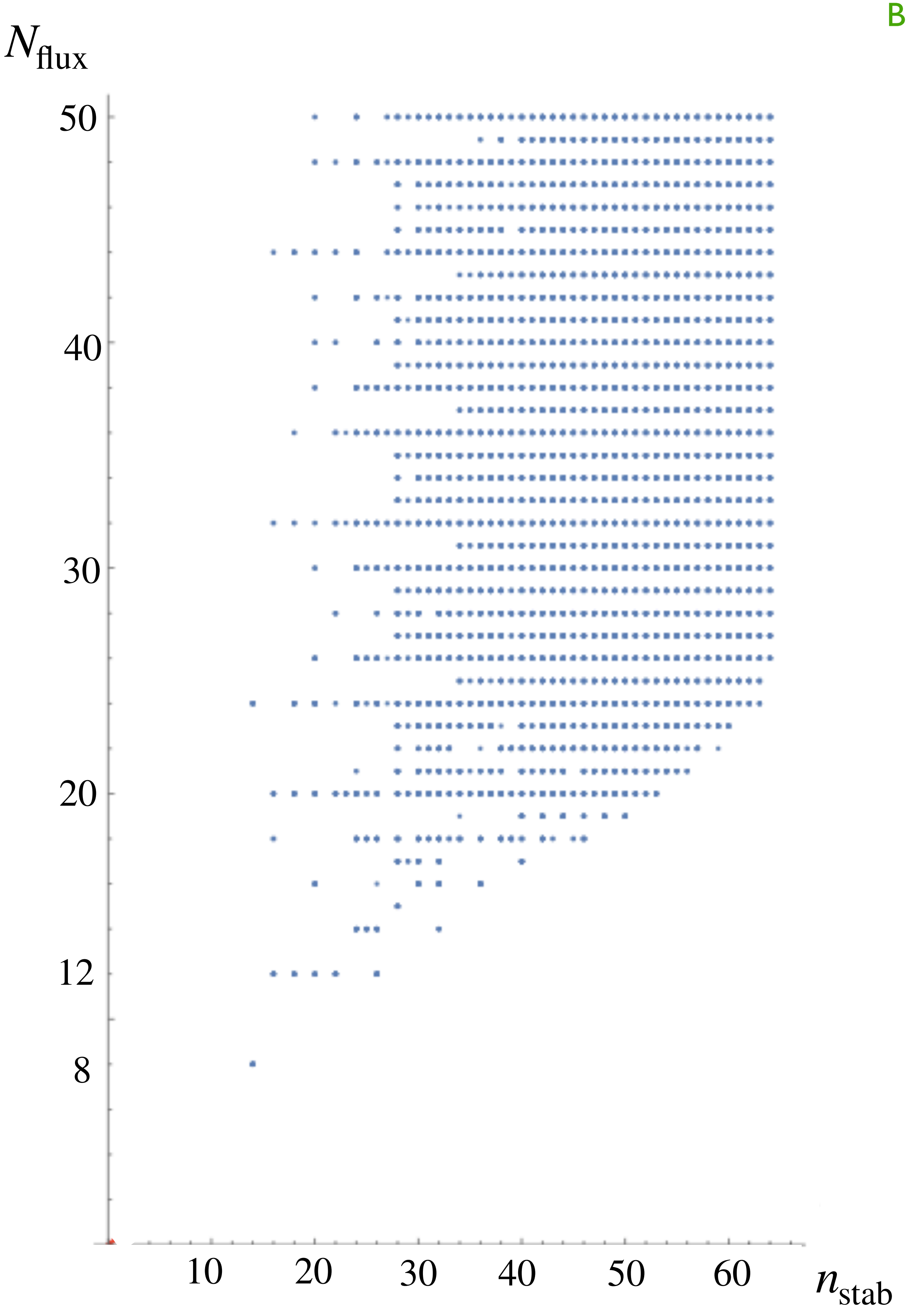

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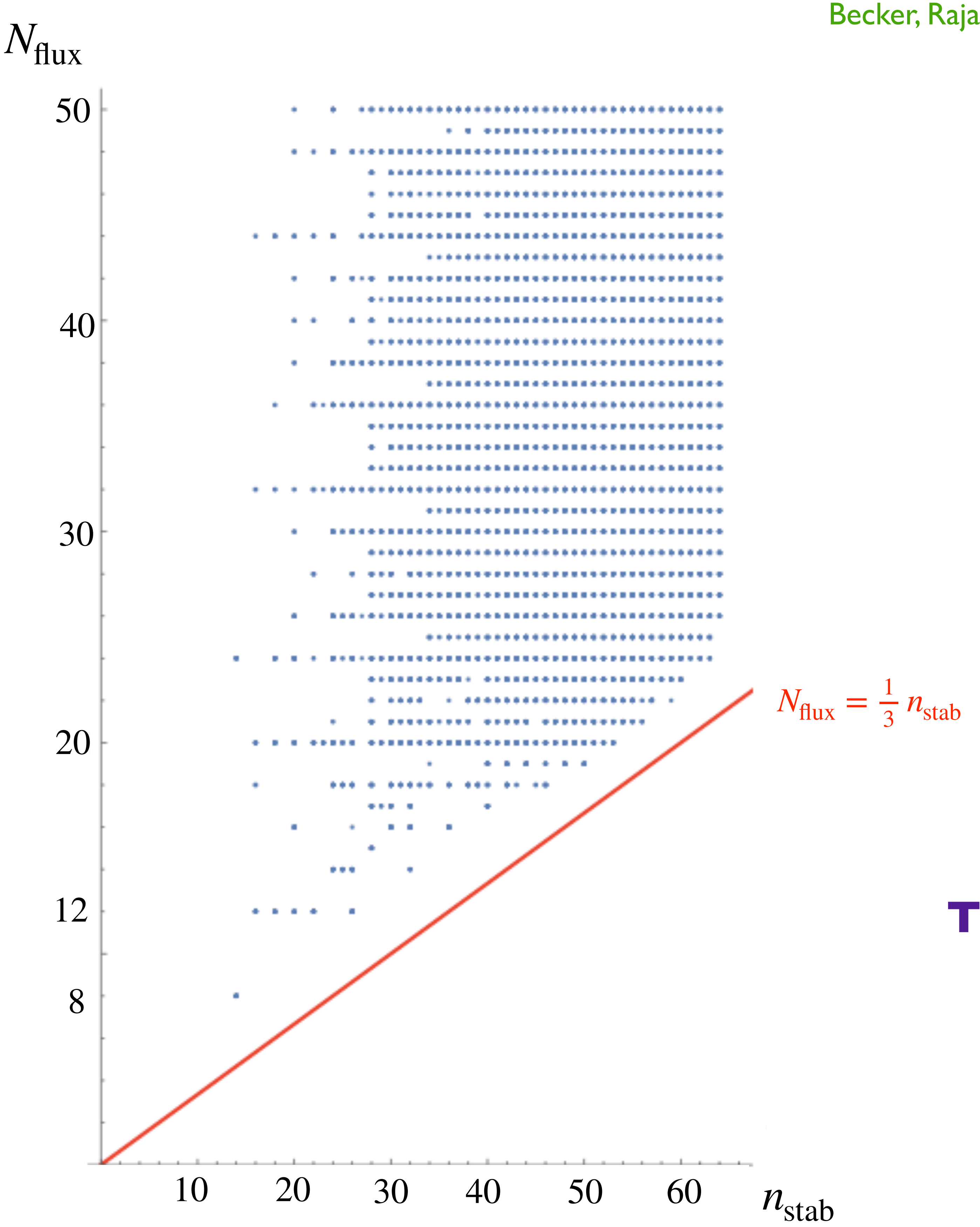

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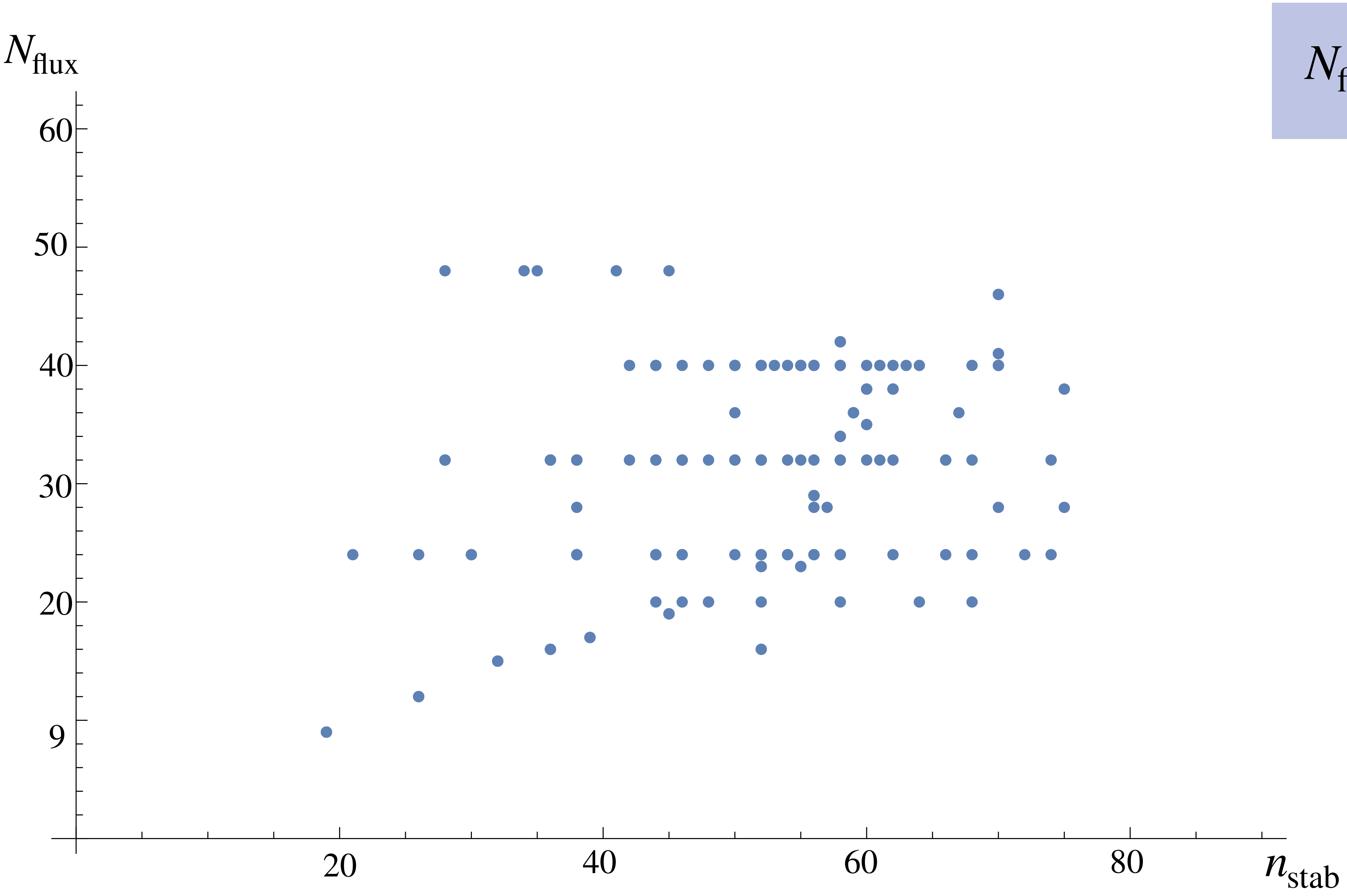

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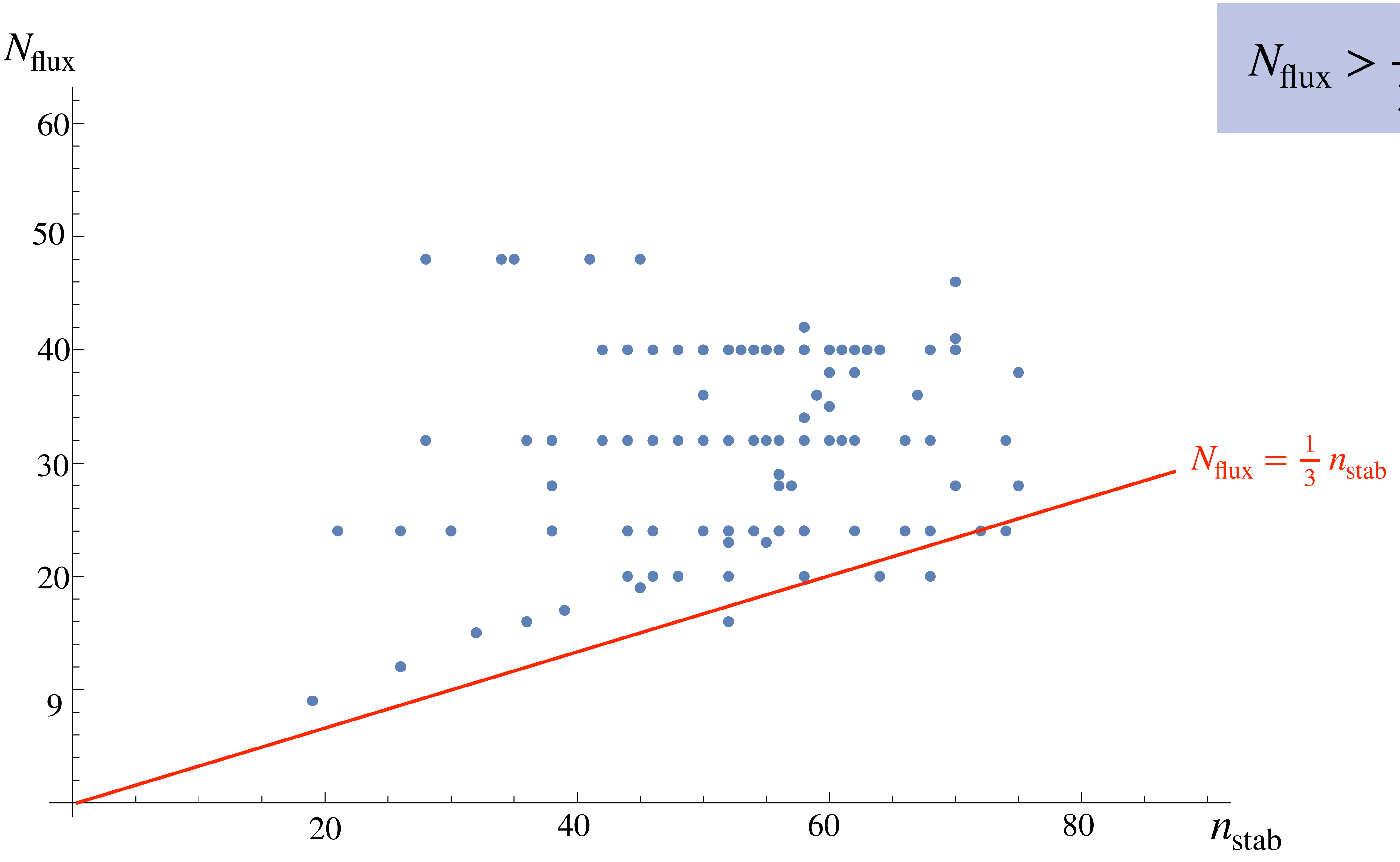

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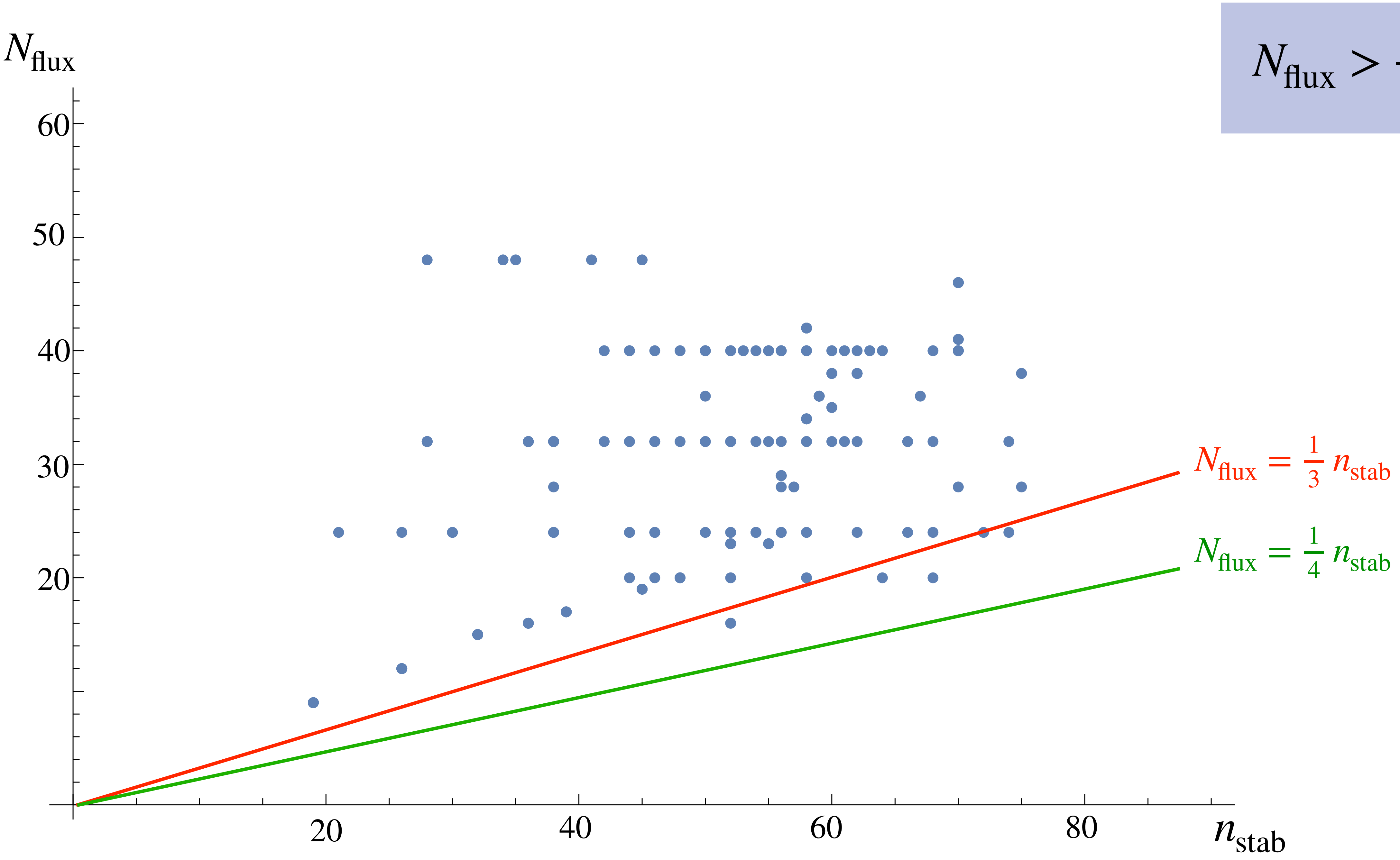

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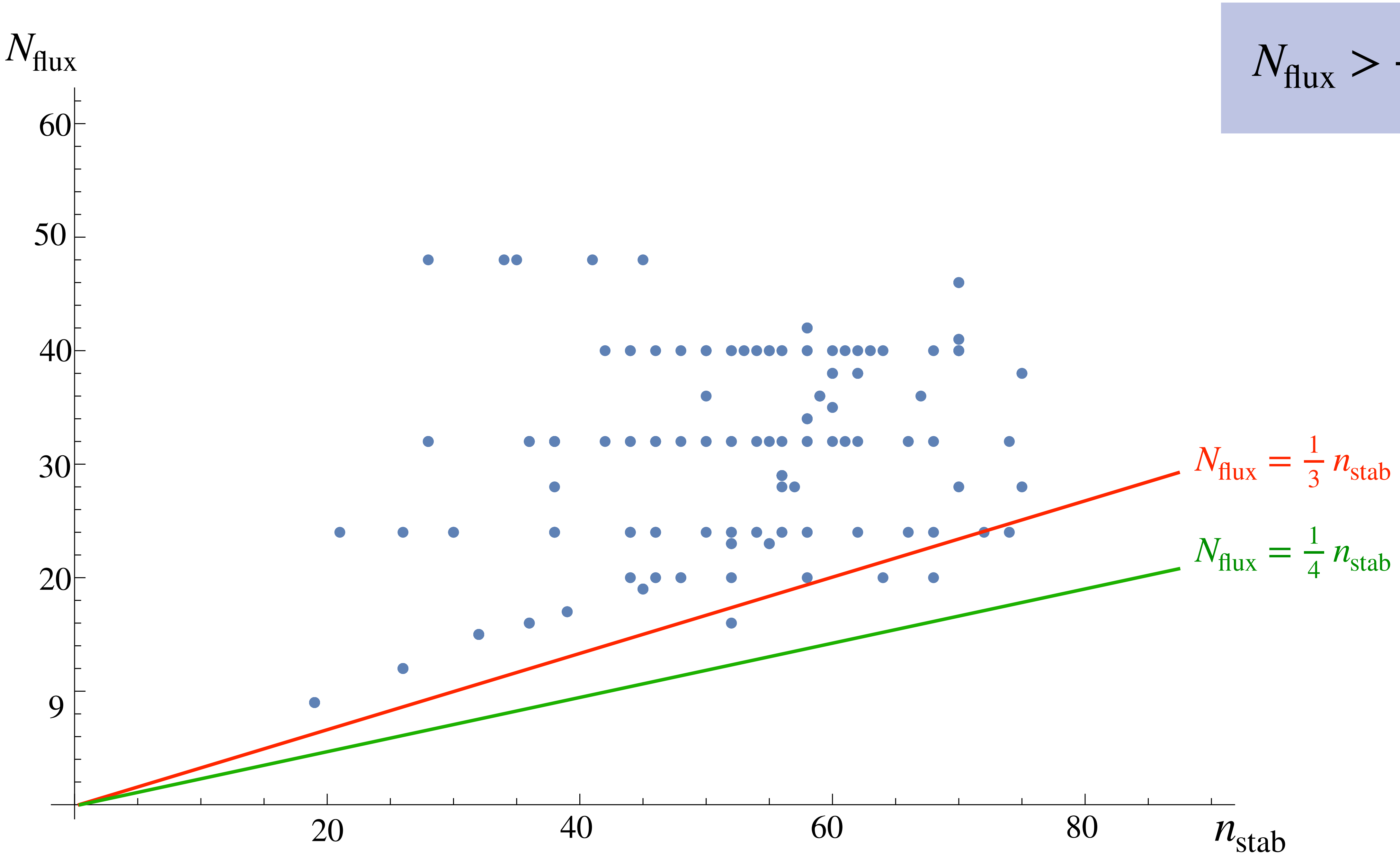

In F-theory tadpole cond: $N_{\text{flux}} \leq \dfrac{\chi}{24} \simeq \dfrac{1}{4} n_{\text{mod}}$

Conclusions

$$N_{\text{flux}} > \alpha\, n_{\text{stab}}$$

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  -Linear behavior (even beyond tadpole bound)

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We/many others checked massive, but results in 1^9 at higher order indicate also true with n_{stab}

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Related story by Grimm

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Probably yes $$M|_{DW=0} = \begin{pmatrix} D_I D_J W & g_{I\bar{J}}\,\bar{W} \\ g_{\bar{I}J}\,W & D_{\bar{I}} D_{\bar{J}} \bar{W} \end{pmatrix}$$

Shown e.g $W_0 \neq 0$ sol at point with discrete symmetry with $\frac{N_{\text{flux}}}{n_{\text{stab}}} = \frac{3}{1052}$

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S.Lust, Wiesner22

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→ What does generic point mean?

A point where non-Abelian gauge symmetries is not generic (K3 x K3)

Bena, Blåbäck, M.G., Lüst 20

Braun, Fraiman, MG, Lust, Parra de Freitas 23

A point with discrete symmetries (Fermat) satisfies tadpole conjecture

Generic = no non-Abelian gauge symmetries?

Conclusions

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THANK YOU!