Can you hear the Planck mass?

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mainly based on 2406.00095 with
De Luca, De Ponti, Mondino

String Phenomenology, Padova, 27 June, 2024

Introduction

- In a compactification, gravitational potential: [e.g. $D = 4 + d \to 4$]

$$U \sim \frac{1}{m_{\text{Pl},4}^2 r} \quad \text{for } r \gg r_{\text{KK}} \qquad U \sim U_D \propto \frac{m_{\text{Pl},D}^{2-D}}{r^{D-3}} \quad \text{for } r \ll r_{\text{KK}}$$

- U_D reproduced by resumming Yukawa contributions from KK masses m_k

In a direct product
$\mathrm{d}s_4^2 + \mathrm{d}s_d^2(X)$

$$m_{\text{Pl},4}^2 = m_{\text{Pl},D}^{D-2} V(X)$$

⇨

Weyl law:

$$m_k \sim 2\pi \left(\frac{k}{\omega_d V(X)} \right)^{1/d}$$

$k \to \infty$

[Weyl '11,... Levitan '52]

$\omega_d = V(B_d)$
unit d-dim. ball

- but for warped compactification

$$e^{2A}(\mathrm{d}s_4^2 + \mathrm{d}s_d^2(X))$$

$$m_{\mathrm{Pl},4}^2 = m_{\mathrm{Pl},D}^{D-2} V_A(X)$$

$$\underline{} \; \equiv \int_X \mathrm{d}^d y \, e^{(D-2)A}\sqrt{g}$$

and yet the Weyl law is still

$$m_k \sim 2\pi \left(\frac{k}{\omega_d V(X)}\right)^{1/d}$$

even in presence of singularities,
if they are nice enough

[Hörmander '68, ...
Ambrosio, Honda, Tewodrose '17,
Zhang, Zhu '17]

- What's going on?

The solution to the puzzle will involve the wavefunctions
and a property we call *weighted quantum ergodicity*

Plan

- Weyl law from gravitational potential — A smeared argument
- (Quantum) ergodicity — A more physical situation
- The warped case — Weighted quantum ergodicity
- The role of singularities — More rigorous results

Weyl law from gravity

- KK masses: spectrum of internal diff. operators

famous review [Duff, Nilsson, Pope '85]

- Model-dependent in general, but universal for spin-two

[Csaki, Erlich, Hollowood, Shirman'00; Bachas, Estes '11]

'weighted Laplacian' $$\Delta_f \psi \equiv -\mathrm{e}^{-f} \nabla^m (\mathrm{e}^f \nabla_m \psi)$$

$$f = (D-2)A$$

$$\mathrm{e}^{2A}(\mathrm{d}s_4^2 + \mathrm{d}s_d^2(X))$$

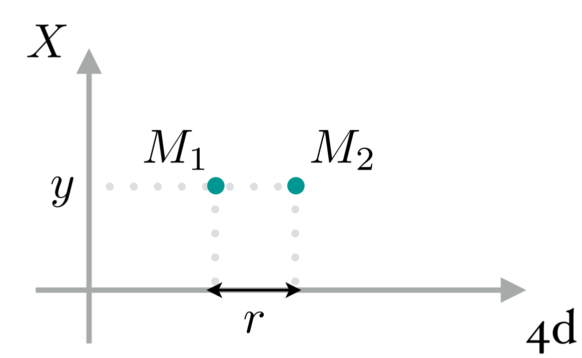
$$\Delta_f \psi_k = m_k^2 \psi_k$$

- Several universal bounds on m_k in string theory and more generally

[De Luca, De Ponti, Mondino, AT '21–23]

- Each mode contributes $\frac{M_1 M_2}{m_{\mathrm{Pl},4}^2 r} \mathrm{e}^{-m_k r} \psi_k^2(y)$ to 4d gravitational potential

[for Minkowski, or well below cosmological scale]



- $A = 0$ for now: unwarped

$$U \propto \frac{1}{m_{\mathrm{Pl},4}^2 r} \sum_k \psi_k^2(y) \mathrm{e}^{-m_k r} \quad \underset{r \to 0}{\overset{\text{expectation:}}{\sim}} \quad \frac{d!\omega_d}{(2\pi)^d} \frac{m_{\mathrm{Pl},D}^{2-D}}{r^{d+1}}$$

$$m_{\mathrm{Pl},4}^2 = m_{\mathrm{Pl},D}^{D-2} V(X)$$

- what shall we do with the wavefunctions ψ_k?

- smear particles on internal space: $\int_X \mathrm{d}^d y \sqrt{g}$ on both sides

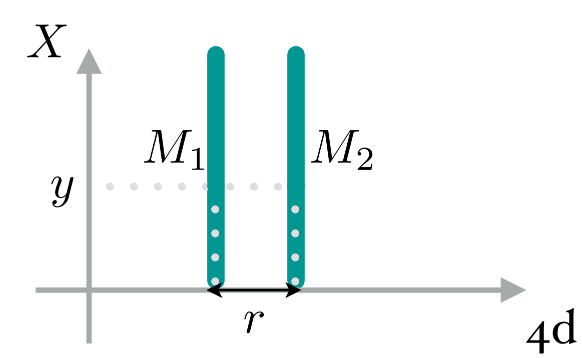
$$\Rightarrow \quad \frac{1}{r} \sum_k \mathrm{e}^{-m_k r} \quad \underset{r \to 0}{\overset{\text{expectation:}}{\sim}} \quad \frac{d!\omega_d}{(2\pi)^d} \frac{V(X)}{r^{d+1}}$$

use 'Karamata's Tauberian theorem'

idea: $\sum_k \mathrm{e}^{-ak^{1/d} r} \sim \frac{1}{(ar)^d} \int_0^\infty \mathrm{d}k \mathrm{e}^{-k^{1/d}} = \frac{d!}{(ar)^d}$

$$\Rightarrow \quad m_k \sim 2\pi \left(\frac{k}{\omega_d V(X)} \right)^{1/d} \quad \checkmark$$

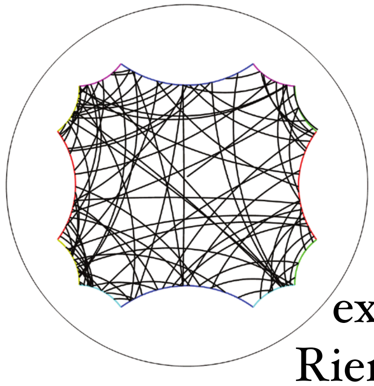
[similar to heat kernel proof]



Ergodicity

- classical ergodicity: for almost all initial conditions

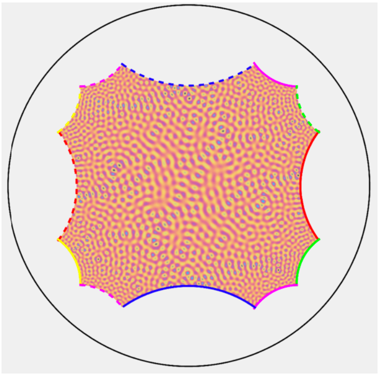
$\Rightarrow$ trajectory dense in phase space



example on a
Riemann surface
pictures: [Dyatlov '21, '23]

$\Downarrow$ [Schnirelman '74, Colin de Verdière '85, Zeldtich '87]

- quantum ergodicity: almost all ψ_k
  oscillate around constant

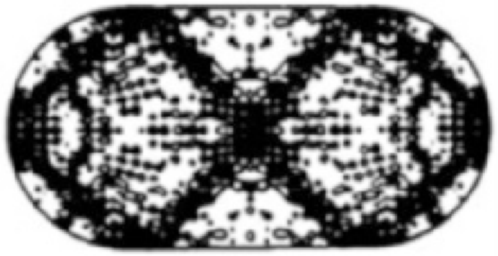


$$\lim_{\substack{k \to \infty \\ k \notin e}} \frac{\int_B \sqrt{g} \psi_k^2}{\int_X \sqrt{g} \psi_k^2} = \frac{V(B)}{V(X)} \qquad \forall B \subset X$$

- occasional eigenfunction can be 'scarred':
  peaked around classical closed trajectory

if there are no scars,
quantum unique ergodicity

example on
'Bunimovich stadium'
picture: [Reichl '92]



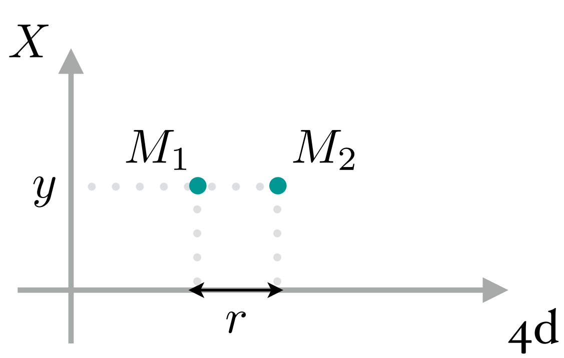
- QE expected to be common. If it holds: Still $A = 0$!

$$U \propto \frac{1}{m^2_{\mathrm{Pl},4} r} \sum_k \psi_k^2(y) \mathrm{e}^{-m_k r} \quad \underset{r \to 0}{\overset{\text{expectation:}}{\sim}} \quad \frac{d!\omega_d}{(2\pi)^d} \frac{m^{2-D}_{\mathrm{Pl},D}}{r^{d+1}}$$

$\int_B \mathrm{d}^d y \sqrt{g}$ on both sides; at large k, $\int_B \psi_k^2 \sim V(B)$

$$\Rightarrow \qquad \sum_k \frac{1}{r} \mathrm{e}^{-m_k r} \quad \underset{r \to 0}{\overset{\text{expectation:}}{\sim}} \quad \frac{d!\omega_d}{(2\pi)^d} \frac{V(X)}{r^{d+1}}$$

and conclude Weyl law as before.



Weighted ergodicity

- Now warped case: $e^{2A}(ds_4^2 + ds_d^2(X))$

$$m^2_{\mathrm{Pl},4} = m^{D-2}_{\mathrm{Pl},D} V_A(X)$$

why doesn't V_A appear in the Weyl law?

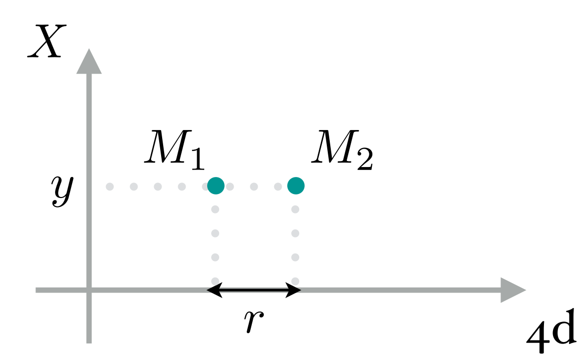
$$\int_X d^d y\, e^{(D-2)A}\sqrt{g}$$

- The expectation $U_4 \underset{r\to 0}{\sim} U_D$ now reads

[De Luca, De Ponti, Mondino, AT '24]

$$e^{(D-2)A}\sum_{k=0}^{\infty}\frac{1}{r}e^{-m_k r}\psi_k^2 \quad \underset{r\to 0}{\sim} \quad \frac{d!\,\omega_d V_A(X)}{(2\pi r)^{d+1}}$$

- Quantum ergodicity can't be right! LHS would depend on point, RHS doesn't



- Weighted quantum ergodicity: almost all ψ_k oscillate around e^{-f}

[De Luca, De Ponti, Mondino, AT '24]

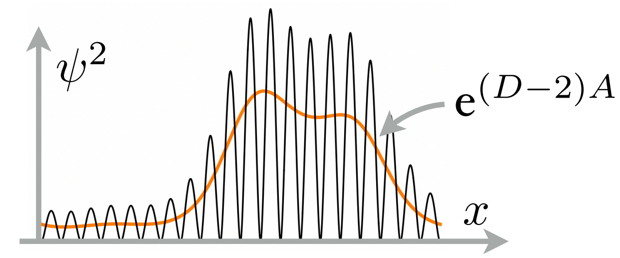
$$\lim_{\substack{k\to\infty \\ k\notin e}} \frac{\int_B \sqrt{g}\mathrm{e}^f \psi_k^2}{\int_X \sqrt{g}\mathrm{e}^f \psi_k^2} = \frac{V(B)}{V(X)} \qquad \forall B \subset X \qquad f = (D-2)A$$

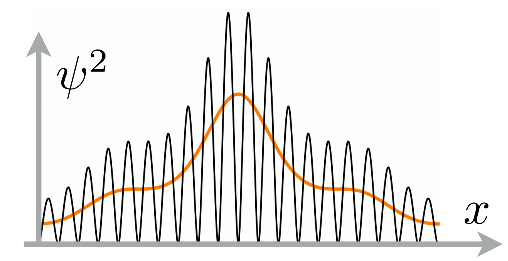
unwarped!

$$\| \qquad \text{our norm.} \quad \frac{\int_B \sqrt{g}\mathrm{e}^f \psi_k^2}{V_A(X)}$$

- also supported by
  - some numerical and analytic models
  - 'analogue Schrödinger' approach:

$$\Delta_f = \mathrm{e}^{-f}\left(\Delta_0 - \underbrace{\mathrm{e}^{-f/2}\Delta_0\,\mathrm{e}^{f/2}}_{\text{'potential'}}\right)\mathrm{e}^f$$





- Now $\int_B \sqrt{g}(U_4 \sim U_D)$:

$$\cancel{V_A(X)}\frac{\cancel{V(B)}}{V(X)}\sum_{k=0}^{\infty}\frac{1}{r}\mathrm{e}^{-m_k r}\psi_k^2 \quad \underset{r\to 0}{\sim} \quad \cancel{V(B)}\frac{d!\,\omega_d \cancel{V_A(X)}}{(2\pi r)^{d+1}}$$

⇨ warping disappears ⇨ $m_k \sim 2\pi\left(\frac{k}{\omega_d V(X)}\right)^{1/d}$

'unwarped' Weyl law.

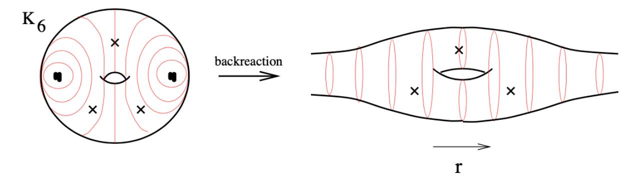
- possible spin-off application of WQE: gravity localization

$$\Vert$$

models where $U_4 \underset{r \gg r_0}{\sim} 1/r$ even when X noncompact

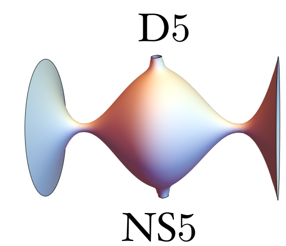
- Famous ex.: Randall–SundrumII, Karch–Randall ($r_0 = L_5$)

- In string theory?
  - RSII: D3-branes



[Verlinde '99, Chan, Paul, Verlinde '00]

  - KR: hol. duals of defects in $\mathcal{N} = 4$ super-YM



[Bachas, Lavdas '17, '18] based on [D'Hoker, Estes, Gutperle '07] [Assel, Bachas, Estes, Gomis '11]

here $m_0 \ll \sqrt{|\Lambda|} \ll m_1$

easy

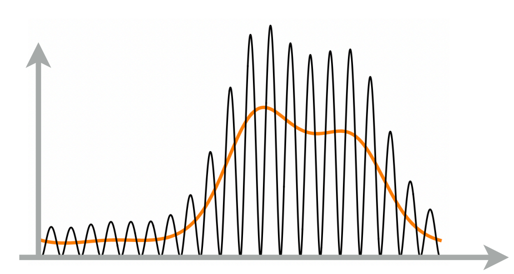
problematic, because:

$$m_k^2 < 150 k^2 \max\left\{m_0^2,\ |\Lambda| + \sigma^2\right\}$$

[De Luca, De Ponti, Mondino, AT '23]

'localization' only up to cosmological scale?

- wave-function suppression could help pushing localization scale down.



Singularities

- We expect $U_4 \underset{r\to 0}{\sim} U_D$ also with physical singularities

[D-branes, O-planes]

- WQE argument: $\int_B$, far from singularities ✓

on the other hand, not fully rigorous [limits vs. integrals…]

- Nasty enough singularities can break Weyl:

∃ 2d examples with
  - $m_k \sim c k^\alpha \quad \forall \alpha < 1/2$
  - $m_k^2 \log m_k \sim 2\pi k$

[Dai, Honda, Pan, Wei '22]

- Weyl law proven for 'RCD' singularities under a certain condition on geodesic balls

[Ambrosio, Honda, Tewodrose '17; Zhang, Zhu '17]

- We proved that this holds for D6, D7, D8

[De Luca, De Ponti, Mondino, AT '24]

[the spectrum is continuous in the presence of D3, D4]

Conclusions

- Gravity compactifications give a perspective on Weyl law
- Physical argument particularly clean if ergodicity holds
- For warped compactifications, a new weighted ergodicity appears
- Rigorous version available also with D-brane singularities