MAX-PLANCK-INSTITUT
FÜR PHYSIK

Reflections on the Emergence Proposal

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in collaboration with
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(parallel session talks)

[arXiv: 2309.11551+2309.11554+2404.01371]

Review: [arXiv:2404.05801]

StringPheno, Padova, June 27, 2024

Reflection on Emergence

Appearance of properties of a system that are novel with respect to other (more fundamental) descriptions of the same system. [Butterfield, (2011)]

Example: 1-loop annulus amplitude for D-branes → tree-level graviton exchange

a.) $g_s \ll 1$ regime: no open strings without closed strings → no emergence

b.) emergence of gravity: consistent QG theory with light D-branes and decoupled open/closed strings, where gravity is solely a quantum effect

(reminiscent: BFSS matrix model [review: W.Taylor. (2001)])

Swampland Distance Conjecture + Species Scale

Moduli space of QG contains infinite distance limits: $\phi \to \infty$

- SDC: in such a limit a tower of states becomes exponentially light

$$m \sim m_0\, e^{-c\phi}$$

(in Planck units!)

[Ooguri, Vafa (2006)]

Examples: weak coupling limit , decompactification limit

- Species scale UV cutoff of quantum gravity: $\tilde{\Lambda} < M_{\rm pl}^{(d)}$

$$\boxed{\tilde{\Lambda} \simeq \frac{M_{\rm pl}}{N_{\rm sp}^{\frac{1}{d-2}}}}$$

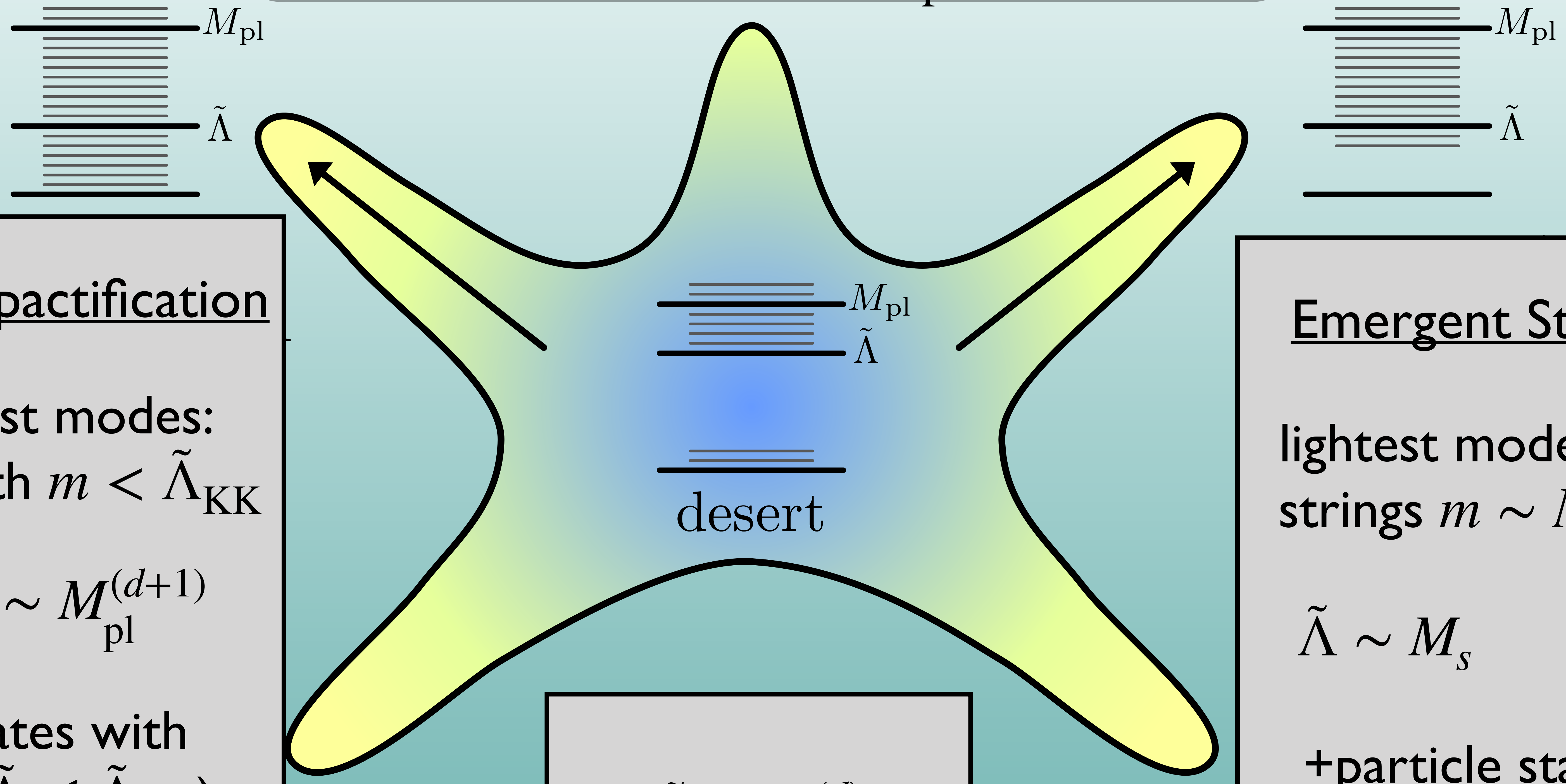
Weak coupling limit: $\tilde{\Lambda} \sim M_s$

Decompact. limit: $\tilde{\Lambda} \sim M_{\rm pl}^{(d+k)}$

Emergent String Conjecture

Strings and KK modes are the only possibilities [Lee, Lerche, Weigand (2019)]

Coscaled limits with $M_{\text{pl}}^{(d)} = \text{const}$



Decompactification

lightest modes:
KK with $m < \tilde{\Lambda}_{\text{KK}}$

$\tilde{\Lambda}_{\text{KK}} \sim M_{\text{pl}}^{(d+1)}$

(+states with
$m \sim \tilde{\Lambda} \leq \tilde{\Lambda}_{\text{KK}}$)

$\tilde{\Lambda} \sim M_{\text{pl}}^{(d)}$

Emergent String

lightest modes:
strings $m \sim M_s$

$\tilde{\Lambda} \sim M_s$

+particle states

Perturbative QG Theories

The SDC and $\tilde{\Lambda}$ are usually interpreted as limitations on validity of an EFT

Working assumption: They also reflect the structure of full perturbative QG theories arising in infinite distance, $t \to \infty$, limits in moduli space

QG in infinite distance limits:

perturbation theory in small parameter $\boxed{g \sim 1/\langle t \rangle \ll 1}$

Hierarchy of towers of states

- Light towers $\boxed{m_{\text{pert}}(n) \sim g^{\alpha}\, n^{\beta}\ \tilde{\Lambda}}$ (fundamental dof)
- Heavy towers $\boxed{m_{\text{NP}}(n) \sim n^{\gamma} \frac{\tilde{\Lambda}}{g^{\delta}}}$ (classical soliton-like=coherent states)

$(\alpha, \ldots, \delta > 0)$

Perturbative QG Theories

Integrate out only the <u>full light towers</u> in the infinite distance regime

Objective 0: Provide evidence for this picture by analyzing

Type IIA - M-theory

emergent string limit

decompact. limit

Emergence Proposal

Emergence Proposal: [Heidenreich, Reece, Rudelius (2018)],[Grimm, Palti, Valenzuela (2018)]
see also [Marchesano, Melotti (2022)] [Castellano, Herráez, Ibáñez (2022)] [Bhg, Gligovic, Paraskevopoulou (2023)]

from review [Palti, (2019)]

> The dynamics (kinetic terms) for all fields are emergent in the infrared by integrating out towers of states down from an ultraviolet scale, which is below the Planck scale.

1-loop correction to gauge coupling: tower $M_n = n\Delta m$ with $M_n, q \leq \Lambda_{UV}$,

$$\frac{1}{g^2_{U(1)}}\bigg|_{1-\text{loop}} \sim \sum_{n=1}^{n_{\max}} Q_n^2 \log\left(\frac{M_n^2}{\mu^2}\right) \sim \frac{1}{g^2_{U(1)}}\bigg|_{\text{class}}$$

Originally motivated at leading order in a (toy) QFT approach with $M_n, q \leq \tilde{\Lambda}$

Emergence Proposal

Could this be a general property of QG?

- was answered negatively in the original work of Ooguri/Vafa [Ooguri, Vafa (2006)]

Objective I: How and where is it realized in quantum gravity (string theory)?

Emergence Proposal

Emergent string limit: Unphysical log-factors appear→sign of no emergence

[Bhg,Gligovic,Paraskevopoulou (2023)]

Decompactification limit: Quantization of M-theory?

Approach

- Collect evidence from 1/2 BPS saturated amplitudes which admit geometric formulation
- Higher derivative R^4-terms in theories with maximal supersymmetry by Green-Gutperle-Vanhove (1997) (and Kiritsis,Obers,Pioline)

  [Bhg, Cribiori, Gligovic, Paraskevopoulou, 2404.01371]
- Topological amplitudes $\mathscr{F}_g$ in 4D with N=2 supersymmetry a la Gopakumar-Vafa (1998)

  [Bhg, Cribiori, Gligovic, Paraskevopoulou, 2309.11551]

  [Hattab, Palti, 2312.15440+2404.05176]

  (Eran's talk at STRINGS'24)

Infinite distance limits

Perturbative fundamental string

- Lighest towers are strings, mass scale M_s, string coupling $g_s \ll 1$
- Accompanied by particle like states of mass $M \sim M_s$, KK + winding
- Species scale $\tilde{\Lambda} \sim M_s$
- All other towers are non-perturbative:

  (classical = coherent quantum states) $m_{Dp} \simeq \dfrac{\tilde{\Lambda}}{g_s}\,, \qquad m_{NS5} \simeq \dfrac{\tilde{\Lambda}}{g_s^2}$

M-theory limit (special decompactification limit)

$$R_{11} \to \lambda R_{11}\,, \qquad M_* \to \frac{M_*}{\lambda^{\frac{1}{d-1}}}\,, \qquad R_I \to \lambda^{\frac{1}{d-1}} R_I,$$

(compactified type IIA: $g_s \to \lambda^{\frac{3(d-2)}{2(d-1)}} g_s\,, \qquad M_s \to \lambda^{\frac{d-4}{2(d-1)}} M_s\,, \qquad R_I \to \lambda^{\frac{1}{d-1}} R_I\,.$)

Light BPS Towers

- Lightest towers of states: D0-branes with [Bhg, Cribiori, Gligovic, Paraskevopoulou, 2309.11554]

$$\boxed{M_{D0}} \sim M_s/g_s \sim M_{\rm pl}^{(d)}/\lambda$$

- For such a KK-like tower, the species scale is

$$\tilde{\Lambda} \sim M_{\rm pl}^{(d)}/\lambda^{1/(d-1)} \sim M_{\rm pl}^{(d+1)} \sim \boxed{M_*}$$

- Room for additional light towers

$$\boxed{M_{D2,NS5}} \sim M_s/g_s^{1/3} \sim M_{\rm pl}^{(d)}/\lambda^{1/(d-1)} \sim \tilde{\Lambda}$$

M-theory: transverse M2 and M5 branes with KK momentum

Emergent String: 1-loop correction to R^4 term

Higher derivative term

$$S_{R^4} \simeq M_s^{d-8}\, V_k \int d^d x \sqrt{-g}\, a_d\, t_8 t_8\, R^4,$$

General expansion in g_s

$$a_d = \frac{c_0}{g_s^2} + \underbrace{\left(c_1 + \mathcal{O}\left(e^{-S_{\text{ws}}}\right)\right)}_{1-\text{loop}} + \mathcal{O}\left(e^{-S_{\text{st}}}\right)$$

with the one-loop contribution

$$a^{(1)}_{d,\text{string}} \simeq \frac{2\pi}{V_k} \overline{\overline{\sum_{m_i, n^i \in \mathbb{Z}}}} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^{\frac{d-6}{2}}}\, e^{-\pi\tau_2 M^2 - 2\pi i \tau_1 m_i n^i} =$$

1/2 BPS: $m_i n^i = 0$

Light towers: KK+ winding

$$M^2 = m_i G^{ij} m_j + n^i G_{ij} n^j$$

undo integral τ_1:

$$a^{(1)}_d \simeq \frac{2\pi}{V_k} \overline{\overline{\sum_{m_i, n^i \in \mathbb{Z}}}} \int_0^\infty \frac{dt}{t^{\frac{d-6}{2}}}\, \delta(\text{BPS})\, e^{-\pi t M^2} =$$

UV divergence $\rightarrow$ regularization

M-theoretic Emergence of R^4 term

[Green,Gutperle,Vanhove (1997)]
[Russo,Tseytlin (1997)]
[Kiritsis,Pioline (1997)]
[De Wit,Lüst (1999)]
[Obers,Pioline (1999)]
[Calderón-Infante, Delgado, Uranga (2023)]

Higher derivative term

$$S_{R^4} \simeq M_*^{d-8}\, r_{11} \mathscr{V}_k \int d^d x \sqrt{-g}\, a_d\, t_8 t_8\, R^4,$$

with the coefficient

$$a_{d,\mathrm{M}}^{(1)} \simeq \frac{2\pi}{r_{11}\mathscr{V}_k} \overline{\overline{\sum_{N^I, m \in \mathbb{Z}}}} \int_0^\infty \frac{dt}{t^{\frac{d-6}{2}}}\, \delta(\mathrm{BPS})\ \exp\left(-\pi t\, N^I \mathscr{M}_{IJ} N^J - \pi t \frac{m^2}{r_{11}^2} \right),$$

KK, M2, M5 transverse wrapping numbers $\quad (N^I) = \left(m_i, n^{ij}, n^{ijklm} \right).$

particle masses $\quad \mathscr{M} = \mathrm{diag}\left(\frac{1}{r_i^2}, t_{ij}^2, t_{ijklm}^2 \right)$ (axions will induce off-diagonal entries)

Emergence of R^4 term

1/2 BPS conditions

$$n^{ij} m_j = 0\,, \qquad \# = k$$

$$n^{[ij} n^{kl]} + m_p n^{pijkl} = 0\,, \qquad \# = \binom{k}{4}$$

$$n^{i[j} n^{klmnp]} = 0\,, \qquad \# = k\binom{k}{6}$$

equivalent to section constraints in ExFT

(analogous truncation of modes)

[Bossard, Kleinschmidt (2015)]

Particle states and 1/2 BPS conditions

d	k	Particles $SL(k)$ reps.	$E_{k(k)}(Z)$	Λ_{E_k}	1/2-BPS: λ_{E_k}
9	1	$[1]_p$	1	1	0
8	2	$[2]_p + [1]_{M2}$	$SL(2)$	3	2
7	3	$[3]_p + [3]_{M2}$	$SL(3) \times SL(2)$	(3,2)	(3,1)
6	4	$[4]_p + [6]_{M2}$	$SL(5)$	10	5
5	5	$[5]_p + [10]_{M2} + [1]_{M5}$	$SO(5,5)$	16	10
4	6	$[6]_p + [15]_{M2} + [6]_{M5}$	E_6	27	27

Note: U-duality $E_{k+1(k+1)}$

Emergence in 9D

Example 9D: Evaluate

$$a_{9,\mathrm{M}}^{(1)} \simeq \frac{2\pi}{r_{11}\, r_1} \sum_{(m,n)\neq(0,0)} \int_\epsilon^\infty \frac{dt}{t^{3/2}}\, e^{-\pi t\left(\frac{m^2}{r_{11}^2}+\frac{n^2}{r_1^2}\right)}\,.$$

invoking Poisson resummation of n and the same regularization procedure

$$a_9 \simeq \boxed{\frac{2\zeta(3)}{g_s^2}} + \frac{2\pi^2}{3}\left(1+\frac{1}{\rho_1^2}\right) + \boxed{\frac{8\pi}{g_s}\sum_{m\neq 0}\sum_{n\geq 1}\left|\frac{m}{n}\right| K_1\left(2\pi|m|n\frac{\rho_1}{g_s}\right)}$$

ED0 brane instanstons

missing (reminiscent to 10D string)

tree level

(more details in the parallel session talk of A. Paraskevopoulou)

Emergence in dim=d

For 8D, new sector with D2-branes and mass $M^2 = n^2 t_{12}^2 + \frac{m^2}{r_{11}^2}$

$$a^{(1)}_{8,\mathrm{M};M2} \simeq \frac{2\pi^2}{3} + \frac{8\pi}{r_{11}t_{12}} \sum_{n_1,n_2\geq 1} \frac{1}{n_2} e^{-2\pi n_1 n_2 r_{11} t_{12}} = -\frac{2\pi}{T}\log\left(\left|\eta(iT)\right|^4\right)$$

The full amplitude from 1-loop Schwinger-integral!

- Full result for 7D, partial results $d \leq 6$
- Transverse M2,M5 yield all instantons ⟶
- Constrained Eisenstein series

$$a^{(1)}_{d,\mathrm{M}}(\mathrm{transv}) = \mathcal{E}^{E_{k(k)}}_{\Lambda_{E_k}\oplus 1, s=\frac{k}{2}-1}$$

Particle states	Instantons
$(D0, \mathrm{KK}_{(k)})$	$ED0_{(k)}$
$(D2_{(ij)}, \mathrm{KK}_{(k)})$	$ED2_{(ijk)}$
$(NS5_{(ijklm)}, \mathrm{KK}_{(n)})$	$ENS5_{(ijklmn)}$
$(D2_{(ij)}, D0)$	$EF1_{(ij)}$
$(NS5_{(ijklm)}, D0)$	$ED4_{(ijklm)}$
$(NS5_{(ijklm)}, D2_{lm})$	$ED2_{(ijk)}$

M-theoretic Emergence Proposal

Integrating out only the light towers in the infinite distance regime,

implies relation:

$$\underbrace{a^{(1)}_{d,\mathrm{M}}(\mathrm{transv})}_{\text{pert. } r_{11} \gg 1} = \underbrace{a^{(1)}_{d,\mathrm{M}}(\mathrm{transv} + \mathrm{longi})}_{\text{desert: } r_{11} = O(1)}$$

proof for $d \geq 4$

[Bossard, Kleinschmidt (2015)]

[Bossard, Pioline (2016)]

R^4-term over IIA moduli space with $M_{\mathrm{pl}} = \mathrm{const}$:

pert. string theory	desert	pert. M-theory
$g_s \ll 1$	$g_s = O(1)$	$g_s \gg 1$

$$a_d = \frac{c_0}{g_s^2} + \underbrace{\left(c_1 + \mathcal{O}\left(e^{-S_{\mathrm{ws}}}\right)\right)}_{1-\mathrm{loop}} + \mathcal{O}\left(e^{-S_{\mathrm{st}}}\right) = \mathcal{E}^{E_{k+1(k+1)}}_{\Lambda_{E_{k+1}}, s=\frac{k}{2}-1} = \mathcal{E}^{E_{k(k)}}_{\Lambda_{E_k} \oplus 1, s=\frac{k}{2}-1}$$

emergence! emergence!

M-theoretic Emergence Proposal

In the infinite distance M-theory limit $M_* R_{11} \gg 1$ with the Planck scale kept fixed, a perturbative QG theory arises whose low energy effective description emerges via **quantum effects** by integrating out the **full infinite towers** of states with a mass scale parametrically not larger than the 11D Planck scale. These are transverse M2-, M5-branes carrying momentum along the eleventh direction (D0-branes) and along any potentially present compact direction

Ultimate test for kinetic terms:

- Emergence of Einstein-Hilbert term
- Space-time is emerging
- Prepotential for N=2 in 4D (1/2 BPS)

N=2 susy in 4D

Type IIA compactified on a CY to 4D with N=2 susy: [Bhg, Cribiori, Gligovic, Paraskevopoulou (2023), 2309.11551]

(more details in the parallel session talk of A. Gligovic)

Prepotential is 1/2 BPS saturated and enjoys an expansion

$$\mathcal{F}_0(T) = -\frac{1}{g_s^2}\left[\frac{1}{3!}\kappa_{ijk}T^iT^jT^k + \frac{\zeta(3)}{2}\chi(X) - \sum_{\beta\in H_2(X,\mathbb{Z})} \alpha_0^\beta \; \mathrm{Li}_3\left(e^{-\beta\cdot T}\right)\right],$$

- determines kinetic terms for vector-multiplets
- Gopakumar-Vafa invariants $\alpha_0^\beta \in \mathbb{Z}$

Gopakumar/Vafa: in the M-theory limit given by Schwinger integrals over D2-D0 bound states

$$\mathcal{F}_0 = \sum_\beta \alpha_0^\beta \sum_{n\in\mathbb{Z}} \int_0^\infty \frac{ds}{s^3}\, e^{-sZ_n(\beta)}\,, \qquad \text{with central charge} \qquad Z_n(\beta) = \frac{2\pi}{g_s}\left(\beta\cdot T + in\right)$$

Gopakumar/Vafa approach

Applying the same regularization method as before:

- for single g=0 curve (resolved conifold)

$$\mathcal{F}_0^{M2} = \frac{1}{g_s^2}\left[-\frac{(2\pi T)^3}{12} + \mathrm{Li}_3(e^{-2\pi T}) - \zeta(3) \right].$$

Alternative method via contour integration [Hattab, Palti 2312.15440] [Hattab, Palti 2404.05176]

- compact CY : unsettled

problem: regularization of sum over $\beta \in H_2(X, \mathbb{Z})$

For CY with $h_{11} = 1$: $\frac{1}{2}\sum_{\beta=1}^{\infty} \beta^3\, \alpha_0^{\beta}\Big|_{\text{reg.}} \overset{!?}{=} \kappa_{111}$ …………(stay tuned)

Conclusions

- Motivated that in infinite distance limits perturbative QG theories arise

- Perturbative towers are those with a typical mass scale below $\tilde{\Lambda}$

- Provided evidence for the realization of the (strong) Emergence Proposal in M-theory limits by integrating out these full light towers of states

- Technically, we evaluated 1/2 BPS saturated one-loop Schwinger integrals providing a working regularization of the UV divergences

[checked for string case $\sum_n \int_{\mathscr{F}} \frac{d\tau^2}{\tau_2^{\frac{d-6}{2}}} = \sum_n \int_\epsilon^\infty \frac{dt}{t^{\frac{d-6}{2}}} \delta(\mathrm{BPS}) \Big|_{\mathrm{reg}}$]

Outlook

- Collect more evidence from BPS amplitudes in theories with lower susy
- Non BPS amplitudes like 10D kinetic terms

    △ Requires quantization of M-theory, i.e. include non-BPS states

    △ Space-time itself has to emerge

    △ Problem: susy might imply vanishing Schwinger integrals

    △ Compute appropriate couplings, like 1-loop(!) graviton scattering in BFSS matrix model

$$V = -\frac{15}{16}\frac{v^4}{r^7} + \ldots\,,$$

(velocity v breaks susy)

- BFSS not complete (misses transverse M5): Can we learn anything new about quantization of M-theory?

Thank you!