An Axion Cosmology Scenario

String Pheno 2024

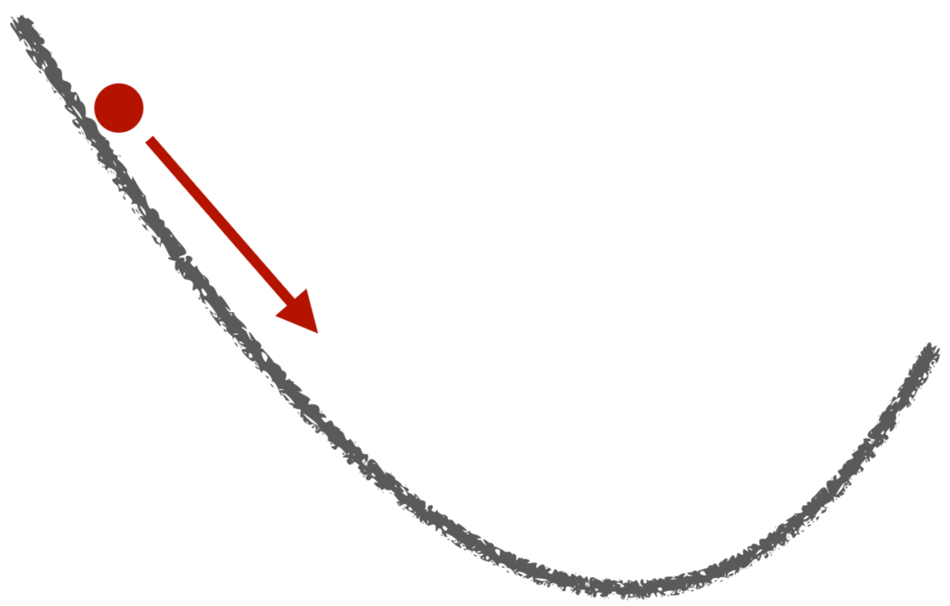
Matt Reece, Harvard University

Introduction

Why Axions?

Bottom up model building:

- QCD axion solves the Strong CP problem
- Misalignment mechanism produces cold dark matter



Top down model building:

- String constructions often predict axions

General principles:

- Axions as gauge fields for (-1)-form instanton number "symmetry"
- No free parameters in quantum gravity

Chern-Weil Global Symmetries

Gauge theories intrinsically have symmetries connected to characteristic classes, e.g., in $d > 4$ we have U(1) $(d-5)$-form **"instanton number symmetry"**:

$$\frac{1}{8\pi^2}\mathrm{d}\left[\mathrm{tr}(F \wedge F)\right] = 0$$

follows from the nonabelian Bianchi identity

$$\mathrm{d}_A F = 0.$$

Quantum gravity: often **gauged** by Chern-Simons terms $\int \frac{1}{8\pi^2} C_{d-4} \wedge \mathrm{tr}(F \wedge F)$. Sometimes **explicitly broken**, e.g., by monopoles.

For instance, for U(1),

$$\mathrm{d}F = 2\pi j_{\mathrm{mon}}$$

implies

$$\mathrm{d}(F \wedge F) \neq 0.$$

w/ Ben Heidenreich, Jake McNamara, Miguel Montero, Tom Rudelius, Irene Valenzuela

arXiv:2012.00009 [hep-th]

Axions from Quantum Gravity Principles

In $d = 4$ the story is a bit different: $\mathrm{d}\left[\mathrm{tr}(F \wedge F)\right] = 0$ is trivial because this is a top form. Does a U(1) "(-1)-form symmetry" have nontrivial meaning?

One definition: such a symmetry $\Rightarrow$ theory admits **consistent coupling to a background axion $\theta(x)$.**

Again this can be **gauged** — coupled to a dynamical axion — or **broken**, e.g., by monopoles, for which the Witten effect obstructs the coupling to $\theta(x)$.

Then "no (-1)-form symmetry in QG" closely related to "**no free parameters** in QG."

Rich story about monopoles, dyon modes, etc.

see my recent symmetry seminar "Instanton Number as a Symmetry"

w/ Daniel Aloni, Eduardo García-Valdecasas, Motoo Suzuki

arXiv:2402.00117 [hep-th] and work in progress

Axion Models at a Glance

	Pseudo-Nambu-Goldstone for 4d $U(1)_{PQ}$	Zero mode of gauge field in higher dimensions
"Pre-inflation" scenario	Quality problem Isocurvature problem	(Quality problem) Isocurvature problem
Post-inflation PQ transition	Quality problem Domain wall problem Stable relic problem	**Not possible** (no linearly realized PQ symmetry to break) (maybe similar late-time physics from other initial conditions?)

Post-Inflation Axion Cosmology

4d U(1) PQ symmetry spontaneously broken after inflation.

- Axion randomized, strings form (Kibble-Zurek)
- QCD phase transition: axion domain walls form
- String-wall network destroys itself ($N_{\rm DW} = 1$)

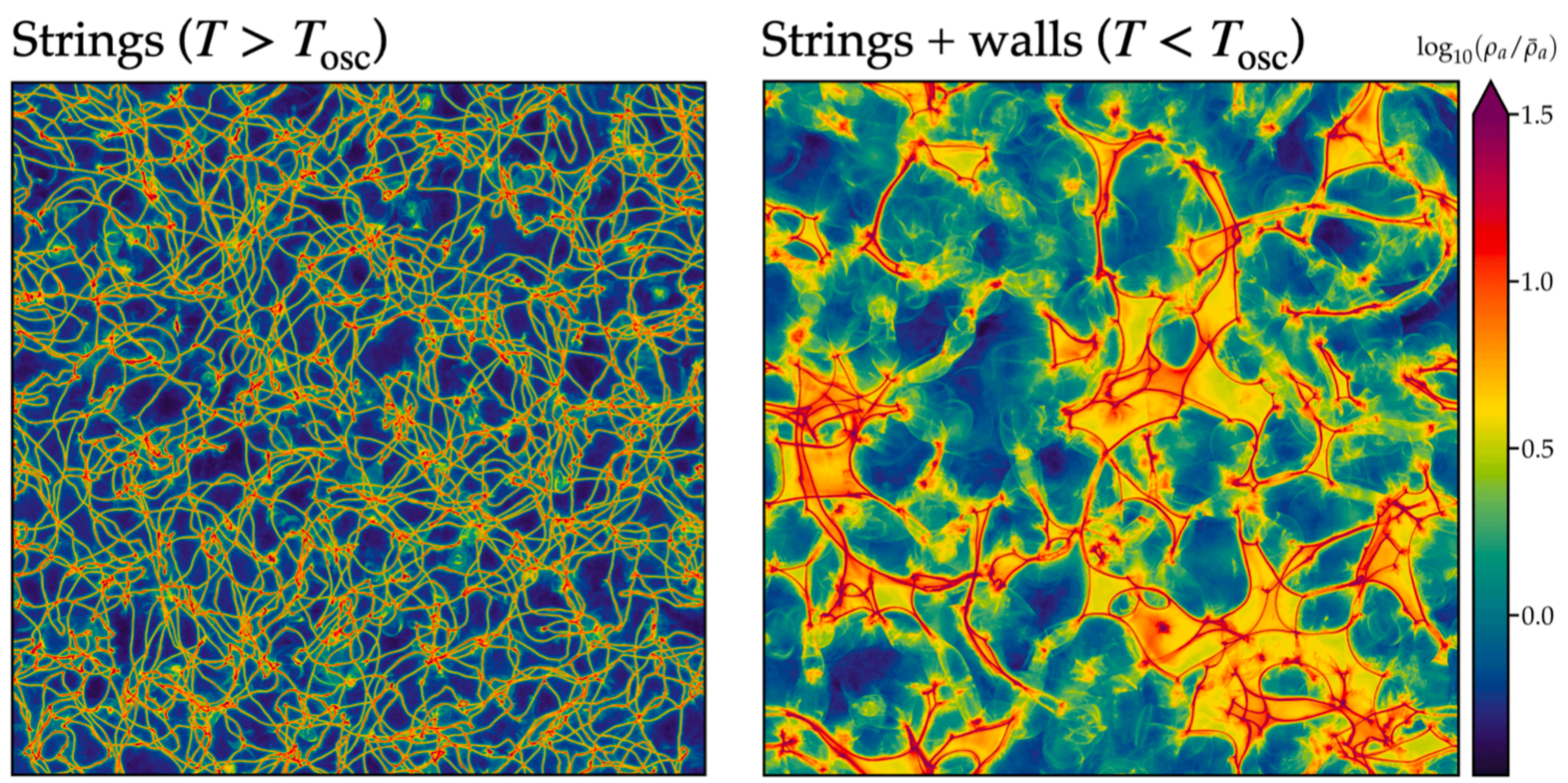


Figure from **Ciaran O'Hare**'s lectures on axion cosmology, arXiv:2403.17697 [hep-ph]
Using code from **Alejandro Vaquero, Javier Redondo, Julia Stadler**, arXiv:1809.09241

Axion dark matter relic abundance dominantly from axion emission from string network, as well as misalignment.

Detailed simulations, e.g., Buschmann, Foster, Hook, Peterson, Willcox, Zhang, Safdi arXiv:2108.05368

Post-Inflation: Axion Domain Wall Problem

Domain walls can end on strings if

$$\int \frac{k_G}{8\pi^2}\theta\, \mathrm{tr}(F \wedge F)$$

has minimal coupling $N_{\mathrm{DW}} = |k_G| = 1.$

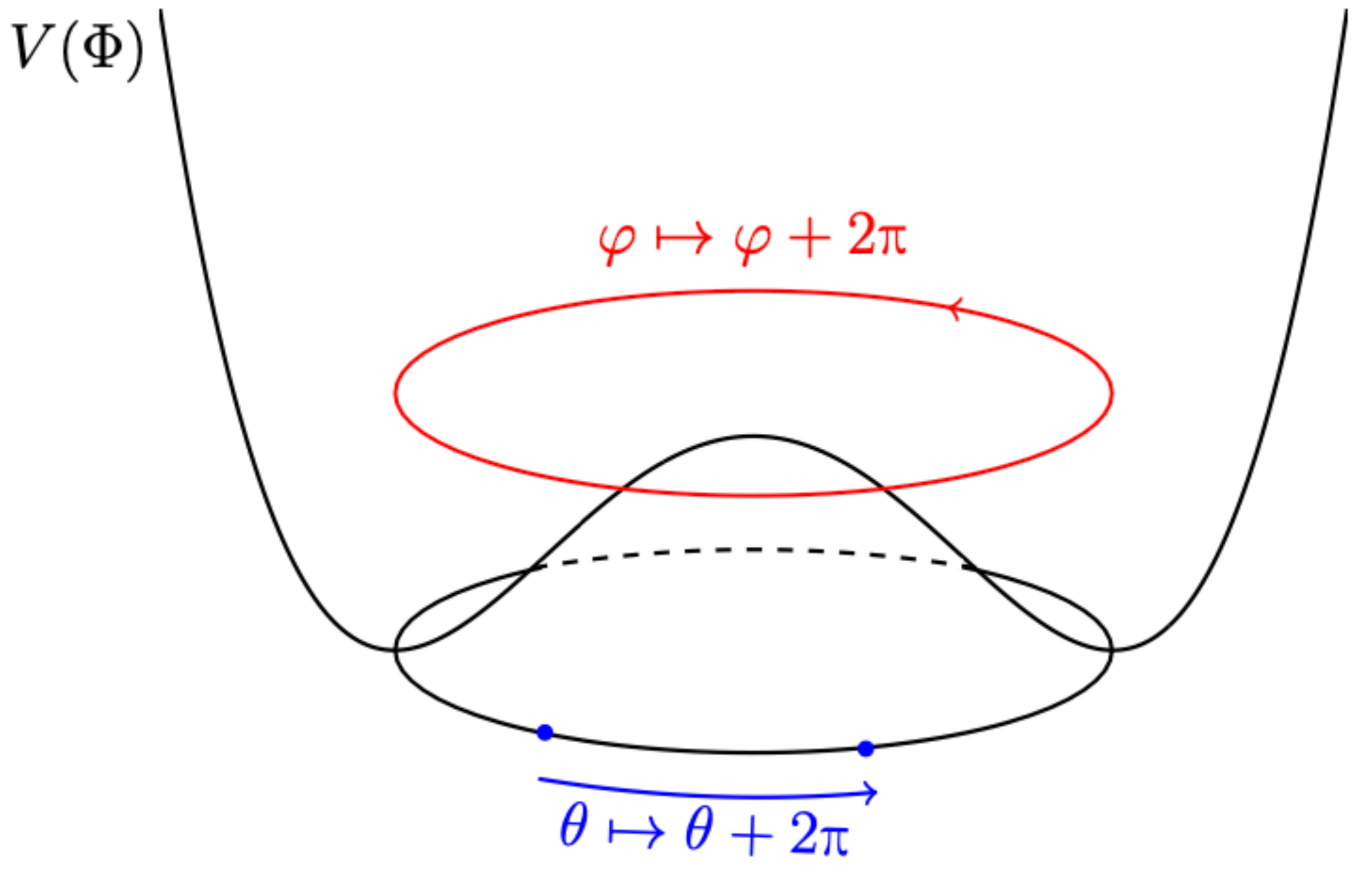
But such strings may not form, or may not be elementary! Tension w/ models for quality problem

Example:

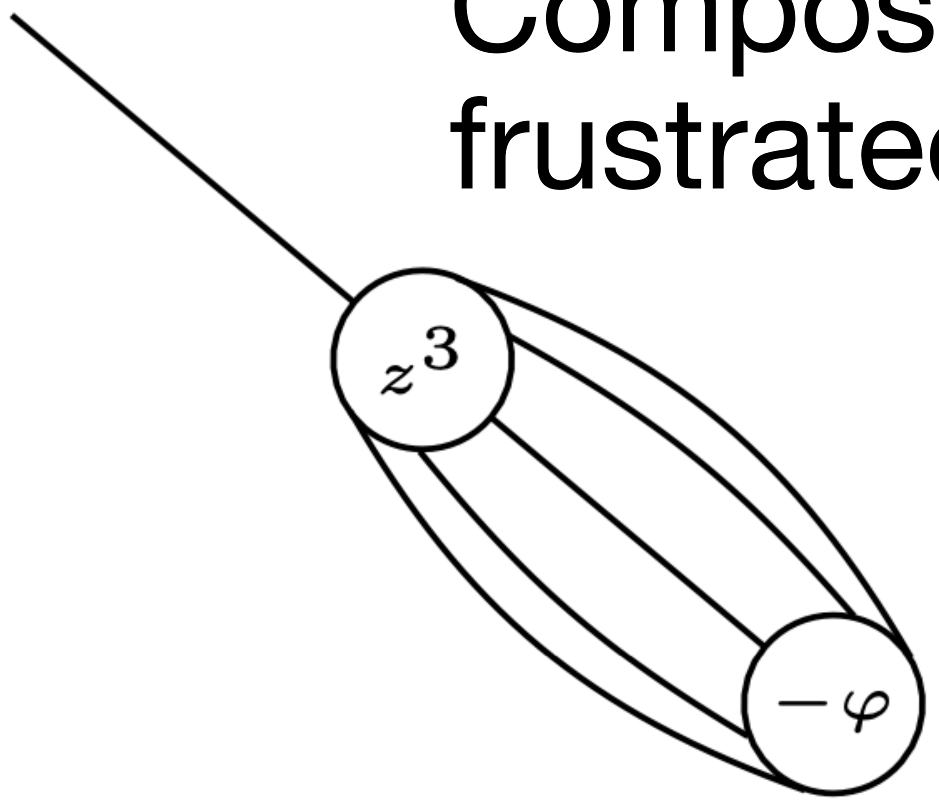
$\mathbb{Z}_p$ symmetry,

$\Phi(x) = fe^{i\varphi(x)} = fe^{i\theta(x)/p}$

Kibble-Zurek: string of winding p



Composite string — frustrated network



Hard to find convincing models!

w/ Qianshu Lu, Zhiquan Sun

arXiv:2312.07650 [hep-ph]

“Pre-Inflation”: Axion Isocurvature Problem

A light scalar during inflation fluctuates by $\delta\varphi \sim H_I/(2\pi)$. Fluctuations independent of inflaton fluctuations ⇒ ***isocurvature***, strongly constrained.

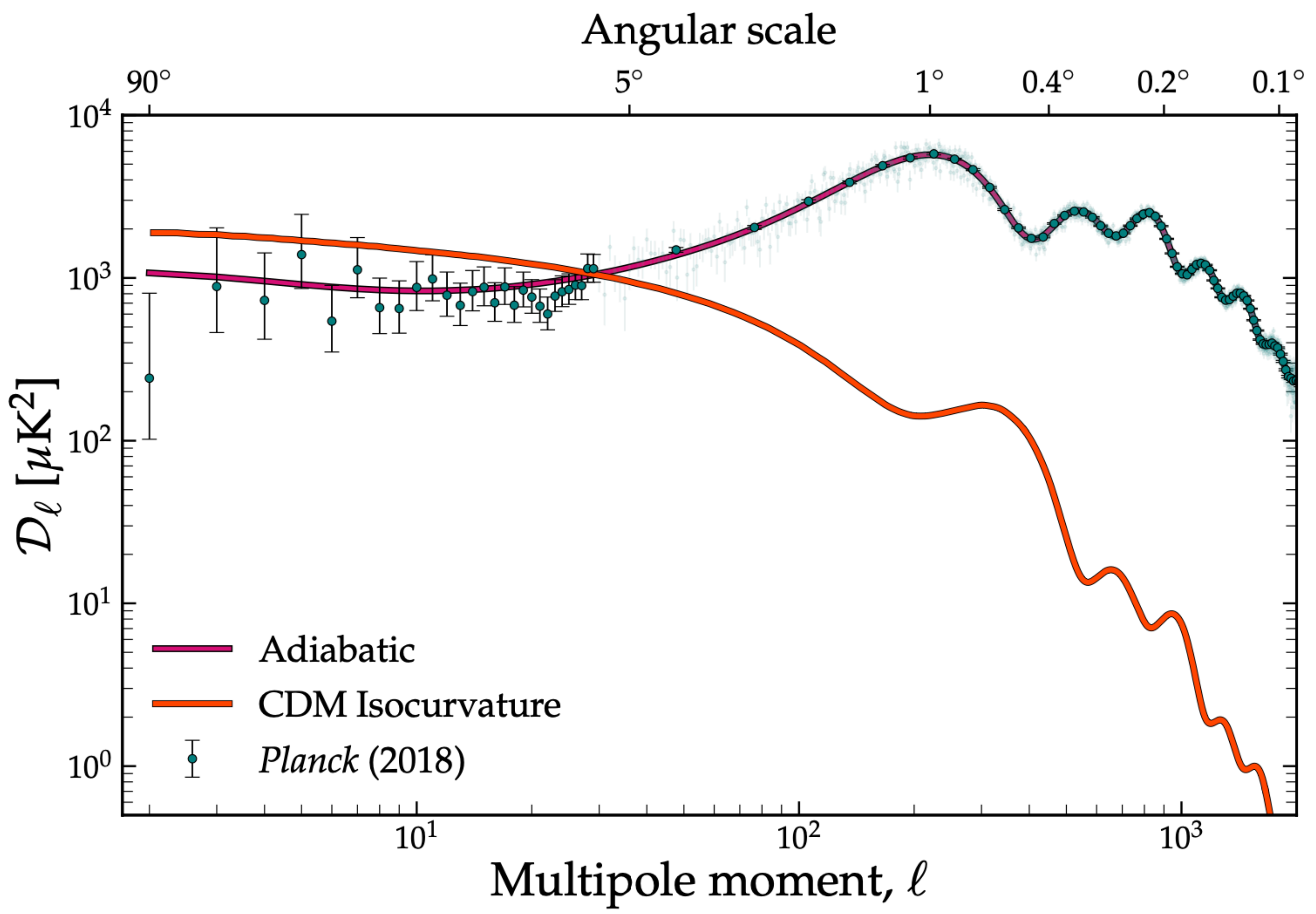


Figure from **Ciaran O’Hare**’s lectures on axion cosmology, arXiv:2403.17697 [hep-ph]

Leads to a bound

$$H_I \lesssim 3 \times 10^7\,\mathrm{GeV}\,\frac{f_I}{10^{12}\,\mathrm{GeV}}$$

which is much stronger than the observational bound from (lack of) tensor modes,

$$H_I \lesssim 10^{13}\,\mathrm{GeV}\,.$$

Is a bound a **problem**? Not a sharp one, but the simplest and most natural inflation models are large-field (hence high-scale).

Solutions to the Axion Isocurvature Problem

Several ideas have been discussed in the literature for opening up a wider range of H_I for a given axion decay constant. Broadly,

- Turn on larger $|\Phi|^2$ term during inflation — back to post-inflation.
- **Dynamical axion mass**, heavier than H_I during inflation, e.g., make QCD very strongly coupled so Λ_{QCD}/f is not small. (Dvali '95, …)

  [Awkward to continuously change *exponentially tiny* number to O(1)!]
- **Dynamical axion decay constant**, $f_I \gg f_a$ to relax bound (Linde/Lyth '90, …)

String pheno: time-varying modulus can lead to both of the last two.

Rest of this talk: a new variation on dynamical axion mass.

Eliminating Axion Isocurvature: A New Approach

w/ Prish Chakraborty, Junyi Cheng, Zekai Wang
expected to appear on arxiv this summer

Monodromy Mass vs. Isocurvature

An axion $\theta \cong \theta + 2\pi$ can get a large ("monodromy") mass from a Chern-Simons coupling to a 4-form field strength $F^{(4)} = \mathrm{d}C^{(3)}$:

$$S = \int -\frac{1}{2}f^2|\mathrm{d}\theta|^2 - \frac{1}{2g^2}|F^{(4)}|^2 + \frac{n}{2\pi}\theta F^{(4)}, \quad n \in \mathbb{Z}\,.$$

[Kallosh, Linde, Linde, Susskind '95; Gabadadze '99; Silverstein, Westphal '08; Kaloper, Sorbo '08; ….]

Axion mass: $m_{\hat{\theta}} = \frac{n}{2\pi}\frac{g}{f}\,.$

Main idea:

If $n \in \mathbb{Z}$ is a ***dynamical integer***, it could be nonzero during inflation (heavy axion, no isocurvature) and zero today (standard axion).

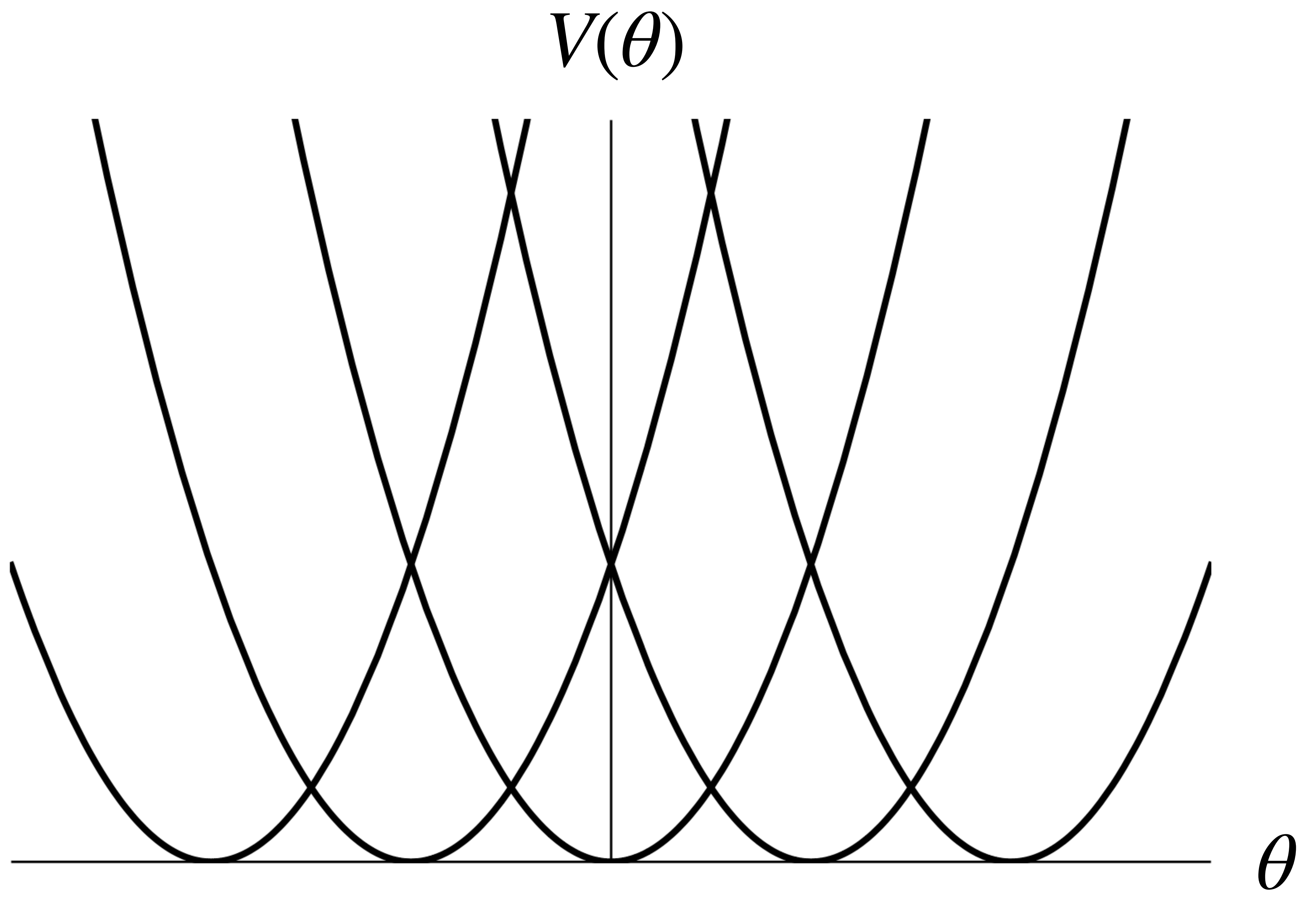
Change between them with a ***first-order phase transition***.

Avoiding confusion

$$S = \int -\frac{1}{2}f^2|\mathrm{d}\theta|^2 - \frac{1}{2g^2}|F^{(4)}|^2 + \frac{n}{2\pi}\theta F^{(4)}, \quad n \in \mathbb{Z}\,.$$

The monodromy potential $V(\theta)$ has infinitely many branches labeled by an integer $j = \frac{1}{e_4^2}\star F^{(4)} - \frac{n}{2\pi}\theta$, and a gauge invariance $\theta \mapsto \theta + 2\pi,\ j \mapsto j - n\,.$

j, the $C^{(3)}$ electric field, is **always dynamical**. It is **not** the dynamical integer n that we wish to change in cosmology.



Making n Dynamical

Idea: the integer n is flux of higher-dimensional gauge field, $n = \frac{1}{2\pi}\int_{\Sigma^{(q+1)}} \mathrm{d}A^{(q)}$

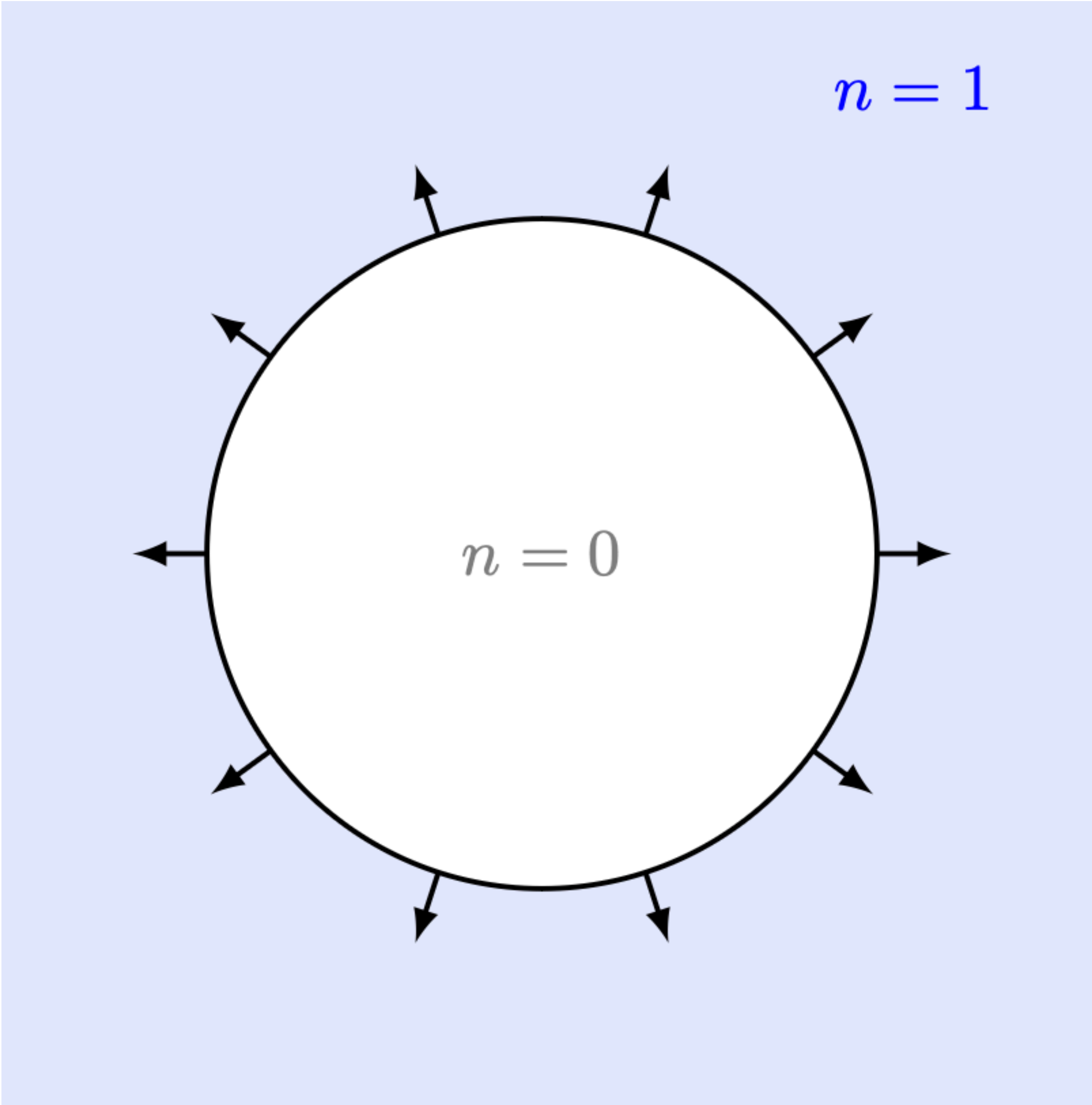
Extra-dimensional axion $\theta = \int_{\Lambda^{(p)}} C^{(p)}$

Chern-Simons in $n = p + q + s + 1$ extra dims:

$$\frac{1}{4\pi^2}\int_{M^{(4)}\times Y^{(n}} C^{(p)} \wedge \mathrm{d}A^{(q)} \wedge \mathrm{d}C^{(3+s)}$$

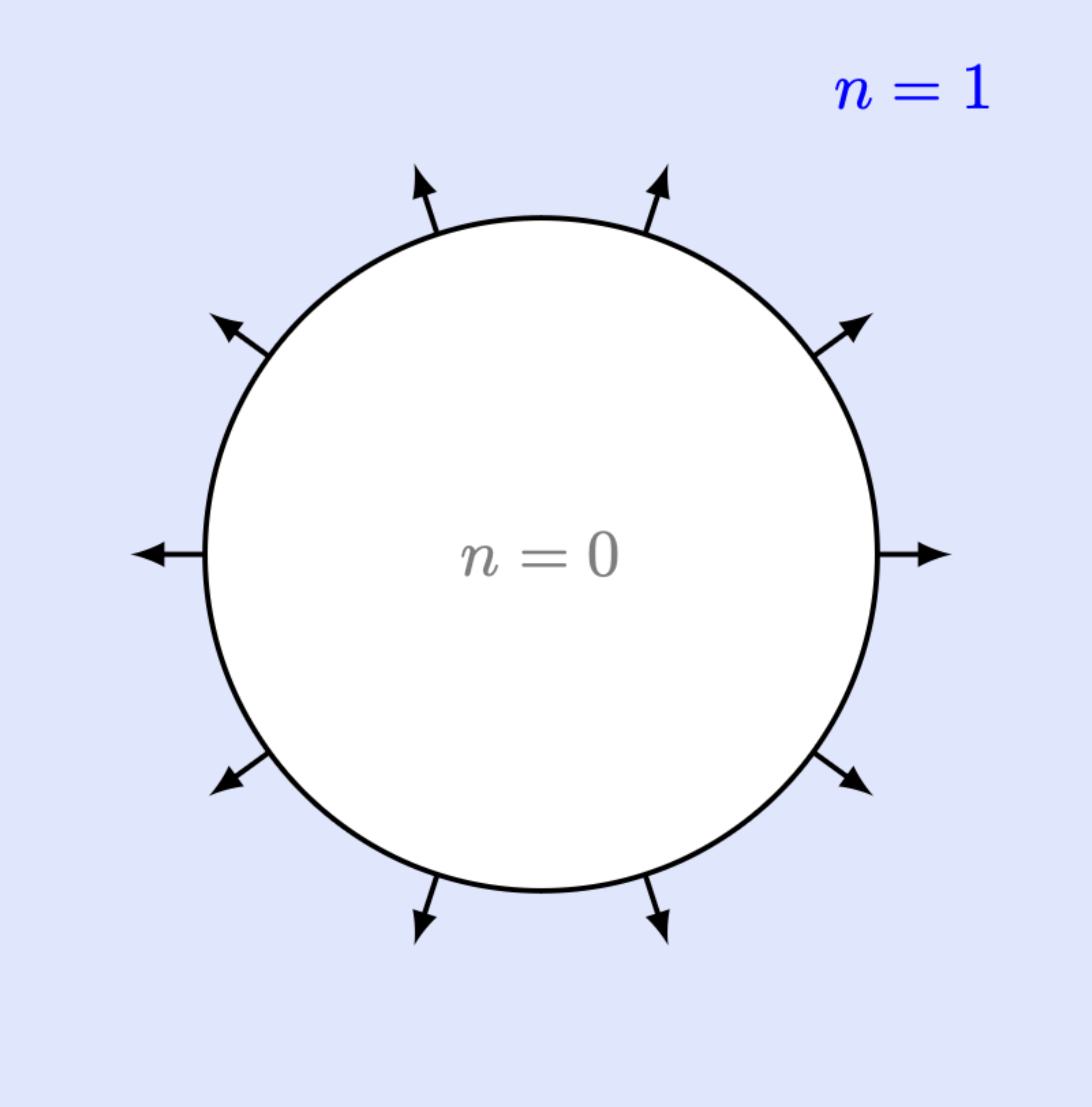
$$\Downarrow$$

$$\int_{M_4} \frac{n}{2\pi}\theta F^{(4)}$$



Flux Tunneling

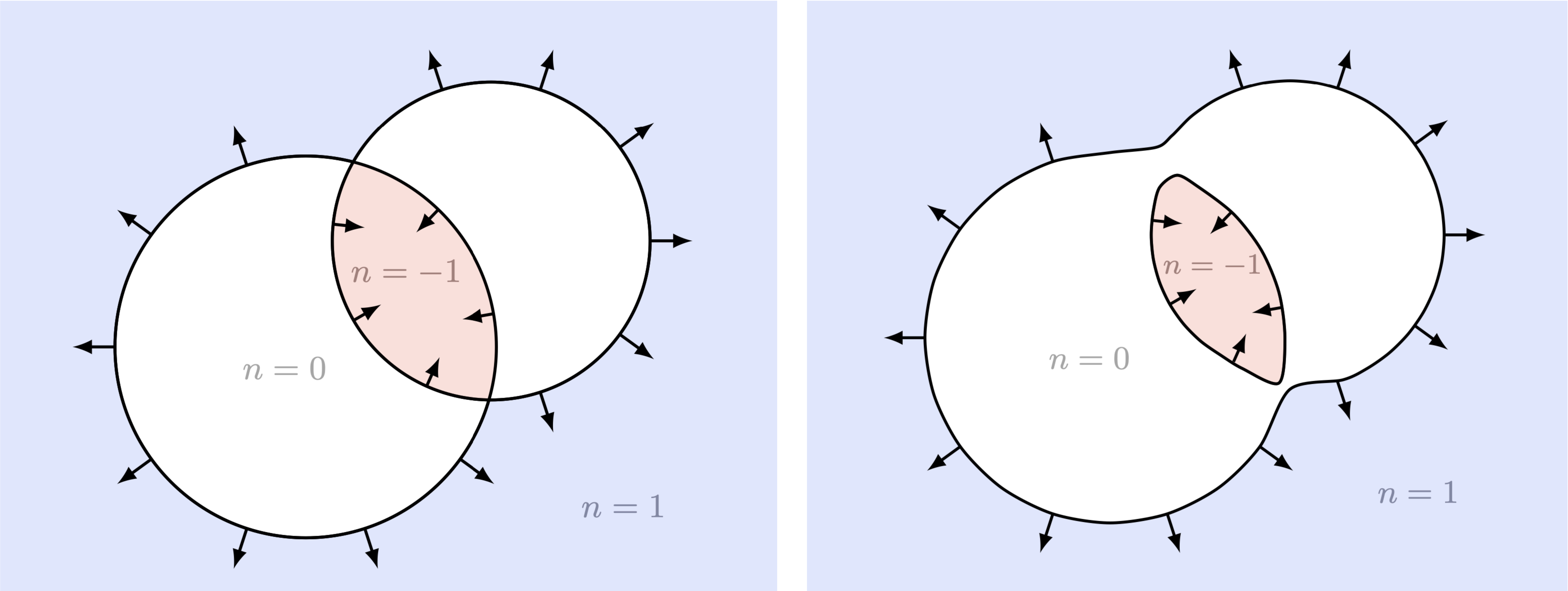
Our tunneling process must change the flux $n = \frac{1}{2\pi} \int_{\Sigma^{(q+1)}} \mathrm{d}A^{(q)}$.



This can only happen by nucleating a dynamical **magnetically charged brane** for $A^{(q)}$. This has $4 + n - q = 3 + r + s$ dimensions. Wrapping the $r + s$ internal dimensions transverse to $\Sigma^{(q+1)}$, we have a **domain wall** in (3+1)d.

(see, e.g., Blanco-Pillado, Schwartz-Perlov, Vilenkin '09, but details differ — we do *not* want a Freund-Rubin compactification, our flux is through a cycle in a larger geometry)

Bubble Mergers



Provided the $n = 0$ state has lowest vacuum energy, we expect colliding branes to reconnect and the $n \neq 0$ regions to collapse.

Flux Tunneling at the End of Inflation?

Need to suppress isocurvature: $|n| > 0$ during inflation. Drops to 0 after.

Vacuum energy contribution $V(n)$ from flux energy density: could that provide the energy driving inflation?

"Graceful exit" problem of old inflation: need to make bubble nucleation rate time-dependent. $\Gamma(t) < H(t)^4$ until some critical time t_*.
Scenarios:

- Inflaton ϕ affects $\Gamma(t)$, e.g., brane tension $\mathcal{T}(\phi)$ dynamical.

- Tunneling as inflation is ending, $H(t)$ starts to drop rapidly.

$$S = \int -\frac{Z_n}{2}|\mathrm{d}\phi|^2 - \frac{1}{2}f_n(\phi)^2|\mathrm{d}\theta|^2 - \frac{1}{2g_n(\phi)^2}|F^{(4)}|^2 + \frac{n}{2\pi}\theta F^{(4)} + V(\phi, n) + h(\phi)\mathcal{T}\delta^{(2)}(M)$$

String Theory Embedding?

All the *ingredients* exist in string theory, e.g.:

Type IIA model with D6 branes, $\theta = \int_\alpha C^{(3)}$, dynamical integer $n = \int_\beta H^{(3)}$, axion mass from $C^{(3)} \wedge \mathrm{d}C^{(3)} \wedge H^{(3)}$.

The bubble wall is an NS5 brane wrapped on β.

Inside the wall: D6's wrapped on β for realizing Standard Model.
Outside the wall: obstructed by $H^{(3)}$ flux on β.

Dynamical emergence of chirality after inflation? (Potential implications for baryogenesis, Festina Lente bound, ….)

Conclusions

- Axions play an important role in quantum gravity.
- Conventional 4d QCD axion models face serious cosmological challenges.
- Extra-dimensional axions primarily face the axion isocurvature problem: difficult to combine with high-scale inflation.
- Possible scenario: time-dependent moduli fields after inflation change the value of the decay constant.
- **Novel scenario:** first-order phase transition from large tree-level axion mass during inflation to zero mass afterward. Implications for reheating, gravitational waves, and more. Can we find a realistic version of this scenario?