String Phenomenology'24
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Higher Global Symmetries from Nesting of Symmetry Theories

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Based on:

M.C., R. Donagi, J. Heckman, M. Hübner, E. Torres, UPR-1328-T, 2406…

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M.C., R. Donagi, J. Heckman, M. Hübner, E. Torres, UPR-1328-T, 240~~6~~7…

[Also earlier work:
M.C., Heckman, Hübner, Torres, 2203.10102;
M.C., Heckman, Hübner, Torres, Zhang, 2305.09665;
M.C., Heckman, Hübner, Torres, 2307.1023…]

Symmetry Theories → active subject
c.f., Cordova's talk, Hackman's talk,…

Focus on new perspective →
Nesting of Symmetry Theories

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Apologies for unintentionally missed references…

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1A) Introduction of Symmetry Topological Field Theories

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Given a QFT in d-dim one can often isolate its symmetry structures into a topological field theory (TFT) in (d+1)-dim → Introduction of the so-called Symmetry TFT (SymTFT)

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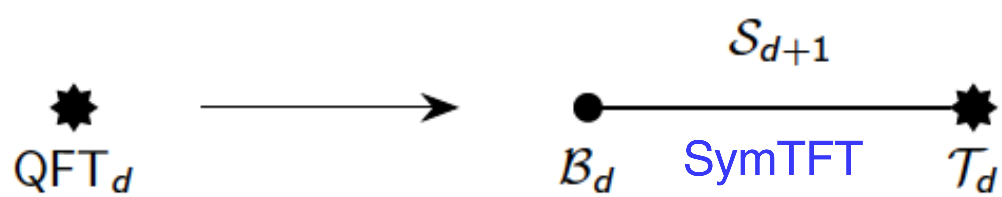

indicating physical boundary conditions (b.c.) at $\mathcal{T}$ and topological b.c. at $\mathcal{B}$ for the SymTFT.

Many symmetry structures depend only on $\mathcal{B}$ and the TFT.

1) Motivation – Global Symmetries in QFT

1B) Symmetry TFT (SymTFT) in isolation (by itself)

$\mathcal{S}_{d+1}$

SymTFT

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Particularly natural perspective in the context of continuous global symmetries where SymTFT is not necessarily TFT anymore and replaced by SymTh.

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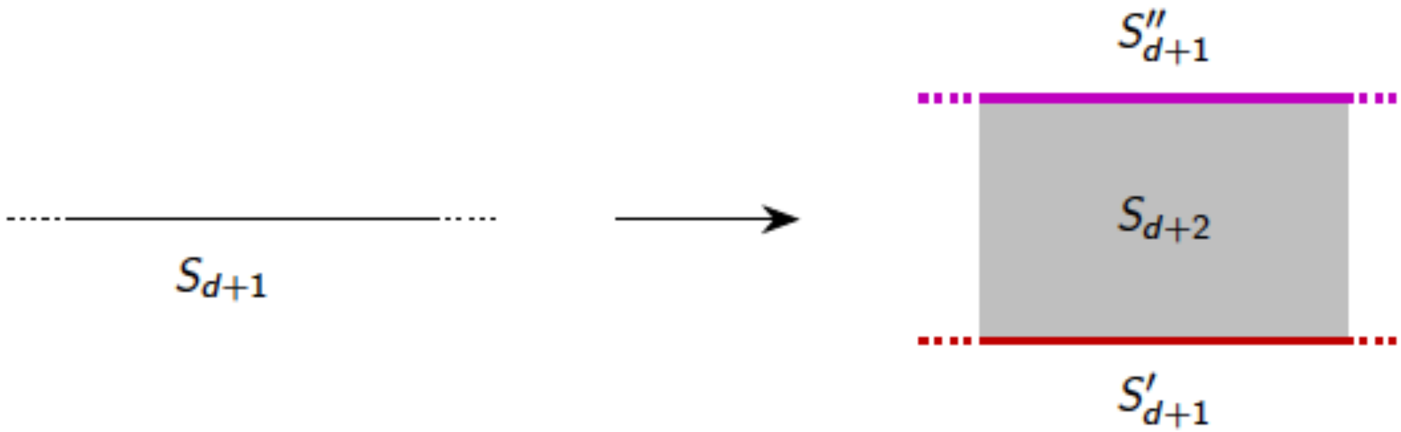
1C) With 1B) we have returned to the starting point of 1A), so we can re-iterate the procedure → construct a SymTh in (d+2)-dim!

Sketch of steps in 1):



1B) ``decompression'' of S_{d+1}

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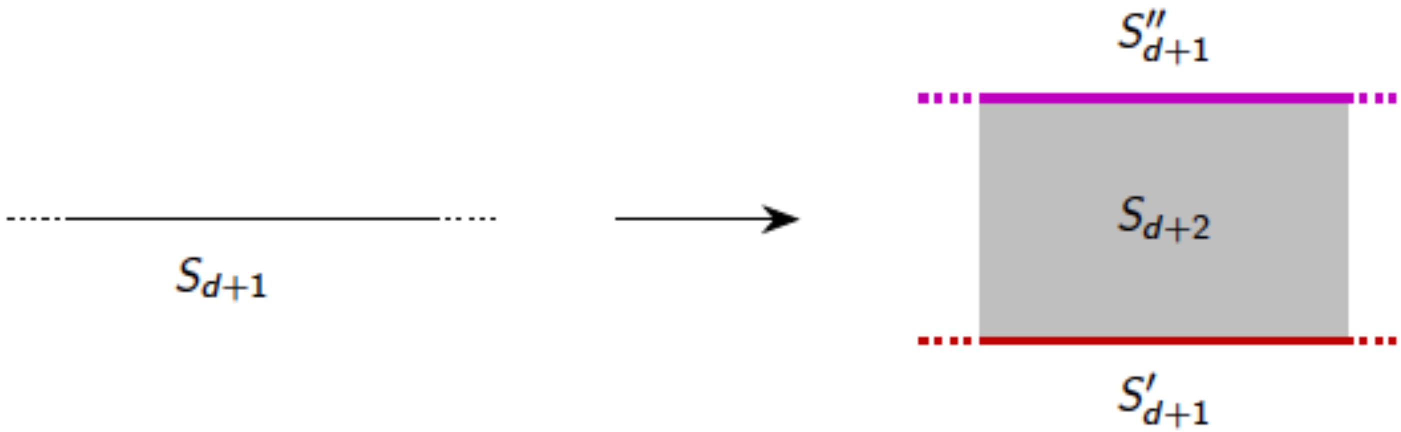

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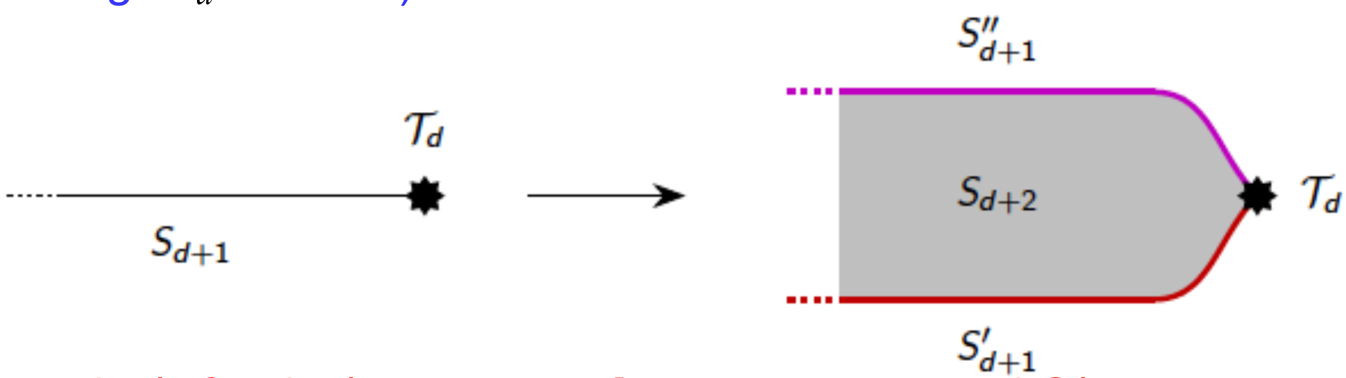

1A) & 1B) → 1C)

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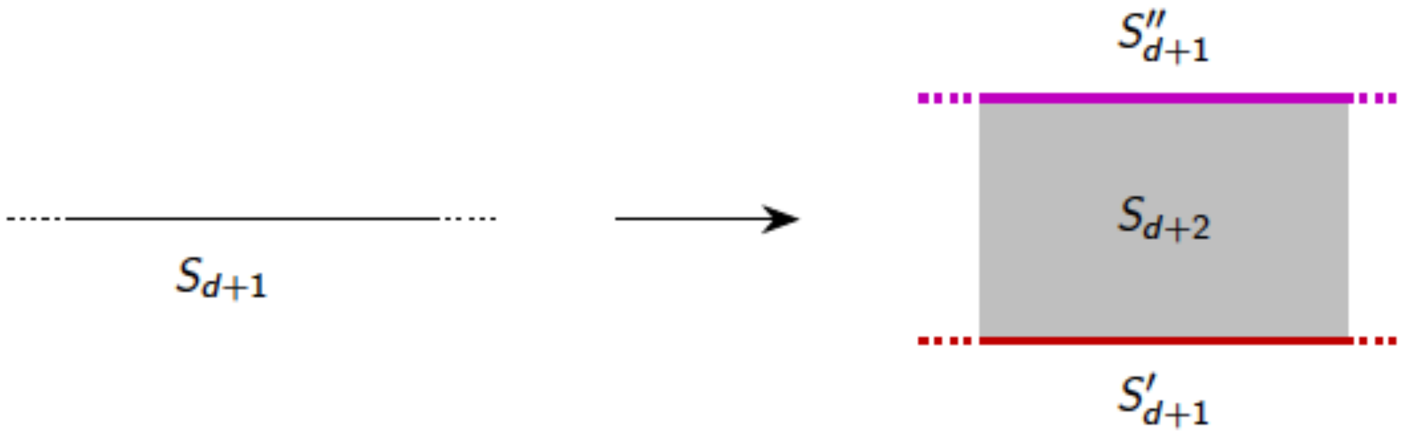

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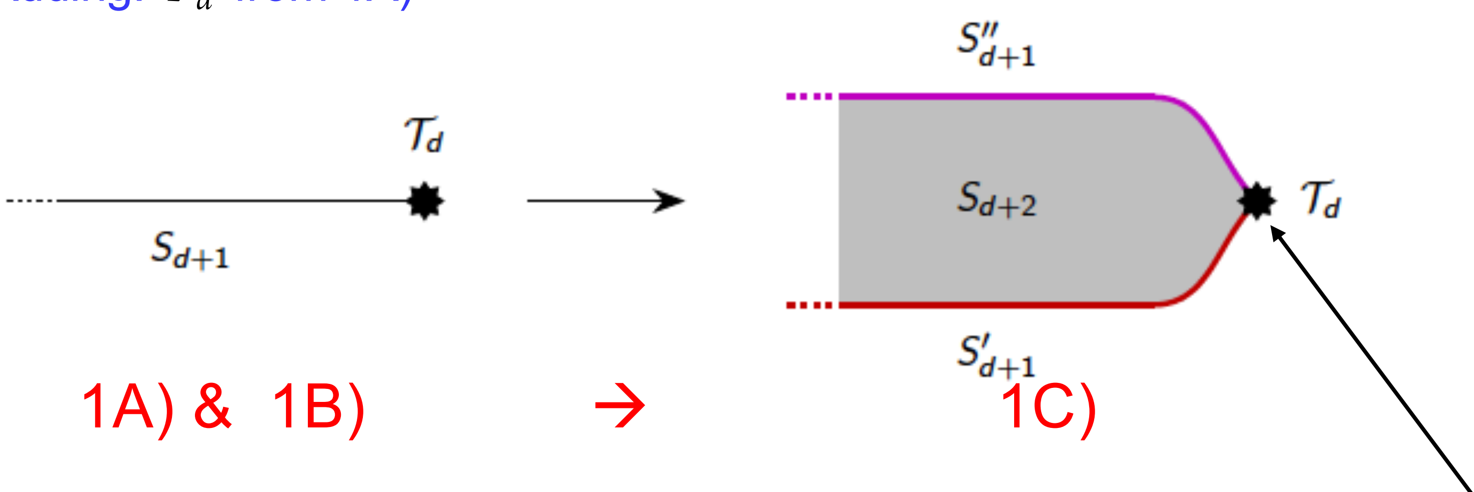

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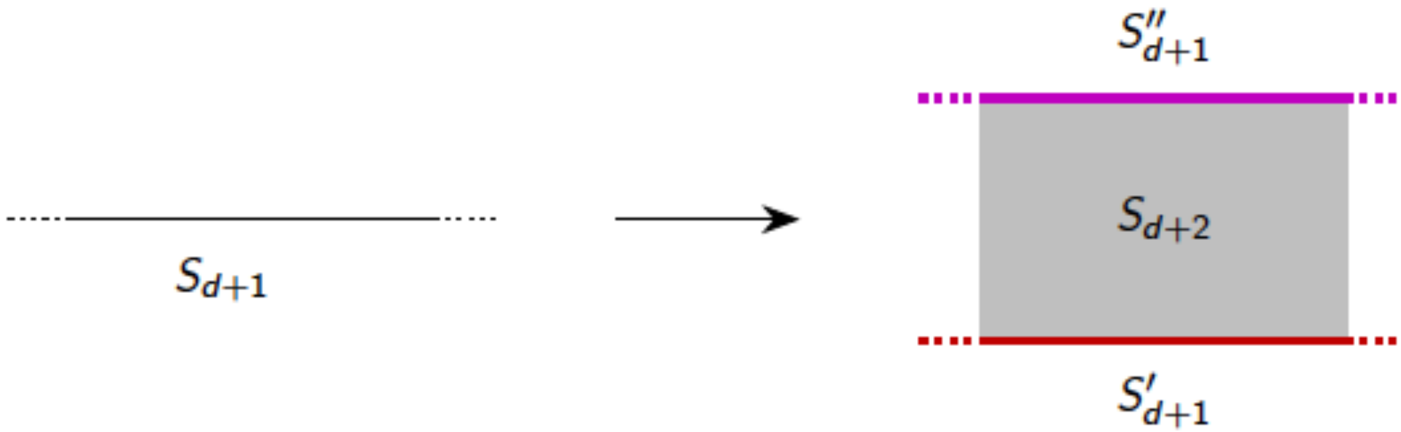

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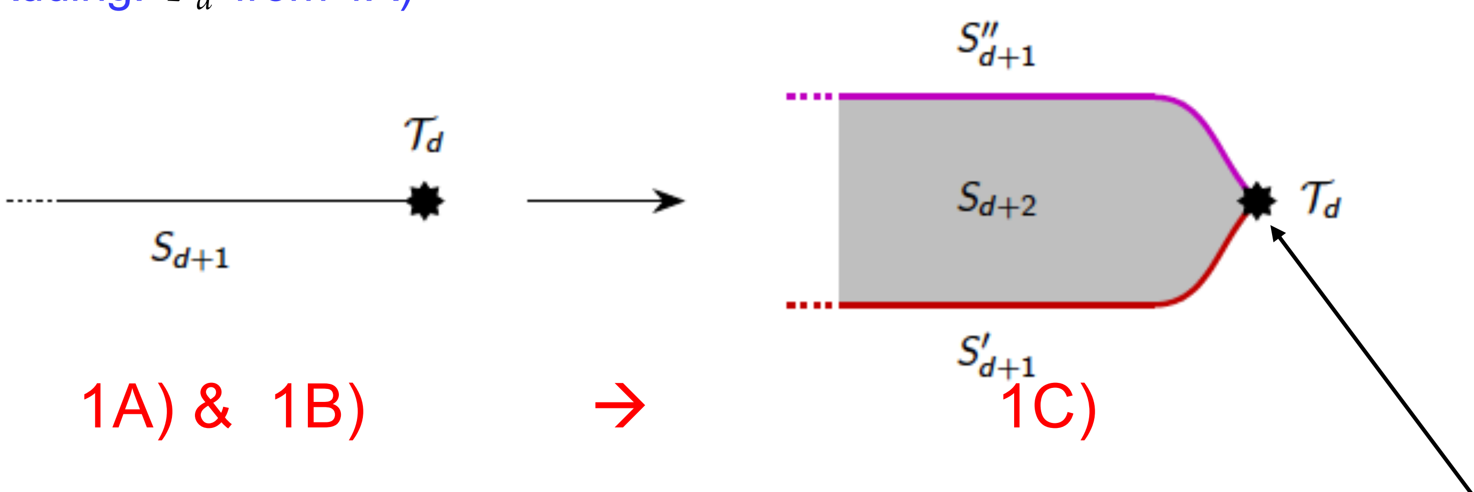

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In principle, this nesting can continue

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Consider a QFT in D dim and 'insert' a defect QFT in d<D dim.
What's the symmetry theory of the combined system?

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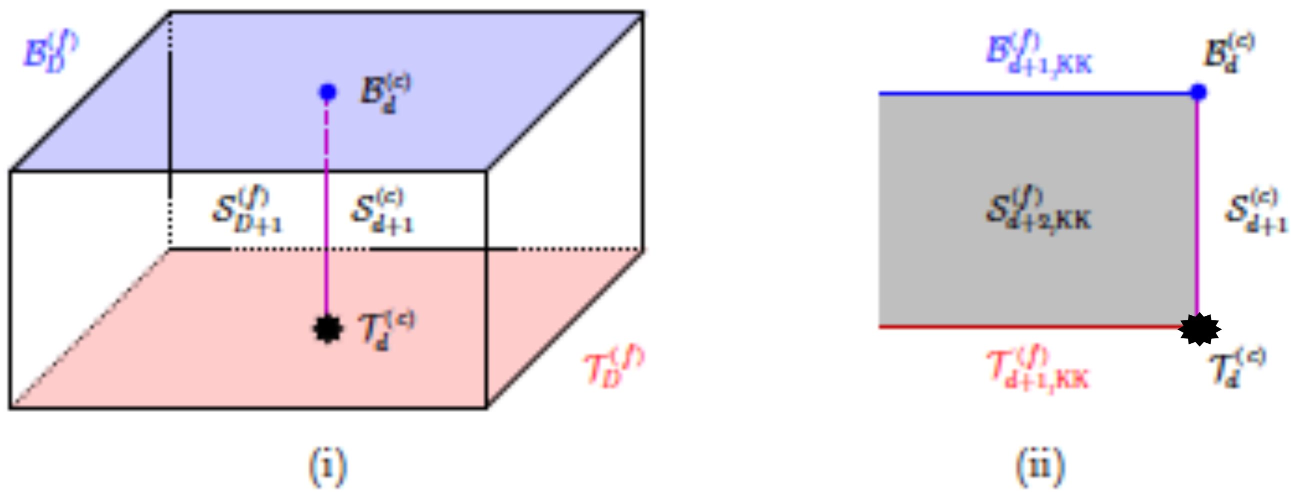

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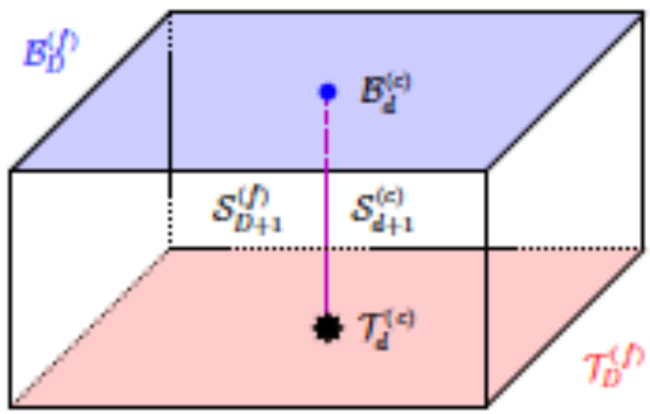

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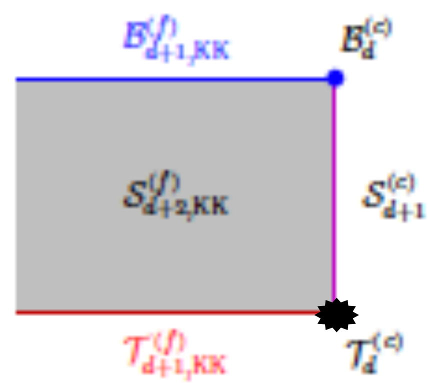

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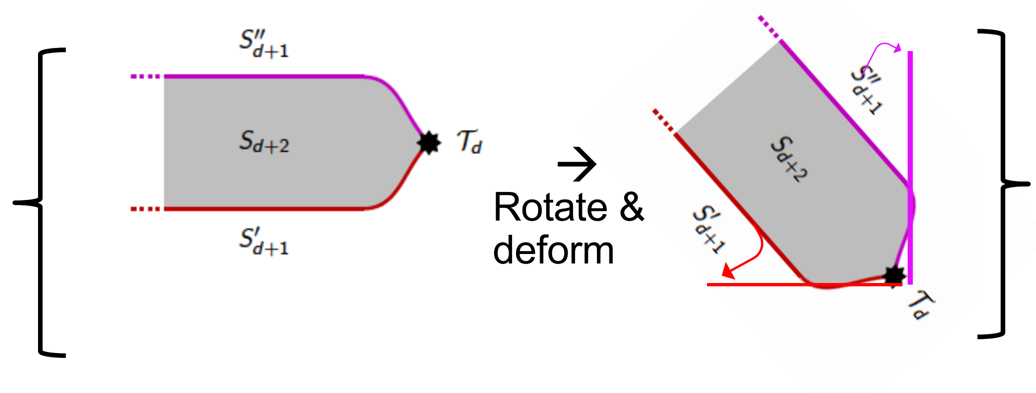
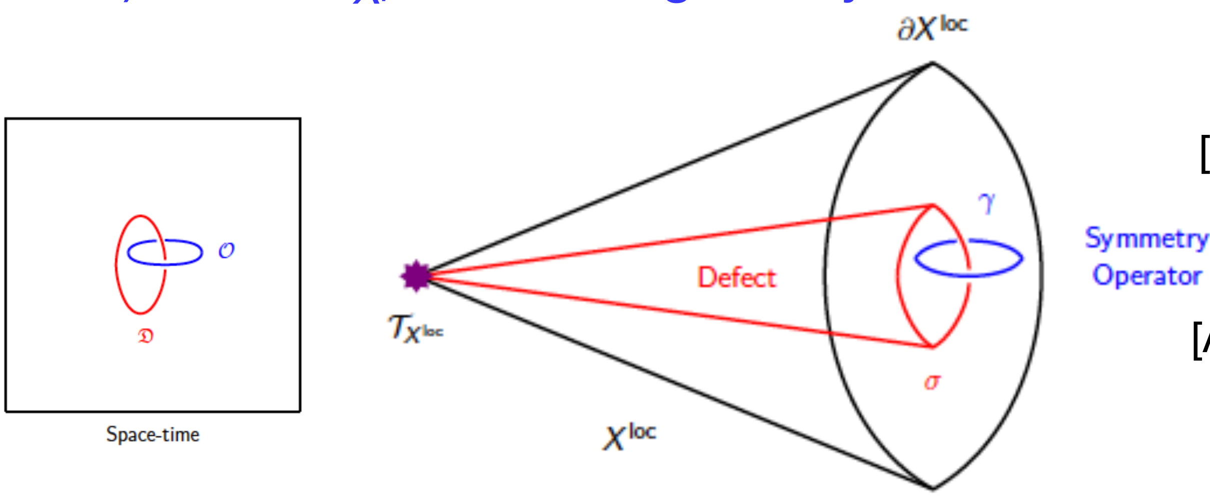


3) SymThs from String Theory (Geometric Engineering) Natural!

Prototype Example (Isolated singularity):

3A) QFT $\mathcal{T}_{X^{loc}}$ with string theory construction on conical non-compact X^{loc}



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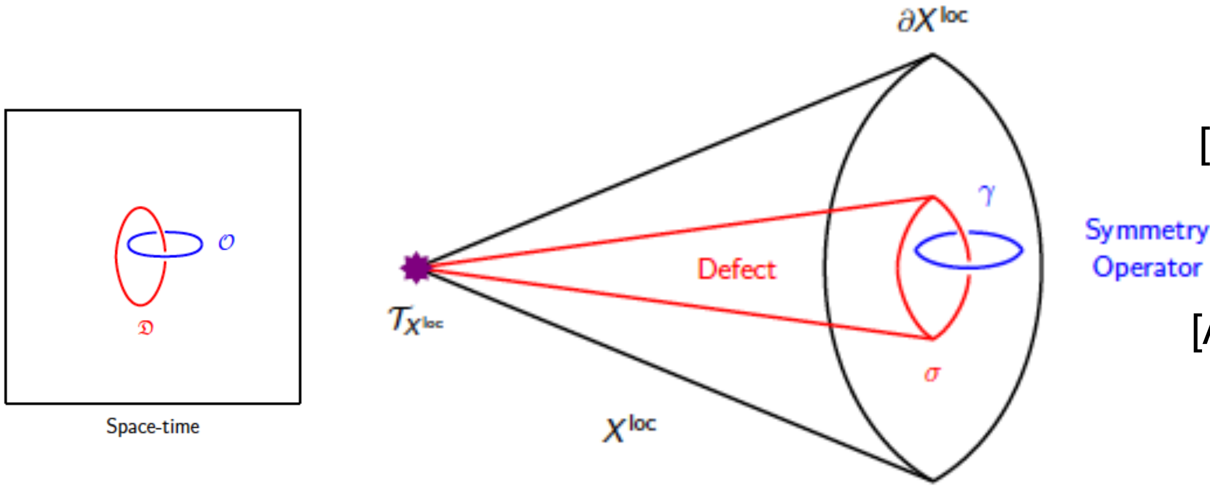

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Its SymTh follows via compactification over radial slices, resulting in 1A):

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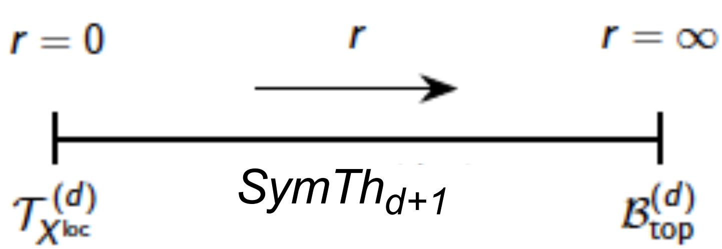

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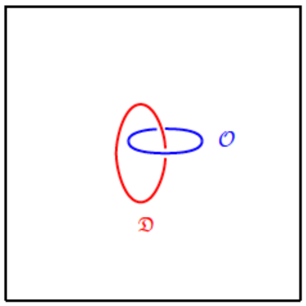
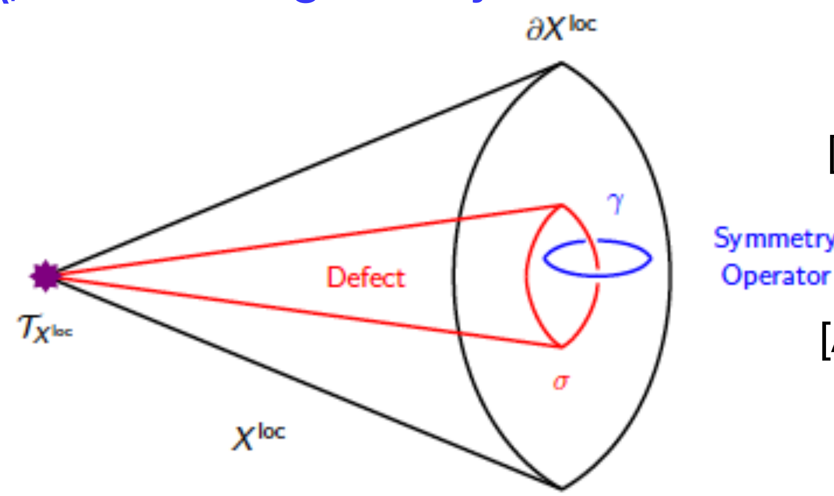


Space-time



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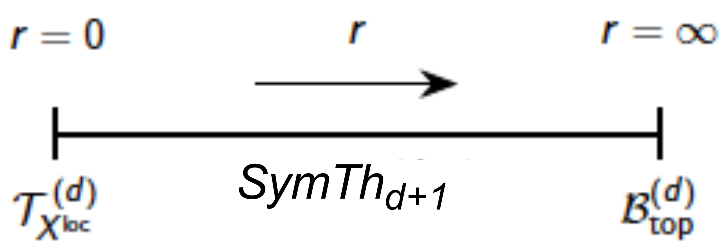

Note: Branes support worldvolume SymTh w/ Non-invertible Fusion Rules

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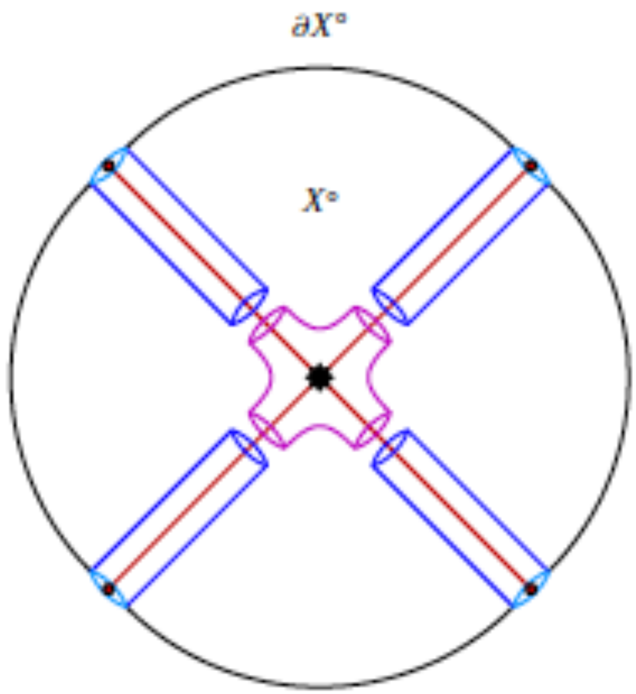
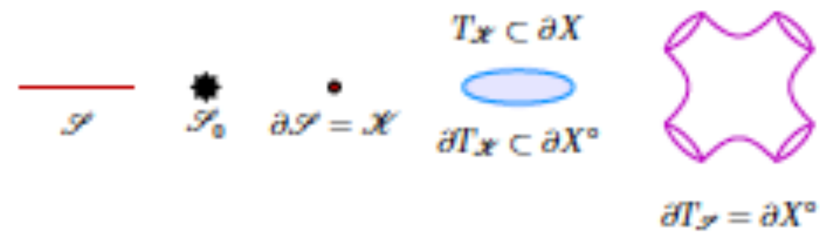
Prototype (non-isolated singularity): A string construction of intersecting flavor branes with defect degrees of freedom at their intersection:



Legend:



(X^{o}, ∂X^{0} - excised, regular spaces)

Two flavor branes (red) intersect at S_0 (star). The tubular neighborhoods of the flavor branes (blue) and their asymptotic boundaries (light blue). The tubes glue along the neighborhood associated with the intersection S_0 (purple) to the tubular neighborhood of the full singular locus $\mathscr{S}$.

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Natural!

- Connection to 2): obvious
- Connection to 1): from string theory prescription one finds the theory on radial slices (associated w/ flavor branes) to be a non-gapped QFT

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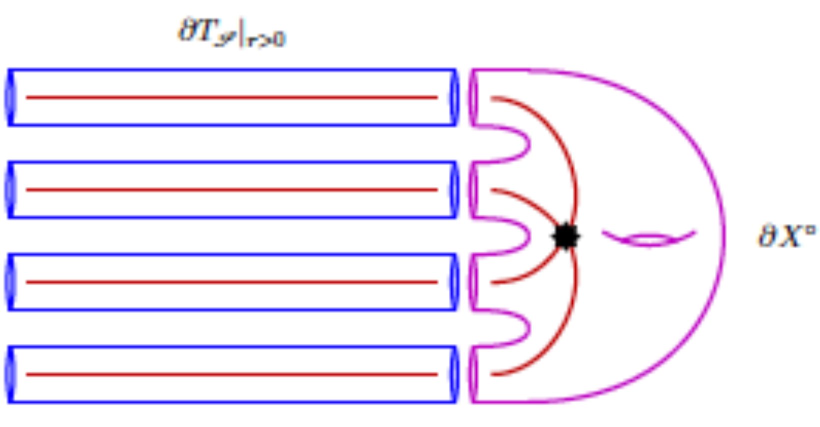
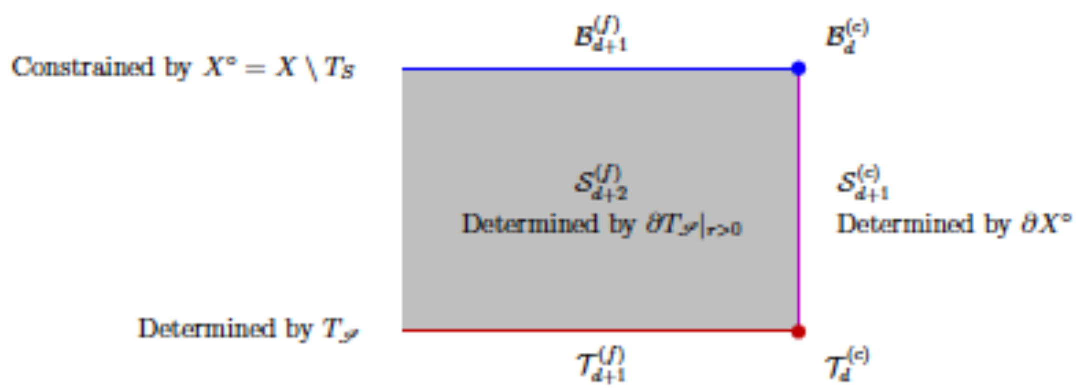
3C) Shift the perspective: how does the symmetry theory of the full system interplay with the symmetry theory of the flavor branes?

- Answer: the defect and its symmetry theory realize an edge-mode to the flavor brane and its symmetry theory, respectively:





The compactification now naturally realizes a corner mode which we know to be the relative defect theory (via string theory).

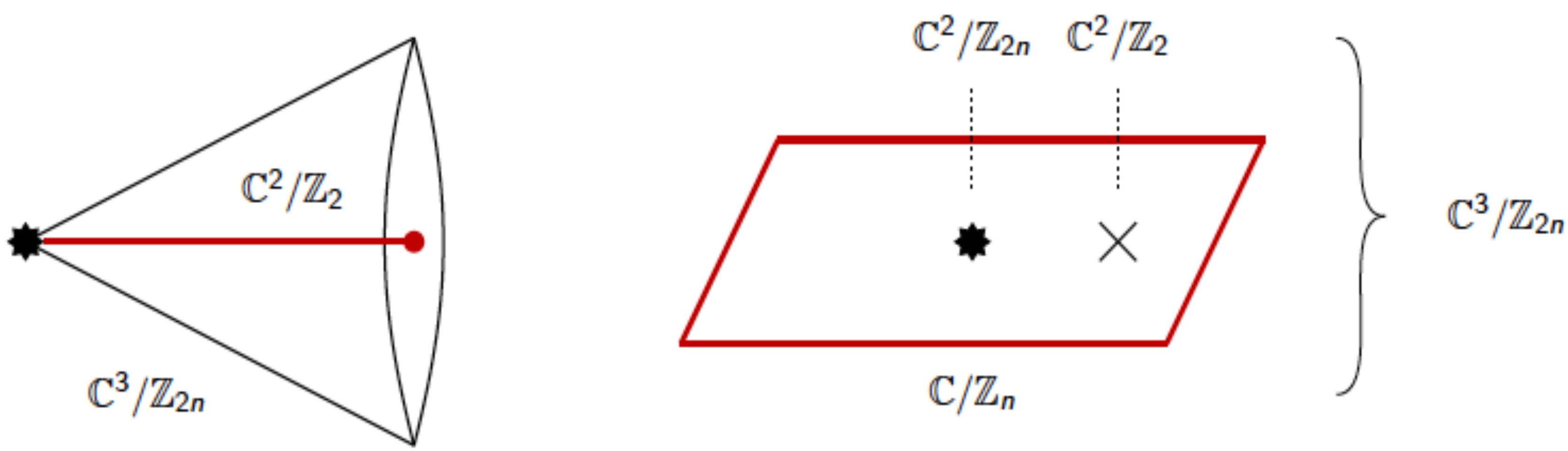
In the rest of the talk:

- Technical details of a concrete example (torsional)

- Outlook & Concluding remarks

Example: 5D SCFT

- Setup: M-theory on $X = \mathbb{C}^3/\mathbb{Z}_{2n}(1,1,2n-2)$ $\mathbb{C}^3 = \{z_1, z_2, z_3\}$
- Two equivalent presentations of the geometry:



(i) Cone (ii) Fibration over z_3

- SU(n) 5D SCFT at $\{z_1, z_2, z_3\} = 0$ → ✸
  SU(2) flavor brane at $\{z_1, z_2\} = 0$ → red line

- SU(n) 5D SCFT as defect within a 7D SU(2) SYM theory

Comment:

[M.C. Heckman, Hübner, Torres, '22]

These examples studied to identify geometric origin of higher-form symmetries (0-form, 1-form & 2-group) by studying the corresponding symmetry defects via algebraic topology [cutting & gluing of singular boundary of the non-compact space].

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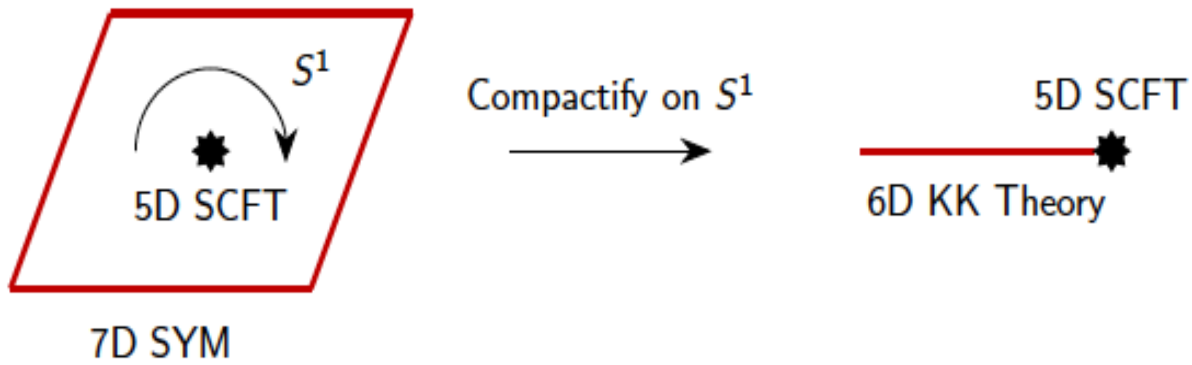
Within the current framework, a concrete construction of the nested SymTh of this joint 7D/5D system. It will reproduce old results, but the framework is now more general as one can study how any topological operator behaves when pushed from the symmetry bulk onto the symmetry boundary.

Example: 5D SCFT Steps toward nested SymTh

- Compactification to KK theory and end-of-world theory:

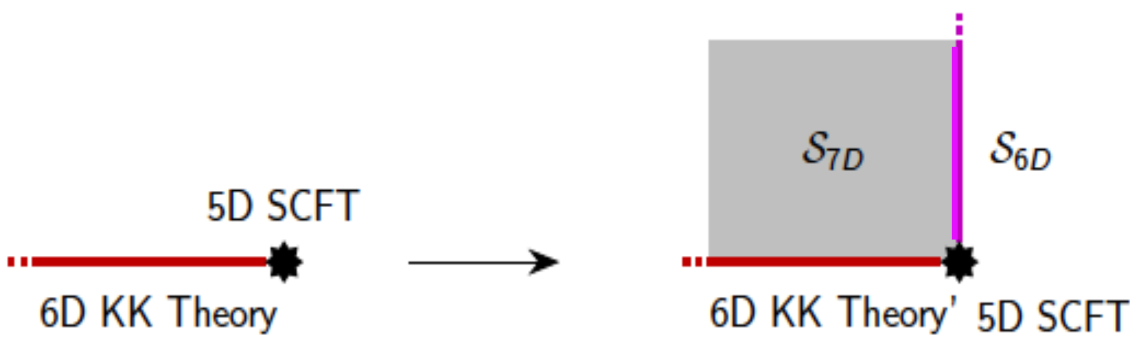
S^1- in the singular locus



- Construct symmetry theory from normal geometry to flavor brane $(S^1 \rtimes \mathbb{C}^2/\mathbb{Z}_2)$ and SCFT $(\mathbb{C}^2/\mathbb{Z}_{2n})$:

[Via reduction of the topological 11D supergravity terms on corresp. radial shells]

... - further b.c.



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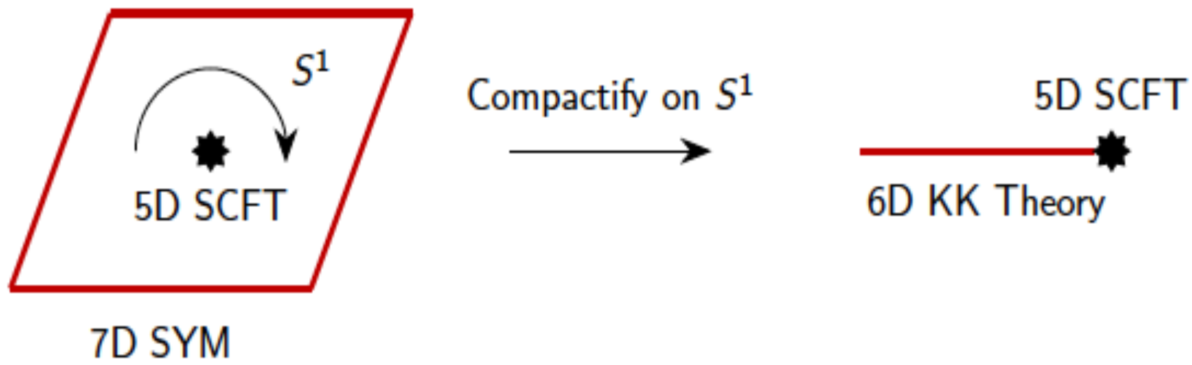

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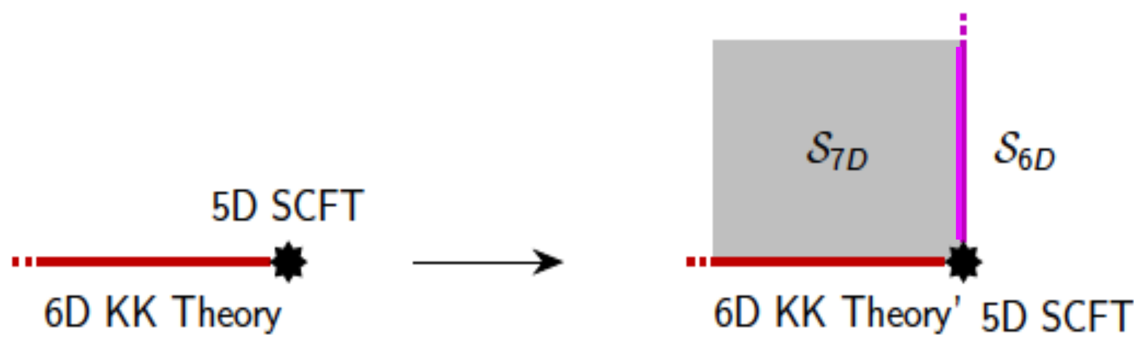


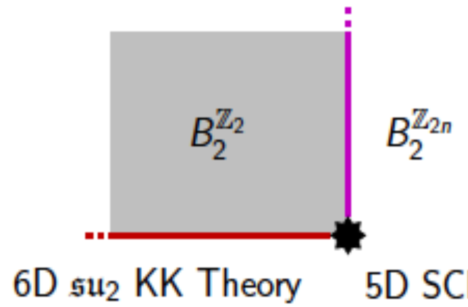
- Result: Bulk flavor brane SymTh S_{7D} with edgemode SymTh S_{6D}

& 5D SCFT a corner mode

Example: 5D SCFT 7D SymTh, its edge 6D SymTh & b.c.

- Field Content: 1-form symmetry backgrouns for 6D flavor brane and 5D SCFT are 2-cocycles $B_2^{\mathbb{Z}_2}$ and $B_2^{\mathbb{Z}_{2n}}$ in 7D and 6D SymTh, respectively:

M-theory 3-form on S^3/Z_2 & S^3/Z_{2n}
→ 2-cocycles B_2^{Z2} & B_2^{Z2n}



- Boundary conditions:

$$B_2^{\mathbb{Z}_2}|_{\text{right edge}} = nB_2^{\mathbb{Z}_{2n}}$$

- $\Rightarrow \mathbb{Z}_n \subset \mathbb{Z}_{2n}$ worth of profiles not fixed by boundary conditions
- $\Rightarrow \mathbb{Z}_n$ 1-form symmetry background field of 5D SCFT

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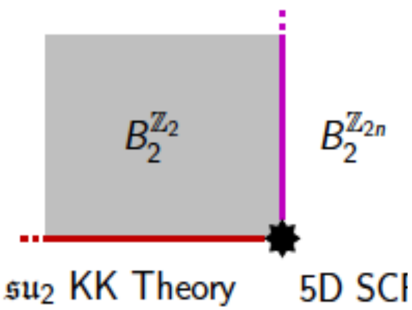

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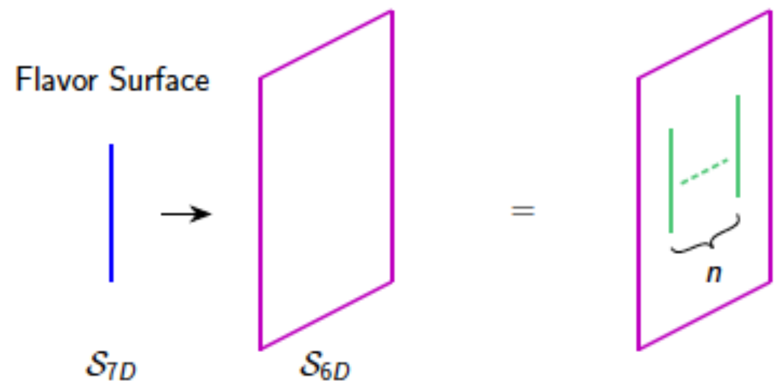
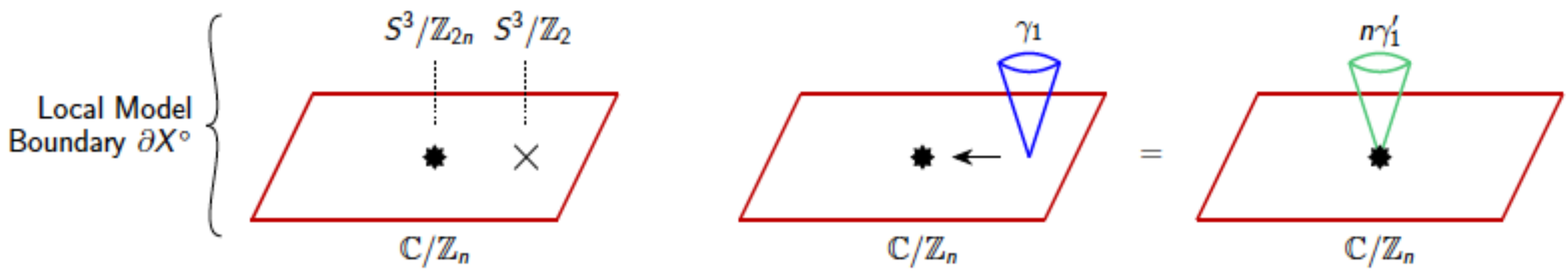
Comments: - The symmetry structures of the corner mode are those of the edge 6D SymTh, modulo the constraints imposed by the flavor 7D SymTh.
- The 1-form symmetry of SCFT is extended by the 1-form symmetry of flavor brane → hallmark of a 2-group.

Example: 5D SCFT Field Theory vs Geometry

- Interpretation of Boundary Condition (Field Theory): Bulk surface of $\mathcal{S}_{7D}$, pushed into $\mathcal{S}_{6D}$, decomposes into n surfaces of $\mathcal{S}_{6D}$.



- Interpretation of Boundary Condition (Geometry): M2-brane wrapping loci fractionate in homology.



(i) (ii) with γ_1, γ_1' the Hopf circles of $S^3/\mathbb{Z}_2$, $S^3/\mathbb{Z}_{2n}$ respectively

Example: 5D SCFT Anomaly Couplings

- The SymTh Lagrangian, characterizing anomaly couplings

This framework offers interesting new extensions:
In the reduction of the topological M-theory terms, one finds a mixed anomaly, a pairing between the flavor 1-form symmetry & that of the SCFT:

- Bulk-boundary mixed anomaly via refinements of triple linkings:

$$\begin{aligned} H^2(\partial X) \times H^2(\partial X) \times H^2(\partial X^\circ) &\to \mathbb{Q}/\mathbb{Z} \\ \mathbb{Z}_n \times \mathbb{Z}_n \times \mathbb{Z}_{2n} &\to \mathbb{Q}/\mathbb{Z} \\ (1,1,1) &\mapsto 1/n \end{aligned}$$

Also:

- Pure 5D 1-form self-anomaly by resticting $H^2(\partial X) \subset H^2(\partial X^\circ)$:

$$\begin{aligned} H^2(\partial X) \times H^2(\partial X) \times H^2(\partial X) &\to \mathbb{Q}/\mathbb{Z} \\ \mathbb{Z}_n \times \mathbb{Z}_n \times \mathbb{Z}_n &\to \mathbb{Q}/\mathbb{Z} \\ (1,1,1) &\mapsto 2/n \end{aligned}$$

Outlook

- Refined calculation of mixed anomalies between flavor edge and corner theory (see example) → work in progress…
- More examples...; Higher order nesting…

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- Symmetry Inheritance: relating defect and symmetry operators between bulk and boundary SymThs (see example) →

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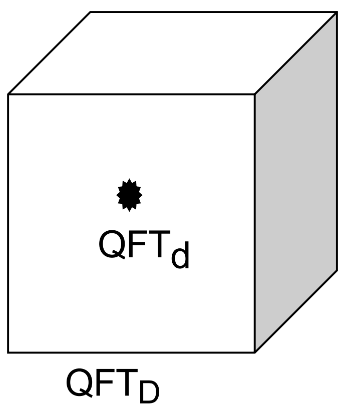

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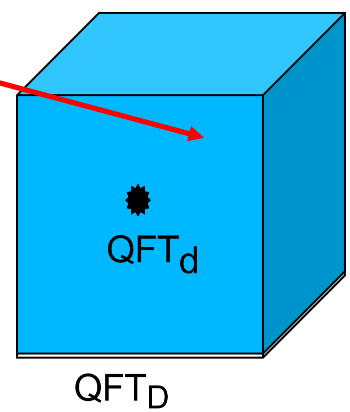

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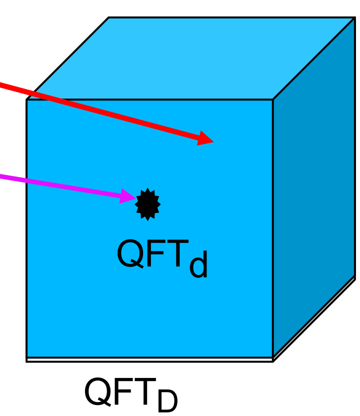

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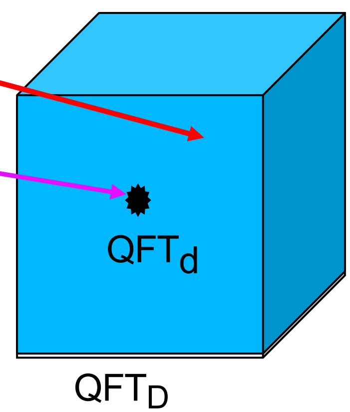

Comments: Could be applied to any defect QFT_d inside an ambient QFT_D ?
Can it be invariantly defined & is there a lower depth?

Outlook Last, but not least

- The fate of global symmetries in compact models with singular strata of varying dimension

[M.C., Heckman, Hübner, Torres ’23]

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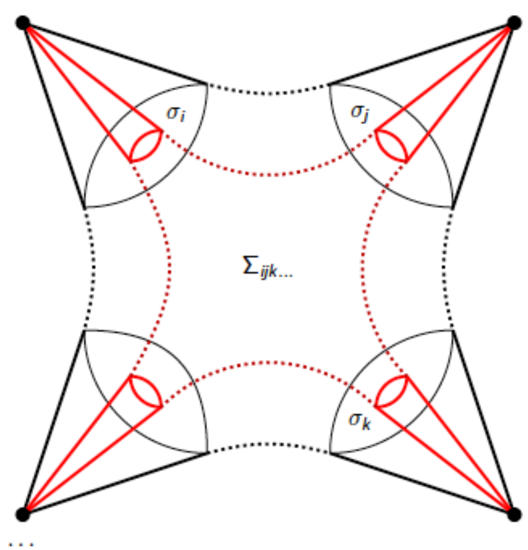
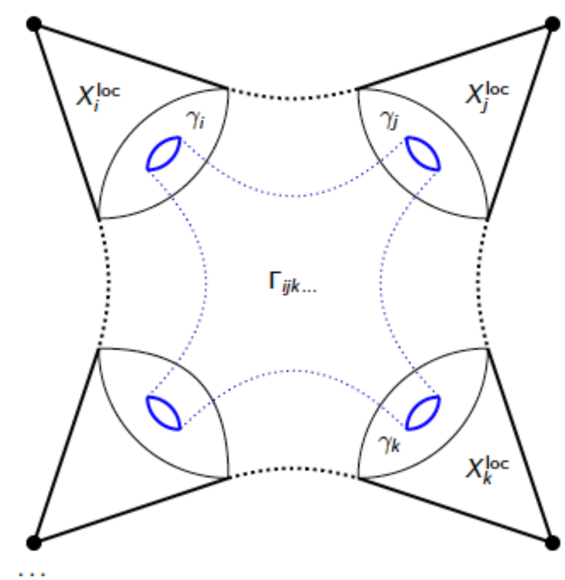
Σ_{ijk} – two-chain glues defects



Defect Operators
(some compactified [massive matter])



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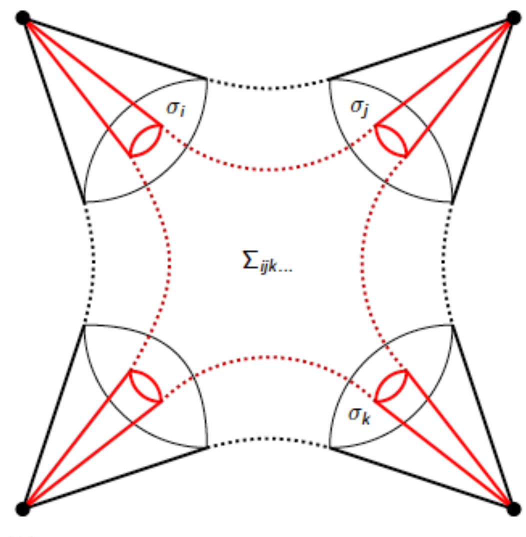

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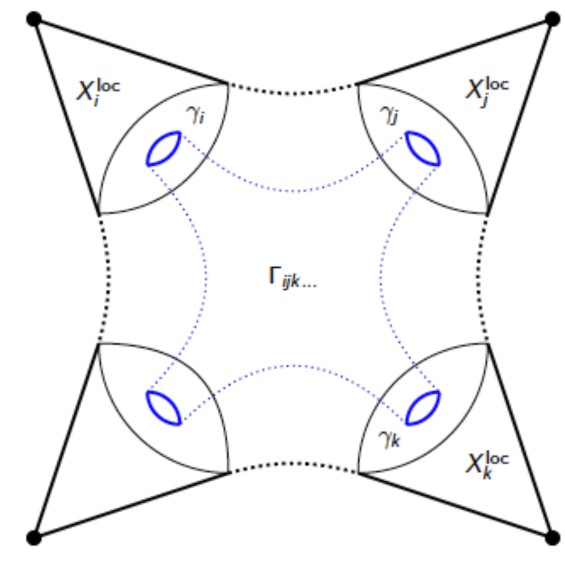

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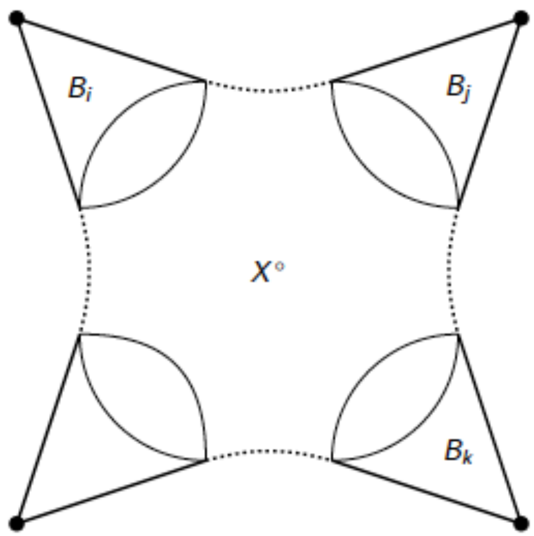
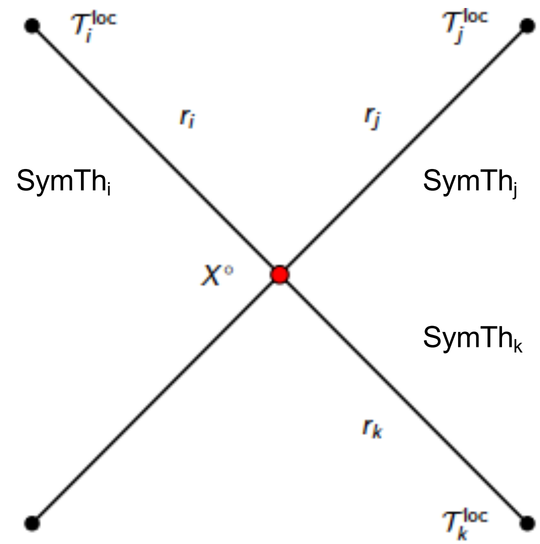
(M-Theory on all T^4 orbifolds & some T^6 orbifolds)

SymThs + Junctions

Gluing local constructions (w/ localized singularity) → assign a junction of SymTh to the field theory sector of M-theory on compact CY theories





SymThs + Junctions = SymTrees

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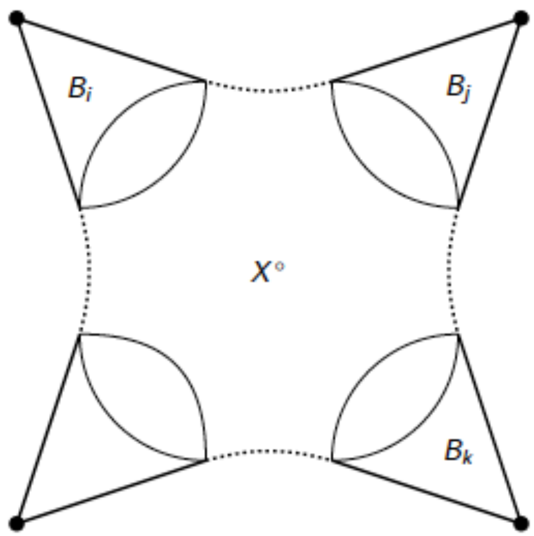

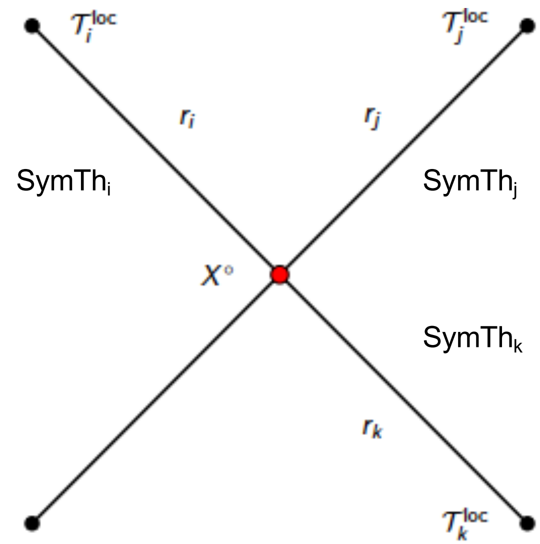

Physical boundary conditions for $\mathcal{T}_i^{\rm loc}$ at $r_i = 0$.

At the central node; partially topological and partially physical b.c.

[M.C., Heckman, Hübner,Torres, '23]

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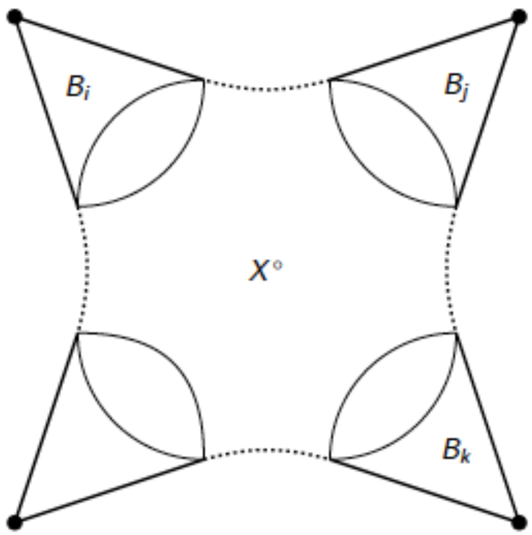

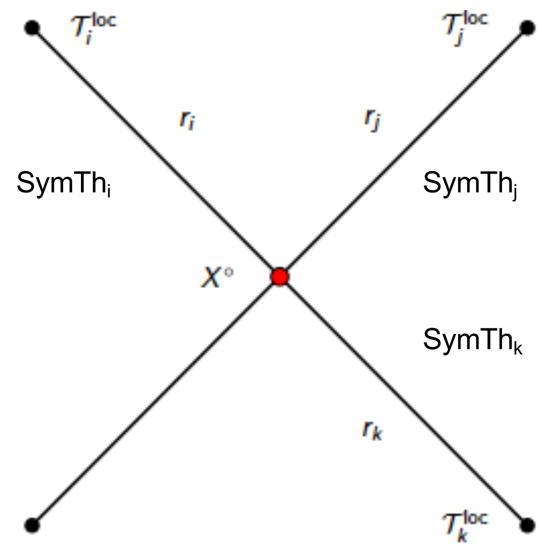

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Inclusion of flavor banes (strata of singularities) →
systematize gluing procedure, leading to nesting of SymTrees →
Further work

Thank you!