Higher Symmetry in Particle Physics

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Higher Symmetry Breaking

[Gaiotto-Kapustin-Seiberg-Willet].

Higher Symmetry

Develop symmetries that involve extended operators. Apply to pheno

Language: p-form global symmetry → charged operators p-dim defects

Ordinary (0-form) symmetry has charged local operators (create particles)

1-form global symmetry has charged line operators (create strings)

2-form global symmetry has charged surface operators (create domain wall)

Currents antisymm with p+1 indices. For 1-form symmetry (Example Maxwell):

$$J_{\mu\nu} = -J_{\nu\mu}\,, \qquad \partial^\mu J_{\mu\nu} = 0$$

1-Form Symmetry Breaking

For ordinary symmetries simply add symmetry violating operators to action

How are higher-form symmetries broken in QFT?

New effect: all local operators are uncharged under higher-form symmetry

Impossible to break them by adding operator deformations. In the leading Euler-Heisenberg deformation of Maxwell theory:

$$S = \frac{1}{e^2}\int d^4x \; F_{\mu\nu}F^{\mu\nu} + \frac{1}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2$$

Bianchi (magnetic): $\partial^\mu \epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma} = 0$, EOM (electric): $\partial^\mu(F_{\mu\nu}+\frac{1}{\Lambda^4}F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}) = 0$

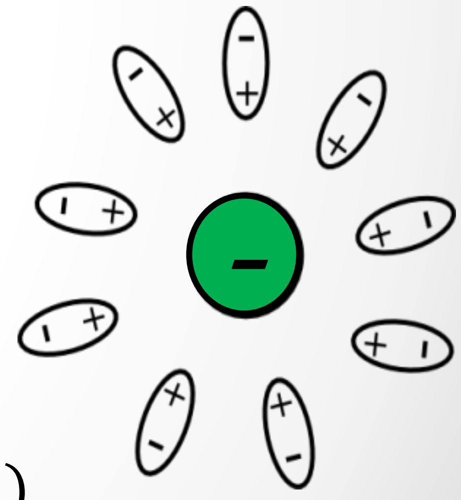
1-Form Breaking & Screening

Break one-form symmetry with **charged matter fields via screening**

$$\partial^\mu F_{\mu\nu} \sim J_\nu^{elec}$$

With mobile charged particles (mass m, charge $n \in \mathbb{Z}$) Gauss' law breaks

vacuum has virtual charges that screen Wilson line charge q



Charge $q(R)$ depends on the distance R from the source

$$q(R)/q(\infty) = 1 - e^2n^2\log(mR) + e^2(const. + O(mr))$$

Lesson

Ordinary approximate symmetries govern coefficients of operators in EFT

Approximate higher symmetries control scale of new propagating modes

- Electric and magnetic particles: dynamically break one-form symmetries
- Propagating axion strings: dynamically break two-form symmetries
- Non-abelian instantons & monopole loops: break non-invertible symmetry

Higher symmetry algebra guides UV models where the symmetry is broken

Selected Applications

- Higher group symmetry breaking and constraints on GUTs [Cordova-Koren]
- Higher group symmetry breaking and constraints on axion strings [Cordova-Brennan, Choi-Lam-Shao]
- Non-invertible symmetry and pion decay in the standard model [Choi-Lam-Shao]
- Non-invertible lepton symmetries and neutrino mass models [Cordova-Hong-Koren-Ohmori]
- Non-invertible PQ symmetry and the solutions of the strong CP problem [Cordova-Hong-Koren]

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Non-Invertible Chiral Symmetry

[Choi-Lam-Shao, CC-Ohmori]

Chiral Symmetry in Massless QED

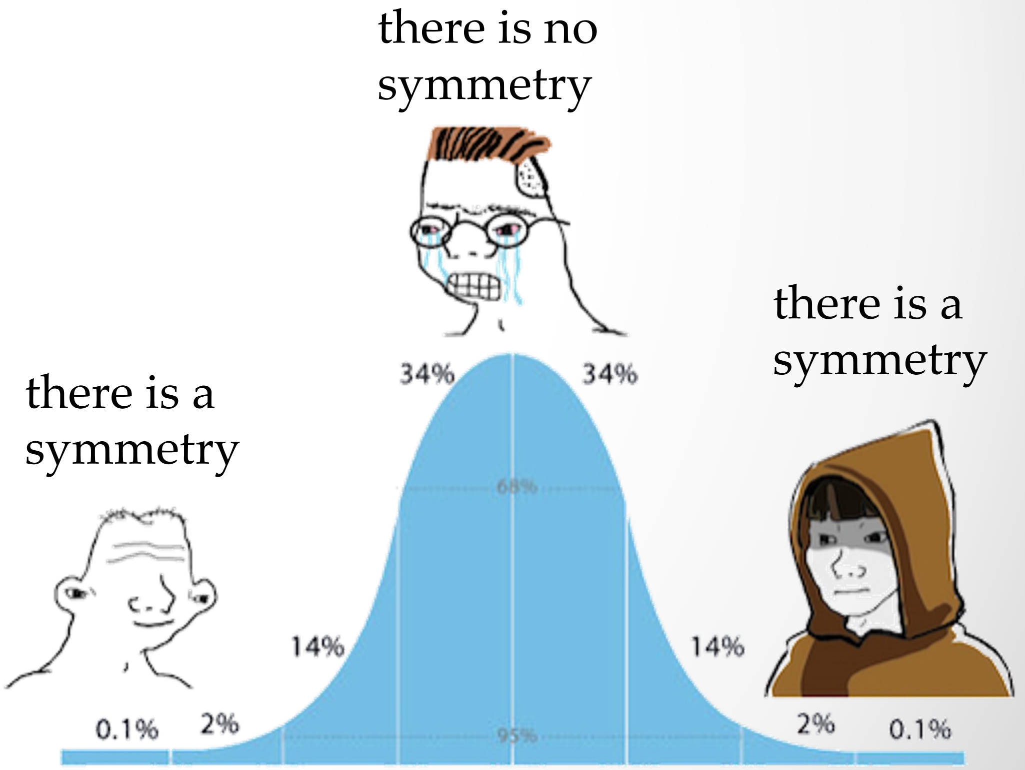
Consider $U(1)$ gauge theory with single light massless fermion $\chi^{\pm}$

J_μ chiral current has anomaly flavor-gauge-gauge: $\partial^\mu J_\mu \sim \epsilon_{\nu\rho\sigma\tau} F^{\nu\rho} F^{\sigma\tau}$

Does J_μ generate a conserved charge?

NO: at quantum level J_μ not conserved

YES: Conservation only violated by abelian Instantons and on S^4 (regulated spacetime) there are no such configurations ['t Hooft]



Define Chiral Symmetry

Can we make sense of ABJ non-perturbatively?

Idea: give up trying to find current. Work with exponentiated charge

Without anomaly this is trivial $\exp\left(2\pi i\alpha \int_{M_3} * J\right)$ (Hodge *)

Defines a topological unitary operator $D_\alpha(M_3)$ on spatial slice M_3

With anomaly naively might want to integrate formal current

$$* J - \frac{1}{8\pi^2} A \wedge F$$

$D_\alpha(M_3)$

Old idea, but not really correct since not gauge invariant

Define Chiral Symmetry

Integral $\exp\left(2\pi i\alpha \int_{M_3}(*J - \frac{1}{8\pi^2} A \wedge F)\right)$ uses effective Chern-Simons level α

Can't make sense of this in general, but can for a rational fraction $\alpha = \frac{1}{k}$

Fractional level is well defined as response to fractional quantum Hall state

Introduce a new $U(1)_k$ Chern-Simons theory (Field C_μ) localized on M_3

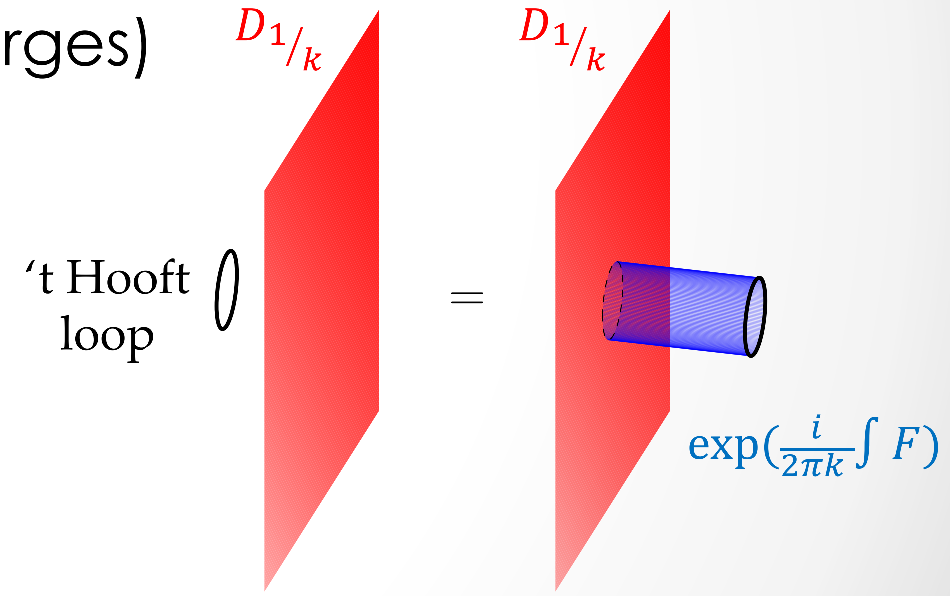
$$D_{1/k}(M_3) = \exp\left(\frac{2\pi i}{k} \int_{M_3} * J\right) \int DC \exp\left(\frac{ik}{4\pi} \int_{M_3} C \wedge dC + \frac{i}{2\pi} \int_{M_3} C \wedge dA\right)$$

Features of the Symmetry

Gives an intrinsic definition of symmetry with abelian ABJ anomaly.

Acts on local operators as chiral rotation e.g. fermion mass $m\,\chi^+\chi^-$ forbidden

Acts unusually on 't Hooft lines (magnetic charges)



magnetic charges create the topology needed to activate abelian instantons

Not unitary (**beyond groups**):

$$D_{1/k} \times \overline{D}_{1/k} \sim \sum_{S \in M_3} \exp\left(\frac{i}{2\pi k}\int_S F\right)$$

Non-Invertible Symmetry Breaking

Chiral symmetry with abelian ABJ anomaly in algebra with magnetic charge

$$D_{1/k} \times \bar{D}_{1/k} \sim \sum_{S \in M_3} \exp\left(\frac{i}{2\pi k} \int_S F\right)$$

Dynamical monopoles break 1-form symmetry $\Rightarrow$ violation of chiral symmetry

Symmetry algebra connects UV propagating particles to size of coeffs in EFT

Monopole Loops and Instantons

Estimate effects of monopoles using U(1) EFT with monopole worldline [Fan-Fraiser-Reece-Stout]

monopole mass $\mathrm{m} \sim v/g$, cutoff $\sim \delta t^{-1} \sim vg$ (v higgs scale, g coupling)

monopole loop action: $\exp(-S) \sim \exp(-m\,\delta t) \sim \exp(-\#/g^2)$

Loops of monopoles will violate chiral symmetry non-perturbatively

Can also view monopole loops as giving an IR description of UV instantons

$$\partial^\mu J_\mu \sim \epsilon_{\nu\rho\sigma\tau} F^{\nu\rho} F^{\sigma\tau} \rightarrow \partial^\mu J_\mu \sim \epsilon_{\nu\rho\sigma\tau} Tr(F^{\nu\rho} F^{\sigma\tau})$$

Non-abelian instantons violate symmetry even in S^4 (regulated spacetime)

Non-Invertible Peccei-Quinn

[CC-Hong-Koren]

Strong CP Problem Redux

Strong CP angle $\bar{\theta}$ measures CP violation in the strong sector.

Takes into account instanton coupling θ as well as phases from Yukawas

$$\bar{\theta} = \arg(e^{-i\theta}\det(y_u y_d))$$

Experimentally constrained to be tiny: $\bar{\theta} \lesssim 10^{-10}$ (likely to improve by orders of magnitude through atomic measurements over the next decades)

BUT, CP violated at order one, by the weak sector $\delta_{CKM} \sim O(1)$

Small $\bar{\theta}$ not technically natural. Runs in SM as $^{d\bar{\theta}}/_{dlog(\mu)} \sim \delta_{CKM}$ (first at 7 loop)

PQ Symmetry and Strong CP

- One possibility is the axion: $\overline{\theta}$ promoted to a periodic scalar

Requires potential, generated by $SU(3)$ instantons, with minimum near zero

The axion has an approximate shift invariance (Peccei-Quinn Symmetry)

- Another possibility: $U(1)_{PQ}$ protecting a quark mass. (e.g. the up quark):

$$U(1)_{PQ}\text{:}\ \ \bar{u} \rightarrow \bar{u}e^{i\alpha}, \qquad \theta \rightarrow \theta - \alpha$$

To if this is an invariance of the action, $\det(y_u) = 0 \Longrightarrow$ No strong CP violation

Not a priori ruled out, need to check the meson and baryon spectrum!

The Massless Quark Reborn

Results of lattice QCD essentially rule out the massless up quark solution!

Possible reincarnation?? Embed QCD as the low energy of UV gauge group

Need UV to give bare quark mass of sufficient size with approx PQ symmetry

Equivalently $e^{-i\theta}\det(y_u y_d) \neq 0$ must be generated nearly on the real axis

Suggests UV instantons should generate one of the yukawas [Agrawal-Howe]

Use non-invertible symmetry to hunt for candidate UV completions

Z' Baryon Family Difference Model

Baryon family $B_1 - B_2$, $B_2 - B_3$ differences are anomaly free global symmetries

Gauge a particular linear combination $H = B_1 + B_2 - 2B_3$

(Obviously must be higgsed at low energies to reproduce the SM)

Special feature of this horizontal gauge group is that its overlaps with color:

$$U(1)_H \supseteq \mathbb{Z}_3^H \cong \mathbb{Z}_3^C \subseteq SU(3)_C$$

Choose gauge group to have non-trivial global structure: $(SU(3)_C \times U(1)_H)/\,\mathbb{Z}_3$

Alternatively gauge full $SU(3)_H$ with total gauge group: $(SU(3)_C \times SU(3)_H)/\,\mathbb{Z}_3$

Fractional Instantons

These models exist because: number of generations = number of colors

Non-trivial quotient leads to fractional instantons

Field configurations with the feature that minimum instanton cannot be saturated in flat space

	$SU(3)_C$	$SU(3)_H$
$\mathbf{Q}$	3	3
$\bar{\mathbf{u}}$	$\bar{3}$	$\bar{3}$
$\bar{\mathbf{d}}$	$\bar{3}$	$\bar{3}$

Examine consequences for classical chiral symmetry $U(1)_{\bar{d}}$:

Broken to $\mathbb{Z}_3^{\bar{d}}$ by usual instanton. Made non-invertible by fractional instanton

$\mathbb{Z}_3^{\bar{d}}$ non-invertible PQ symmetry forbids down-type Yukawas (kills $\bar{\theta}$)

Yukawas from Color-Flavor Unification

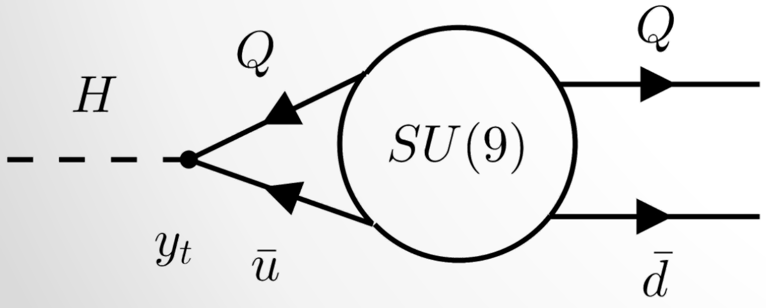
Find UV where this symmetry is broken by monopole loops/small instantons

One possibility is a UV $SU(9)$ model with quarks in fundamentals

	$SU(9)$
$\mathbf{Q}$	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$

UV Lagrangian includes: $\mathcal{L} \supseteq y_t \widetilde{H} Q \bar{u} + \frac{\theta_9}{16\pi^2} Tr(F \wedge F)$

$\mathbb{Z}_3^{\bar{d}}$ now violated explicitly by instanton processes. Generates y_d



$$\mathcal{L} \supseteq y_t^* \, e^{-\frac{2\pi}{\alpha} + i\,\theta_9} H Q \bar{d}$$

aligned: $\overline{\theta} = -3\theta_9 + \arg(\det(|y_t|^2 e^{i\theta_9})) = 0$

Conclusions

Theme: algebra between higher symmetry and ordinary symmetry

Gave interplay between symmetry breaking effects:

local operator deformations (ordinary symmetry)	$\Leftrightarrow$	dynamical charged particles (one-form symmetry)

Symmetry is THE way (old school) to understand questions of naturalness

Perhaps these ideas give new views on old problems in pheno and beyond

Thanks for Listening!

Higgsing Back to SM

$SU(9)$ higgsed to $(SU(3)_C \times SU(3)_H)/\mathbb{Z}_3$ by 3-index symmetric scalar:

$$\langle \Phi^{ABC} \rangle = \Lambda_9 \epsilon^{ijk} \epsilon^{abc}$$

Next we must generate flavor structure via higgsing $SU(3)_H$. (Options exist)

One possibility is to use a complex adjoint scalar Σ with a potential:

$$V(\Sigma) \sim \eta_1 \mathrm{Tr}(\Sigma^4) + \eta_2 \mathrm{Tr}(\Sigma^2) + h.c. + \text{real terms}$$

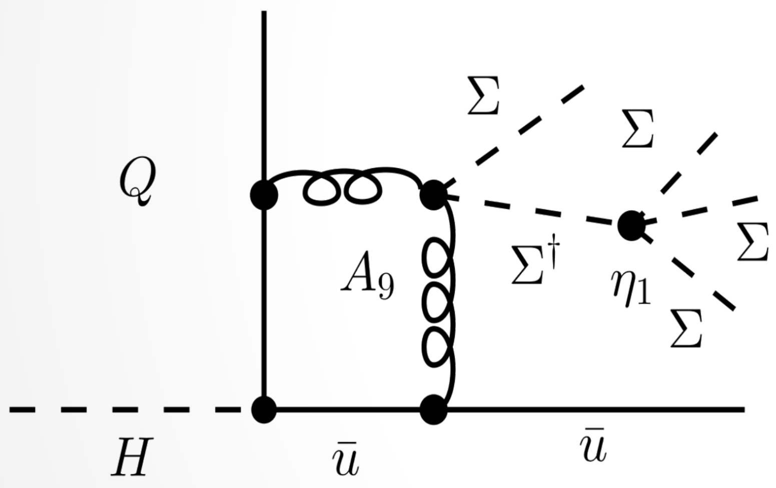
Parameter $\eta_1^* \eta_2$ violates CP at order one. So $V(\Sigma)$ can generate CKM matrix

Note: this is NOT a predictive theory of flavor. Potential $V(\Sigma)$ put in by hand

Protecting $\overline{\theta} \approx 0$

With $V(\Sigma)$ have explicit CP violation. Natural worry that $\overline{\theta}$ will be generated

Not the case! Yukawa corrections via $SU(9)$ bosons come in conjugate pairs



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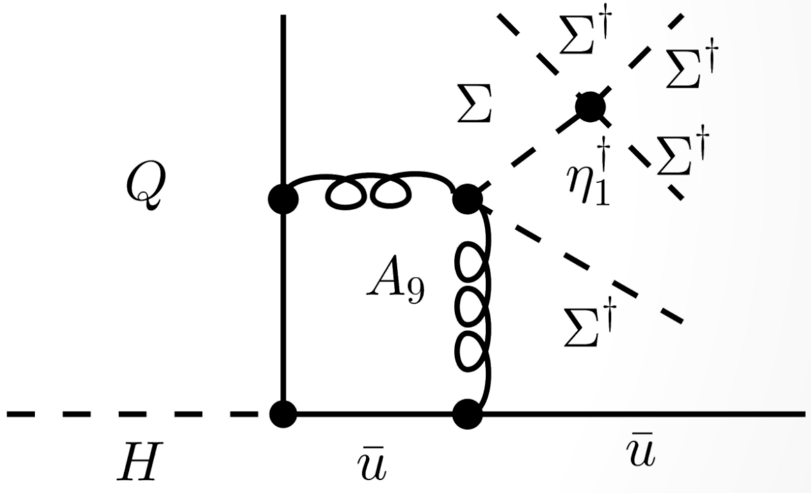

Keep quark yukawas Hermitian preserving $\overline{\theta} = 0$

Independent of size of generated yukawas or scale hierarchies

Quality Problems?

Both axions and the massless quark solution require an approximate PQ symmetry. Is it really robust?

For axion case one must protect against Planck suppressed corrections to the axion potential:

$$\Delta\mathcal{L} \sim \theta^n / M_p^{n-4}$$

Cannot move minimum too far from the origin! Tuning problem

In the massless quark model corrections are $\Delta\mathcal{L} \sim \widetilde{H} Q \Sigma \bar{d} / M_p$

Can have order one coefficient as long as $\langle\Sigma\rangle \sim \Lambda_3 \ll \bar{\theta}\, M_p$. Physics nearby!