On 4D dS Solutions of 6D Supergravity and String Theory

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String Phenomenology 2024

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C.P. Burgess, F. Muia and FQ 2407…

A Commercial

Talks by

Anshuman Maharana

Gonzalo Villa

String Thermodynamics and Gravitational Waves

Also

Mario Ramos-Hamud

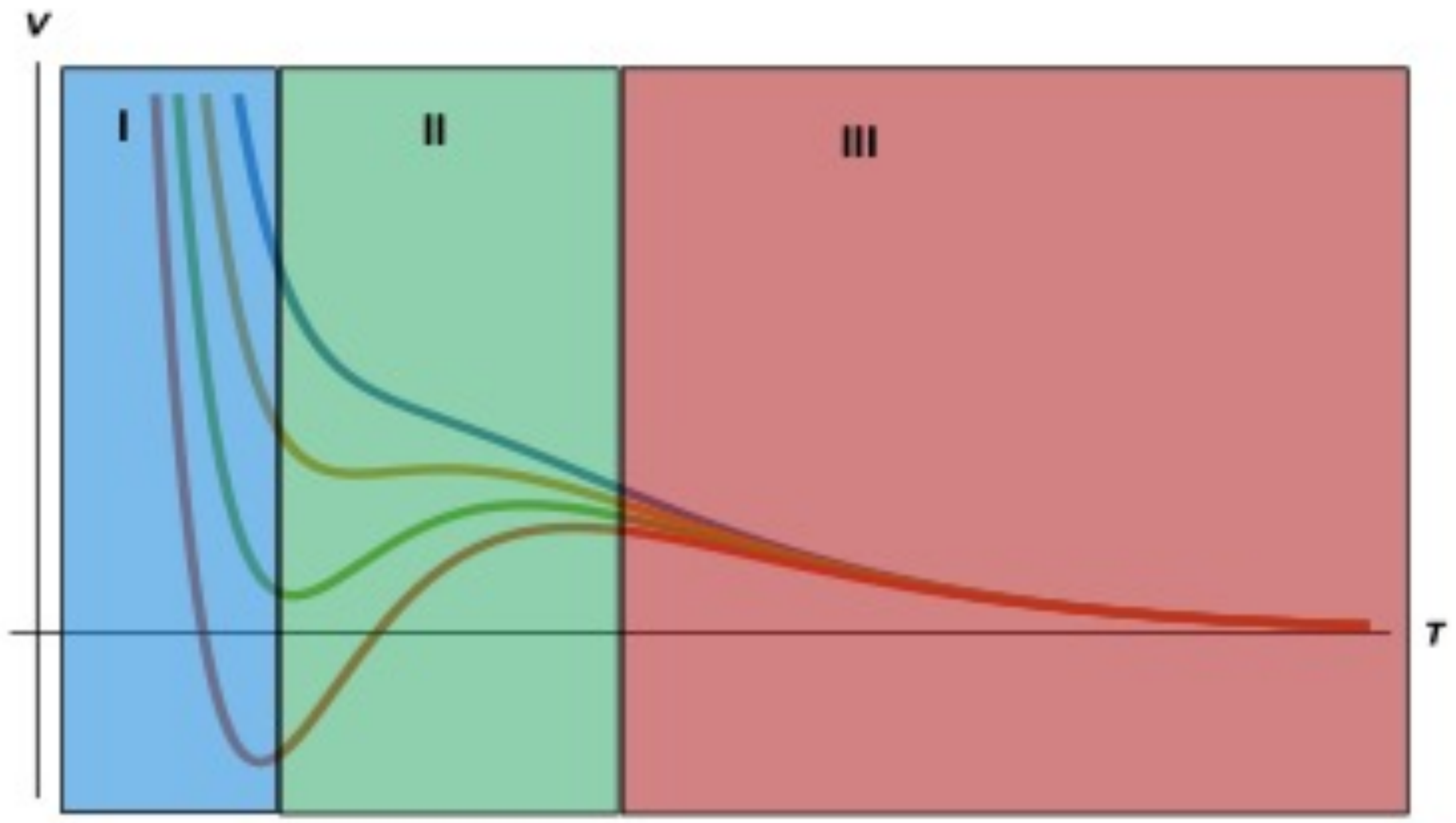
Ahmed Kamal

Realistic Brane Inflation

Obstacles for dS from UV Theory

- Classical No-Go Theorems

- Dine Seiberg problem



Different approaches

- **String flux compactification EFTs** **(e.g. KKLT, LVS)**

  Review: L. McCallister and FQ, 2310.20559

  (See also talks by Schachner and McAllister)

- **Classical solutions?** **(evading no-go theorems)**

  e.g. Cordova, Tomasiello et al.

Classical de Sitter solutions

Classical no-go theorem

Gibbons, De Wit, Maldacena-Nunez…

- The gravity action does not contain higher curvature corrections.
- The potential is non-positive, $V \leq 0$.
- The theory contains massless fields with positive kinetic terms.
- The d dimensional effective Newton's constant is finite.

$$ds_D^2 = \Omega^2(y)\left(dx_d^2 + \hat{g}_{mn}dy^n dy^m\right)$$

$$\frac{1}{(D-2)\Omega^{D-2}}\nabla^2\Omega^{D-2} = \quad R \ + \Omega^2(-T^\mu{}_\mu + \frac{d}{D-2}T^L{}_L) \geq 0$$ **For de Sitter** $R \geq 0$

But integrating: $$\int d^{(D-d)}y\sqrt{\hat{g}}\left(\hat{\nabla}\Omega^{(D-2)}\right)^2 \leq 0$$ **So no de Sitter**

Ways out

- Quantum effects,…
- Relax assumptions (e.g. $V \leq 0$)

De Sitter from 6D (1,0) Gauged Supergravity

Matter content

- *Gravity multiplet.* Metric g_{MN} a self-dual antisymmetric tensor B^{+}_{MN}, one left-handed gravitino Ψ^{α}_{M}.
- *Tensor multiplet.* One anti self-dual antisymmetric tensor B^{-}_{MN}, one scalar ϕ, one right-handed fermion ψ (tensorino).
- *Vector multiplet.* One vector A_M and one fermion λ (gaugino).
- *Hypermultiplet:* Two complex scalars q^1, q^2 and one right-handed Weyl fermion ξ (hyperino) .

In general n_T tensor, n_V vector and n_H hyper multiplets

$$n_H - n_V + 29 n_T = 273$$

Scalar fields

- **From tensor multiplets n_T real scalars**

$$SO(1,n_T)/SO(n_T)$$

$$j^\alpha \qquad \alpha = 1,...n_T+1 \qquad \Omega_{\alpha\beta} j^\alpha j^\beta = 1 \qquad g_{\alpha\beta} = 2 j_\alpha j_\beta - \Omega_{\alpha\beta}$$

n_T=1 $\qquad j^0 = \sinh\varphi\,, \qquad j^1 = \cosh\varphi$

- **From hypermultiplets**

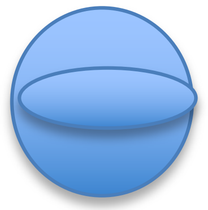
$$q^U \; (U = 1,...,4n_H)$$

Quaternionic manifold

6D Supergravity (Salam-Sezgin)

$$\mathcal{L}_6 = -\sqrt{-g}\left[\frac{1}{2\kappa^2}\, g^{MN}\left(R_{MN} + \partial_M\varphi\,\partial_N\varphi\right) + \frac{1}{4}e^{-\varphi}F_{MN}F^{MN} + \frac{1}{12}e^{-2\varphi}H_{MNP}H^{MNP} + \frac{2g^2}{\kappa^4}\,e^{\varphi}\right]$$

- **Positive potential (evades Maldacena-Nunez theorem)**
- **Chiral**
- **No maximally symmetric solution in 6D (Dine-Seiberg problem in 6D?)**
- **Maximally symmetric in 4D**
- **Maximally symmetric smooth solution: S^2 x Minkowski, N=1 SUSY.**



x $\mathbf{R}^{1,3}$

Field Equations:

$$\Box_6\varphi + \frac{\kappa^2}{4}e^{-\varphi}F_{MN}F^{MN} + \frac{\kappa^2}{6}e^{-2\varphi}H_{MNP}H^{MNP} - \frac{2g^2}{\kappa^2}e^{\varphi} = 0$$

$$\nabla_M\left(e^{-\varphi}F^{MN}\right) + \kappa e^{-2\varphi}H^{PNQ}F_{PQ} = 0\,, \qquad \nabla_M\left(e^{-2\varphi}H^{MNP}\right) = 0$$

$$R_{MN} + \partial_M\varphi\,\partial_N\varphi + \kappa^2 e^{-\varphi}F_{MP}F_N^{\ P} + \frac{1}{2}(\Box_6\varphi)\,g_{MN} = 0\,,$$

Scaling symmetry

$$g_{MN} \to c\,g_{MN} \quad \text{and} \quad e^{-\varphi} \to c\,e^{-\varphi} \qquad \text{imply} \qquad \mathcal{L}_6 \to c^2\mathcal{L}_6\,,$$

Salam-Sezgin solution

$$\mathrm{d}s^2 = \eta_{\mu\nu}(x)\,\mathrm{d}x^\mu\mathrm{d}x^\nu + \rho^2\left(\mathrm{d}\phi^2 + \sin^2\phi\,\mathrm{d}\theta^2\right) \qquad F_{mn} = f\,\epsilon_{mn}$$

$$e^{-\varphi}f = 2g/\kappa^2 \qquad R_{mn} = -e^{-\varphi}F_{mp}F_n^{\ p} = -f^2e^{-\varphi}g_{mn} = -\frac{g_{mn}}{\rho^2} \implies \rho^2e^{\varphi} = \left(\frac{\kappa^2}{2g}\right)^2$$

4D EFT: **(Aghababie et al 2003)**

$$V = \frac{2g^2e^{\varphi}}{\rho^2}\left(1 - \frac{\kappa^4}{4g^2e^{\varphi}\rho^2}\right)^2$$

General 4D Solutions

Gibbons et al 2004
Burgess et al 2005

$$\mathcal{L}_6 = R * \mathbf{1} - *d\phi \wedge d\phi - \frac{1}{2} e^{-\varphi} * F_{(2)} \wedge F_{(2)} - \frac{1}{2} e^{-2\varphi} * H_{(3)} \wedge H_{(3)} - 8g^2 e^{\varphi} * \mathbf{1}$$

Runaway potential! 6D Dine-Seiberg problem?

$$\mathrm{d}s^2 = \hat{g}_{MN}\, \mathrm{d}x^M\, \mathrm{d}x^N = W^2(y)\, g_{\mu\nu}(x)\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu + \tilde{g}_{ij}(y)\, \mathrm{d}y^i \mathrm{d}y^j$$

$$\hat{g}_{\mu\nu} = W^2 g_{\mu\nu}\,, \qquad \hat{R}_{\mu\nu} = R_{\mu\nu} + \frac{1}{n}(W^{2-n}\tilde{\nabla}^2 W^n)\, g_{\mu\nu} \quad \text{and} \quad \hat{\Box}\varphi = W^{-n}\tilde{\nabla}_i(W^n \tilde{g}^{ij}\partial_j \varphi)\,,$$

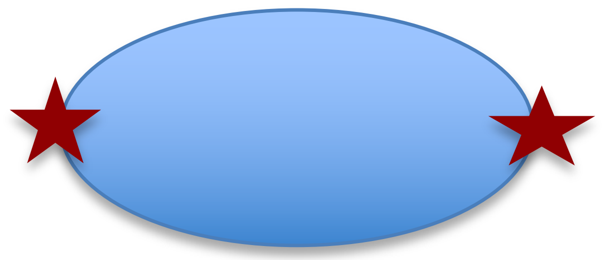
$$\frac{1}{n}\int_M \mathrm{d}^d y\, \sqrt{\tilde{g}}\, W^{n-2}\, R = -\sum_\alpha \int_{\Sigma_\alpha} \mathrm{d}^{d-1}y\, \sqrt{\tilde{g}}\, N_i \left[W^n \tilde{g}^{ij}\partial_j \left(\ln W + \frac{2\,\varphi}{D-2} \right) \right]$$

No singularities/boundaries imply $R=H^2=0$ e.g. $S^2 \times R^{1,3}$

(uniqueness of Salam-Sezgin solution)

General Solutions

Burgess et al 2005



x $(A)dS_4$

★ 3-Branes

Asymptotic near brane solutions (n=4, d=2):

$$\varphi \approx q \ln r \quad \text{and} \quad \mathrm{d}s^2 \approx r^{2w}\, g_{\mu\nu}(x)\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu + \mathrm{d}r^2 + r^{2\alpha} f_{ab}(z)\, \mathrm{d}z^a \mathrm{d}z^b\,,$$

$$nw + \alpha(d-1) = 1. \qquad nw^2 + \alpha^2(d-1) + q^2 = 1.$$

Kasner constraints
(BKL: Belinsky et al)

$$-\frac{1}{\sqrt{n}} \le w \le \frac{1}{\sqrt{n}}\,, \quad -\frac{1}{\sqrt{d-1}} \le \alpha \le \frac{1}{\sqrt{d-1}} \quad \text{and} \quad -1 \le q \le 1\,.$$

Flat Solutions

Gibbons et al.

$$ds^2 = \hat{g}_{MN}\,dx^M dx^N = W^2 q_{\mu\nu}\,dx^\mu dx^\nu + a^2 d\theta^2 + a^2 W^8 d\eta^2,$$

$$\begin{aligned}
e^{\varphi} &= W^{-2} e^{-\lambda_3 \eta} \\
W^4 &= \left(\frac{Q\lambda_2}{4g\lambda_1}\right) \frac{\cosh[\lambda_1(\eta-\eta_1)]}{\cosh[\lambda_2(\eta-\eta_2)]} \\
a^{-4} &= \left(\frac{gQ^3}{\lambda_1^3\lambda_2}\right) e^{-2\lambda_3\eta} \cosh^3[\lambda_1(\eta-\eta_1)]\cosh[\lambda_2(\eta-\eta_2)] \\
F &= \left(\frac{Qa^2}{W^2}\right) e^{-\lambda_3\eta}\, d\eta \wedge d\theta\,.
\end{aligned}$$

Numerical de Sitter solution

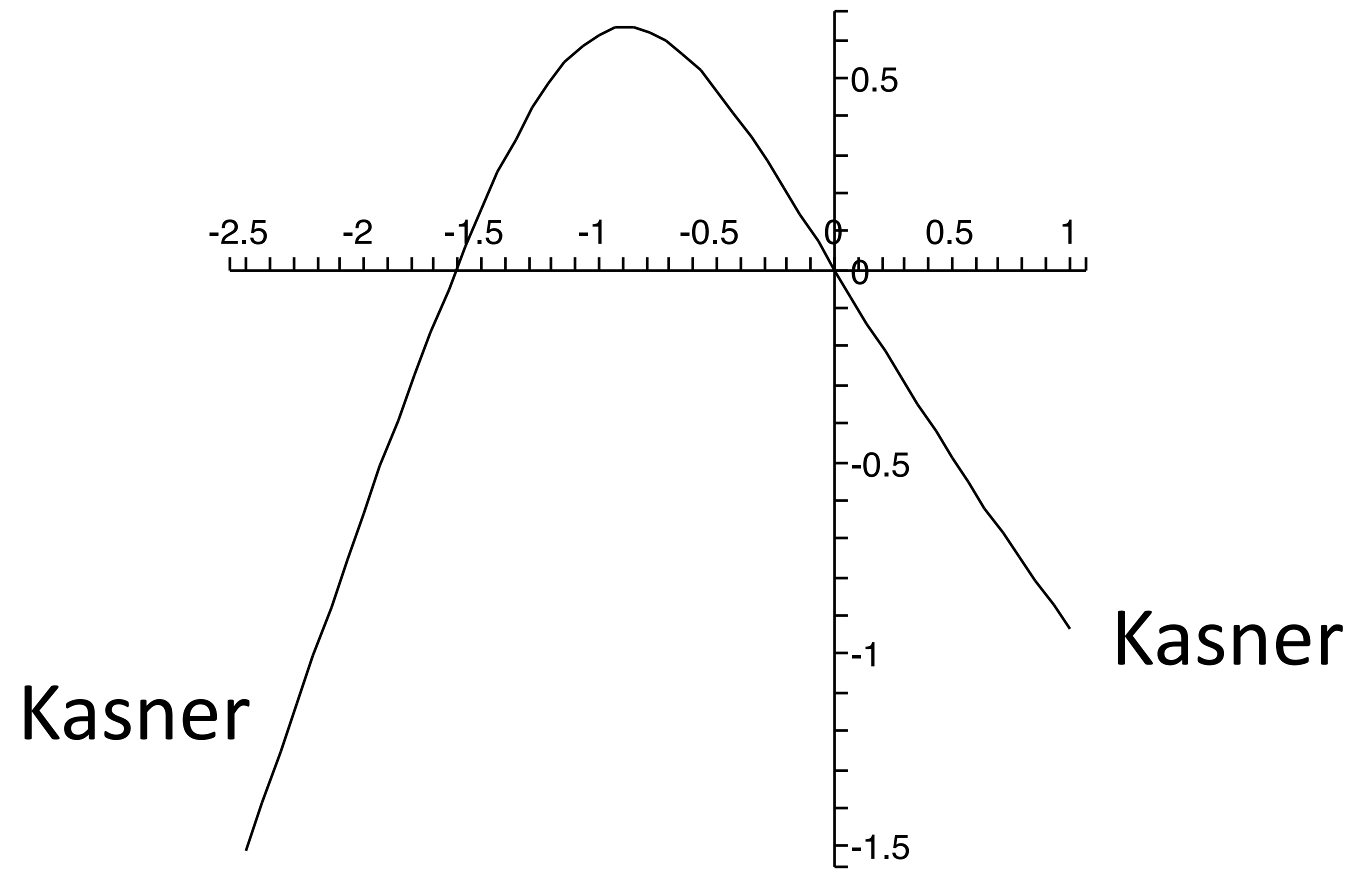
$$X'' + e^{2X} = 0$$
$$Y'' + e^{2Y} - \epsilon e^{2Y+Z} = 0$$
$$Z'' + \frac{\epsilon}{2} e^{2Y+Z} = 0\,,$$

$$e^{-X} = \lambda_1^{-1} \cosh[\lambda_1(\eta - \eta_1)].$$

X,Y,Z linear combinations of log W, log a, φ

$\epsilon = H^2$

Burgess et al 2005



Solutions stable under small perturbations!

6D (1,0) Supergravity From String Theory?

- M-theory/IIA on hyperbolic manifold $H^{(2,2)}$

$x_1^2+x_2^2-x_3^2-x_4^2=\rho^2$

Consistent truncations give Salam-Sezgin theory

Cvetic, Gibbons, Pope hep-th/0308026

- F-theory on elliptic Calabi-Yau

Grimm, Pugh 1302.3223

10D String on $H^{(2,2)}$ x S^1

(ρ, α, β) (z)

Any solution to the 6D equations from:

$$\mathcal{L}_6 = R * \mathbf{1} - *d\phi \wedge d\phi - \frac{1}{2} e^{-\varphi} * F_{(2)} \wedge F_{(2)} - \frac{1}{2} e^{-2\varphi} * H_{(3)} \wedge H_{(3)} - 8g^2 e^{\varphi} * \mathbf{1}$$

Can be uplifted to solutions of 10D (string) equations:

$$\mathcal{L}_{10} = \hat{R} \hat{*} \mathbf{1} - \frac{1}{2} \hat{*} d\hat{\phi} \wedge d\hat{\phi} - \frac{1}{2} e^{-\hat{\phi}} \hat{*} \hat{F}_{(3)} \wedge \hat{F}_{(3)}$$

From:

$$d\hat{s}_{10}^2 = (\cosh 2\rho)^{1/4} \left[e^{-\phi/4} ds_6^2 + e^{\phi/4} dz^2 + \frac{e^{\phi/4}}{2\bar{g}^2} \left(d\rho^2 + \frac{\cosh^2 \rho}{\cosh 2\rho} (D\alpha)^2 + \frac{\sinh^2 \rho}{\cosh 2\rho} (D\beta)^2 \right) \right]$$

$$\hat{F}_{(3)} = H_{(3)} + \frac{\sinh 2\rho}{2\bar{g}(\cosh 2\rho)^2} d\rho \wedge D\alpha \wedge D\beta + \frac{1}{2\bar{g} \cosh 2\rho} F_{(2)} \wedge \left(\cosh^2 \rho \, D\alpha - \sinh^2 \rho \, D\beta \right)$$

$$e^{\hat{\phi}} = (\cosh 2\rho)^{-1/2} \; e^{\varphi}$$

Cvetic et al 2003

Then the 6D de Sitter solutions can be uplifted to 10D !!!

Non-compactness?

6D Supergravity from F-theory

Grimm et al 2013

11D M-theory to 5D on elliptically fibred CY_3 and uplift to D=6

h_{12} +1 hypermultiplets, h_{11}-1 tensor multiplets

$$S^{(6)} = \int_{\mathcal{M}_6} \Big[\frac{1}{2}\hat{R}\hat{*}1 - \frac{1}{4}\hat{g}_{\alpha\beta}\hat{G}^\alpha \wedge \hat{*}\hat{G}^\beta - \frac{1}{2}\hat{g}_{\alpha\beta}d\hat{j}^\alpha \wedge \hat{*}d\hat{j}^\beta - \frac{1}{2}\hat{h}_{UV}\hat{D}\hat{q}^U \wedge \hat{*}\hat{D}\hat{q}^V$$
$$- 2\Omega_{\alpha\beta}\hat{j}^\alpha b^\beta C_{IJ}\hat{F}^I \wedge \hat{*}\hat{F}^J - \Omega_{\alpha\beta}b^\alpha C_{IJ}\hat{B}^\beta \wedge \hat{F}^I \wedge \hat{F}^J - \hat{V}^{(6)}\hat{*}\hat{1}\Big] ,$$

6D potential from D7 fluxes

$$\hat{V}^{(6)}_{\text{flux}} = \frac{1}{32\Omega_{\alpha\beta}\hat{j}^\alpha b^\beta \hat{\mathcal{V}}^2} C^{-1ij}\theta_i\theta_j \, .$$

From 6D to 4D

$$ds^2 = W(r)^2 q_{\mu\nu} dx^\mu dx^\nu + a(r)^2 d\theta^2 + dr^2 = e^{2\Gamma(r)} q_{\mu\nu} dx^\mu dx^\nu + e^{2\Omega(r)} d\theta^2 + dr^2$$

Field equations

$$\begin{aligned}
\ddot{\varphi} + \left(\dot{\Omega} + 4\dot{\Gamma}\right)\dot{\varphi} &= \tilde{V} e^{\varphi - 2\chi} - 2C\dot{\Delta}^2 e^{-\varphi + 2\Delta - 2\Omega} \\
\ddot{\chi} + \left(\dot{\Omega} + 4\dot{\Gamma}\right)\dot{\chi} &= -\frac{k^2}{4} e^{-2\chi + 2\Delta - 2\Omega} - 4\tilde{V} e^{\varphi - 2\chi} \\
\ddot{\Gamma} + \left(\dot{\Omega} + 4\dot{\Gamma}\right)\dot{\Gamma} &= 3H^2 e^{-2\Gamma} - \frac{1}{2}\left(\ddot{\varphi} + \left(\dot{\Omega} + 4\dot{\Gamma}\right)\dot{\varphi}\right) \\
\ddot{\Omega} + \left(\dot{\Omega} + 4\dot{\Gamma}\right)\dot{\Omega} &= -4C\dot{\Delta}^2 e^{-\varphi + 2\Delta - 2\Omega} - \frac{k^2}{8} e^{-2\chi + 2\Delta - 2\Omega} - \frac{1}{2}\left(\ddot{\varphi} + \left(\dot{\Omega} + 4\dot{\Gamma}\right)\dot{\varphi}\right) \\
\ddot{\Delta} + \left(\dot{\Omega} + 4\dot{\Gamma}\right)\dot{\Delta} &= \dot{\Delta}\dot{\varphi} + 2\dot{\Omega}\dot{\Delta} - \dot{\Delta}^2 + \frac{k^2}{32C} e^{\varphi - 2\chi}
\end{aligned}$$

Constraint

$$6H^2 e^{-2\Gamma} - 4\dot{\Omega}\dot{\Gamma} - 6\dot{\Gamma}^2 + \frac{1}{2}\dot{\varphi}^2 + \frac{1}{4}\dot{\chi}^2 + 2Ce^{-\varphi - 2\Omega + 2\Delta}\dot{\Delta}^2 - \tilde{V} e^{\varphi - 2\chi} - \frac{k^2}{16} e^{-2\chi - 2\Omega + 2\Delta} = 0$$

χ =log volume, Γ =log W, Ω = log a , Δ = log A

$H^2>0$ de Sitter

$$\left\{\varphi, a, W, H\right\} \to \left\{\varphi + \varphi_0, a\, e^{-\varphi_0/2}, W, H\, e^{\varphi_0/2}\right\}$$

Flat direction

Asymptotic solutions

Near brane solutions:

$$\begin{aligned}\varphi &= q\ln r + \ln u\\ \chi &= s\ln r + \ln v\\ \Gamma &= w\ln r + \ln x\\ \Omega &= \alpha\ln r + \ln y\\ \Delta &= \delta\ln r + \ln z\end{aligned}$$

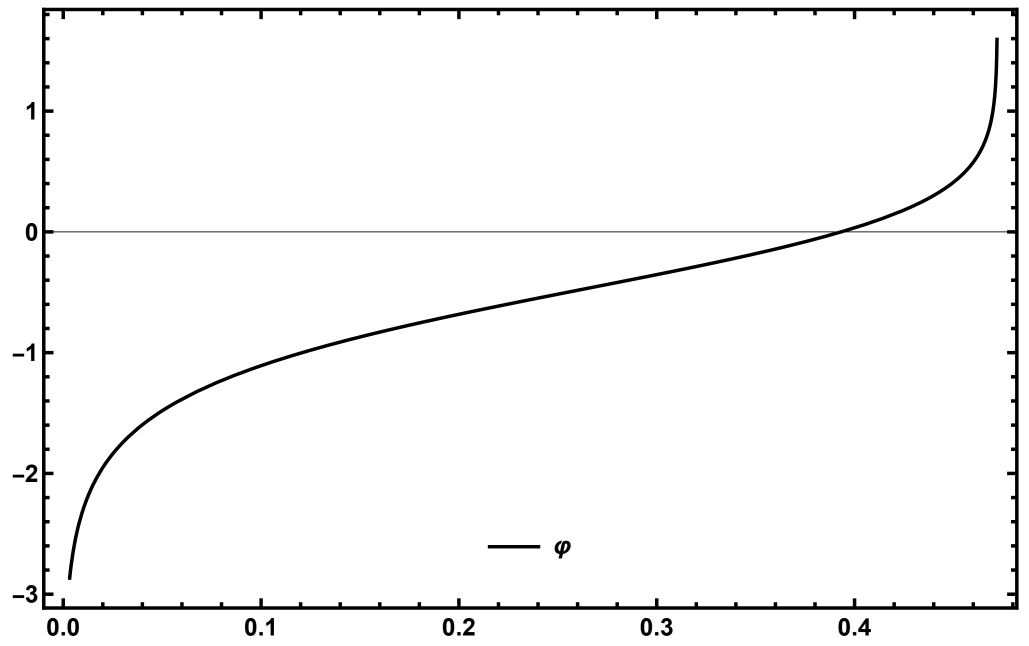
$$\begin{aligned}\frac{1}{2}q^2 + \frac{1}{4}s^2 - 6w^2 - 4\alpha w + \frac{6}{x^2}H^2 + \frac{2Cz^2}{uy^2}\delta^2 - \frac{u}{v^2}\tilde{V} &= 0\\ (\alpha + 4w - 1)q - \frac{u}{v^2}\tilde{V} + \frac{2Cz^2}{uy^2}\delta^2 &= 0\\ (\alpha + 4w - 1)s + \frac{4u}{v^2}\tilde{V} &= 0\\ (\alpha + 4w - 1)w - \frac{3}{x^2}H^2 + (\alpha + 4w - 1)\frac{q}{2} &= 0\\ (\alpha + 4w - 1)\delta - (q + 2\alpha - \delta)\delta &= 0\\ 2\delta - q - 2\alpha &= 0\\ q - 2s + 2 &= 0\\ w - 1 &= 0\end{aligned}$$

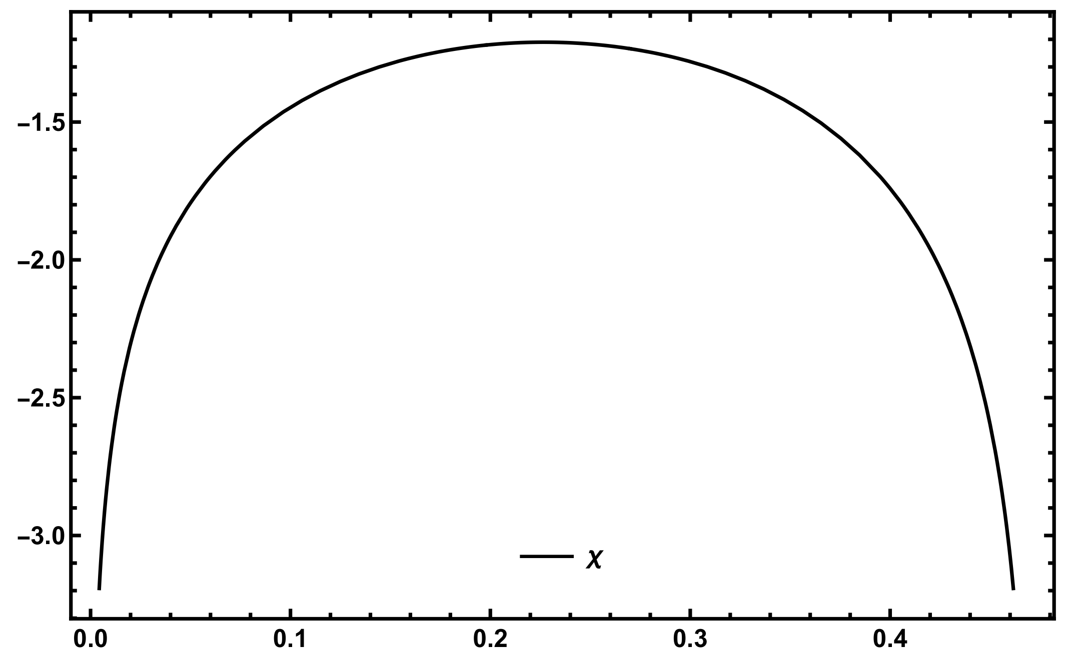
Kasner constraints
(BKL: Belinsky et al)

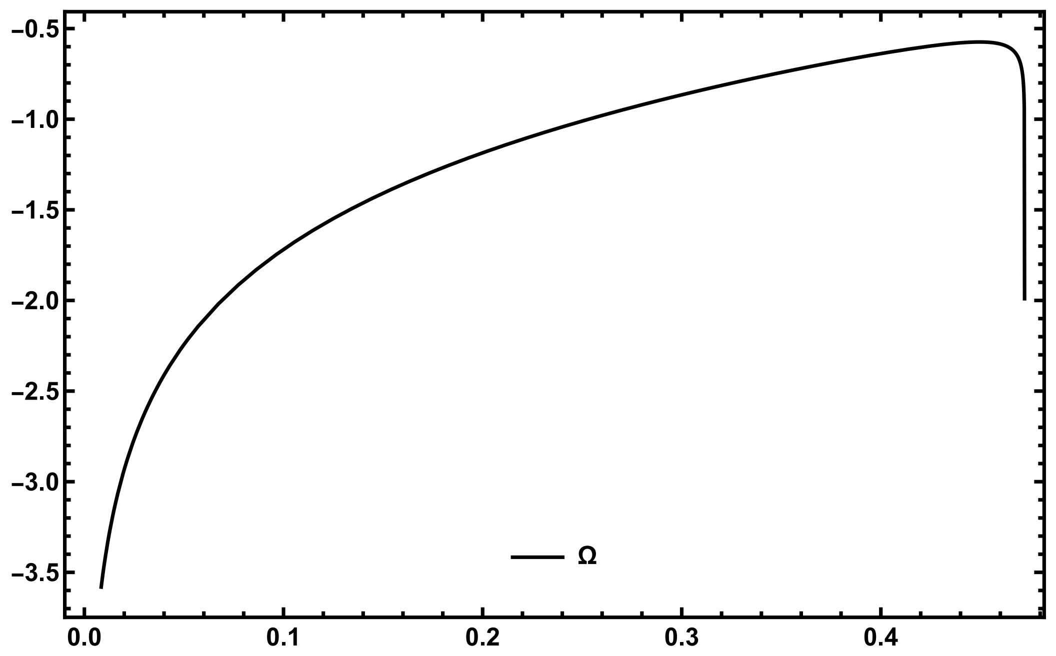
Solution

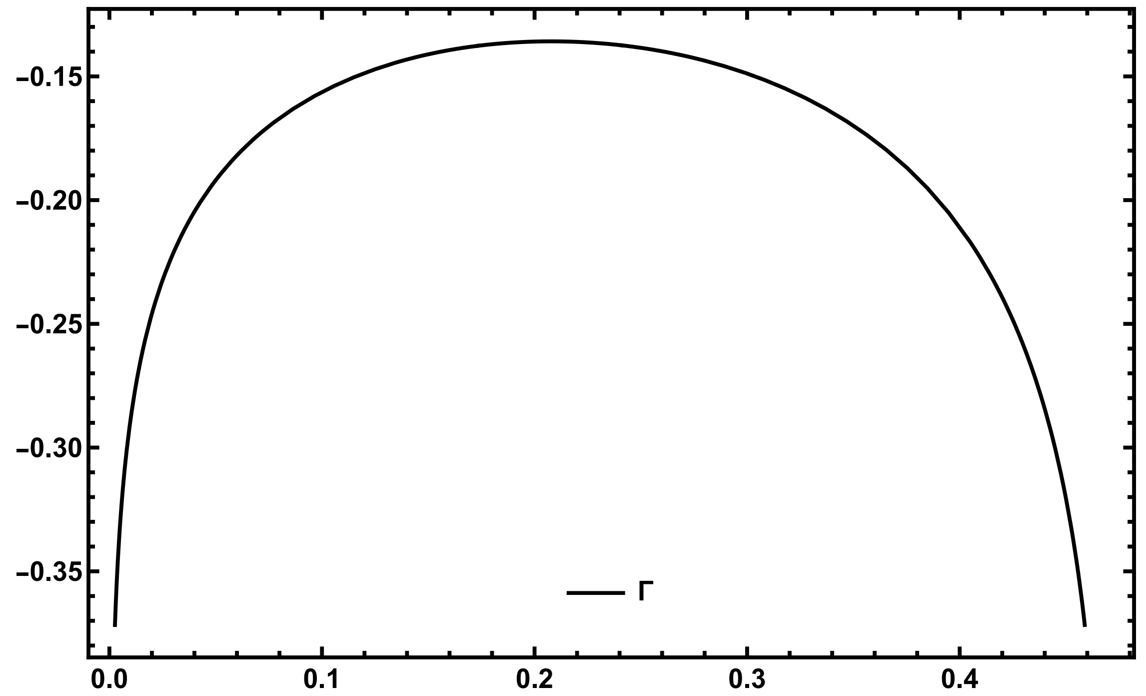
$$q = -\frac{2}{9}, \quad s = \frac{8}{9}, \quad \alpha = \frac{1}{9}, \quad w = \frac{1}{9}, \quad \delta = 0, \quad \frac{u}{v^2}\tilde{V} = \frac{8}{81}$$

Numerical Solutions $H^2 = 0$

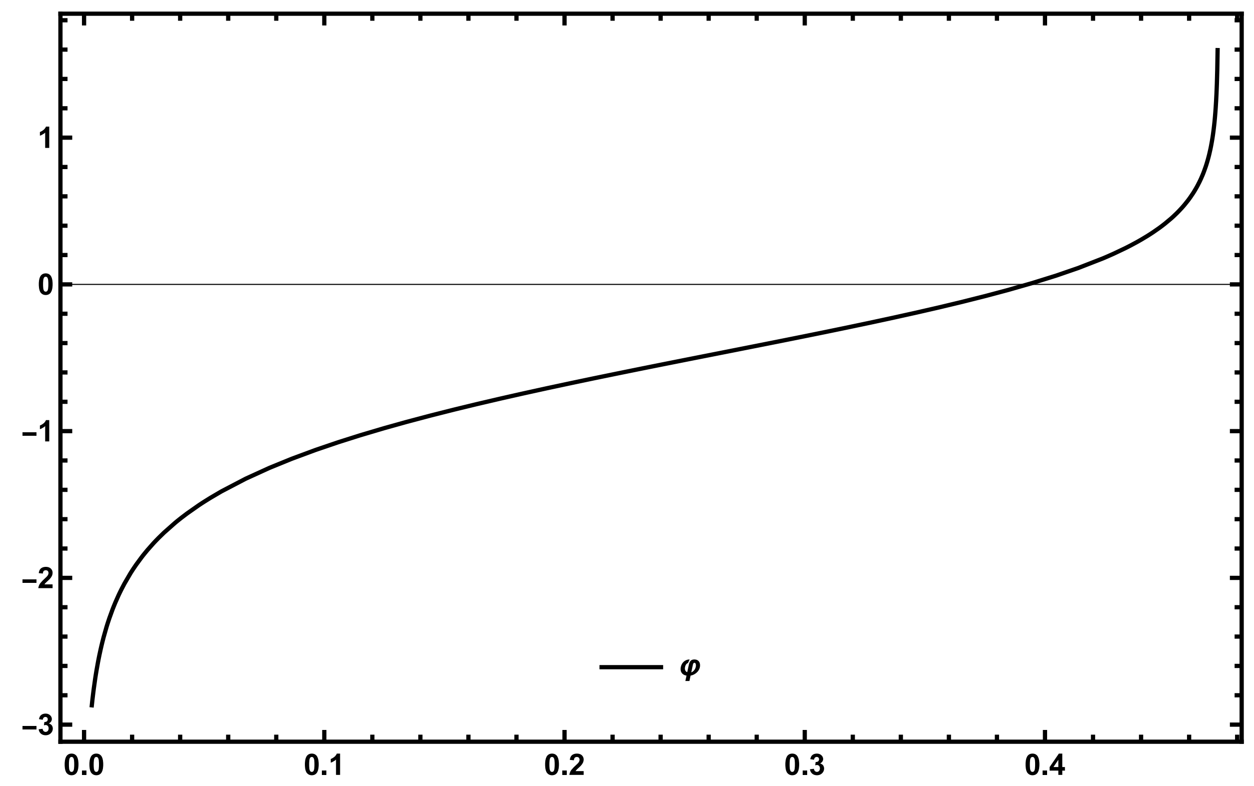


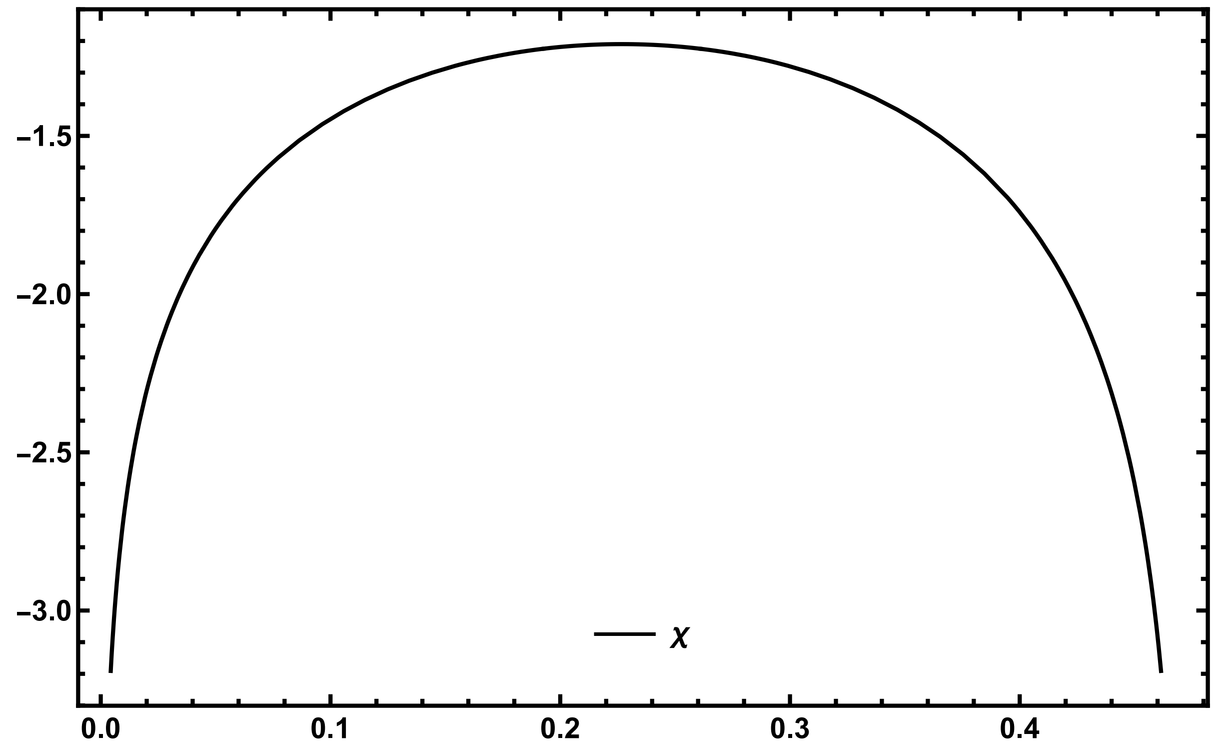


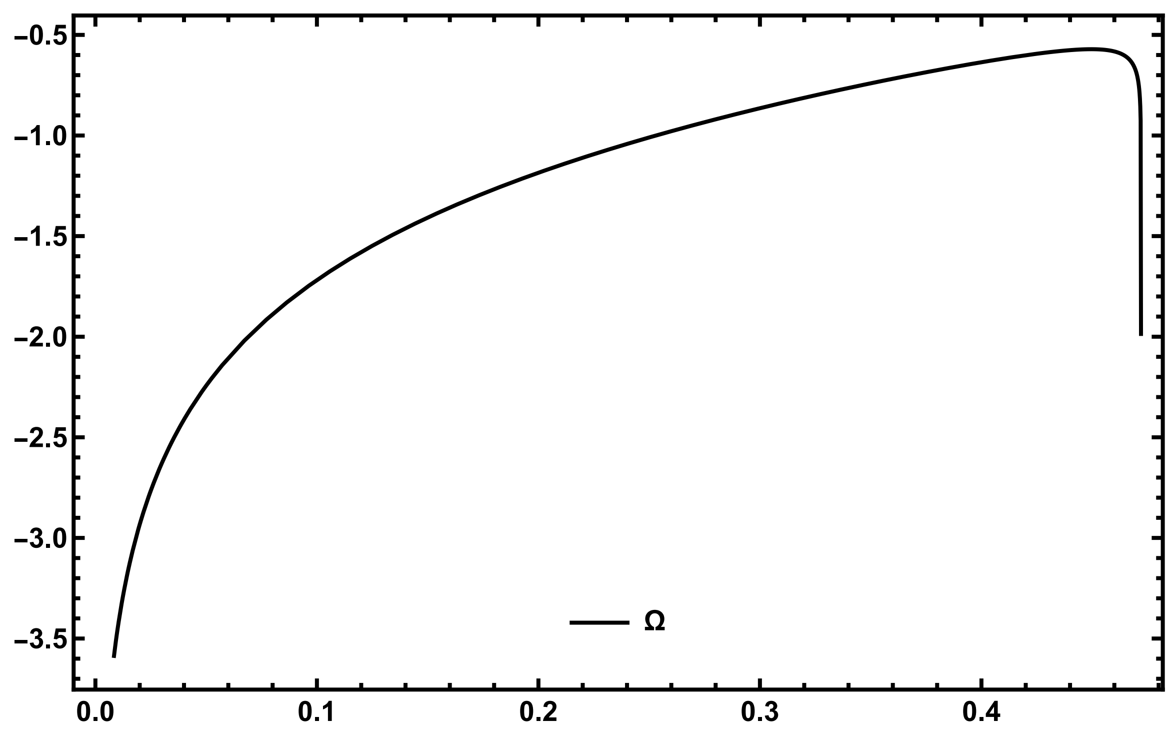


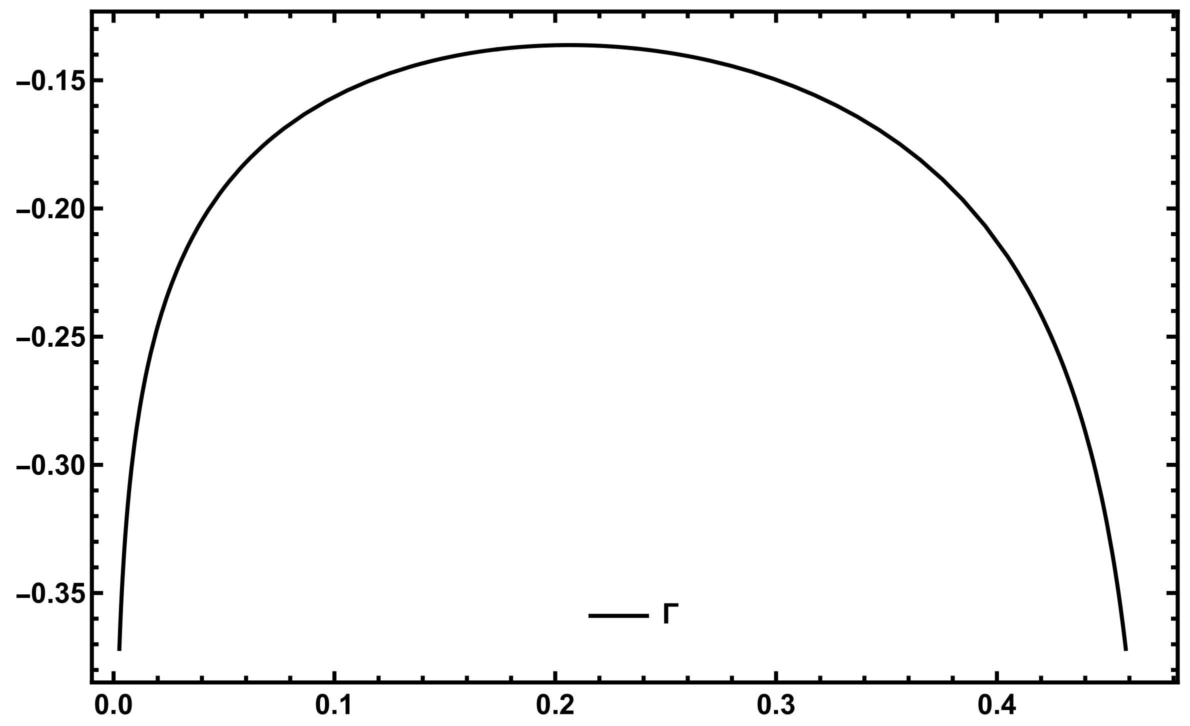


Numerical AdS Solutions $H^2 \leq 0$

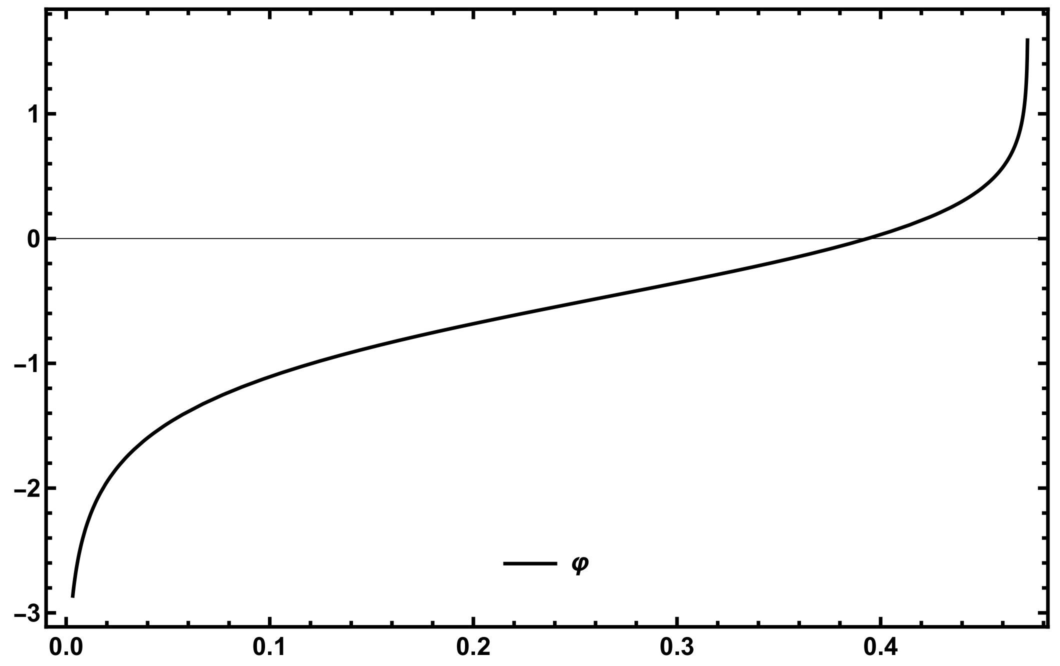


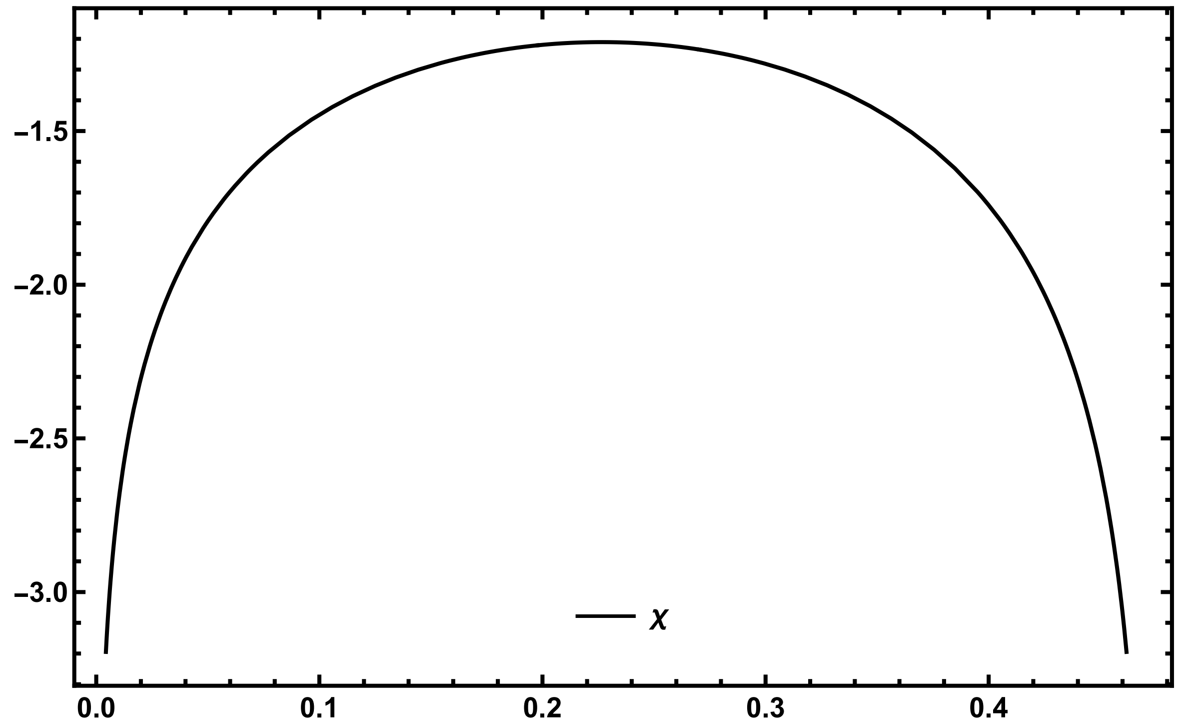


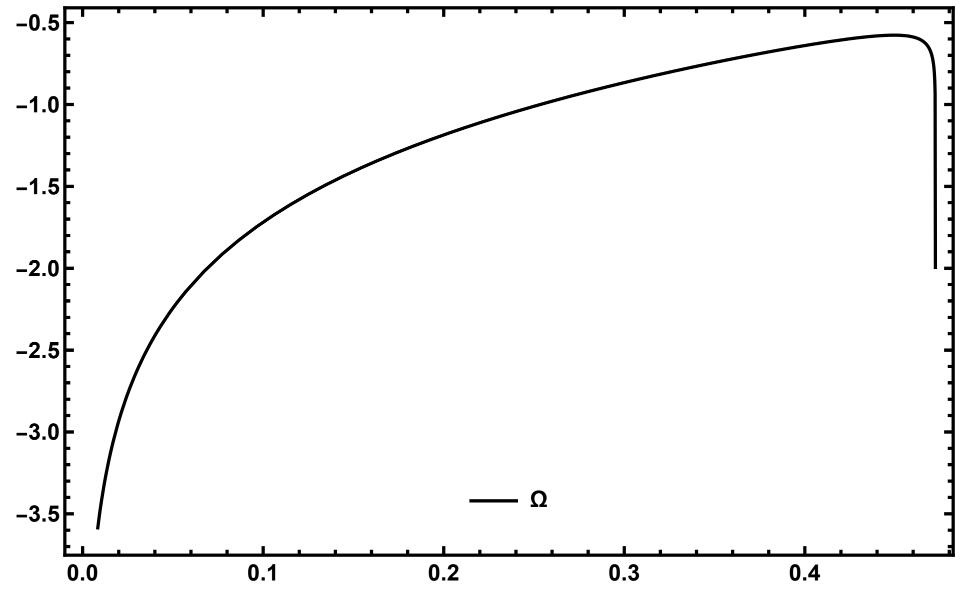


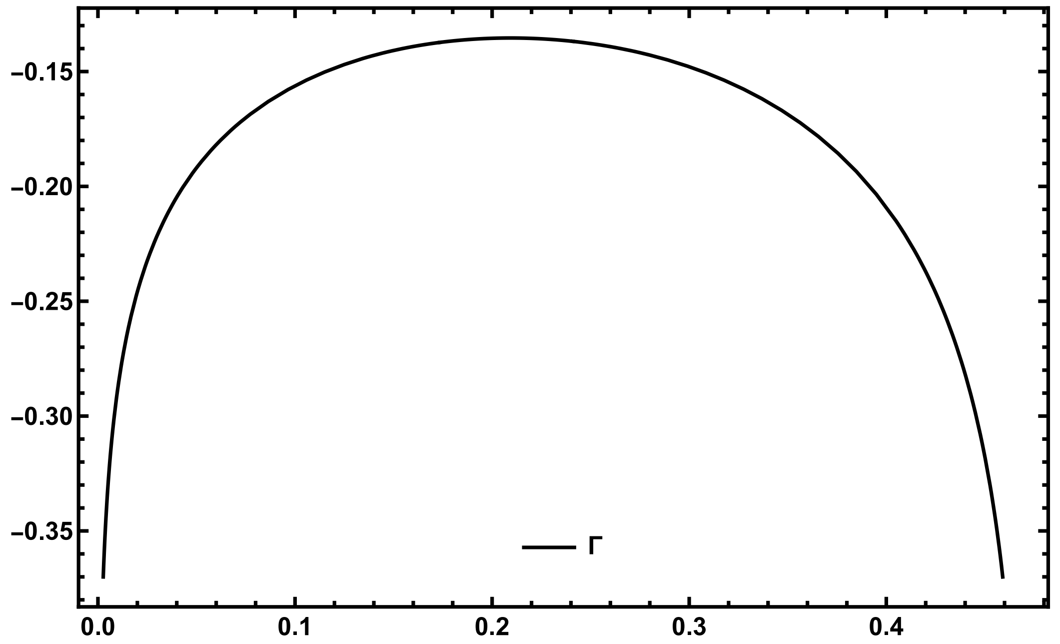


Numerical dS Solutions $H^2 \geq 0$









Singularities?

Effective field theory of localized objects

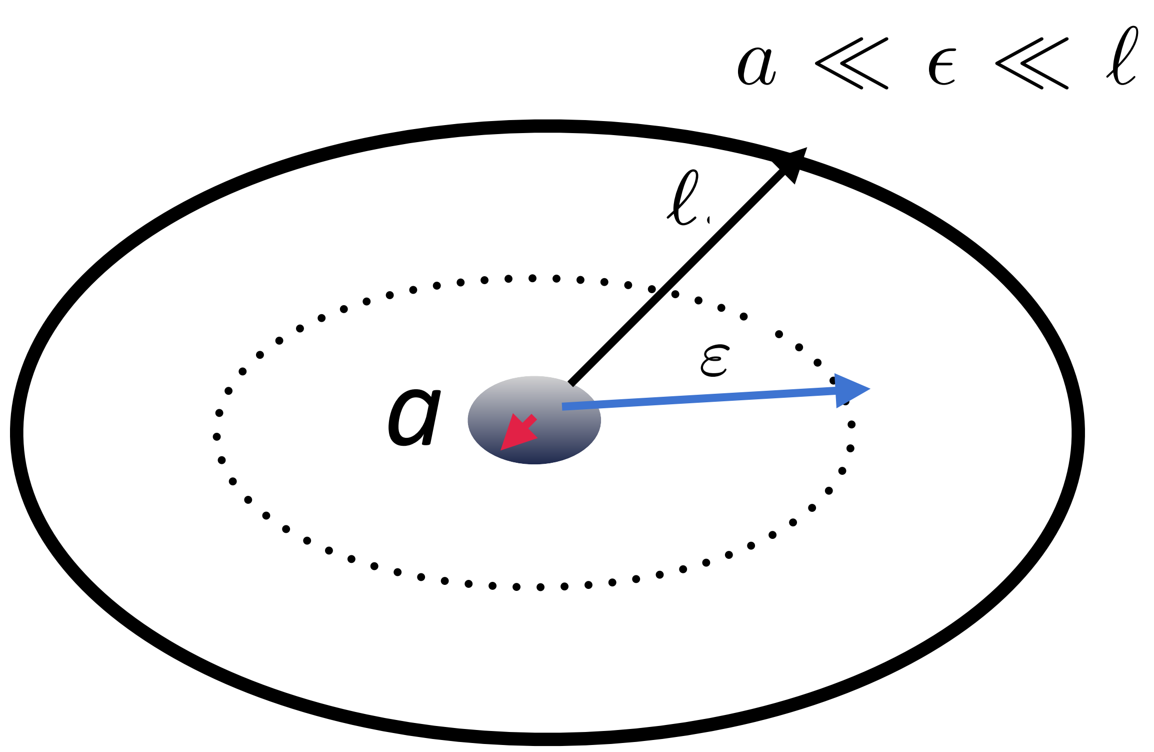
e.g. atomic nucleus!

*Field

$$S_{Ren} = S_{QED} + S_\Phi = -\int \mathrm{d}^4x \, \left\{ \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \overline{\Psi}\left[\not{D} + m\right]\Psi + \overline{\Phi}\left[\not{D} + M\right]\Phi \right\}.$$

$$S_{NRen} = -\int \mathrm{d}^4x \, \left\{ \frac{\tilde{c}_d}{2}\left(\overline{\Phi}\gamma^{\mu\nu}\Phi\right)F_{\mu\nu} + \tilde{c}_s\left(\overline{\Psi}\Psi\right)\ \left(\overline{\Phi}\Phi\right) + \tilde{c}_v\left(\overline{\Psi}\gamma^\mu\Psi\right)\left(\overline{\Phi}\gamma_\mu\Phi\right) + \cdots \right\}$$

$$S = S_{Ren} + S_{NRen}$$



*Localised object

PPEFT

Goldberger, Wise,

Burgess et al

$$S_{Nucl} = -\int_{\mathcal{P}} \mathrm{d}s \left\{ \sqrt{-\dot{y}^2}M - Ze\dot{y}^\mu A_\mu + c_s\sqrt{-\dot{y}^2}\left(\overline{\Psi}\Psi\right) + ic_v\dot{y}^\mu\left(\overline{\Psi}\gamma_\mu\Psi\right) + \cdots \right\}$$

$$S = S_{QED} + S_{Nucl}$$

Applications to precision atomic levels, Helium 4…

Gauge field

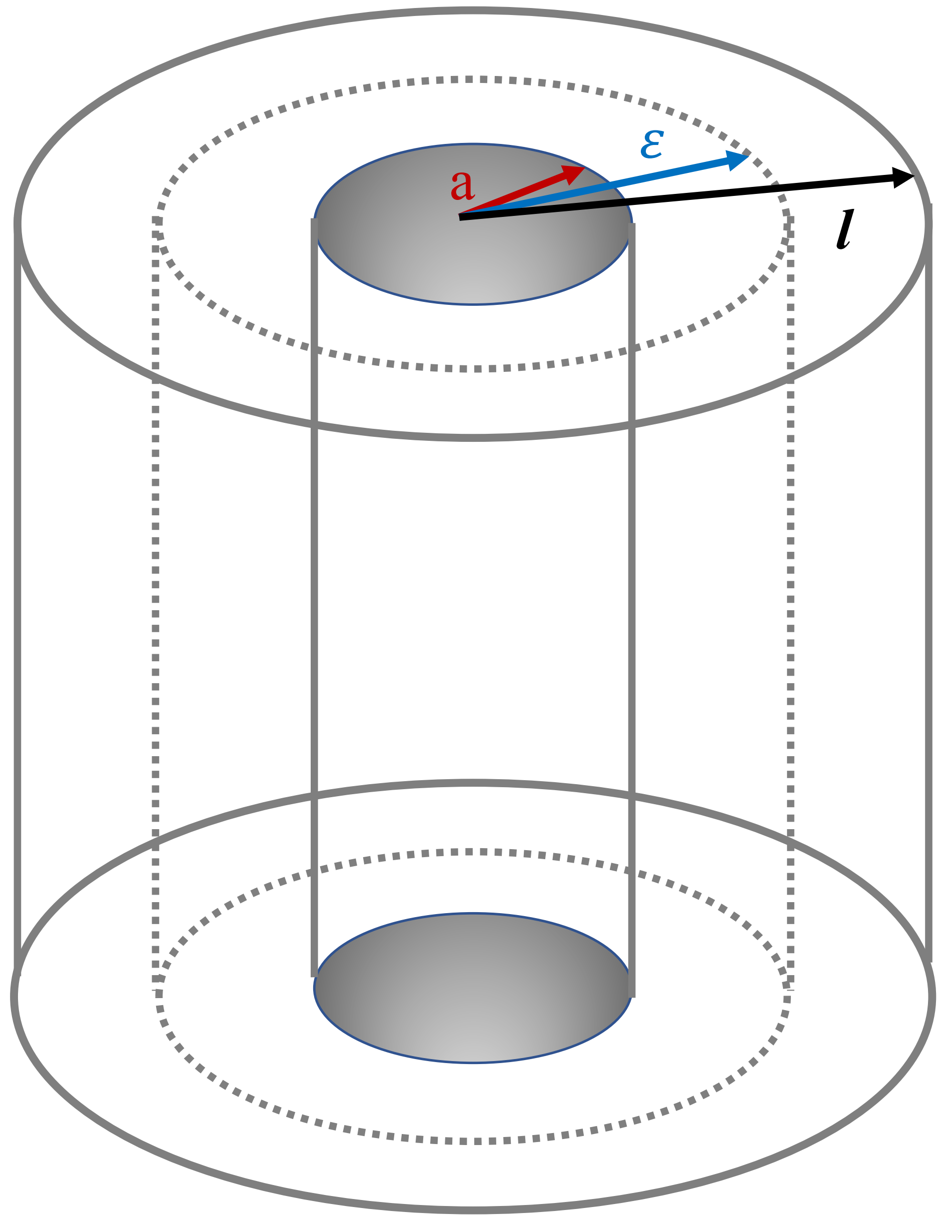
$$\oint_{C_\epsilon} \mathrm{d}\theta \left[n_N \sqrt{-g}\, f(\phi)\, F^{NM} \right] = -\left(\frac{\delta S_b}{\delta A_M} \right)_\epsilon ,$$

Scalar field

$$\oint_{C_\epsilon} \mathrm{d}\theta \left[\frac{1}{\kappa^2} \sqrt{-g}\, \mathcal{G}_{AB} n_M \partial^M \phi^B \right] = -\left(\frac{\delta S_b}{\delta \phi^A} \right)_\epsilon ,$$

Gravity

$$\lim_{r\to 0} \oint_{C_\epsilon} \mathrm{d}\theta \left[\frac{1}{2\kappa^2} \sqrt{-g}\, \left(K^{ij} - K g^{ij} \right) - (\text{flat}) \right] = -\left(\frac{\delta S_b}{\delta g_{ij}} \right)_\epsilon ,$$



$$a \ll \varepsilon \ll \ell .$$

Source back reaction:

Boundary conditions from pillbox

$$S_b = -\int \mathrm{d}^4x\sqrt{-g}\; W_b^4 L_b(\varphi,\chi) = -\int \mathrm{d}^4x\sqrt{-g}\; T_b(\varphi,\chi)\,.$$

$$b = \pm.$$

$$\frac{2\pi}{\kappa^2}\left[aW^4\,\partial_r\,\varphi\right]_{r=\epsilon} = \frac{2\pi}{\kappa^2}\left[a_b W_b^4 r^{\alpha_b+4w_b-1}\,q_b\right]_{r=\epsilon} = \frac{\partial}{\partial\varphi}\Big[\,W_b^4\,L_b\Big]\,,$$

$$\frac{2\pi}{\kappa^2}\left[aW^4\,\partial_r\,\chi\right]_{r=\epsilon} = \frac{2\pi}{\kappa^2}\left[a_b W_b^4 r^{\alpha_b+4w_b-1}\,s_b\right]_{r=\epsilon} = \frac{\partial}{\partial\chi}\Big[\,W_b^4\,L_b\Big]\,,$$

$$-\frac{2\pi}{\kappa^2}\left\{W^4\left[a\left(3\frac{\partial_r W}{W}+\frac{\partial_r a}{a}\right)-1\right]\right\}_{r=\epsilon} = W_b^4 L_b(\varphi)\,,$$

$$3\omega_\pm + \alpha_\pm - W_\pm^4 = -\frac{\kappa^2 T_\pm}{2\pi}$$

Positive tensions!

$$\frac{2\pi}{\kappa^2}\left[aW^4\,\partial_r\,W\right]_{r=\epsilon} = U_b(\phi)\,,$$

$$\mathcal{U}_b = \frac{1}{3}\left[(W_b^4-\mathcal{T}_b) - \sqrt{(W_b^4-\mathcal{T}_b)^2 - \frac{3}{4}\,(\mathcal{T}_{b\,,\varphi})^2 - \frac{3}{8}\,(\mathcal{T}_{b\,,\chi})^2}\,\right]\,.$$

$$V_{\rm eff}(\varphi_0) = -\sum_b\left(U_b + \frac{T_b'}{2}\right)$$

Conclusions

- Explicit (numeric) (A)dS solutions of 6D Supergravity
- Embedding of gauge 6D supergravity in String Theory
- Solutions uplifted to string solutions
- Singularities have brane-like properties (power of PPEFT)
- Open questions


STRING PHENOMENOLOGY 2024
Thank you !!!