On 4D dS Solutions of 6D Supergravity and String Theory

- **Fernando Quevedo**
- **University of Cambridge and CERN**
	- **String Phenomenology 2024**
		- **Padova June 2024**

C.P. Burgess, F. Muia and FQ 2407…

A Commercial A Commercial

Talks by

Anshuman Maharana

Gonzalo Villa

String Thermodynamics and Gravitational Waves

Mario Ramos-Hamud

Ahmed Kamal

Realistic Brane Inflation

Also

Obstacles for dS from UV Theory

• **Classical No-Go Theorems**

• **Dine Seiberg problem**

Different approaches

Review: L. McCallister and FQ, 2310.20559

(See also talks by Schachner and McAllister)

• **String flux compactification EFTs (e.g. KKLT, LVS)**

• **Classical solutions? (evading no-go theorems)**

e.g. Cordova, Tomasiello et al.

Classical de Sitter solutions

- The gravity action does not contain higher curvature corrections. • The gravity action does not contain higher curvature corrections. showing a negative international behavior constant or it constants. \bullet The gravity action does not contain higher curvature correct consider now that we have a considered given a contribution similar to a potential and \sim
- The potential is non-positive, $V \leq 0$. IIA supergravity which has a positive cosmological cosmological cosmological cosmological constant so we treat • The potential is non-positive, $V \leq 0$. \prod_{α} on the scalars but it can not be positive $\frac{1}{\alpha}$ at least in the range of values of $\frac{1}{\alpha}$ at $\frac{1}{\alpha}$ and $\frac{1}{\alpha}$ at $\frac{1}{\alpha}$ and $\frac{1}{\alpha}$ at $\frac{1}{\alpha}$ and $\frac{1}{\alpha}$ and $\frac{1}{\alpha}$ are $\frac{1}{\$ $s = 110$ potential is non-positive, $v = 0$. Γ les, n ϵ these is Γ as Γ and positivity is non-positive, $v \geq 0$. \mathbb{R}^n • The bolential is non-positive, $v \geq 0$.
- The theory contains massless fields with positive kinetic terms depend on the scalars but it cannot be positive (at least in the range of values of $\bullet\,$ The theory contains massless fields with positive kinetic terms. \mathbf{C} , if \mathbf{C} is not probably given a contribution similar to a point \mathbf{C} and \mathbf{C} LITO UICOLY CONUMING INCOUNCION INTER WITH PO \bullet The theory contains massless fields with positive kinetic terms.
- The d dimensional offective Newton's experient is finite The d dimensional effective Newton's constant is finite. $\bullet\,$ The d dimensional effective Newton's constant is finite. $\begin{array}{ccc} \cdot & & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot \\ \end{array}$ veN \bullet The dimensional effective Newton's constant is finite

Classical no-go theorem We consider a D dimensional gravity theory, with D > 2, compactified down to d dimensions. We dimensions. We denote by M, N, L, ... the D dimensional indices. We denote by M, N, M dimension ν, ν, ρ, ... the d dimensional indices and by m, n, l, the D − d dimensional indices. We We consider a D dimensional gravity theory, with D > 2, compactified down to d dimensions. We dimensions. We denote by M, and the D dimensional indices. We denote by α will assume that the D dimensional gravity theory satisfies the following conditions. The following conditions
The following conditions the following conditions the following conditions. The following conditions of the fo • The gravity action does not contain higher curvature corrections. \mathbf{S} depend on the scalars but it cannot be positive (at least in the range of values of values of values of values depend on the scalars but it cannot but it cannot be positive (at least in the range of values of values of va
The scalars but it cannot be positive (at least in the range of values of values of values of values of values scalar fields that is explored in the solution under consideration).

$$
ds_D^2 = \Omega^2(y) \left(dx_d^2 + \hat{g}_{mn} dy^n dy^m \right)
$$

$$
\frac{1}{(D-2)\Omega^{D-2}} \nabla^2 \Omega^{D-2} = R + \Omega^2 (-T^{\mu}_{\ \mu} + \frac{d}{D-2} T^L) \ge 0 \qquad \text{For de Sitter} \quad R \ge 0
$$

But integrating:
$$
\int d^{(D-d)} y \sqrt{\hat{g}} \left(\hat{\nabla} \Omega^{(D-2)} \right)^2 \le 0 \qquad \text{So no de Sitter}
$$

Gibbons, De Wit, Maldacena-Nunez... \sim . The dimensional indices and by m, n, l, the D $-$ dimensional indices. We have \sim dimensional indices. We have \sim ersume that the D dimensional Gibbons, De Wit, Maldacena-Nunez...
Exponential is non-positions of the potential is non-positional in the following conditions. scalar fields that is explored in the solution under consideration). Gibbons, De Wit, Maldacena-Nunez...

 $\prod_{k=1}^n$ superigravity which has a positive constant $\frac{1}{n}$ is $\frac{1}{n}$ case $\frac{1}{n}$. The gravity ection does not contain higher qurveture corrections. rie gravity action does not contain inglier curvature corrections.

 $\mathcal{L} \left(30\right)$

for d $\frac{1}{200D-2}\nabla^2\Omega^{\nu-2} = -R + \Omega^2(-T^\mu_{\;\;\mu} + \frac{1}{D-2}T^\mu_{\;\;L}) \geq 0$ For de Sitter $R \geq 0$ consider n<D, if n = D it would give a contribution similar to a potential and $\frac{1}{\sqrt{2}}\nabla^2\Omega^{D-2} = R + \Omega^2(-T^{\mu} + \frac{d}{dr}T^L) > 0$ μ $D-2$ μ $D-2$ μ \leq We have so that the kinetic terms are positively defined the kinetic vector μ \int_{a}^{b} (D, i) \int_{a}^{b} (D, i) $\frac{1}{2}$ $de^2 = \Omega^2(\omega) \left(dx^2 + \hat{a} \omega^2 du^m \right)$ $\left(-T^{\mu}_{\mu} + \frac{d}{D}T^{L}_{L}\right) > 0$ ≥ 0 Far do Sittor $R > 0$ \geq 0 Son \overline{d} $D-2$ $T^L_{L})$ $+\frac{d}{D-2}T^L$ ≥ 0 **For de Sitter** $R≥0$ \overline{d} $2^{-L/2}$ $+\frac{u}{D-2}T^L_L$ ≥ 0 where the hat denotes covariant derivatives and contraction of indices with respect to $ds_D^2 = \Omega^2(y) \left(dx_d^2 + \hat{g}_{mn} dy^n dy^m \right)$ 1
1
1 $(L) \geq 0$ For de Sitter $R \geq 0$ T^2 0 **For de Sitter** R **e Sitter** $R ≥ 0$ $(D-d)$ $\left(\hat{p} \left(\hat{\theta} \right) \cap (D-2) \right)^2$ so **Components of Fitting**

de Sitter

Ways out

• **Quantum effects,…**

• **Relax assumptions (e.g. V**≤ ")

De Sitter from 6D (1,0) Gauged Supergravity

Matter content We organise this article as follows. The contract of the state as follows.

- Ψ_M^{α} .
- *Tensor multiplet.* One anti self-dual antisymmetric tensor B_{MN}^- , one scalar ϕ , one right-handed fermion ψ (tensorino). $\text{fermion }\psi \text{ (tensorino)}.$
- *Vector multiplet.* One vector A_M and one fermion λ (gaugino). denoting the number of the number of α and α the number of α the number of α $\begin{pmatrix} 1 & \frac{1}{2} & \frac{$
	-

theory is connection in the matter of the matter may be appeared to the Standard Model, and the Standard Model, it is also subject to anomalies. Contrary to 4D in which anomaly cancelation conditions are very In general n_T tensor, n_V vector and n_H hyper multiplets

$$
n_{\scriptscriptstyle H}-n_{\scriptscriptstyle V}+ \nonumber \\
$$

• *Gravity multiplet*. Metric g_{MN} a self-dual antisymmetric tensor B^+_{MN} , one left-handed gravitino

• *Hypermultiplet:* Two complex scalars q^1, q^2 and one right-handed Weyl fermion ξ (hyperino). \bullet *Hypermultiplet:* Two complex scalars q^2 , q^2 and one right-handed weyl lermion ξ (hyperino).

 $n_H - n_V + 29n_T = 273$

Scalar fields Scalar fields januar 1994 where α = 1, ...n^T + 1.⁷ The scalars coming from the n^T tensor multiplets parameterize and the substraints in the subject to the substraints of the constraints of the constraints of the constraints we use and the scalar fields and the strength strength strength strength strength strength strength strength s

• From tensor multiplets n_T real scalars where a series a series a series and the scalars communications of the scalars communications of the new series parameterize parameterize parameterize parameterize parameterize parameterize parameterize parameterize parame $SO(1,n_T)/SO(n_T)$ • From tensor multiplets n_T real scalars nT + 1 scalar fields ja. The metric T scalar fields ja. The metric T is a mostly minus Lorentzian signature (1, nT), α $SO(1, n) / SO(n)$ \sim \sim $($ - $,$ \cdot \cdot I) $/$ \sim \sim $($ \cdot \cdot I $)$ $O(1,n_T)/SO(n_T)$ \mathbf{P} Let us recall the definition of the Chern-Simons three-form ˆ λ $\frac{1}{2}$

$$
j^{\alpha} \qquad \alpha = 1,...n_T + 1 \qquad \qquad \Omega_{\alpha\beta} j^{\alpha} j^{\beta} = 1 \qquad \qquad g_{\alpha\beta} = 2j_{\alpha}j_{\beta} - \Omega_{\alpha\beta}
$$

 $q^{U}\ (U=1,...,4n_{H})$ Quaternionic manifold Ωαβj^αj^β = 1 . (3.2) ^α in a SO(1, n^T) covariant way, as we will see in equation (3.21). two-form reads \mathcal{A} as vectors are concerned, in this section we consider a supergravity model we consider a supergravity model we consider a supergravity model with \mathcal{A} $\left(1, ..., 4n_H\right)$ Quaternionic manifold ˆ <mark>aterni</mark>g **ic** nic n ˆ nan
 if da
Da $q^U\ (U = 1, ..., 4n_H)$ Quaternionic i **Quaternionic manifold**

 \mathbf{S} It is coset scalar manifold by means of a vielbein formalism. The vielbein formalism \mathbf{r}

$$
\mathbf{n}_{7} = 1 \qquad \qquad j^{0} = \sinh \varphi \,, \qquad j^{1} = \cosh \varphi
$$

$$
j^{\alpha} \hspace{0.5cm} \alpha = 1,...n_T+1 \hspace{0.5cm} \Omega_{\alpha \beta} j^{\alpha} j^{\beta} = 1 \hspace{0.5cm} g_{\alpha \beta} =
$$

• **From hypermultiplets and the metric Capacity minus Lorentzian signals in the metric** \mathbf{r} **and** \mathbf{r} **is a signature (1, nT), and the metric** \mathbf{r} **is a signature (1, nT), and** \mathbf{r} **is a signature (1, nT),** α constant α constant α constant α constant α where the figure is negligible to write down the figure is negligible to write \mathbf{p}_i Moreover, the scalar manifold is endowed with another non-constant, positive definite g_{max} on f_{min} and g_{max} we use anti-Hermitian generators, and the expression for the non-Abelian field strength rmultiplets we get an overall minus sign on the right hand sign of Eq. (2.4) with respect to \mathbb{R}^2

$$
\bigodot \times R^{1,3}
$$

6D Supergravity (Salam-Sezgin) A. The Field Equations only a single nonzero background gauge field, chosen to be the gauge potential, *AM*, that gauges the specific *UR*(1) symmetry for which the gravitino field carries nonzero charge. The action for this gauge

$$
\mathcal{L}_6 = -\sqrt{-g} \left[\frac{1}{2\kappa^2} g^{MN} \Big(R_{MN} + \partial_M \varphi \, \partial_N \varphi \Big) + \frac{1}{4} e^{-\varphi} F_{MN} F^{MN} + \frac{1}{12} e^{-2\varphi} H_{MNP} H^{MNP} + \frac{2g^2}{\kappa^4} e^{\varphi} \right]
$$

-
- Δ \mathbf{h} \mathbf{h} and \mathbf{h} The field equations obtained from this action are $\mathcal{L}^{\mathcal{L}}$ and $\mathcal{L}^{\mathcal{L}}$ action are: • **Chiral** ²The function *U*(*q*) depends on which groups are gauged but it takes generically the form *U*(*q*) = *A* + P
- **<u>dximally</u>** symme iC ur solution in op (dine-3;
 • No maximally symmetric solution in 6D (Dine-Seiberg problem in 6D?) from the maximum of populations both the conventions in the Ventius
	- Maximally symmetric in 4D \sim (c) and \sim 0 (proform) \sim 0 (proform) (2) (p
	- mally symmetric sr 1 both solution 2 1: S² x Minkowski, N=1 SU
△ • Maximally symmetric smooth solution: S^2 x Minkowski, N=1 SUSY.

• Positive potential (evades Maldacena-Nunez theorem) ¹In general for *ⁿ^T >* 1 the coset *SO*(1*, n^T*)*/SO*(*n^T*) is parametrized by fields *^j*↵, with ↵ = 1*, ··· , n^T* + 1, constrained \bullet **PUSILIVE PULENTIAI (EVAUES IVIAIUALENA-IVUNEZ LIIEUIENI)**

in which (2.4) has been used to rewrite (2.6) so that the terms proportional to *gMN* involve only the 6D scaling symmetry under which the replacements of the replacement o **Scaling symmetry** 2.2.1 Salam-Sezgin solution S sumples spacetime spacetime s

The bosonic equations of motion are equations of motion are equations of motion are equations of motion are equ
The bosonic equations of motion are equations of motion are equations of motion are equations of motion are eq **Field Equations:** cannot vanish. The same is not true for solutions that are maximally symmetric only in 4D, however, since although these still require *HMNP* = 0 the gauge field can be nonzero if restricted to the two extra dimensions: *Fmn* with *m, n* = 4*,* 5. *^gMN* ! *c gMN* and *^e*' ! *c e*' imply *^L*⁶ ! *^c*²*L*⁶ *,* (2.7) for constant *c*. Although not a symmetry of the action this transformation does leave the equations cannot vanish. The same is not true for solutions that are maximally symmetric only in 4D, however, since although these still require *HMNP* = 0 the gauge field can be nonzero if restricted to the two

$$
\Box_6 \varphi + \frac{\kappa^2}{4} e^{-\varphi} F_{MN} F^{MN} + \frac{\kappa^2}{6} e^{-2\varphi} H_{MNP} H^{MNP} - \frac{2g^2}{\kappa^2} e^{\varphi} = 0
$$

$$
\nabla_M \left(e^{-\varphi} F^{MN} \right) + \kappa e^{-2\varphi} H^{PNQ} F_{PQ} = 0, \qquad \nabla_M \left(e^{-2\varphi} H^{MNP} \right) = 0
$$

$$
R_{MN} + \partial_M \varphi \, \partial_N \varphi + \kappa^2 e^{-\varphi} F_{MP} F^P_N + \frac{1}{2} (\Box_6 \varphi) \, g_{MN} = 0,
$$

$$
g_{MN} \to c \, g_{MN} \quad \text{and} \quad e^{-\varphi} \to c \, e^{-\varphi}
$$

Salam-Sezgin solution $ds^2 = \eta_{\mu\nu}(x) dx^{\mu} dx^{\nu} + \rho^2 (d\phi^2 + \sin^2 \theta)$ equation (2.5) then implies $\mathcal{E}^{\mathcal{E}}$ is a constant. For such a geometry $\mathcal{E}^{\mathcal{E}}$ $\overline{\nu}$ ¹

$$
ds^{2} = \eta_{\mu\nu}(x) dx^{\mu} dx^{\nu} + \rho^{2} (d\phi^{2} + \sin^{2} \phi d\theta^{2})
$$

$$
F_{mn} = f \epsilon_{mn}
$$

$$
e^{-\varphi}f = 2g/\kappa^2 \qquad \qquad R_{mn} = -e^{-\varphi}F_{mp}F_n^p = -f^2e^{-\varphi}g_{mn} = -\frac{g_{mn}}{\rho^2} \implies \rho^2e^{\varphi} = \left(\frac{\kappa^2}{2g}\right)^2
$$

ding Symmetry
Papa *Papa range range range to be possible because the system of the system* $\frac{2}{3}$ $g_{MN} \rightarrow c \, g_{MN} \quad \text{and} \quad e^{-\varphi} \rightarrow c \, e^{-\varphi} \qquad \text{imply} \qquad \mathcal{L}_6 \rightarrow c^2 \mathcal{L}_6 \, ,$ $g_{MN} \to c \, g_{MN} \quad \text{and} \quad e^{-\varphi} \to c \, e^{-\varphi} \qquad \text{imply} \qquad \mathcal{L}_6 \to c^2 \mathcal{L}_6 \, ,$ $F^{-\varphi}$ the simplest solutions spacetime has a product metric, ρ and ρ χ_{Gauss} and $e^{-\varphi} \to e e^{-\varphi}$ imply $\mathcal{L}_{\text{G}} \to e^2 \mathcal{L}_{\text{G}}$ resulting nonsingular geometry is ^R¹*,*³ ⇥ *^S*² with ^R¹*,*³ denoting 4D Minkowski spacetime and *^S*² the $p^{-\varphi}$ $q^{-\varphi}$ imply $q^{-1/2}$

4D EFT: (Aghababie et al 2003)
$$
V = \frac{2g^2e^{\varphi}}{\rho^2}\left(1-\frac{\kappa^4}{4g^2e^{\varphi}\rho^2}\right)^2
$$

General 4D Solutions r and Solutio will ignore all of the fermion degrees of freedom and restrict to the simplest case *n^T* = 1 and set the scalars of the hypermultiplets to be constant which in practice means *n^H* = 0¹. We will relax this

, and the $\frac{1}{2}$ symmetric coordinates, $\frac{1}{2}$ and $\frac{1}{2}$ **Gibbons et al 2004 Burgess et al 2005** condition later on when we discuss F-theory and one hypermultiplet cannot be set to constant. This is the action that was considered in the original paper of Salam and Sezgin. The Lagrangian

The significance of equation seed it is most easily seen on the compact distributions of the compact distr (uniqueness of Salam-Sezgin solution)

$$
\mathcal{L}_6=R\ast \mathbf{1}-\ast d\phi\wedge d\phi-\frac{1}{2}e^{-\varphi}\ast F_{(2)}\wedge F_{(2)}-\frac{1}{2}e^{-2\varphi}\ast H_{(3)}\wedge H_{(3)}-8g^2e^{\varphi}\ast \mathbf{1}
$$

$$
ds^{2} = \hat{g}_{MN} dx^{M} dx^{N} = W^{2}(y) g_{\mu\nu}(x) dx^{\mu} dx^{\nu} + \tilde{g}_{ij}(y) dy^{i} dy^{j}
$$

$$
\hat{g}_{\mu\nu} = W^2 g_{\mu\nu} \,, \qquad \hat{R}_{\mu\nu} = R_{\mu\nu} + \frac{1}{n} (W^{2-n} \tilde{\nabla}^2 W^n) g_{\mu\nu} \quad \text{and} \quad \hat{\Box} \varphi = W^{-n} \tilde{\nabla}_i (W^n \tilde{g}^{ij} \partial_j \varphi) \,,
$$

$$
\frac{1}{n} \int_M \mathrm{d}^d y \sqrt{\tilde{g}} W^{n-2} R = - \sum_{\alpha} \int_{\Sigma_{\alpha}} \mathrm{d}^{d-1} y \sqrt{\tilde{g}} N_i \left[W^n \tilde{g}^{ij} \partial_j \left(\ln W + \frac{2 \varphi}{D-2} \right) \right]
$$

$\sigma_{\rm N}$ No singularities/boundaries imply R=H²=0 e.g. S^2 X R^{1,3} $\sum_{n=1}^{\infty}$ $\sum_{n=1}^{\infty}$ represents the main result of $\sum_{n=1}^{\infty}$ and $\sum_{n=1}^{\infty}$ $\sum_{n=1}^{\infty}$ $\sum_{n=1}^{\infty}$ dimensions of a similar result in 6 dimensions of a similar result in 6 dimensions derived in ref. [12].
The first in ref. [12]. The first in r then the surface terms must include spacelike surfaces in the remote future and past, for which \mathbf{N} us singularities/ Doundaries iniply \mathbf{N} –n $\mathbf{-U}$ e.g. s **No singularities/boundaries imply R=H2=0 e.g. S2 X R1,3** with *^FMN* defined as *^F*(2) ⁼ *^FMN dx^M* ^ *dx^N* , etc. No singularities/boundaries imply R=H²=0 e.g. S² X l

Runaway potential! 6D Dine- \mathbb{R} and \mathbb{R} in the maximal symmetry in planet Ry , these equations allows allows allows allows allows allows allows allows allows a symmetry in planet Ry , the symmetry in planet Ry , the symmetry in \mathbb{R} eq. (6) to be simplified to be simp **Runaway potential! 6D Dine-Seiberg p** The bosonic equations of motion are $\frac{1}{2}$ **Runaway potential! 6D Dine-Seiberg problem?**

General Solutions $General$ Solutions

Asymptotic near brane solutions (n=4, d=2):

$$
\varphi \approx q \ln r
$$
 and $ds^2 \approx r^{2w} g_{\mu\nu}(x) dx^{\mu} dx^{\nu} + dr^2 + r^{2\alpha} f_{ab}(z) dz^a dz^b$,

$$
nw + \alpha(d-1) = 1.
$$
 $nw^2 + \alpha^2(d-1) + q^2 = 1.$ **Kasner constraints**
(**PVI** • **Pali**els: **et** = 0

$$
-\frac{1}{\sqrt{n}}\leq w\leq \frac{1}{\sqrt{n}}\,,\quad -\frac{1}{\sqrt{d-1}}\leq \alpha\leq \frac{1}{\sqrt{d-1}}\quad \text{and}\quad -1\leq q\leq 1\,.
$$

λ and λ λ (A) λ ₄ and λ 3-Branes **x (A)dS4 3-Branes**

(BKL: Belinsky et al)

Burgess et al 2005

$$
ds^2 = \hat{g}_{MN} dx^M dx^N = W^2 q_{\mu\nu} da
$$

Gibbons et al.

 $\alpha x + a \alpha b + a \quad W \alpha \eta$, $= W^2 q_{\mu\nu} dx^{\mu} dx^{\nu} + a^2 d\theta^2 + a^2 W^8 d\eta^2$ $,$

 $e^{-2\lambda_3\eta}\cosh^3[\lambda_1(\eta-\eta_1)]\cosh[\lambda_2(\eta-\eta_2)]$

 \wedge dθ.

$$
e^{\varphi} = W^{-2} e^{-\lambda_3 \eta}
$$

\n
$$
W^4 = \left(\frac{Q\lambda_2}{4g\lambda_1}\right) \frac{\cosh[\lambda_1(\eta - \eta_1)]}{\cosh[\lambda_2(\eta - \eta_2)]}
$$

\n
$$
a^{-4} = \left(\frac{gQ^3}{\lambda_1^3\lambda_2}\right) e^{-2\lambda_3 \eta} \cosh^3[\lambda_1(\eta - \eta_1)] \cosh[\lambda_2(\eta - \eta_2)]
$$

\n
$$
F = \left(\frac{Qa^2}{W^2}\right) e^{-\lambda_3 \eta} d\eta \wedge d\theta.
$$

Flat Solutions 2 \blacksquare

Numerical de Sitter solution where $\mathcal{L} = \mathcal{L} \left(\mathcal{L} \right)$ for definitions. We now integrate the equations. We now integrate the equations. We now integrate the equations. We now integrate the equations in the equations. We now integrate the equa it is a second to obtain the extra-dimension to obtain extra-dimensional can be discussed to give α

along with the constraint and the constraint $\frac{1}{2005}$ **Burgess et al 2005**

$$
X'' + e^{2X} = 0
$$

$$
e^{-X} = \lambda_1^{-1} \cosh[\lambda_1(\eta - \eta_1)].
$$

$$
V'' + e^{2Y} = \epsilon e^{2Y + Z} - 0
$$

 ϵ is decoupled to ϵ integrated the other variables. It is equation can be directly integrated to give $e^{2Y+Z} = 0$, of log W, log a, φ **X,Y,Z linear combinations** $\epsilon = H^2$

$\mathsf S$ Solutions stable under small perturbations! **Solutions stable under small perturbations!**

6D (1,0) Supergravity From String Theory?

• **M-theory/IIA on hyperbolic manifold H(2,2)**

 $x_1^2 + x_2^2 - x_3^2 - x_4^2 = \rho^2$

 Consistent truncations give Salam-Sezgin theory

Cvetic, Gibbons, Pope hep-th/0308026

• **F-theory on elliptic Calabi-Yau**

Grimm, Pugh 1302.3223

$10D$ String on $H^{(2,2)} \times S^1$ and uplift to 6D \pm (ρ, α, β) (z) 4.1 String theory on Hyperbolic space *^H*²*,*² ⇥ *^S*¹ The 6D theory is obtained by a series of dimensional reductions and consistent truncations. At the end \blacksquare 4.1 String theory on Hyperbolic space *^H*²*,*² ⇥ *^S*¹ ^ˆ⇤*d*^ˆ ^ *^d*^ˆ ¹ $(a \alpha R)$ parametrise and (2.2) we change $(1 - 1)$ will ignore all of the fermion degrees of freedom and restrict to the simplest case *n^T* = 1 and set the (ρ, α, β) (z)

where $\frac{1}{2}$ is the 3D hyperbolic space determined by the 4D hyperbolic space determined by the 4D hyperb

Cvetic et al 2003 $\overline{1}$

Any solution to the 6D equations from: \blacksquare **it and solution to the 6D equations from:** \blacksquare We can start directly from the relevant heterotic part of the relevant heterotic/type I bosonic part of the La
The Lagrangian (with the L Δ ny solution to the 6D equations from the Δ hatted quantities being 10D): Any solution to the 6D equations from:

$$
\mathcal{L}_6 = R* \mathbf{1} - *d\phi \wedge d\phi - \frac{1}{2} e^{-\varphi}*F_{(2)} \wedge F_{(2)} - \frac{1}{2} e^{-2\varphi}*H_{(3)} \wedge H_{(3)} - 8 g^2 e^{\varphi}* \mathbf{1}
$$

it amounts to a dimensional reduction of the e↵ective action of type I/heterotic strings on *^H*(2*,*2) ⇥ *^S*¹

Can be uplifted to solutions of 10D (string) equations: $\mathcal{L}_{10} = \hat{R}$ * $\mathbf{1} - \frac{1}{2}$ 2 $\hat{R} = \hat{R} * \mathbf{1} - \frac{1}{2} * d\hat{\phi} \wedge d\hat{\phi} - \frac{1}{2} e^{-\hat{\phi}} * \hat{F}_{(3)} \wedge \hat{F}_{(3)}$ $e^{-\hat{\phi}}$ $\hat{*} \hat{F}$ $\hat{f}_{(3)} \wedge \hat{F}$ with 0 ⇢ 1, 0 ↵*, <* ²⇡ and use *^z* as the *^S*¹ coordinate. The solutions found in [2] correspond *y*¹ + *iy*² = cosh ⇢*eⁱ*↵; *y*³ + *iy*⁴ = sinh ⇢*eⁱ* (4.2) ${\cal L}_{10}=R\ddot{*}\mathbf{1}-\frac{1}{2}\ddot{*}d\phi\wedge d\phi-\frac{1}{2}e^{-\phi}\ddot{*}F_{(3)}\wedge F_{(3)}$ *<u>can be</u> e/*⁴ 2¯*g*² $\ddot{\mathbf{c}}$ \mathbf{r} olutions of 10^I \mathbf{z} (string) equations: with *F*(2) = *dA*(1), *H*(3) = *dB*(2) + ¹ ²*F*(2) ^ *A*(1). **Can be aphrece to solutions of** $-\frac{1}{2}$ * $a\varphi$ /* $d\phi - \frac{1}{2}e^{-\phi}$

From: $From:$

$$
d\hat{\phi}-\frac{1}{2}e^{-\hat{\phi}}\hat{*}\hat{F}_{(3)}\wedge\hat{F}_{(3)}
$$

$$
d\hat{s}_{10}^{2} = (\cosh 2\rho)^{1/4} \left[e^{-\phi/4} ds_{6}^{2} + e^{\phi/4} dz^{2} + \frac{e^{\phi/4}}{2\bar{g}^{2}} \left(d\rho^{2} + \frac{\cosh^{2}\rho}{\cosh 2\rho} (D\alpha)^{2} + \frac{\sinh^{2}\rho}{\cosh 2\rho} (D\beta)^{2} \right) \right]
$$

$$
\hat{F}_{(3)} = H_{(3)} + \frac{\sinh 2\rho}{2\bar{g}(\cosh 2\rho)^{2}} d\rho \wedge D\alpha \wedge D\beta + \frac{1}{2\bar{g}\cosh 2\rho} F_{(2)} \wedge (\cosh^{2}\rho D\alpha - \sinh^{2}\rho D\beta)
$$

$$
e^{\hat{\phi}} = (\cosh 2\rho)^{-1/2} e^{\varphi}
$$

Cvetic et al 2003

$$ \overline{z} **he 6D de Sitter sol** <u>utions</u> Then the 6D de Sitter solutions can be uplifted to 10D!!! where **Mon-compactness?** Finally the dilaton in 10D ˆ (which from the I truncation of type IIA strings determines the string *e*^ˆ = (cosh 2⇢) *e*' (4.5) so we also we consider the two and on the type in the type I truncation to \blacksquare \blacksquare Than the 6D de Sitter colutions can he unlifted to *^gMN* ! !*gMN* ; *^e*' ! !¹*e*'; *^L*⁶ ! !²*L*⁶ (2.4) which leaves the equations of motion invariant. Notice also that Einstein's equations get much sim-**Then the 6D de Sitter solutions can be uplifted to 10D !!! Non-compactness?**

6D Supergravity from F-theory We will dimensionally reduce this theory on a circle and then determine the couplings by comparison with the M-theory reduction.

11D M-theory to 5D on elliptically fibred CY, and uplift to D=6 four scalars ˆq^U each. The bosonic components of the 6D vector multiplets contain only the vectors **11D M-theory to 5D on elliptically fibred CY₃ and uplift to D=6**

 h_{12} +1 hypermultiplets, h_{11} -1 tensor multiplets

$$
S^{(6)} = \int_{\mathcal{M}_6} \left[\frac{1}{2} \hat{R} \hat{*} 1 - \frac{1}{4} \hat{g}_{\alpha\beta} \hat{G}^{\alpha} \wedge \hat{*} \hat{G}^{\beta} - \frac{1}{2} \hat{g}_{\alpha\beta} d\hat{j}^{\alpha} \wedge \hat{*} d\hat{j}^{\beta} - \frac{1}{2} \hat{h}_{UV} \hat{D} \hat{q}^U \wedge \hat{*} \hat{D} \hat{q}^V \right. \\ - 2 \Omega_{\alpha\beta} \hat{j}^{\alpha} b^{\beta} C_{IJ} \hat{F}^I \wedge \hat{*} \hat{F}^J - \Omega_{\alpha\beta} b^{\alpha} C_{IJ} \hat{B}^{\beta} \wedge \hat{F}^I \wedge \hat{F}^J - \hat{V}^{(6)} \hat{*} \hat{1} \right],
$$

$$
\hat{V}_{\text{flux}}^{(6)} = \frac{1}{32\Omega_{\alpha\beta}\hat{j}^{\alpha}b^{\beta}\hat{\mathcal{V}}^2}C^{-1ij}\theta_i\theta_j.
$$

6D potential from D7 fluxes

form figure strengths need to be imposed by the following Strengths need to be imposed by hand after variation

From 6D to 4D $F_{\mathbf{F}}$ **y** h ²D 2

 \overline{a} . ⇣ $i = log$ \overline{a} χ = log volume, Γ = log W, Ω = log a , Δ = log A de Sitter

> ϵ ² $\left\{^2,W,H\,e^{\varphi_0/2}\right\}$ \overline{r} \overline{r} $\begin{pmatrix} \varphi_0/2 \end{pmatrix}$ \mathbf{H} *^e*2+22⌦ ¹ $\overline{\mathcal{L}}$ **Flat direction**

\blacksquare **From 6D** which *n^m* is the unit normal (pointing out of the source). With these choices the Maxwell equation (2.5) integrates to give

a $\overline{\mathbf{u}}$ ⇣ can be written as: **Constraint** functions *a*, *W* and ' forward in ⌘ given initial conditions at some ⌘ = ⌘0. These initial conditions

$$
ds^{2} = W(r)^{2} q_{\mu\nu} dx^{\mu} dx^{\nu} + a(r)^{2} d\theta^{2} + dr^{2} = e^{2\Gamma(r)} q_{\mu\nu} dx^{\mu} dx^{\nu} + e^{2\Omega(r)} d\theta^{2} + dr^{2}
$$

$\overline{}$ **Field equations**
and the asymptotic behaviour at large simplify the numerical solutions.

$$
\ddot{\varphi} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\varphi} = \tilde{V}e^{\varphi - 2\chi} - 2C\dot{\Delta}^2 e^{-\varphi + 2\Delta - 2\Omega}
$$
\n
$$
\ddot{\chi} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\chi} = -\frac{k^2}{4} e^{-2\chi + 2\Delta - 2\Omega} - 4\tilde{V}e^{\varphi - 2\chi}
$$
\n
$$
\ddot{\Gamma} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\Gamma} = 3H^2 e^{-2\Gamma} - \frac{1}{2} (\ddot{\varphi} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\varphi})
$$
\n
$$
\ddot{\Omega} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\Omega} = -4C\dot{\Delta}^2 e^{-\varphi + 2\Delta - 2\Omega} - \frac{k^2}{8} e^{-2\chi + 2\Delta - 2\Omega} - \frac{1}{2} (\ddot{\varphi} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\varphi})
$$
\n
$$
\ddot{\Delta} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\Delta} = \dot{\Delta}\dot{\varphi} + 2\dot{\Omega}\dot{\Delta} - \dot{\Delta}^2 + \frac{k^2}{32C} e^{\varphi - 2\chi}
$$

CONSTITAIT

$$
6H^2e^{-2\Gamma} - 4\dot{\Omega}\dot{\Gamma} - 6\dot{\Gamma}^2 + \frac{1}{2}\dot{\varphi}^2 + \frac{1}{4}\dot{\chi}^2 + 2Ce^{-\varphi - 2\Omega + 2\Delta}\dot{\Delta}^2 - \tilde{V}e^{\varphi - 2\chi} - \frac{k^2}{16}e^{-2\chi - 2\Omega + 2\Delta} = 0
$$

 $=$ $\overline{0}$ volume $\mathbf{P} = \log \mathbf{W}, \ \mathbf{\Omega} = \mathbf{W}$ 2 blume, $I = \log w$, $\Omega = \log a$, Δ $\mathsf{me},\ \mathsf{\Gamma}=\!\mathsf{log}\ \mathsf{W},\ \mathsf{\Omega}=\mathsf{log}\ \mathsf{a}$, $\mathsf{\Delta}=\mathsf{lo}$

$$
\left\{\varphi, a, W, H\right\} \to \left\{\varphi + \varphi_0, a \, e^{-\varphi_0/2}, W, H \, e^{\varphi_0/2}\right\}
$$

$$
\varphi\,=\,q\ln r+\ln u
$$

$$
\chi\,=\,s\ln r+\ln v
$$

$$
\Gamma = w \ln r + \ln x
$$

$$
\Omega = \alpha \ln r + \ln y
$$

$$
\Delta = \delta \ln r + \ln z
$$

1

2

Asymptotic solutions *H*². From the third equation in (4.40) we can see two branches depending on the values of , either = 0 or = *q* + 2↵ = *q* 8*w* + 2. For = 0 the constraints become *q* + 2 *<* 8*w* and *s <* 4*w*. The intersections of these two planes with the interior of the ellipsoid define the allowed parameter space. where *u, v, x, y, z* are constants. Instead of assuming that the derivative terms dominate we can consider under which conditions the non-derivative terms are of the same order as the derivative $\mathbf{A} \cdot \mathbf{A}$ **m** 2*Cz*² *uy*² ² *^u ^v*² *^V*˜ = 0 (↵ + 4*^w* 1)*^q ^u* 2*Cz*²

$\overline{\text{oluti}}$ \mathbf{J} **, solution** and solutions assume that all terms scale as 1 *^r* at the same rate, unlike the approximate solutions for which the derivative terms dominate.

Near brane solutions:

\n
$$
\varphi = q \ln r + \ln u \qquad \qquad \frac{1}{2}q^{2} + \frac{1}{4}s^{2} - 6w^{2} - 4\alpha w + \frac{6}{x^{2}}H^{2} + \frac{2Cz^{2}}{uy^{2}}\delta^{2} - \frac{u}{v^{2}}\tilde{V} = 0
$$
\n
$$
\chi = s \ln r + \ln v \qquad (\alpha + 4w - 1)q - \frac{u}{v^{2}}\tilde{V} + \frac{2Cz^{2}}{uy^{2}}\delta^{2} = 0
$$
\n
$$
\Omega = \alpha \ln r + \ln y \qquad (\alpha + 4w - 1)s + \frac{4u}{v^{2}}\tilde{V} = 0
$$
\n
$$
\Delta = \delta \ln r + \ln z \qquad (\alpha + 4w - 1)w - \frac{3}{x^{2}}H^{2} + (\alpha + 4w - 1)\frac{q}{2} = 0
$$
\n
$$
(\alpha + 4w - 1)\delta - (q + 2\alpha - \delta)\delta = 0
$$
\n
$$
2\delta - q - 2\alpha = 0
$$
\n**Kasner constraints**

\n
$$
q - 2s + 2 = 0
$$
\n**(BKL: Belinsky et al)**

\n
$$
w - 1 = 0
$$

Ka (BKL: Belinsky et al)

$$
q = -\frac{2}{9}, \quad s = \frac{8}{9}, \quad \alpha = \frac{1}{9}, \quad w = \frac{1}{9}, \quad \delta = 0, \quad \frac{u}{v^2}\tilde{V} = \frac{8}{81}
$$

terms. That means all or some of them scale as 1*/r*² Assuming *k* = 0 and that all scale as 1*/r*² we rane *q*² + 1 *^s*² ⁶*w*² ⁴↵*^w* ⁺ (↵ + 4*^w* 1)*^w* ³ (↵ + 4*w* 1)*s* + **Near brane solutions:**

$$
,\quad \delta=0,\quad \frac{u}{v^2}\tilde{V}=\frac{8}{81}
$$

Numerical Solutions H²= 0

Numerical AdS Solutions H²≤ 0

Numerical dS Solutions H²≥ 0

Singularities? Singularities Physical diverge both at 2 *^µ*1*...µⁿ* ⌫1*...*⌫*ⁿ* ⁼ ^X *µ*(1)*...µ*(*n*) ⌫1*...*⌫*ⁿ* ⁼ ^X Figure 3: The four solutions of the system in Eq. (??) all diverge both at *r* = 0 and at *r* = *r*e.

where the Dirac matrices and the Dirac matrices and the Dirac matrices α is α is α is α and α we will adopt here for extended objects rather than point particles for which the generalization \mathbf{r} **Applications to precision atomic levels, Helium 4...** e↵ective couplings *gi*(✏) and ✏ separately. on the Raggiica only the Characterize that characterize the entire RG trajectory (α **Applications to precision atomic levels, Helium 4…**

$$
S_{Nucl} = -\int_{\mathcal{P}} ds \left\{ \sqrt{-\dot{y}^2} M - Ze\dot{y}^\mu A_\mu + c_s \sqrt{-\dot{y}^2} \left(\overline{\Psi} \Psi \right) + ic_v \dot{y}^\mu \left(\overline{\Psi} \gamma_\mu \Psi \right) + \cdots \right\}
$$
\nGoldberger

\n
$$
S = S_{QED} + S_{Nucl}
$$
\nSuppose

 $\overline{1}$

remain unchanged as we do so because the size of the Gaussian pillboxes are arbitrary after all,

Gravity

Γ outled the singularities in the bulk solutions can also the action of the gravitation of the gravitati whose back-reaction is responsible for the singular behaviour, along the lines also given in §2. whose back-reaction is reaching the singular behaviour, along the singular behaviour, along the lines also given in §2. Source pack reaction: Boundary containing the fewer of the fewer of the sources of the sources of unbent of the formula for unbent of the formula of the sources of the formula for unbent of the formula for unbent of the fo d⁴*x* p*g W*⁴ \mathbf{I} **Source back reaction:**

$$
\frac{2\pi}{\kappa^2} \Big[aW^4 \partial_r W \Big]_{r=\epsilon} = U_b(\phi) \,,
$$

$$
\frac{2\pi}{\kappa^2} \left[aW^4 \partial_r \varphi \right]_{r=\epsilon} = \frac{2\pi}{\kappa^2} \left[a_b W_b^4 r^{\alpha_b + 4w_b - 1} q_b \right]_{r=\epsilon} = \frac{\partial}{\partial \varphi} \left[W_b^4 L_b \right],
$$

$$
\frac{2\pi}{\kappa^2} \left[aW^4 \partial_r \chi \right]_{r=\epsilon} = \frac{2\pi}{\kappa^2} \left[a_b W_b^4 r^{\alpha_b + 4w_b - 1} s_b \right]_{r=\epsilon} = \frac{\partial}{\partial \chi} \left[W_b^4 L_b \right],
$$

$$
\mathcal{U}_b = \frac{1}{3} \left[(W_b^4 - \mathcal{T}_b) - \sqrt{(W_b^4 - \mathcal{T}_b)^2 - \frac{3}{4} (\mathcal{T}_{b,\varphi})^2 - \frac{3}{8} (\mathcal{T}_{b,\chi})^2} \right].
$$

. (5.42)

$$
-\frac{2\pi}{\kappa^2} \left\{ W^4 \left[a \left(3\frac{\partial_r W}{W} + \frac{\partial_r a}{a} \right) - 1 \right] \right\}_{r=\epsilon} = W_b^4 L_b(\varphi), \qquad 3\omega_{\pm} + \alpha_{\pm} - W_{\pm}^4 = -\frac{\kappa^2 T_{\pm}}{2\pi}
$$

$$
S_b = -\int d^4x \sqrt{-g} \, W_b^4 L_b(\varphi, \chi) = -\int d^4x \sqrt{-g} \, T_b(\varphi, \chi) \, . \qquad \qquad b = \pm.
$$

p*g Tb*('*,*)*.* (5.36) **Dournal** y conditions from pinson **The see what the see what the see what the subset of** *Boundary* **conditions from pillbox** *q*2 *^b* = 4!*b*(2↵*^b* + 3!*b*)*,* (2.28) **Boundary conditions from pillbox**

$$
V_{\text{eff}}(\varphi_0) = -\sum_b \left(U_b + \frac{T_b'}{2} \right)
$$

Positive tensions! source couplings do not break the shift symmetry (2.7) and so cannot lift the degeneracy in (2.21). If the source is a codimension-two object then we must ask *a* ! 0 as it is approached (so that circles of proper radius *r* that surround it also have circumferences that shrink as *r* ! 0). In the *a* ! 0

Conclusions

•Explicit (numeric) (A)dS solutions of 6D Supergravity

•Embedding of gauge 6D supergravity in String Theory

•Solutions uplifted to string solutions

•Singularities have brane-like properties (power of PPEFT)

•Open questions

