A Bumpy Ride: Distances Beyond Minkowski space

Irene Valenzuela

CERN

StringPheno 2024, Padova

Based on work in collaboration with members of my group:

Ignacio Ruiz (Nacho)

José Calderón-Infante

Alessandra Grieco

Amineh Mohseni

and other amazing collaborators:

Alberto Castellano
Muldrow Etheredge
Ben Heidenreich
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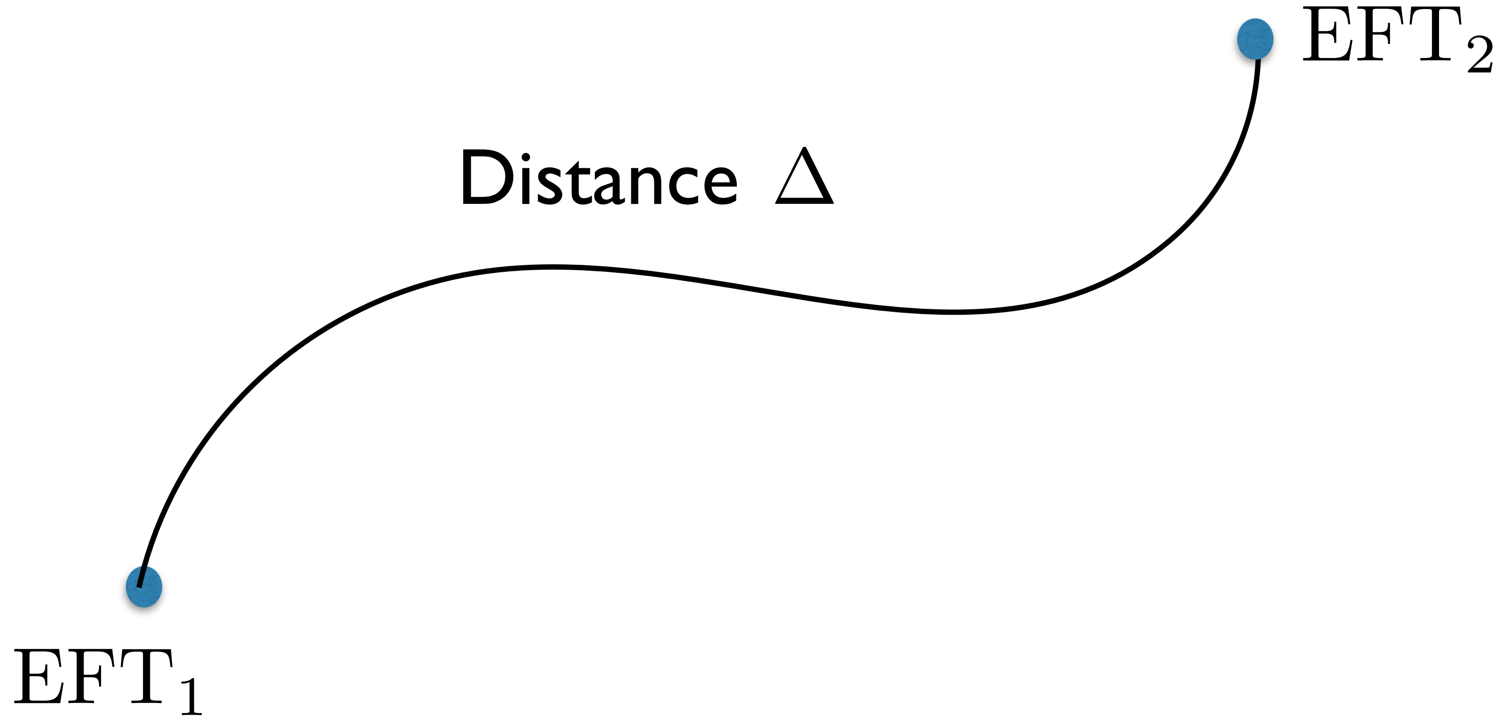

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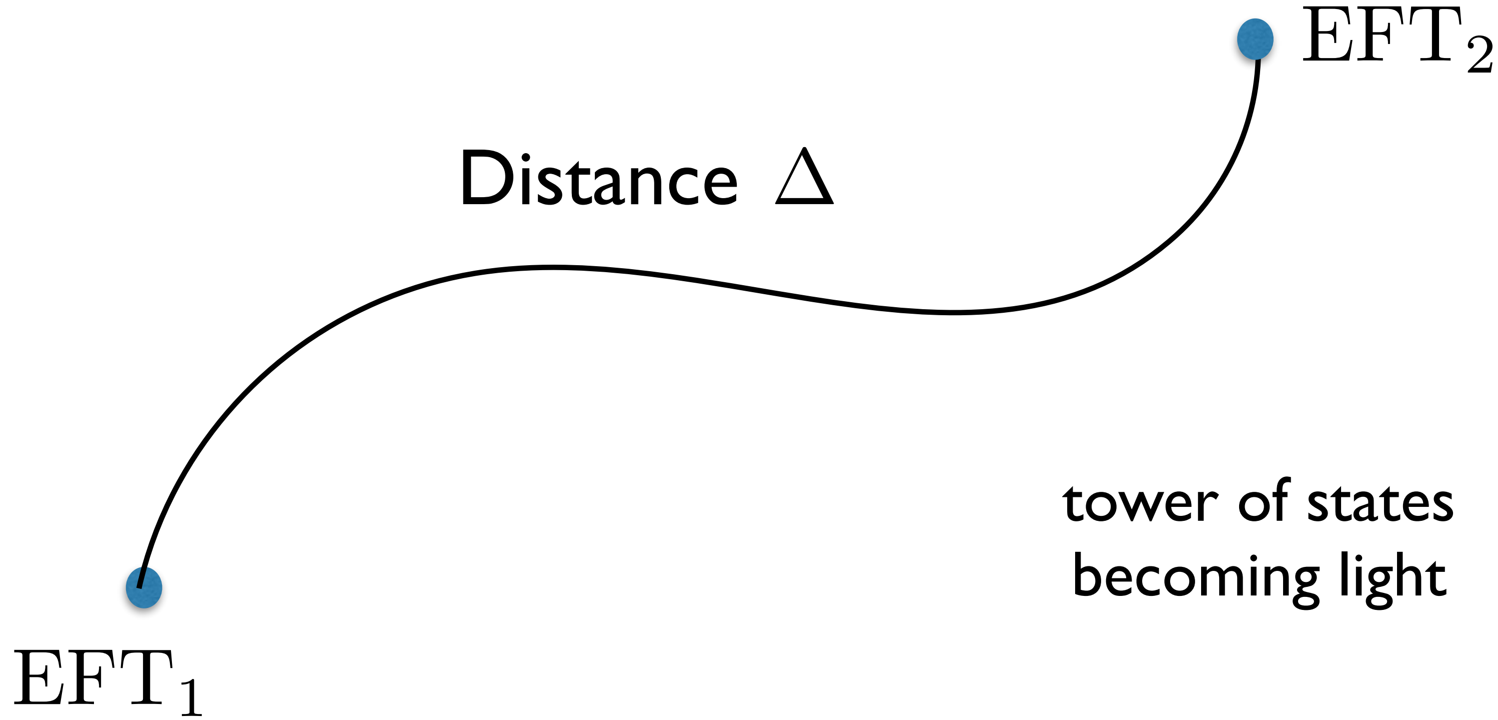
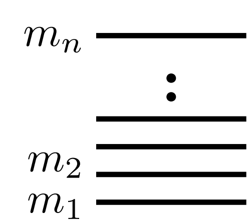


tower of states
becoming light



$$m \sim m_0 e^{-\alpha\Delta} \quad \text{as } \Delta \to \infty$$

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Minkowski space
(EFTs connected by a moduli space)

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Distances in AdS/CFT

- New corners: Non-critical strings
- Absence of scale separation in AdS

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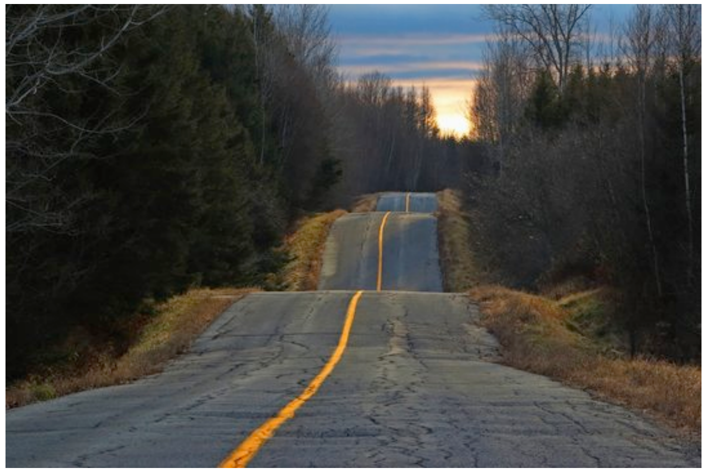


Generalised distance beyond moduli spaces

1) Minkowski space

$$\mathcal{L} = \frac{M_{\mathrm{pl},d}}{2}^{d-2}\left(R - \frac{1}{2}g_{ij}(\phi)(\partial\phi)^2 + \dots\right)$$

field metric

Moduli space distance: $\Delta\phi = \int_Q^P \sqrt{g_{ij}\frac{d\phi^i}{ds}\frac{d\phi^j}{ds}}ds$

[Castellano,Ruiz,IV'23]

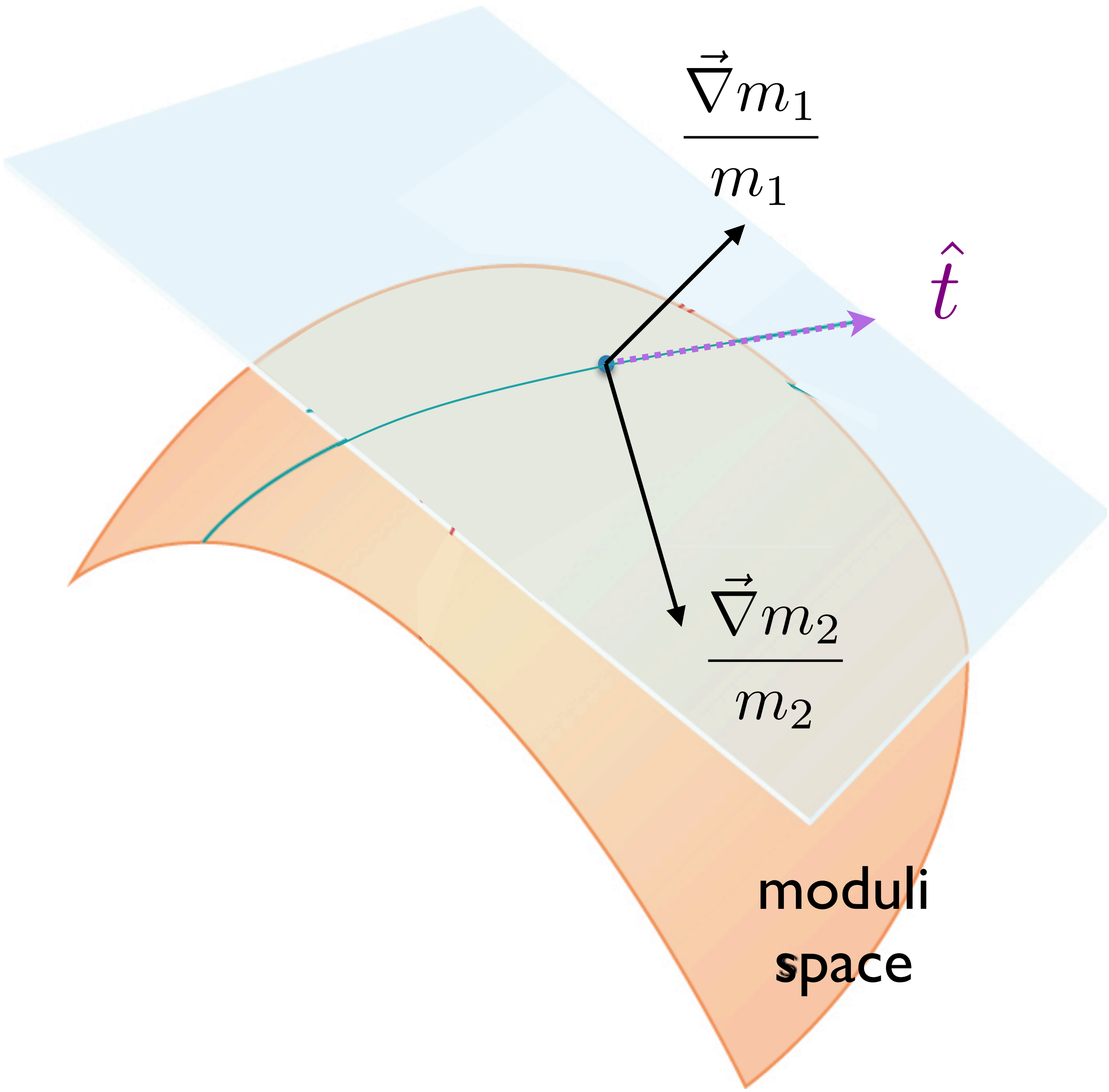
[Etheredge,Heidenreich,Rudelius,Ruiz,IV'24]

Tower and species vectors

Story of success for the Distance conjecture: $m \sim m_0 e^{-\alpha\Delta}$ [Ooguri-Vafa'06]

Tower vector: $\dfrac{\vec{\nabla} m_a}{m_a}$

a : labels number of light towers



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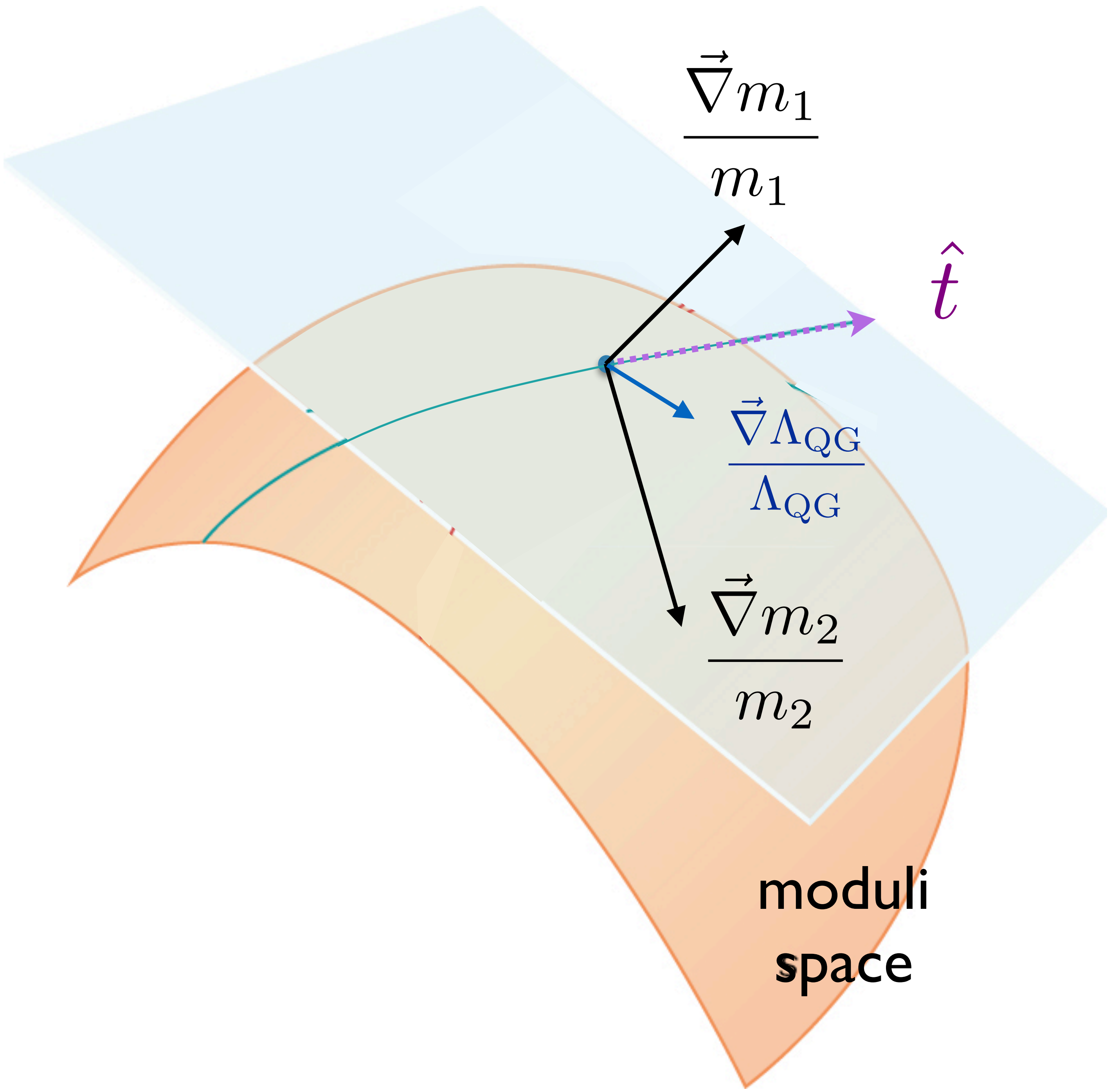
scale at which the local QFT description breaks down



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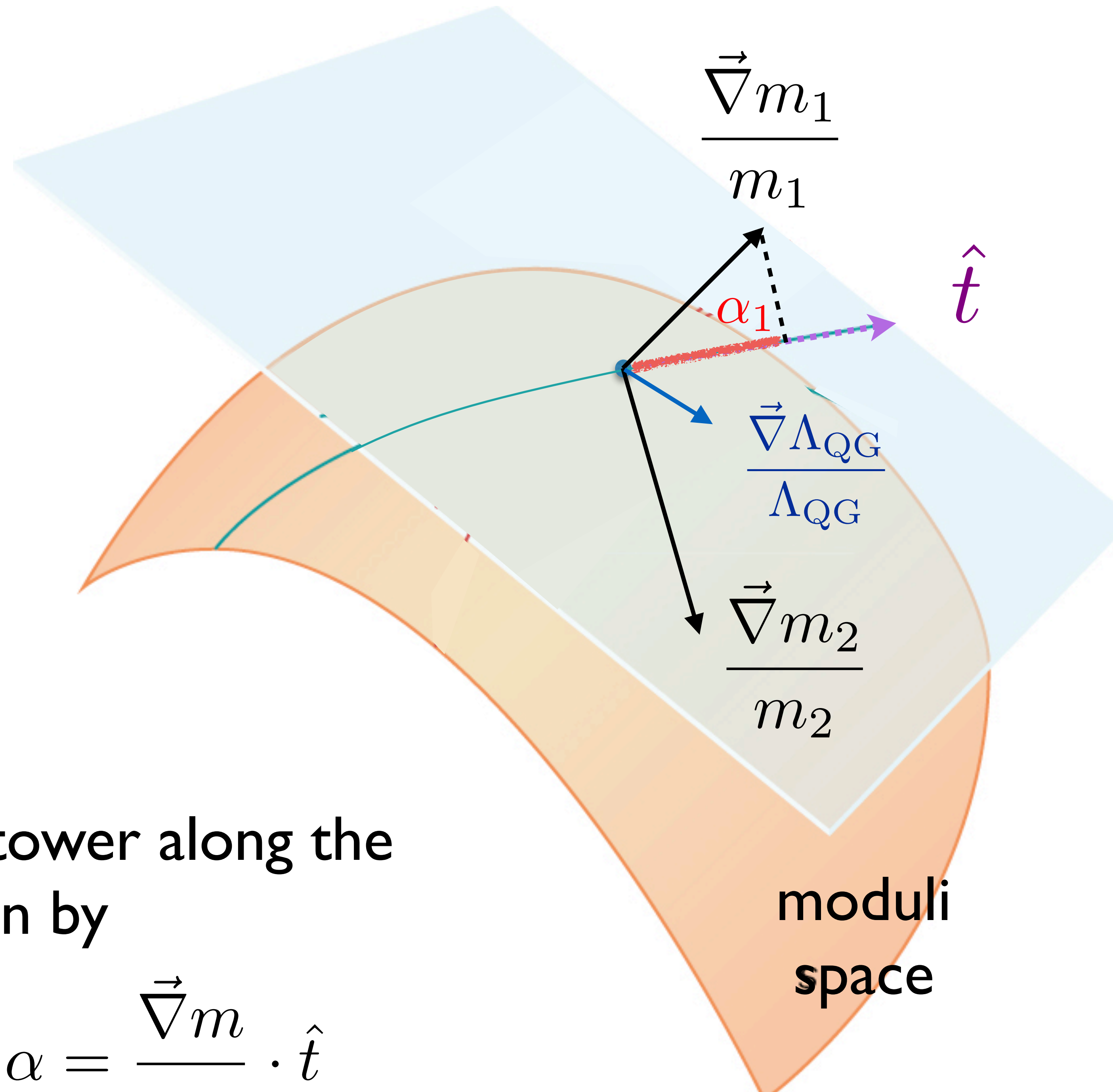

The exponential rate of the tower along the direction $\hat{t}$ is given by

$$m_{\rm tower} \sim m_0 e^{-\alpha\Delta\phi} \qquad \alpha = \frac{\vec{\nabla} m}{m}\cdot\hat{t}$$

Towers in Asymptotically Minkowski Space

So far, all string theory examples satisfy the Emergent String Conjecture, so the leading tower is (in some dual frame) either: [Lee,Lerche,Weigand'19]

- Tower of Kaluza-Klein states
- Tower of string oscillator modes of a critical string

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If so, for every **regular*** infinite distance limit:

$$\frac{\vec{\nabla} m_a}{m_a} \cdot \frac{\vec{\nabla} m_b}{m_b} = \frac{1}{d-2} + \frac{1}{n_a}\delta_{ab} \qquad \frac{\vec{\nabla} m}{m} \cdot \frac{\vec{\nabla} \Lambda_{\rm QG}}{\Lambda_{\rm QG}} = \frac{1}{d-2}$$

$$\left|\frac{\vec{\nabla} \Lambda_{\rm QG}}{\Lambda_{\rm QG}}\right|^2 = \frac{1}{d-2} - \frac{1}{D-2}$$

$d =$ space-time dimension
$n =$ # of extra dimensions

Taxonomy rules

[Etheredge,Heidenreich,Rudelius,Ruiz,IV'24]

*regular: 1) Leading tower is non-degenerate
2) The decompactification manifold is asymptotically empty

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Pattern observed in [Castellano,Ruiz,IV'23]

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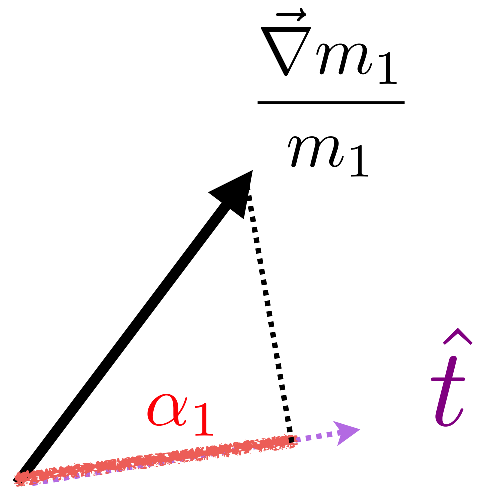

KK tower: $$\left|\frac{\vec{\nabla} m}{m}\right| = \sqrt{\frac{d+n-2}{n(d-2}}$$

(without warping)

string tower: $$\left|\frac{\vec{\nabla} m}{m}\right| = \frac{1}{\sqrt{d-2}}$$

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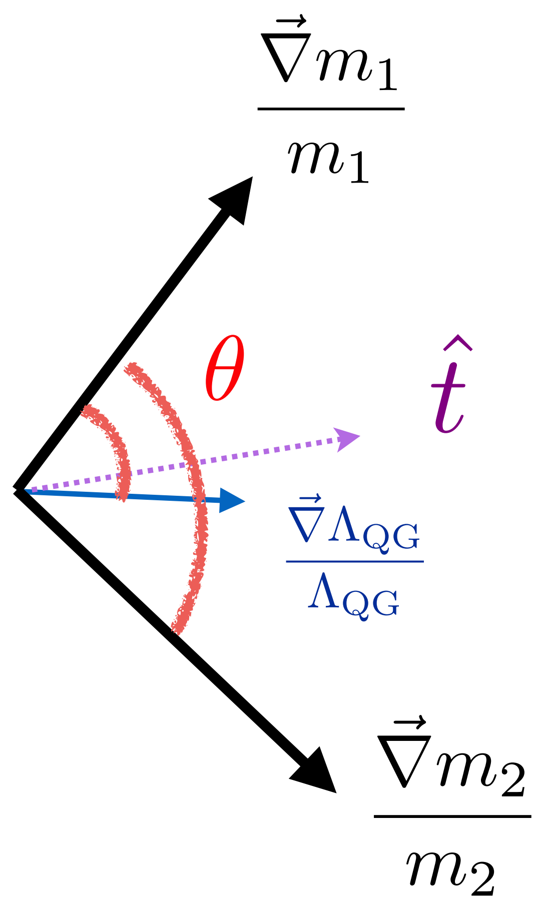
The microscopic nature of the tower constrains the lengths of the tower vectors (and therefore, the exponential rate of the masses)

but also the angles and the full geometry of their convex hull (frame simplex)

[Etheredge et al'24]



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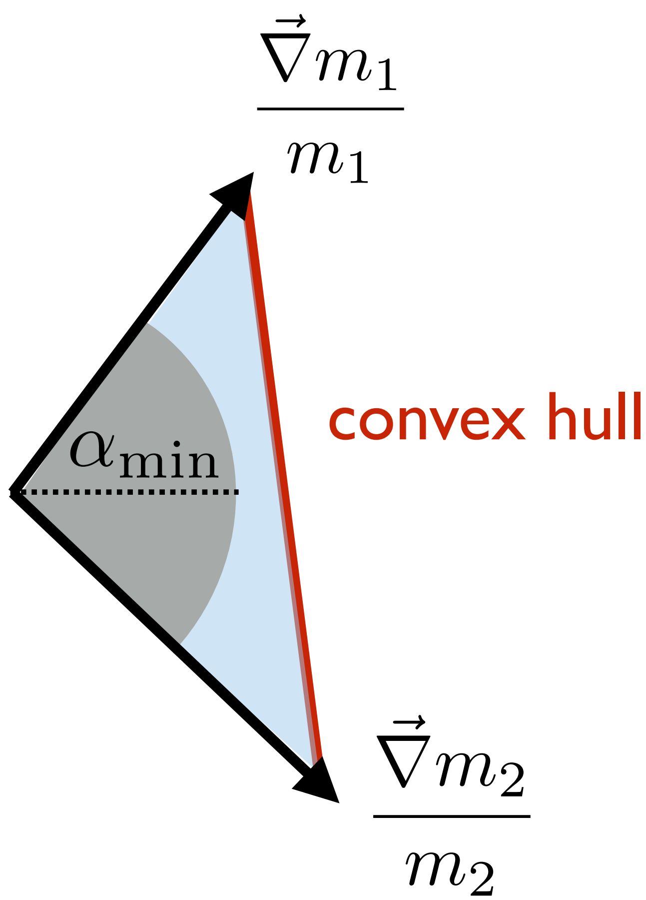

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Convex Hull Distance Conjecture with $\alpha_{\rm min} = \frac{1}{\sqrt{d-2}}$

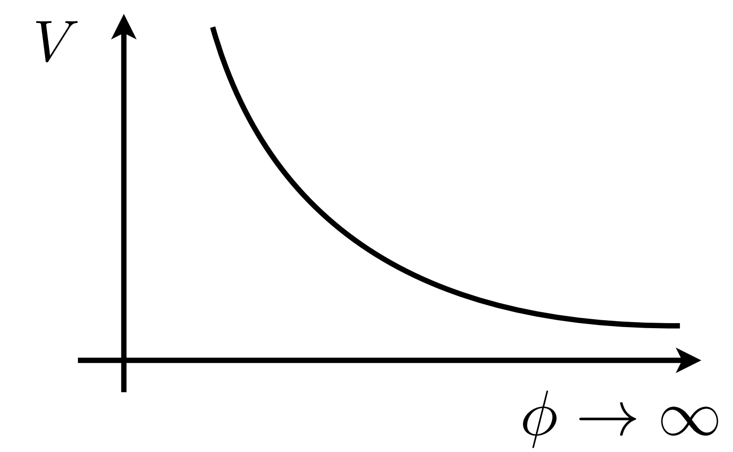
[Calderon-Infante,Uranga,IV'20]
[Etheredge et al'22]

(i.e., $\exists$ tower with exponential rate $\alpha \geq \alpha_{\rm min}$ along any geodesic **of this plane**)

Towers in Asymptotically Minkowski Space

These constraints also hold for asymptotically Minkowski spaces (even if there is a runaway potential):

We are studying the convex hull of the towers in 4d N=1 string compactifications



Interesting interplay with BPS EFT strings

[Grieco,Ruiz,IV'ongoing] see Alessandra Grieco's talk (Thursday)

Novel cosmological scenarios of transient acceleration using the towers of states

[Casas,Montero,Ruiz'24] see Ignacio Ruiz's talk (today)

2) AdS space

[Calderon-Infante,IV' ongoing]

Distance Conjecture in CFTs

Consider AdS_d/CFT_{d-1}

Bulk moduli space $\longleftrightarrow$ Conformal manifold (space of exactly marginal couplings)

field metric $\longleftrightarrow$ Zamolodchikov metric $|x-y|^{2d}\langle O_i(x)O_j(y)\rangle = g_{ij}(t^i)$

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Distance conjecture implies:

$\exists$ infinite tower of operators saturating the unitarity bound at every infinite distance limit measured by Zamolodchikov metric, such that

$$\gamma_J \sim e^{-\alpha d(\tau,\tau')} \quad \text{as} \quad d(\tau,\tau') \to \infty$$

[Perlmutter,Rastelli,Vafa,IV'21]
[Baume,Calderon-Infante'21]

anomalous dimension

$\gamma_J = \Delta - \Delta_{\text{unitarity}}$

distance measured by Zamolodchikov metric

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Infinite distance $\rightleftarrows$ Infinite tower

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For $d = 3$: Tower of scalar modes [Kontsevich,Soibelman'00] [Acharya,Douglas'06]

For $d > 3$: Tower of higher spin modes "CFT Distance conjecture"

[Perlmutter,Rastelli,Vafa,IV'21] (see also [Baume,Calderon-Infante'21])

Classification of Towers in d>3

Let us focus on 4d SCFT $(\mathcal{N}=1, \mathcal{N}=2, \mathcal{N}=4)$ (see though [Bobev et al'23])

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$g_{YM} \to 0$ ➡ $\mathrm{CFT}_{\mathrm{free}} \times \mathrm{CFT}'$ [Perlmutter,Rastelli,Vafa,IV'21]

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$\Downarrow$ [Maldacena, Zhiboedov'11]

Higher spin tower

$$m_{\mathrm{tower}} \sim m_0 e^{-\alpha \Delta\phi} \quad \text{with} \quad \boxed{\alpha = \sqrt{\frac{2c}{\mathrm{dim}G}}}$$

c: central charge

G: gauge group getting free

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Do the higher spin fields always correspond to a critical string becoming tensionless in the bulk?

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Exponential rate of the higher spin tower: $\alpha = \sqrt{\frac{2c}{\dim G}} = \frac{1}{\sqrt{4a/c - 2}}$

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see Jose Calderon-Infante's talk

We compute the thermal partition function to show that they have a different Hagedorn-like density of states (different Haggedorn temperature)

$$\rho(E) \sim e^{E/T_H}\ , \quad T_H = T_H(a/c) = T_H(\alpha)$$

[Calderon-Infante,IV' ongoing]

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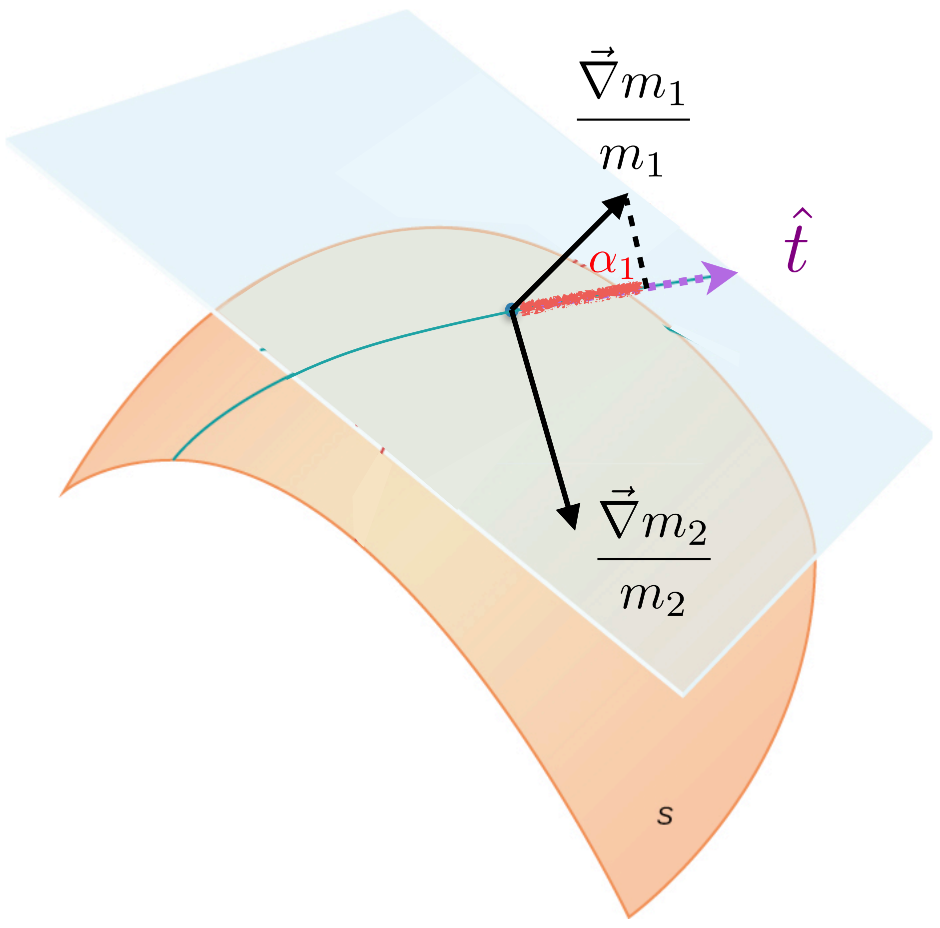
- Quantization of the string changes in AdS
- $\alpha = \frac{\vec{\nabla} m}{m} \cdot \hat{t}$ is the projection on the conformal manifold

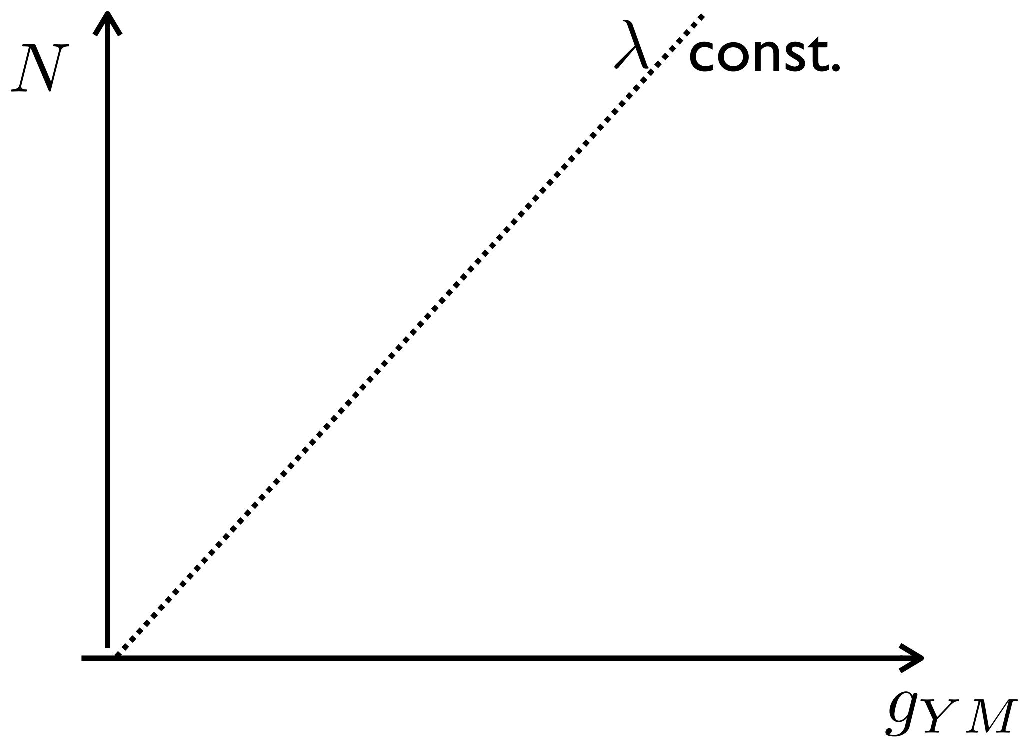
but $m(R_{S^5}, g_s) \longleftrightarrow m(N, g_{YM})$



Convex Hull in N=4 SYM

Let us focus on N=4 SYM / $AdS_5 \times S^5$ [Calderon-Infante,IV' ongoing]

$$R = M_{p,5}^{-1} N^{2/3}$$
$$g_s = g_{YM}^2$$
$$\lambda = g_{YM}^2 N$$



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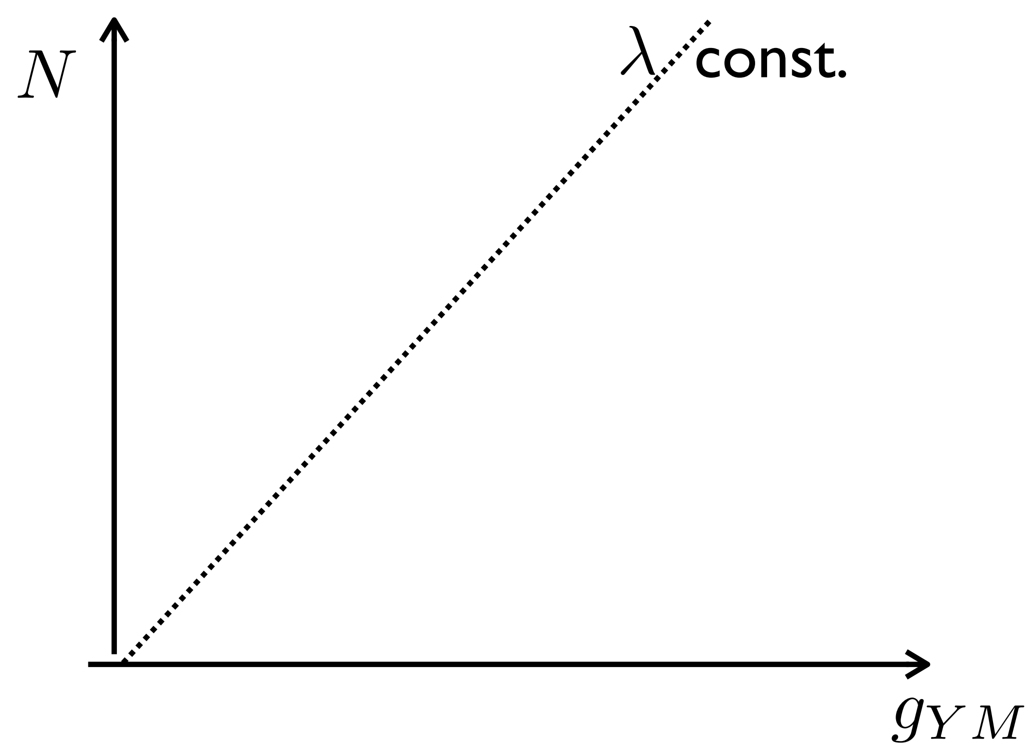

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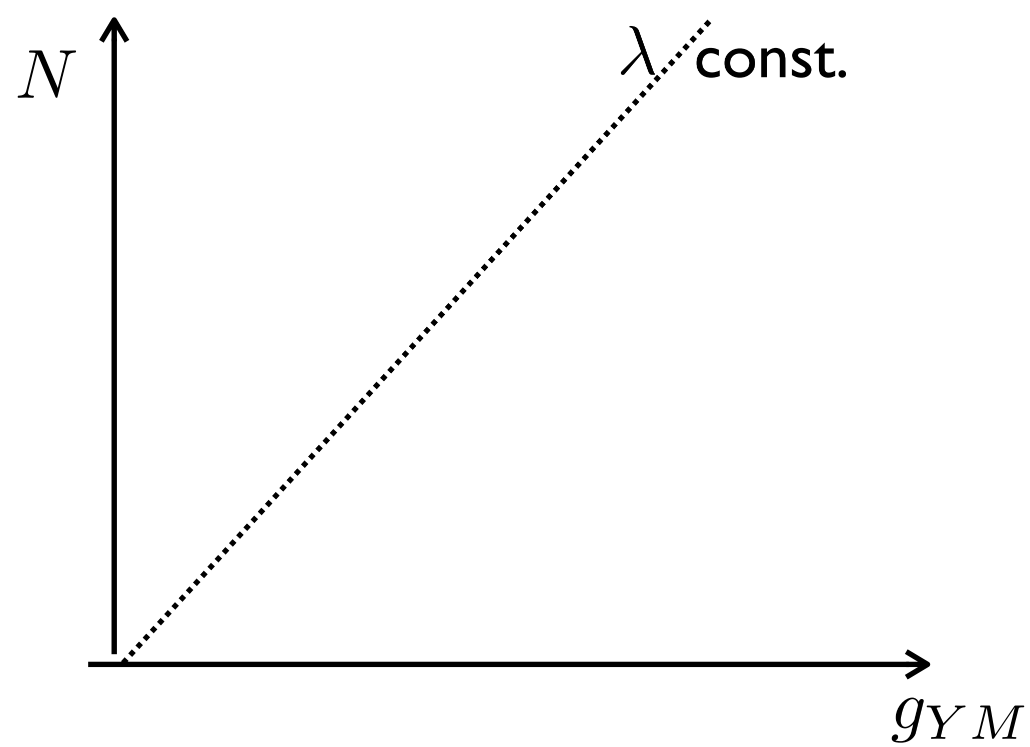

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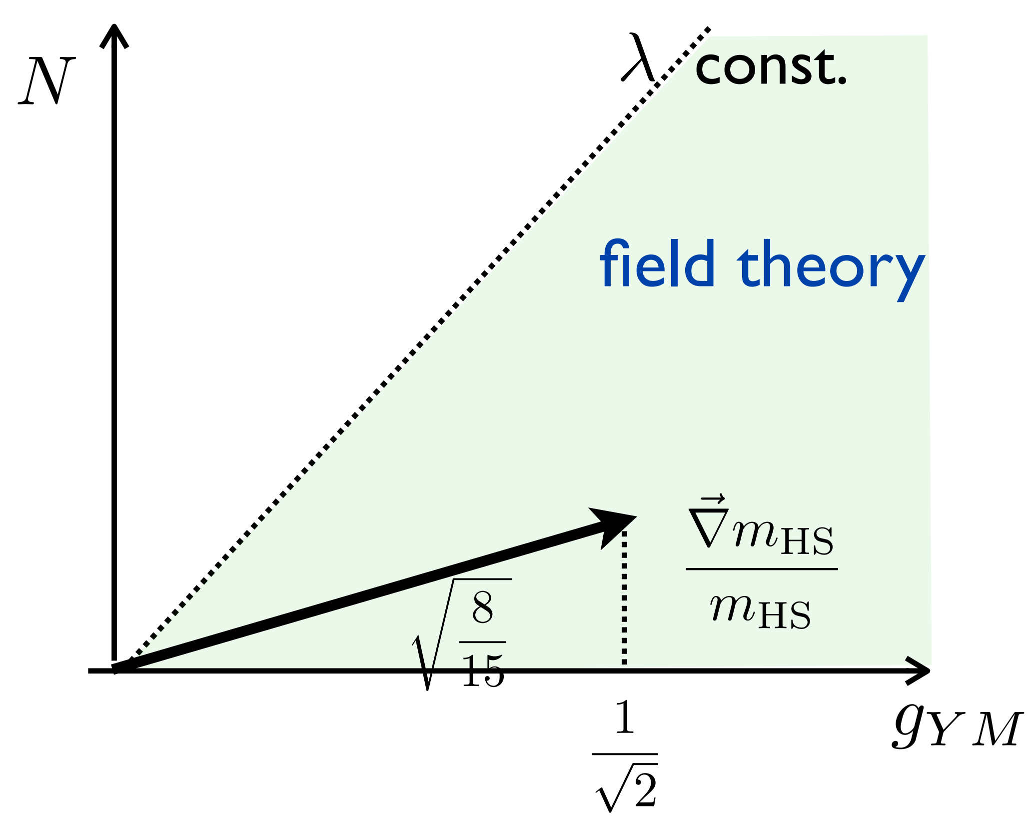

Convex Hull in N=4 SYM

Let us focus on N=4 SYM / $AdS_5 \times S^5$ [Calderon-Infante,IV' ongoing]

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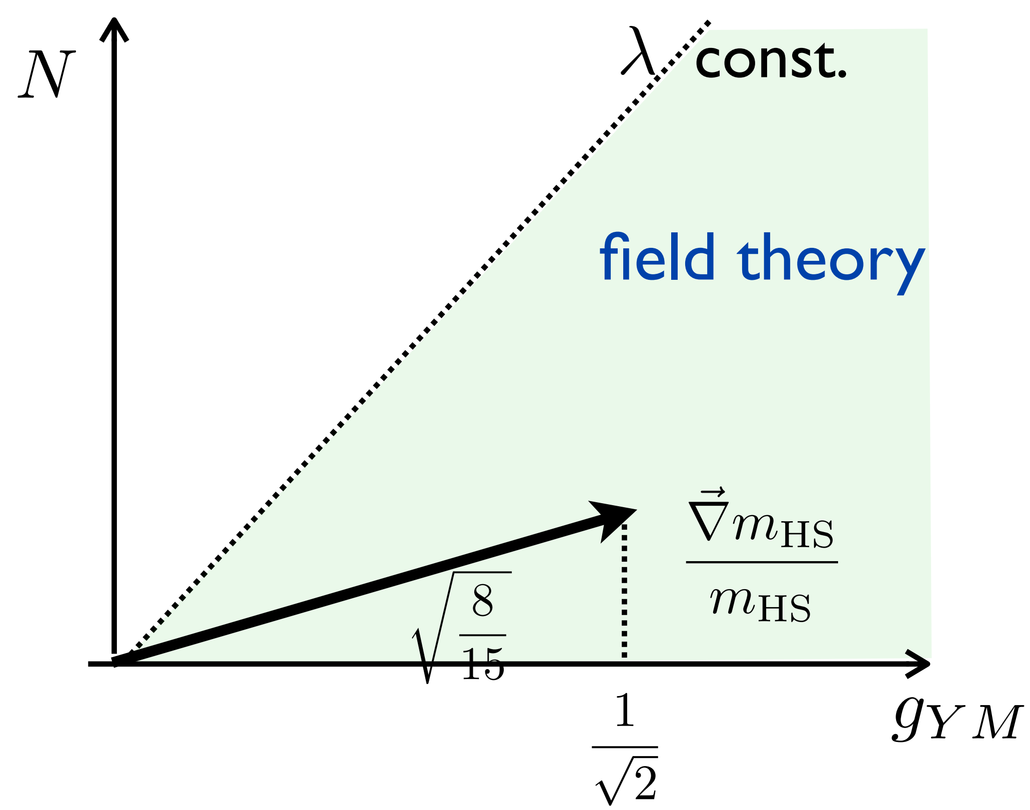

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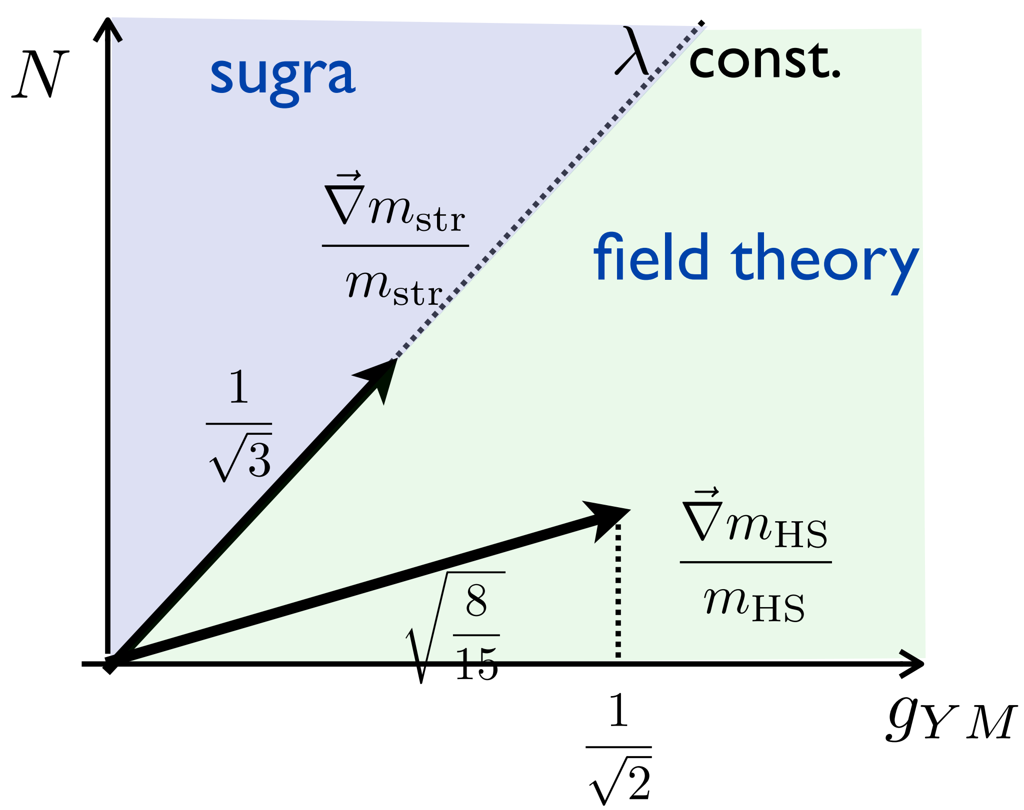

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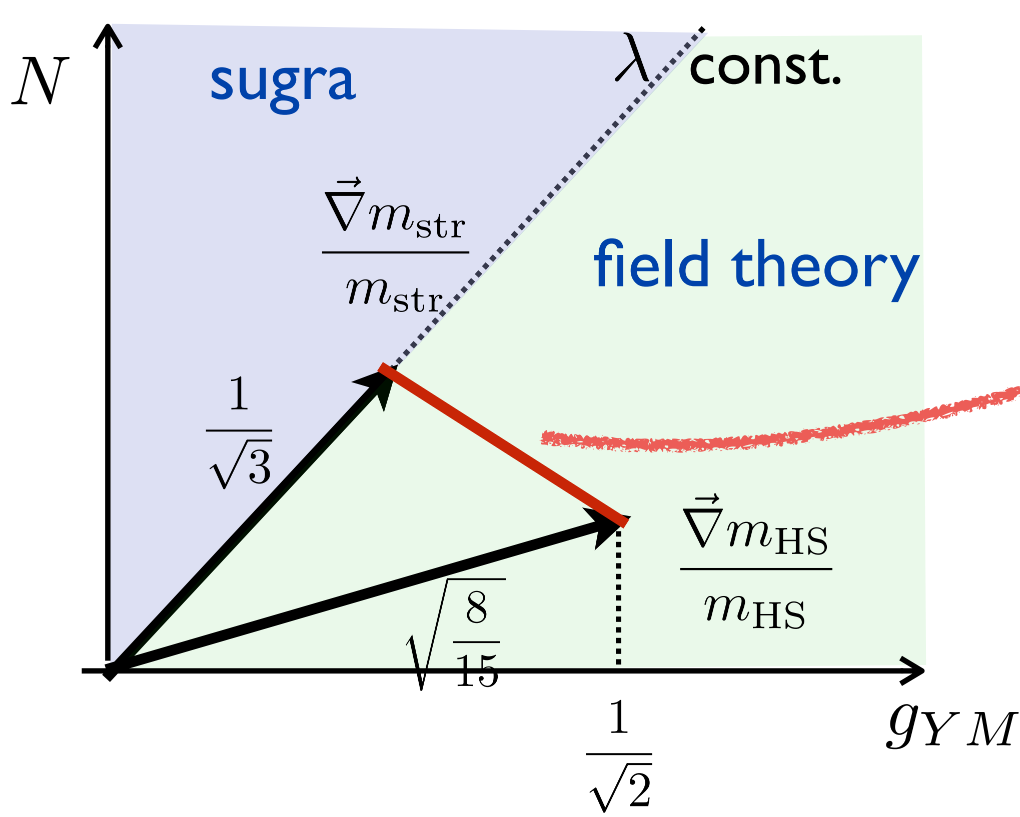


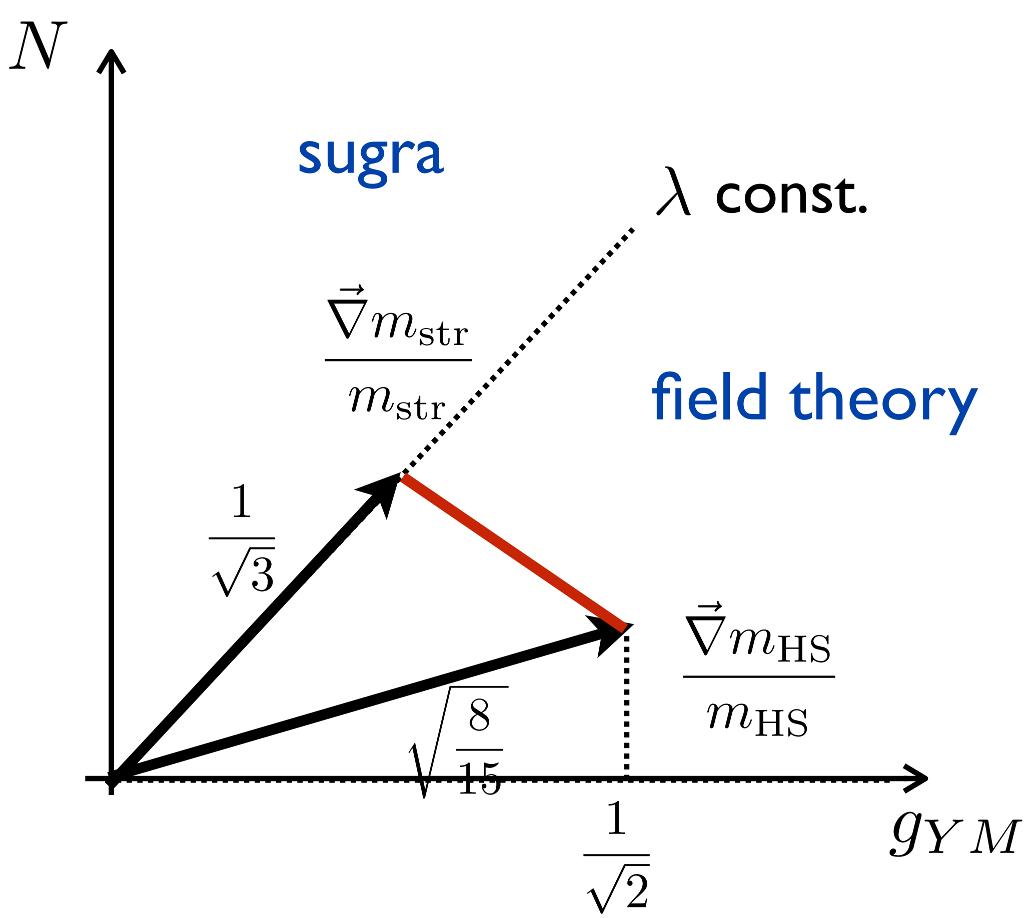
Results from integrability as λ changes

$$m \sim \frac{\sqrt{\gamma_{\rm cusp}(\lambda)}}{N^{2/3}}$$

see e.g. [Dorigoni, Hatsuda'15]

Convex Hull and Scale Separation

Let us plot the convex hull of all the light towers, including the KK towers



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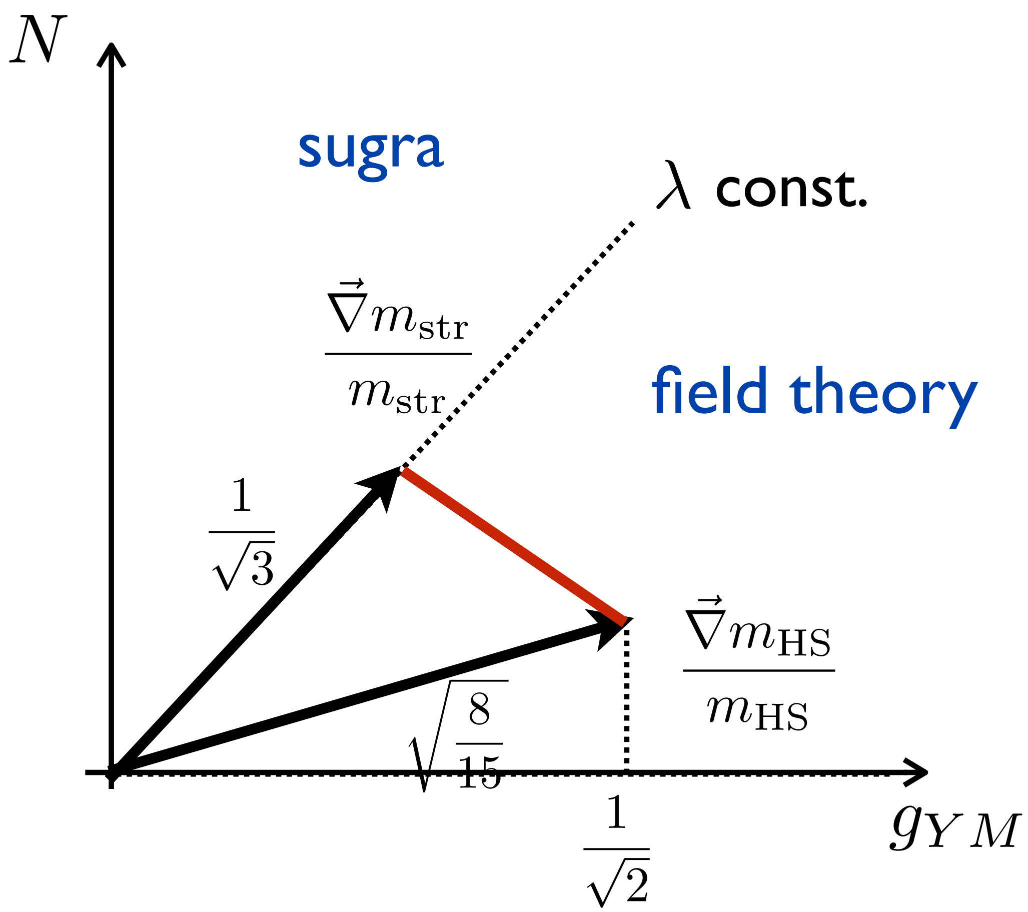

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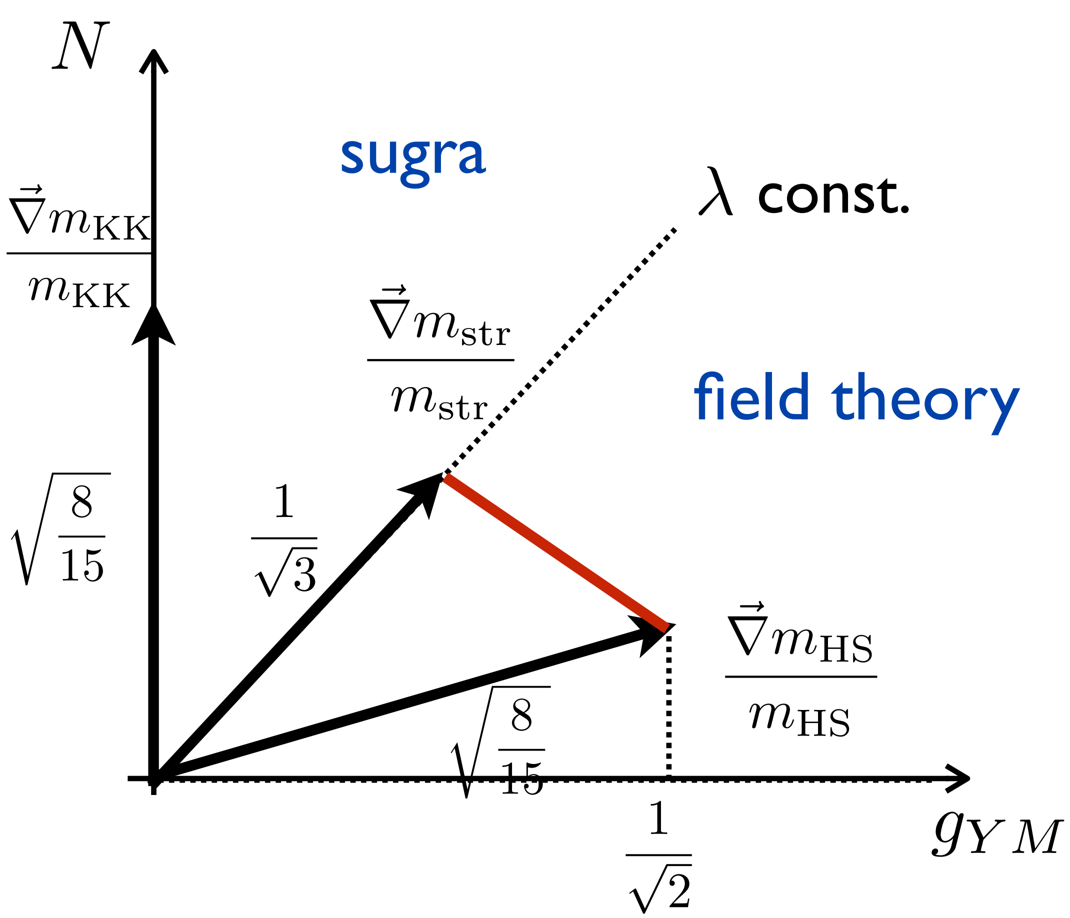

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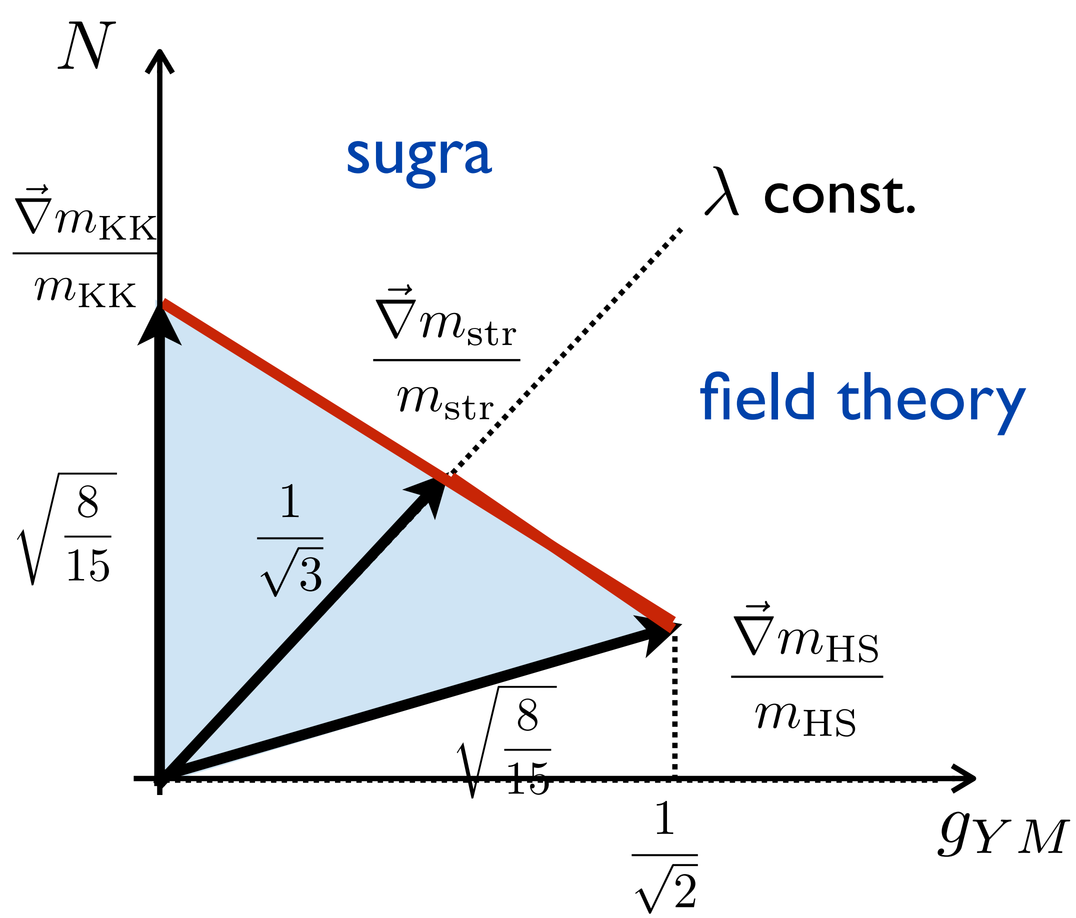

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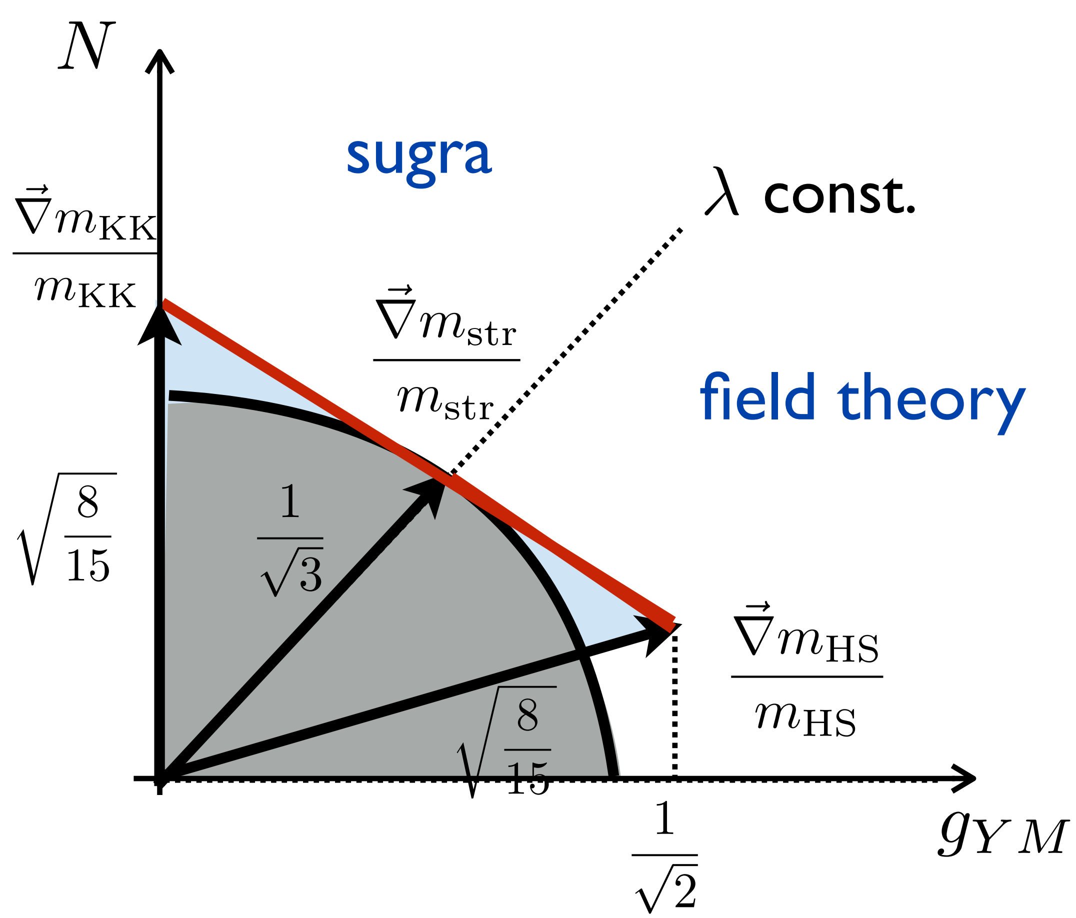

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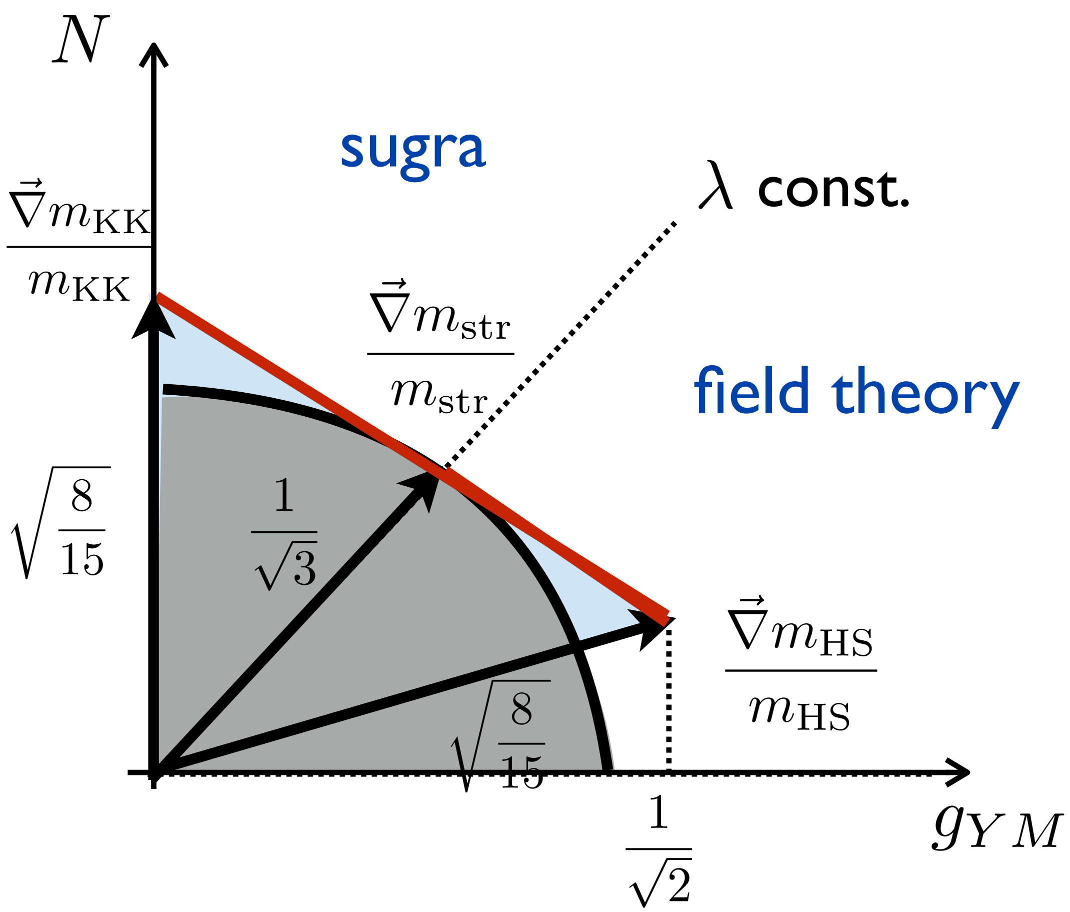

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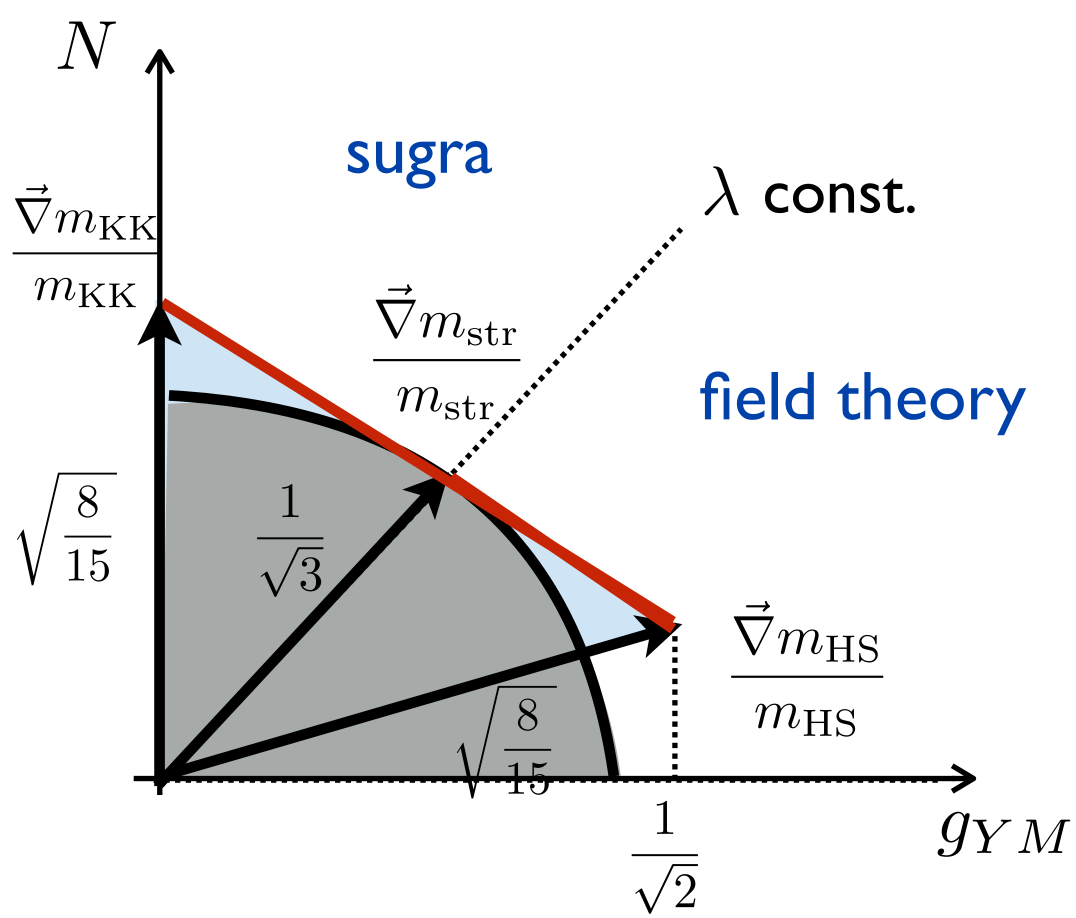

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It would be violated if the AdS vacuum was scale-separated $R_{KK} \ll R_{AdS}$

Convex Hull and Scale Separation

The Convex Hull Distance Conjecture with $\alpha \geq \frac{1}{\sqrt{d-2}}$

implies that holographic AdS spaces with a conformal manifold

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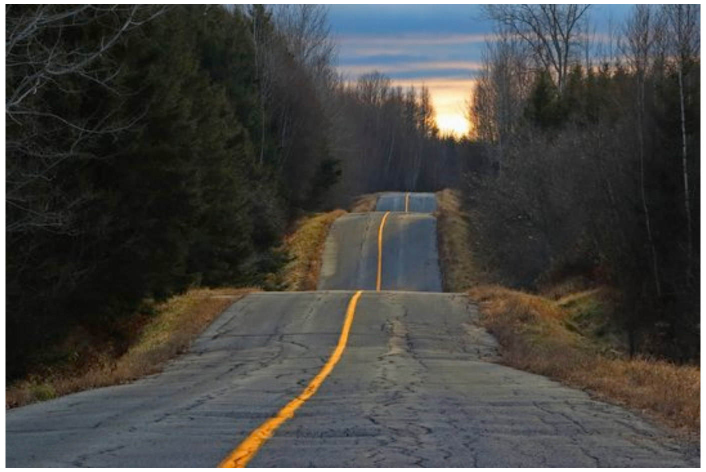
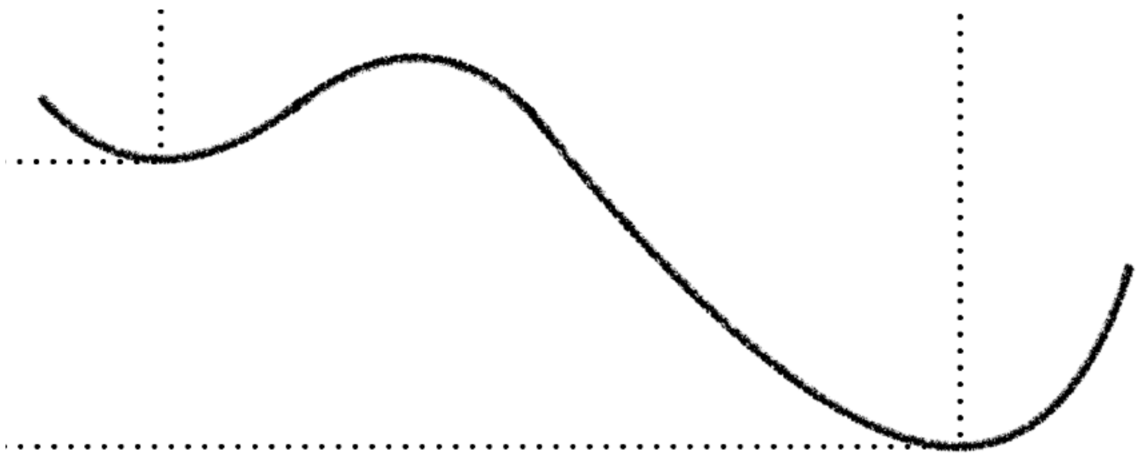
but can we generalize the notion of distance including a scalar potential?

3) Generalized distance with a potential



[Mohseni,Montero,Vafa,IV'ongoing]

Distance between theories



Very challenging, many attempts…

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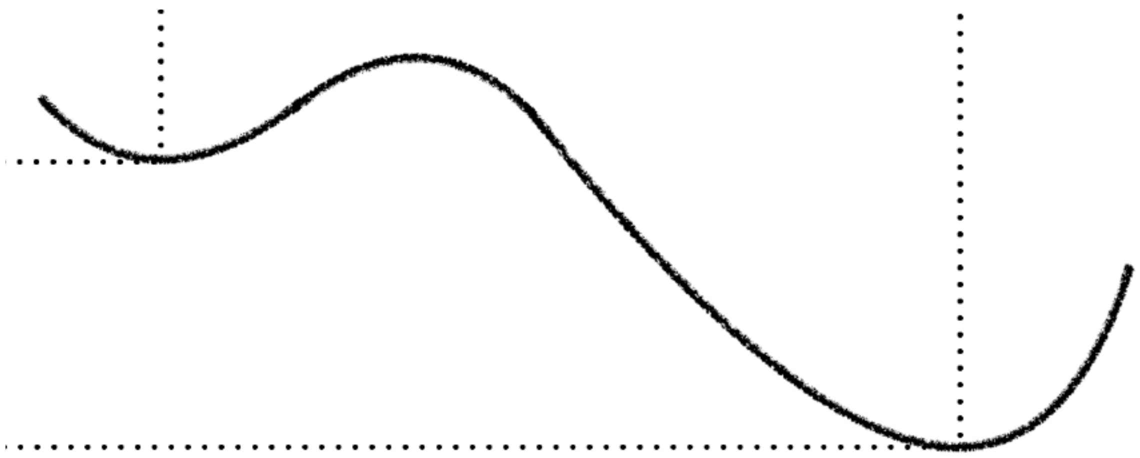

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Wish list:

- A notion of metric and geodesics in the space of theories
- It recovers moduli space distance if $V=0$

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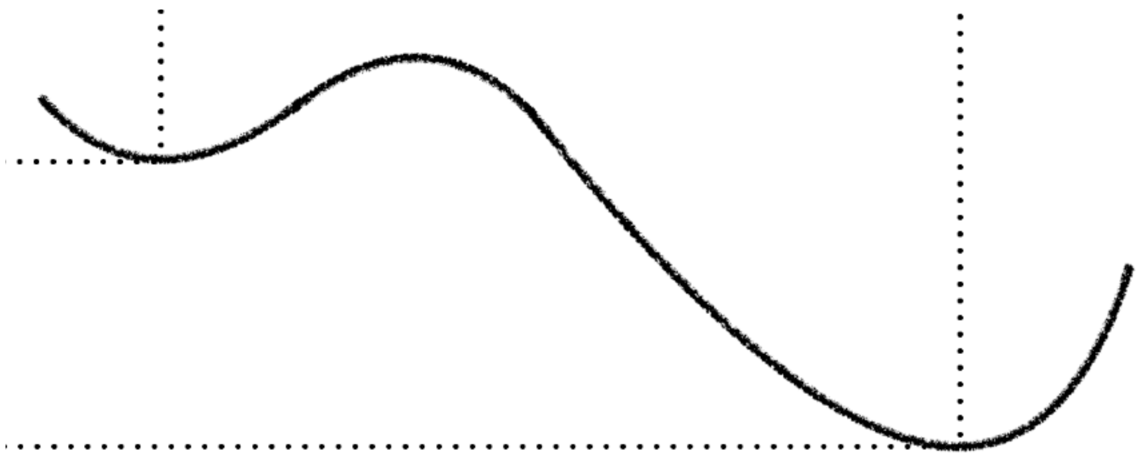

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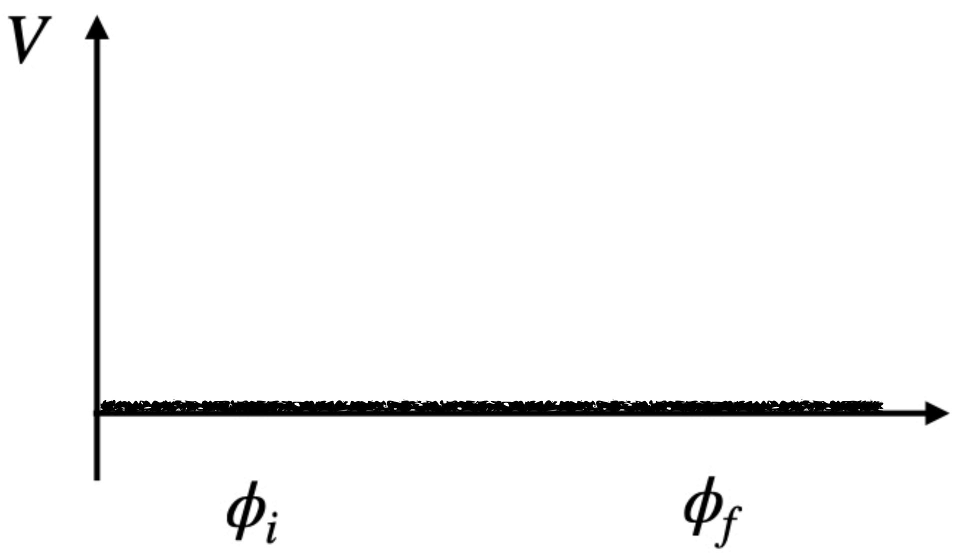
Let us take inspiration on the cobordism conjecture: [McNamara,Vafa'19]

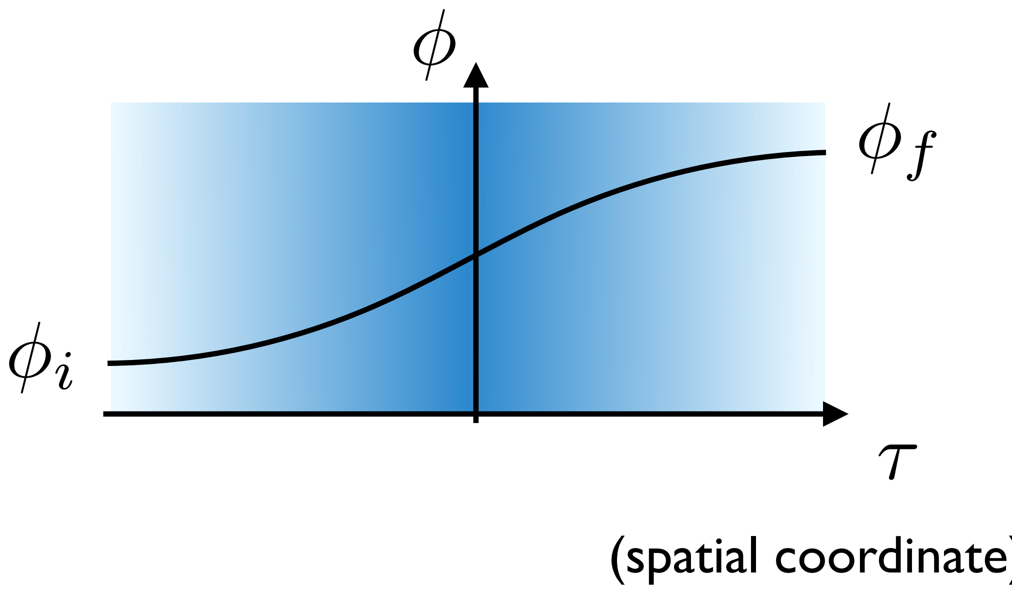
Any two d-dim theories are connected by a finite energy domain wall

[Mohseni,Montero,Vafa,IV'ongoing]

Distance between theories

Let us start with V=0 (a moduli space)





$$T_{DW} = \int (\partial_\tau \phi)^2 d\tau = \frac{(\Delta\phi)^2}{\Delta\tau} \to 0$$

minimized for $\phi(\tau) = \phi_0 + \sqrt{2\rho_E}\tau$

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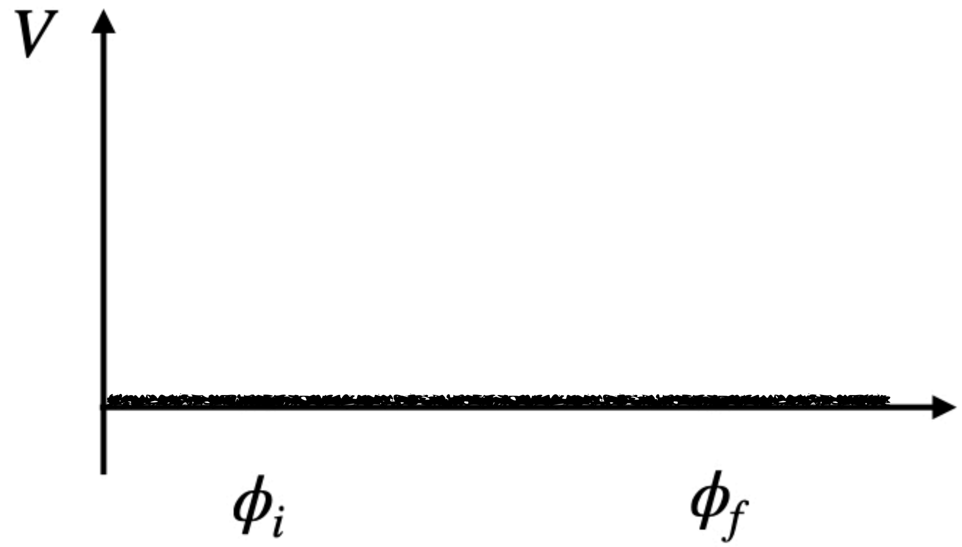

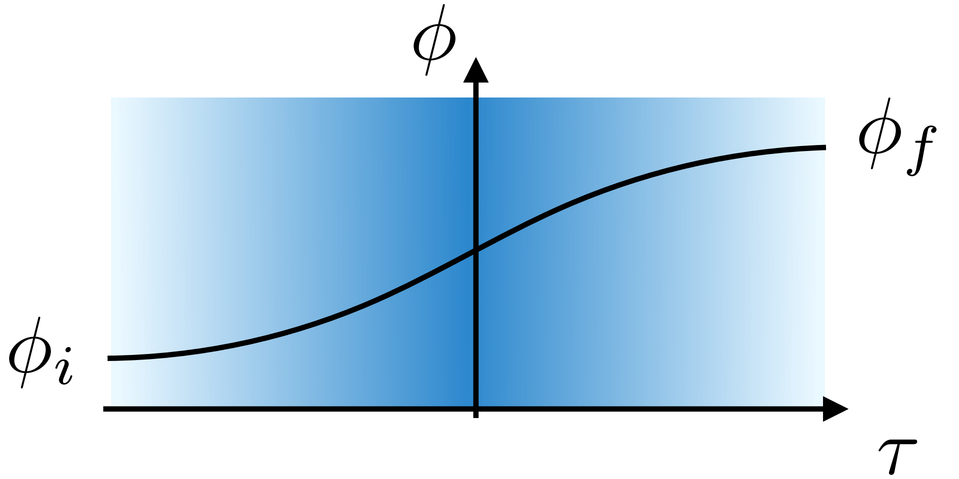

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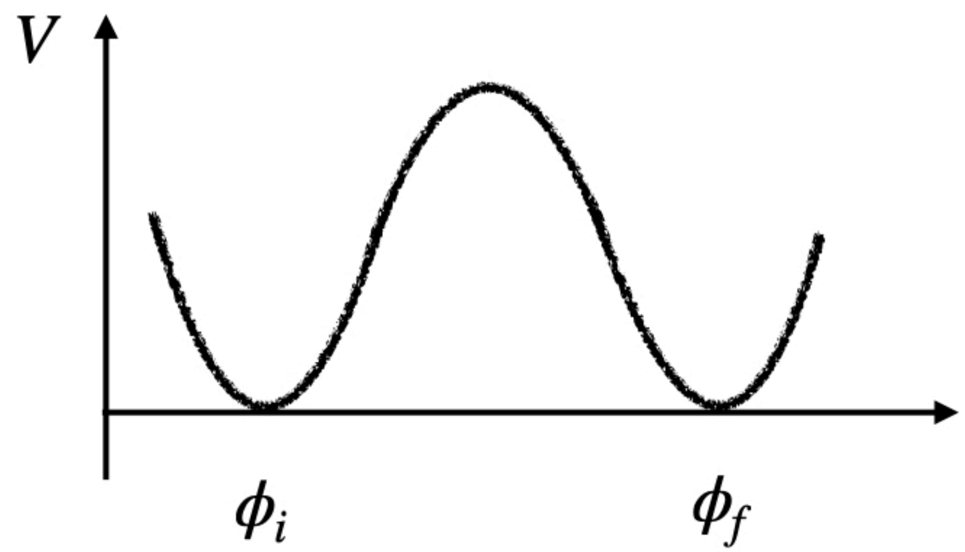
Define $\boxed{\Delta \equiv \frac{T_{DW}}{\sqrt{2\rho_E}} = \Delta\phi}$ recovers moduli space distance!

Distance between theories

Let us add a potential between two Minkowski vacua (ignoring gravitational effects):



$$\Delta = \frac{T(\rho_E)}{\sqrt{2\rho_E}} = \frac{1}{\sqrt{2\rho_E}} \int_{\phi_i}^{\phi_f} \sqrt{2(V+\rho_E)}d\phi$$

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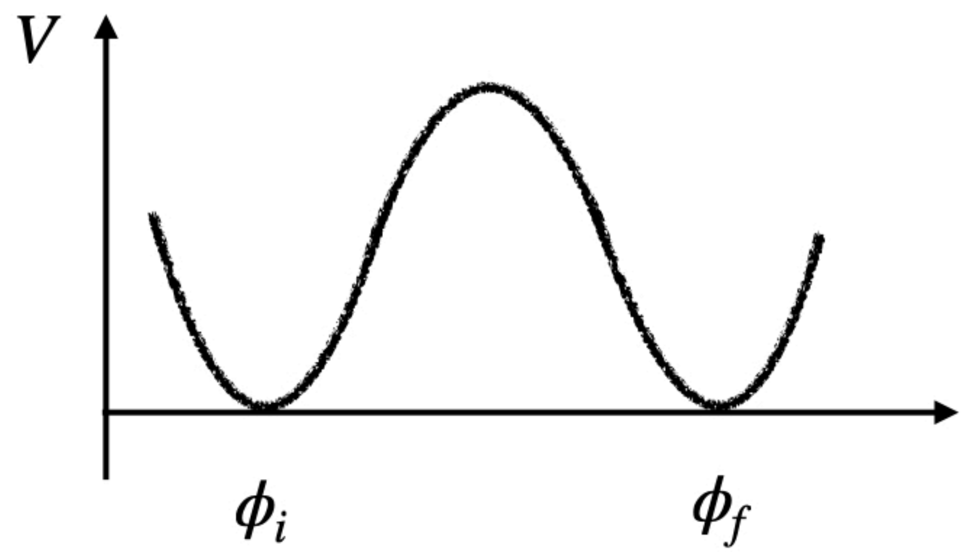

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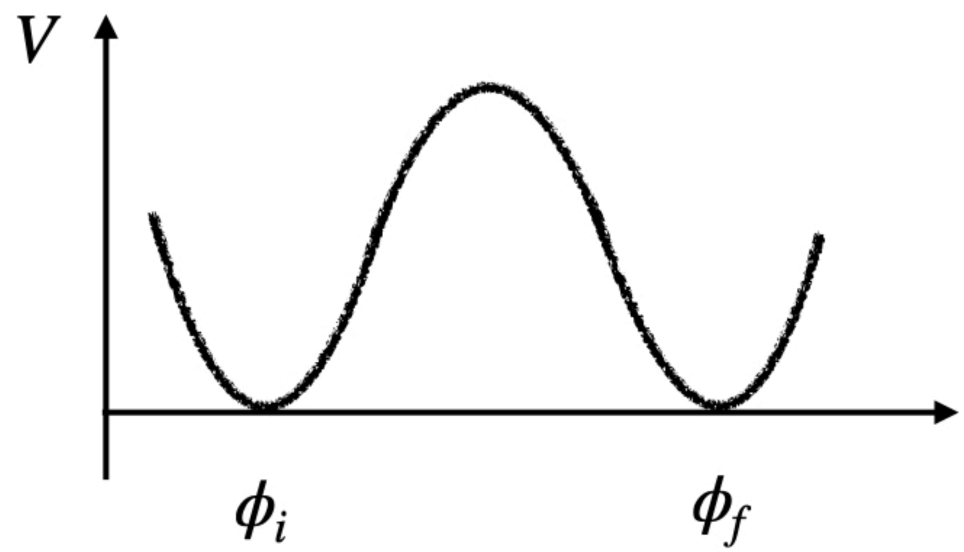

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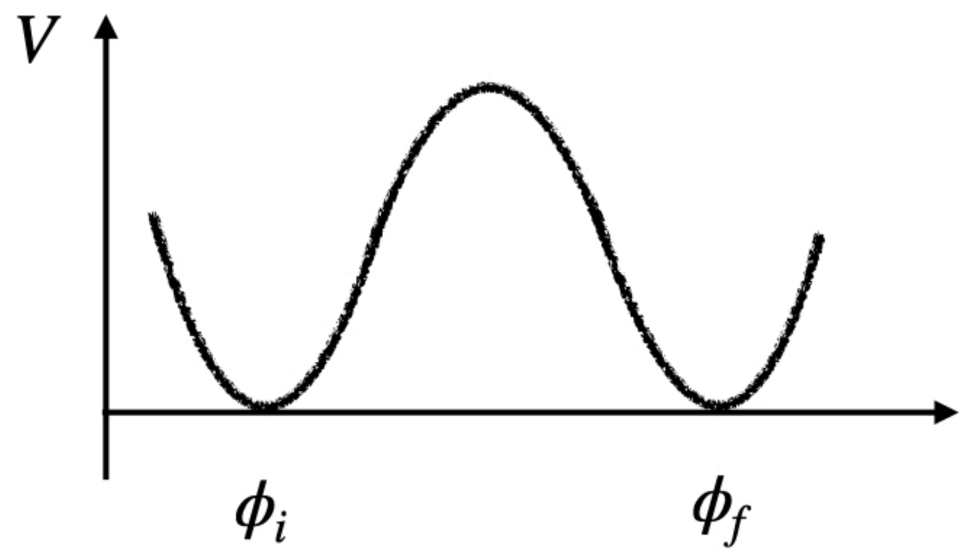

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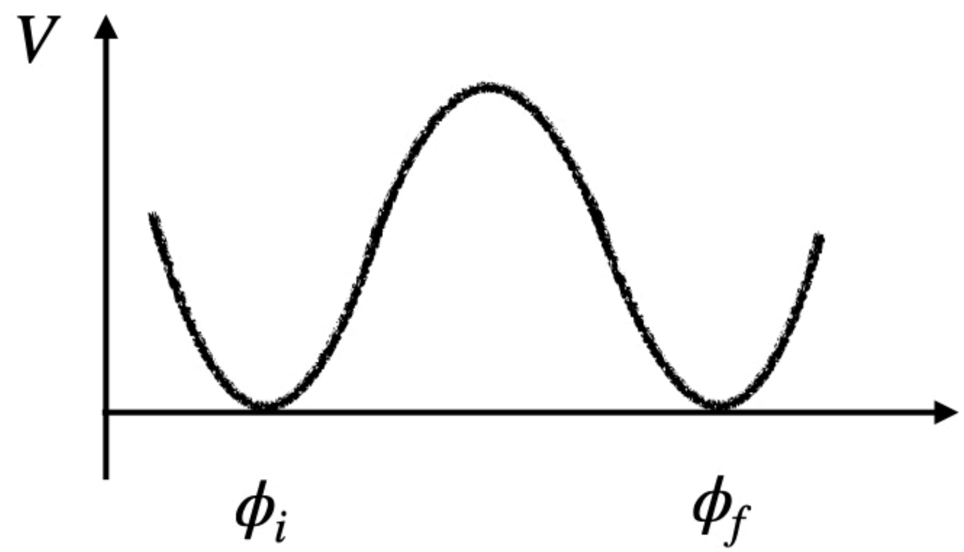

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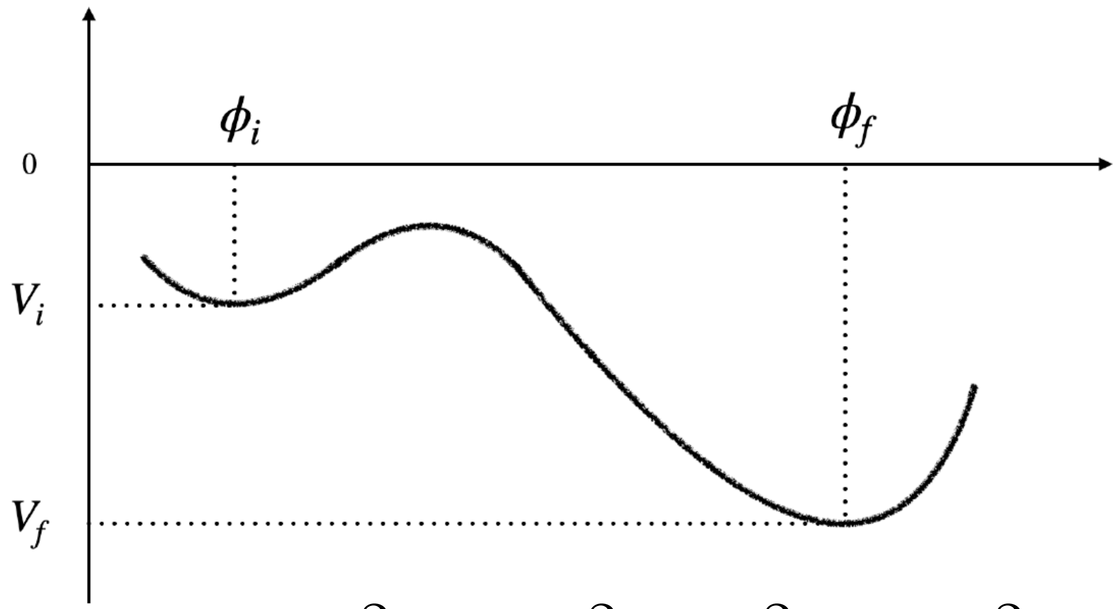
- It depends on the euclidean energy scale ρ_E such that: $\begin{cases} \text{If} \quad \rho_E \to \infty : \Delta \to \Delta\phi \\ \text{If} \quad \rho_E \to 0 : \Delta \to \frac{T_{DW}}{\sqrt{2\rho_E}} \to \infty \end{cases}$

Distance between theories

Let us include gravitational effects and consider e.g. two AdS vacua:



$$ds^2 = d\tau^2 + a^2(\tau)d\sigma^2$$

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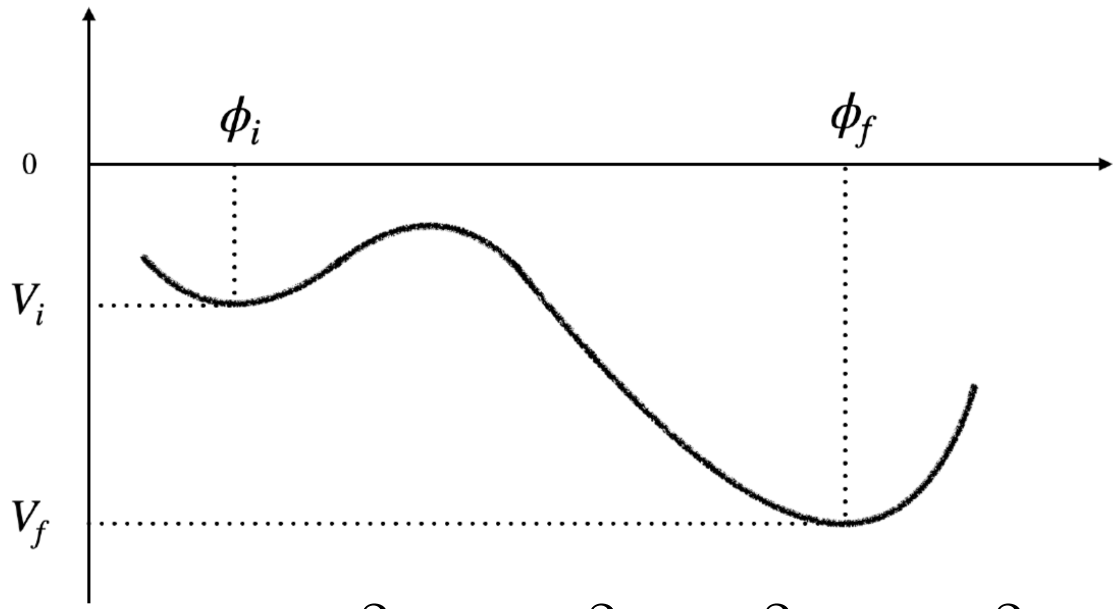

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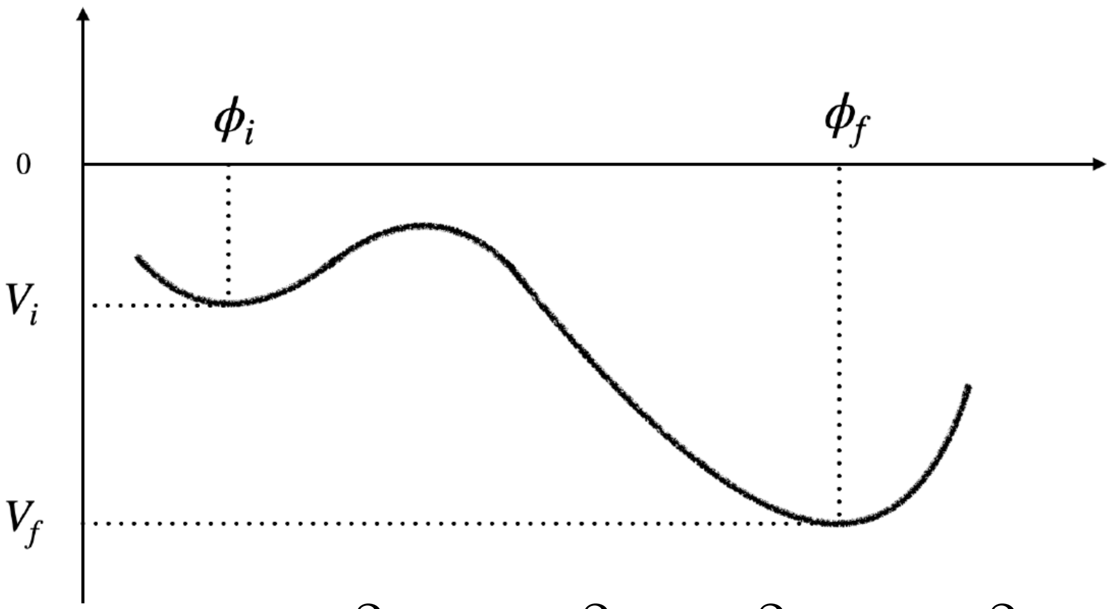

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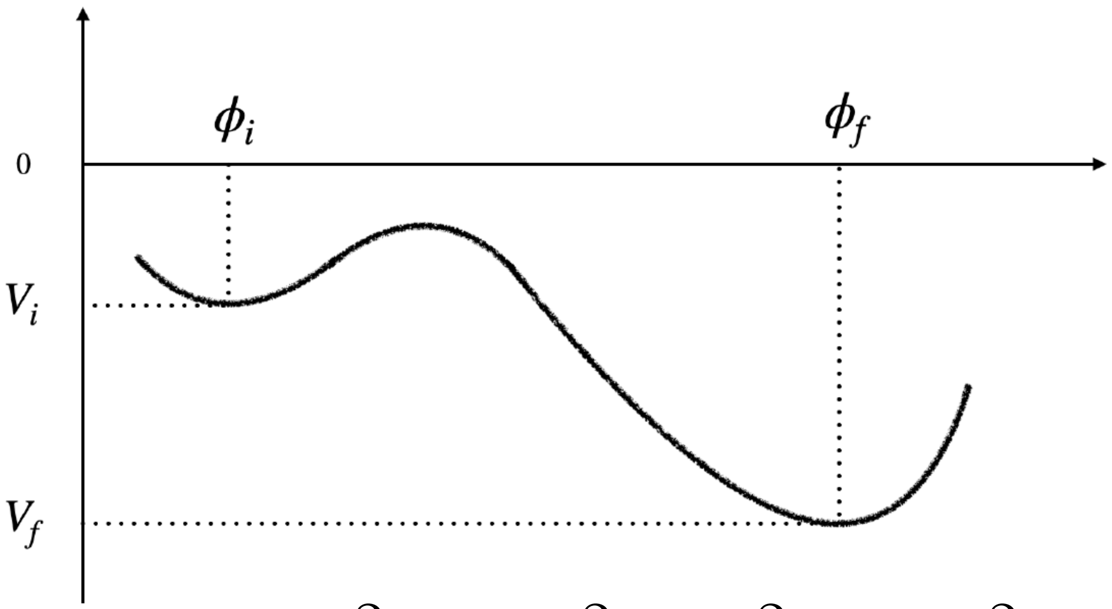

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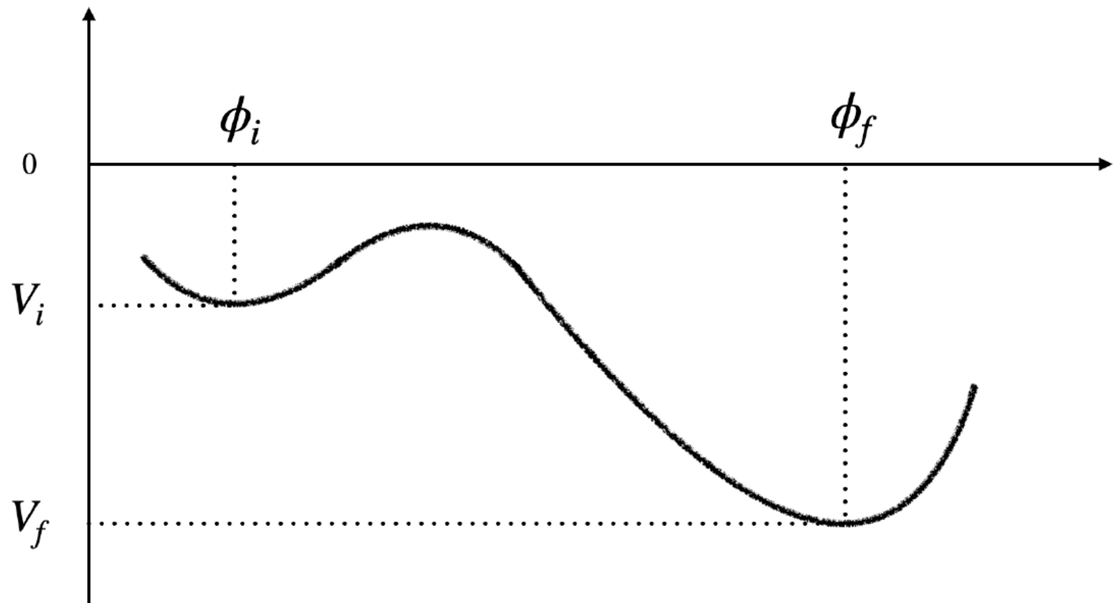


$$ds^2 = d\tau^2 + a^2(\tau) d\sigma^2$$

$$\Delta = \int_{\phi_i}^{\phi_f} \sqrt{1 - \frac{V(\phi)}{\Lambda(\phi)}} d\phi \;=\; \frac{1}{\sqrt{\alpha}} \log\left(\frac{\Lambda_f}{\Lambda_i}\right)$$

$$\frac{1}{2}\dot{\phi}^2 - V = \rho_E(\phi) \neq \text{const.} \qquad \alpha = 4\frac{d-1}{d-2}$$

$$= -\Lambda(\phi) \propto \frac{\dot{a}^2}{a^2}$$

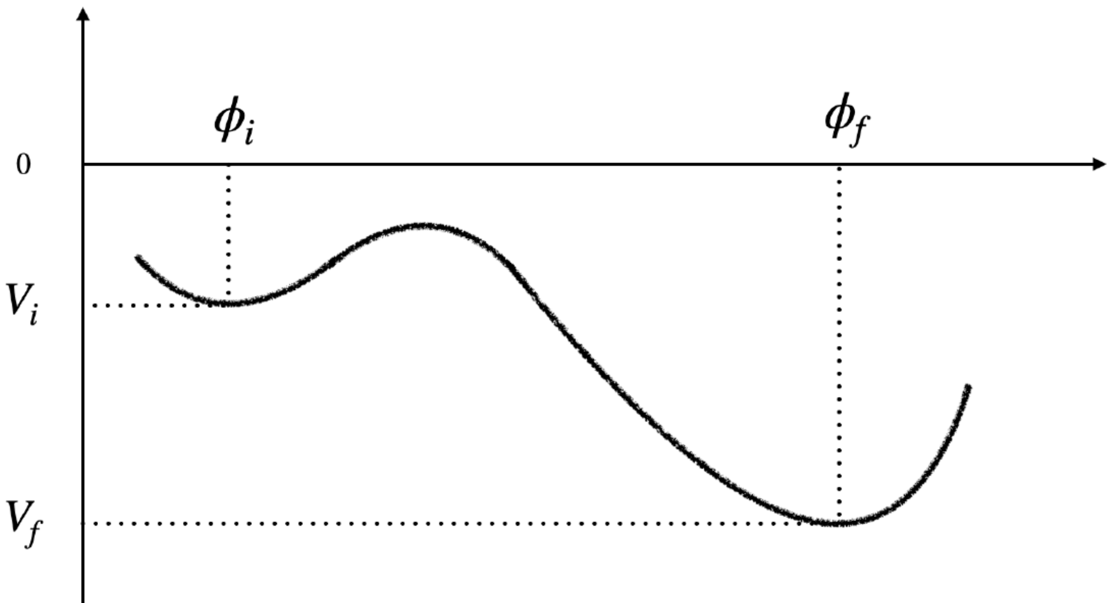
Euclidean Einstein eqs + eom for scalar: $\frac{\Lambda'}{\Lambda} = \sqrt{\alpha(1 - V/\Lambda)}$ with $\Lambda(\phi_i) = \Lambda_i \geq V(\phi_i)$

- It depends on the scale Λ_i
  - If $\Lambda_i \gg V$: $\Delta \to \Delta\phi$ recovers moduli space distance
  - If $\Lambda_{i,f} = V_{i,f}$: $\Delta \to \frac{1}{\sqrt{\alpha}} \log\left(\frac{V_f}{V_i}\right)$ recovers AdS distance

[Luest,Palti,Vafa'19]

Distance between theories

Let us include gravitational effects and consider e.g. two AdS vacua:



$$ds^2 = d\tau^2 + a^2(\tau)d\sigma^2$$

$$\Delta = \int_{\phi_i}^{\phi_f} \sqrt{1 - \frac{V(\phi)}{\Lambda(\phi)}} d\phi = \frac{1}{\sqrt{\alpha}} \log\left(\frac{\Lambda_f}{\Lambda_i}\right)$$

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[Luest,Palti,Vafa'19]

- Not a distance in the mathematical sense

(unless in the SUSY case where there is a DW solution with $\Lambda_{i,f} = V_{i,f}$)

Conclusions

- The microscopic nature of the tower constrains the value of the exponential rate of the towers and how different limits fit together in moduli space

- We start a classification of infinite distance limits in the conformal manifold of 4d SCFTs, obtaining non-critical tensionless string limits

- The Convex Hull Distance Conjecture with $\alpha \geq \frac{1}{\sqrt{d-2}}$ in holographic AdS spaces (with a conformal manifold) if they are not scale separated

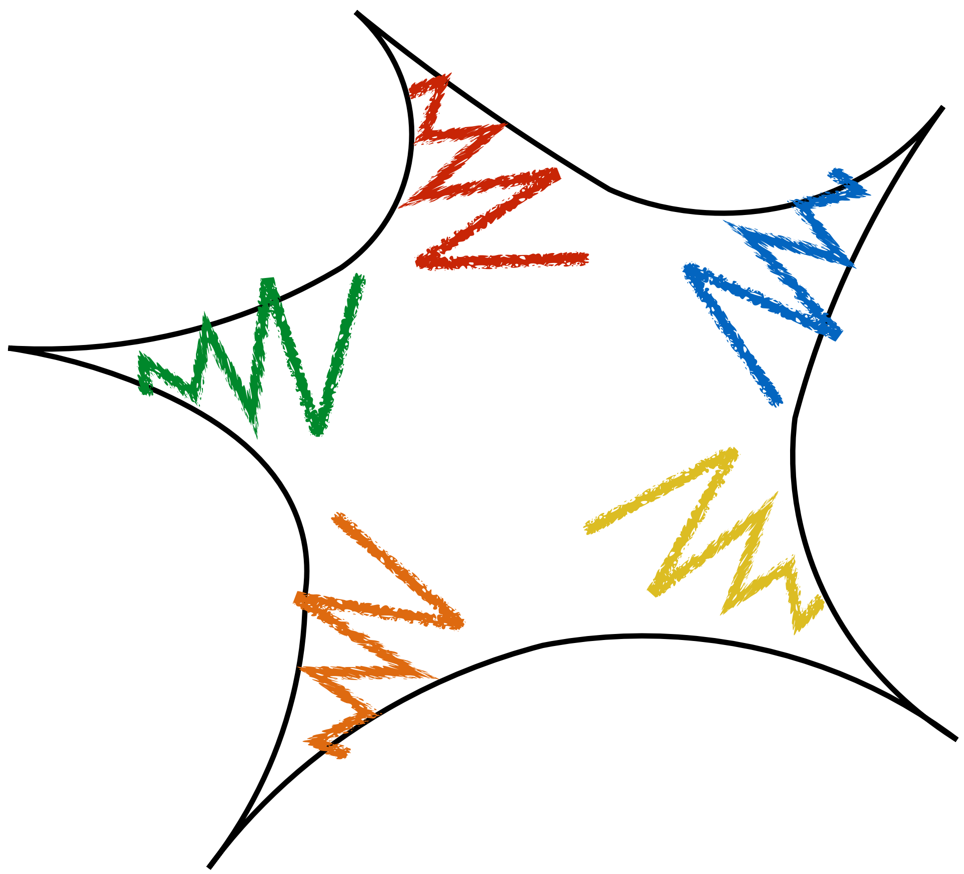
- We propose a notion of distance in the presence of a scalar potential inspired by the cobordism domain wall

Thank you!

back-up slides

Dualities

Different regions of the moduli space described by different perturbative descriptions related by dualities



tower of states becoming all light

m_n : m_2 m_1

$$m(P) \sim m(Q) e^{-\alpha \Delta \phi}$$

(Distance conjecture)

[Ooguri-Vafa'06]

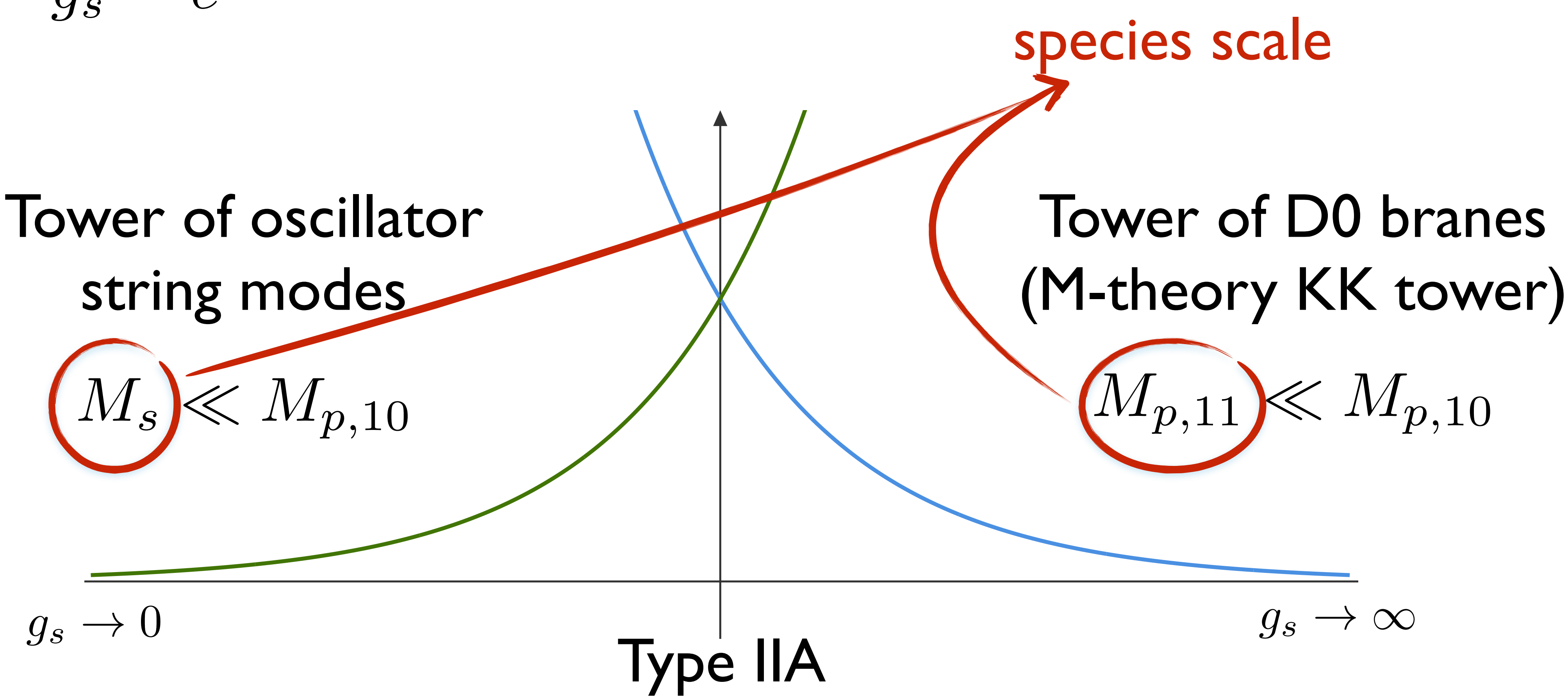
What are the possible descriptions?

How can they be combined? What are the possible dualities?

Example: 10d Type IIA

One dimensional moduli space: $S = M_p^2 \int d^{10}x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{4}(\partial\phi)^2 + \dots\right)$

$g_s = e^{-\phi}$



As $g_s \to 0$ $\quad M_s = M_{p,10}\, g_s^{1/4} = M_{p,10} \exp\left(-\frac{1}{\sqrt{8}}\Delta\phi\right)$

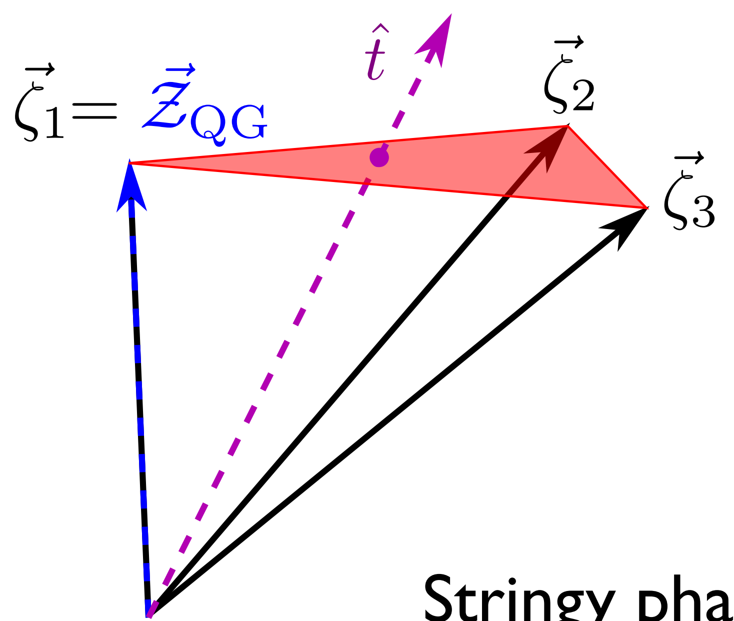
As $g_s \to \infty$ $\quad m_{\mathrm{KK}} = \frac{M_s}{g_s} = M_{p,10} \exp\left(-\frac{3\sqrt{2}}{4}\Delta\phi\right) \; ; \quad M_{p,11} \sim m_{\mathrm{KK}}^{1/8}$

Frame simplex

The towers generate a frame simplex (convex hull of the tower vectors) to which the species scale is orthogonal

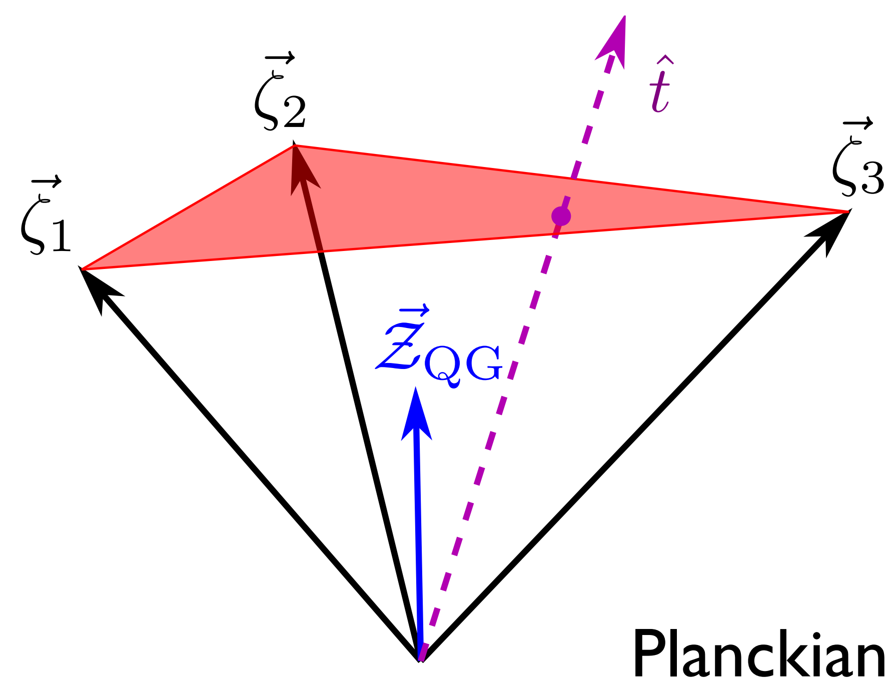
$$\vec{\zeta}_i \equiv \frac{\vec{\nabla} m_i}{m_i}$$

$$\vec{\mathcal{Z}}_{\rm QG} \equiv \frac{\vec{\nabla}\Lambda_{\rm QG}}{\Lambda_{\rm QG}}$$



Stringy phase

$\Lambda_{\rm QG} = M_s$



Planckian phase

$\Lambda_{\rm QG} = M_{p,D}$

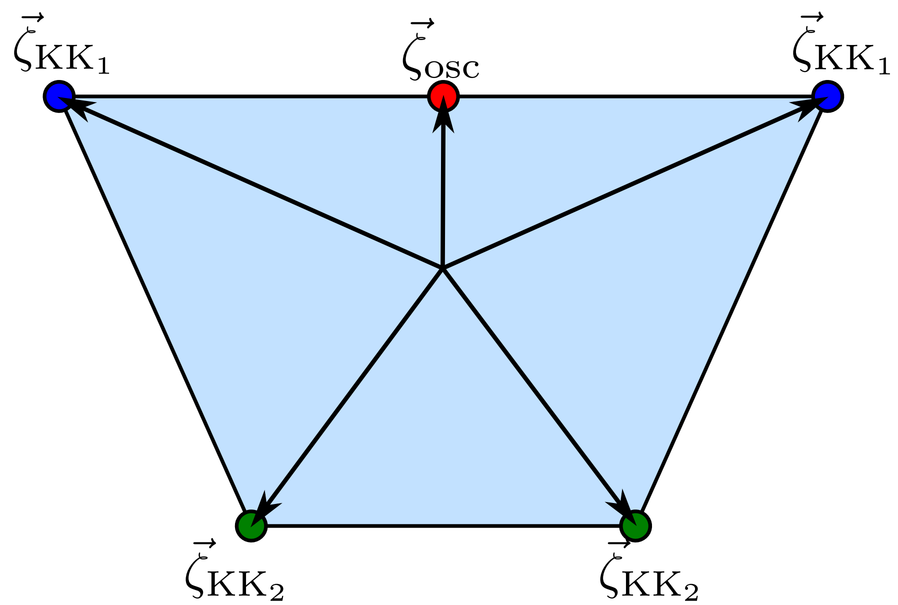
The geometry is rigid under variations of the direction of the infinite distance limit

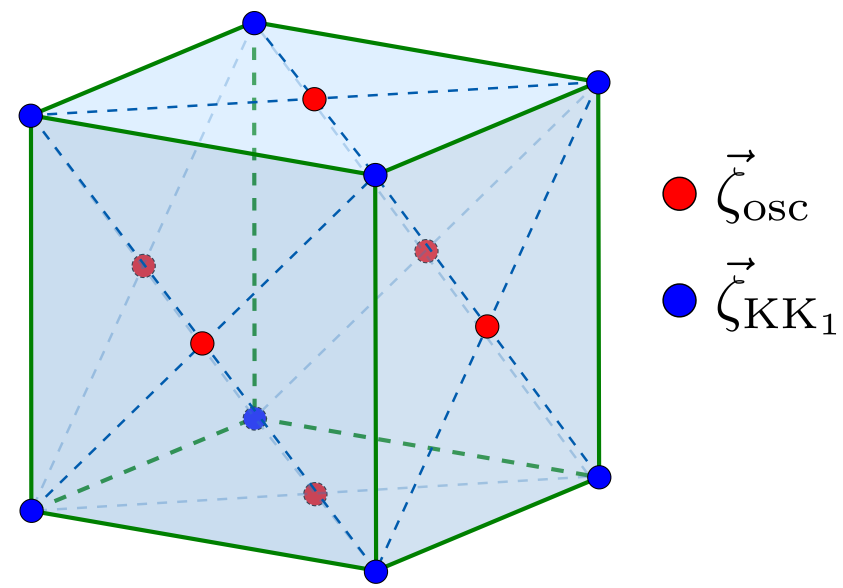
(as long as we do not break the assumption of regularity and we stay inside the cone)

We can associate a frame simplex to each duality frame

Tower polytopes

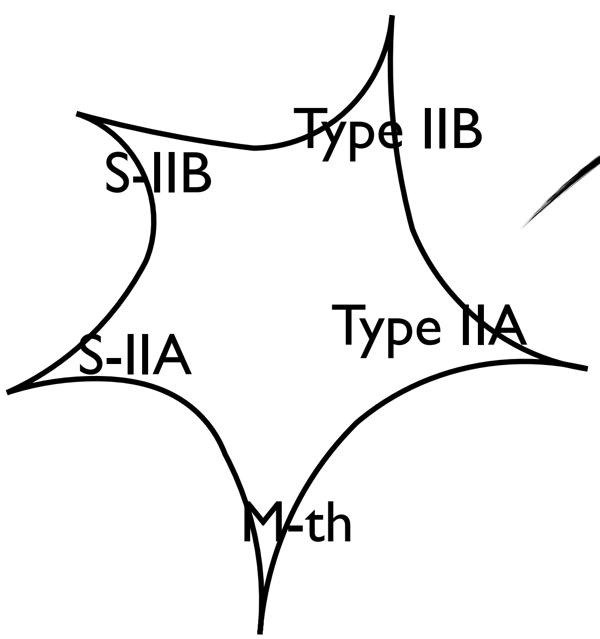
In certain cases (e.g. if there is an asymptotically flat slice of the moduli space) we can combine duality frames by gluing individual frame simplices to form a full polytope

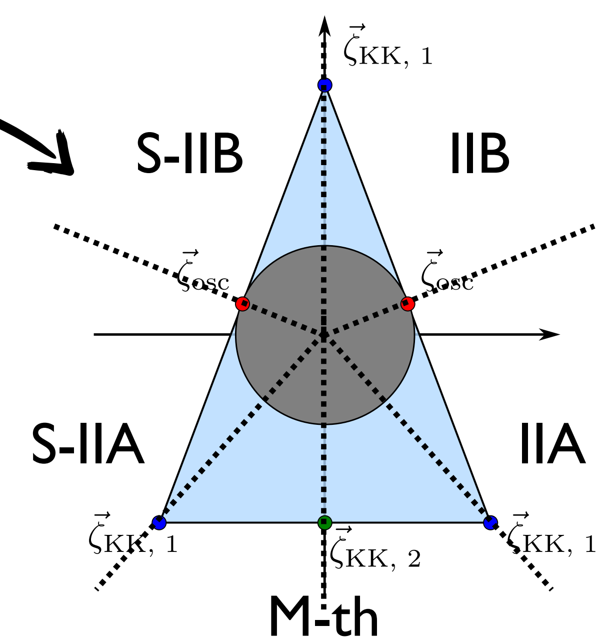




For example:

M-theory on T^2



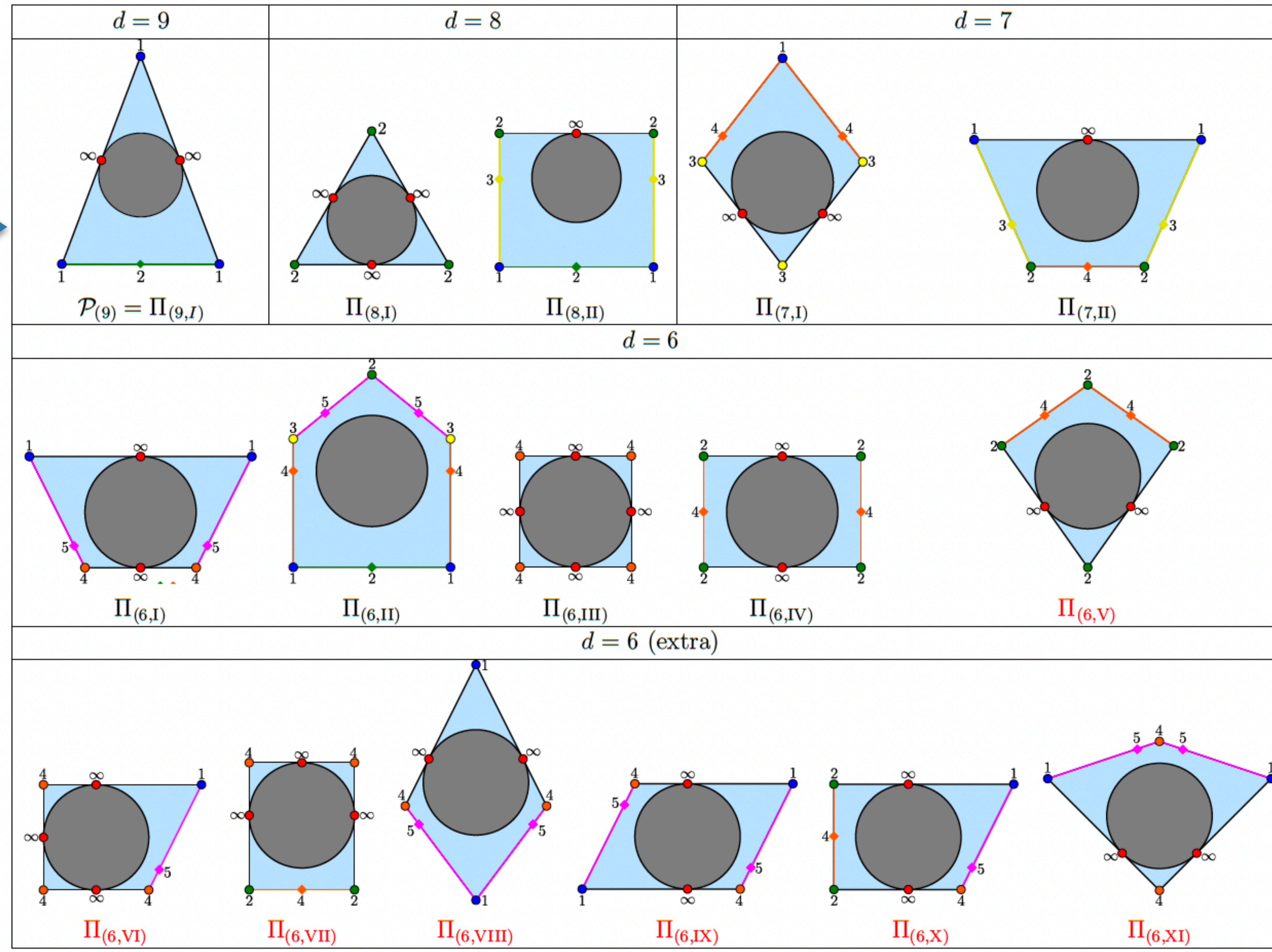


Classification of 2d polytopes

This puts constraints on the structure of dualities of the moduli space

Example:

Classification of 2d polytopes (with $D_{\max} = 11$)



The sharpened DC is satisfied

$$\alpha \geq \frac{1}{\sqrt{d-2}}$$

[Etheredge et al'22]

following the Convex Hull Distance Conjecture

[Calderon-Infante,Uranga,IV'20]

Many arise in string theory but others are new!

(similar story for species polytopes)

Classification of Infinite Distance Limits in d>2

Let us focus on AdS_{d+1}/CFT_d with $d > 2$

[Perlmutter,Rastelli,Vafa,IV'21]
(see also [Baume,Calderon-Infante'21])

In the free limit $g_{YM} \to 0$ $\longrightarrow$ $\mathcal{O}_\tau = \mathrm{Tr}(F^2 + \dots)$ $\quad \tau = \frac{4\pi i}{g_{YM}^2} + \frac{\theta}{2\pi}$

By perturbation theory:

$$ds^2 = \beta^2 \frac{d\tau d\bar{\tau}}{(\mathrm{Im}\tau)^2} \quad \text{as} \quad \mathrm{Im}\tau \to \infty \qquad \beta^2 = 24\ \mathrm{dim}G$$

gauge group getting free

$$\gamma_J \sim f(J) g_{YM}^2 \sim f(J) \exp\left(-\frac{d(\tau,\tau')}{\beta}\right)$$

If there is a weakly coupled AdS dual, it implies:

Infinite distance limits at fixed AdS$_5$ radius $\Longleftrightarrow$ Tower of higher spin fields with an exponential rate:

$$\alpha = \sqrt{\frac{2c}{\mathrm{dim}G}} \quad \begin{aligned} &\geq \frac{1}{\sqrt{3}} \quad \text{for 4d N=2} \\ &\geq \frac{1}{2} \quad \text{for 4d N=1} \end{aligned}$$

Lower bound for α !

Classification of Infinite Distance Limits in d>2

[Bhardwaj, Tachikawa'13] [Razamat, Sabag, Zafrir'20]

Consider the full classification of 4d SCFTs with large N and simple factor for the gauge group $G = SU(N), USp(2N), SO(N)$

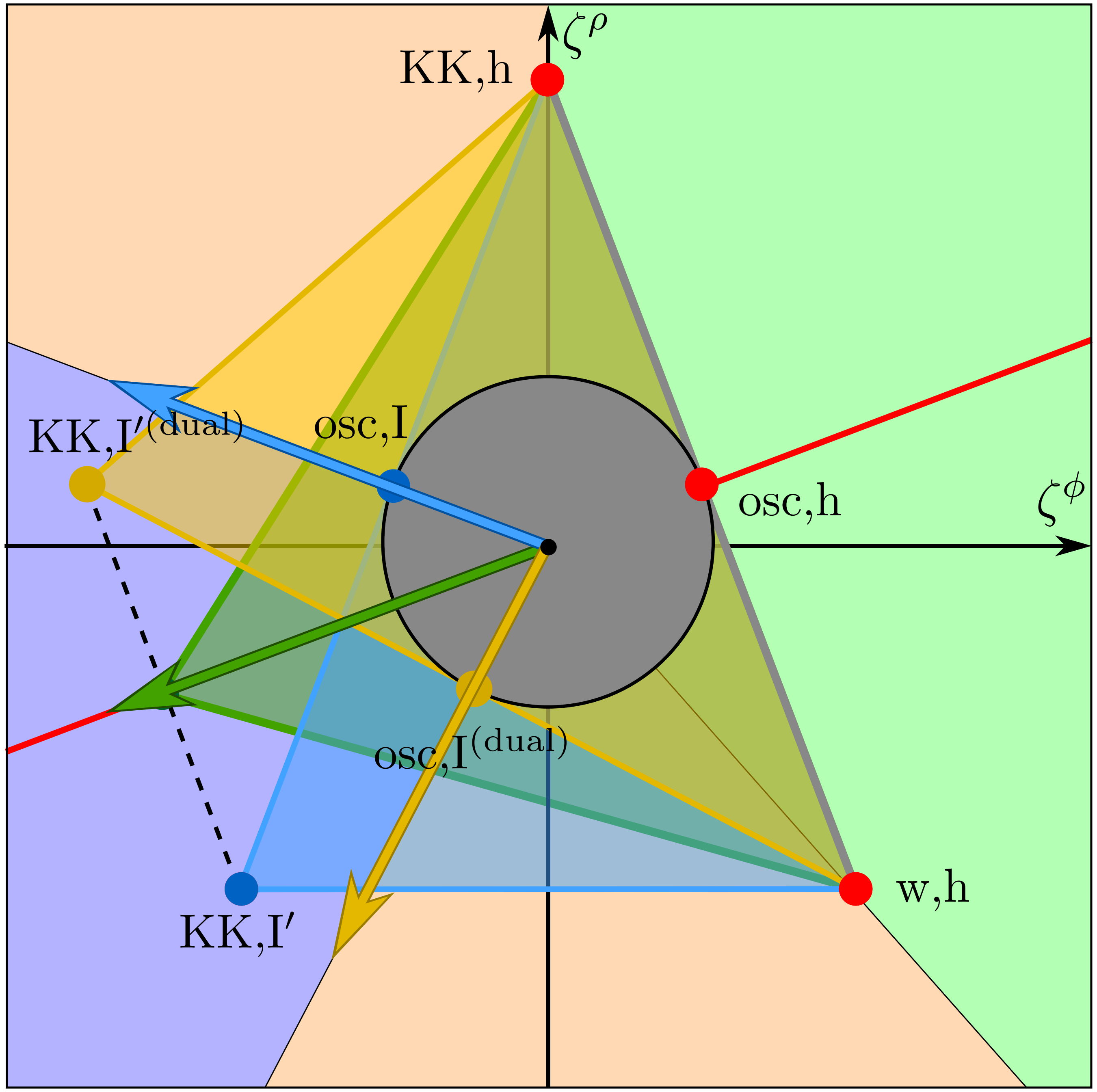
$\mathcal{N}=2$

G	Hypermultiplets	c	α
$SU(N)$	$2N$ **fund**	$\frac{1}{6}(2N^2-1)$	$\sqrt{\frac{2}{3}}$
$SU(N)$	1 **asym**, $N+2$ **fund**	$\frac{1}{24}(7N^2+3N-4)$	$\sqrt{\frac{7}{12}}$
$SU(N)$	2 **asym**, 4 **fund**	$\frac{1}{12}(3N^2+3N-2)$	$\frac{1}{\sqrt{2}}$
$SU(N)$	1 **asym**, $N-2$ **fund**	$\frac{1}{24}(7N^2-3N-4)$	$\sqrt{\frac{7}{12}}$
$SU(N)$	1 **sym**, 1 **asym**	$\frac{1}{12}(3N^2-2)$	$\frac{1}{\sqrt{2}}$
$USp(2N)$	$4N+4$ $\frac{1}{2}$ **fund**	$\frac{1}{6}N(4N+3)$	$\sqrt{\frac{2}{3}}$
$USp(2N)$	1 **asym**, 4 **fund**	$\frac{1}{12}(6N^2+9N-1)$	$\frac{1}{\sqrt{2}}$
$SO(N)$	$N-2$ **vect**	$\frac{1}{12}N(2N-3)$	$\sqrt{\frac{2}{3}}$

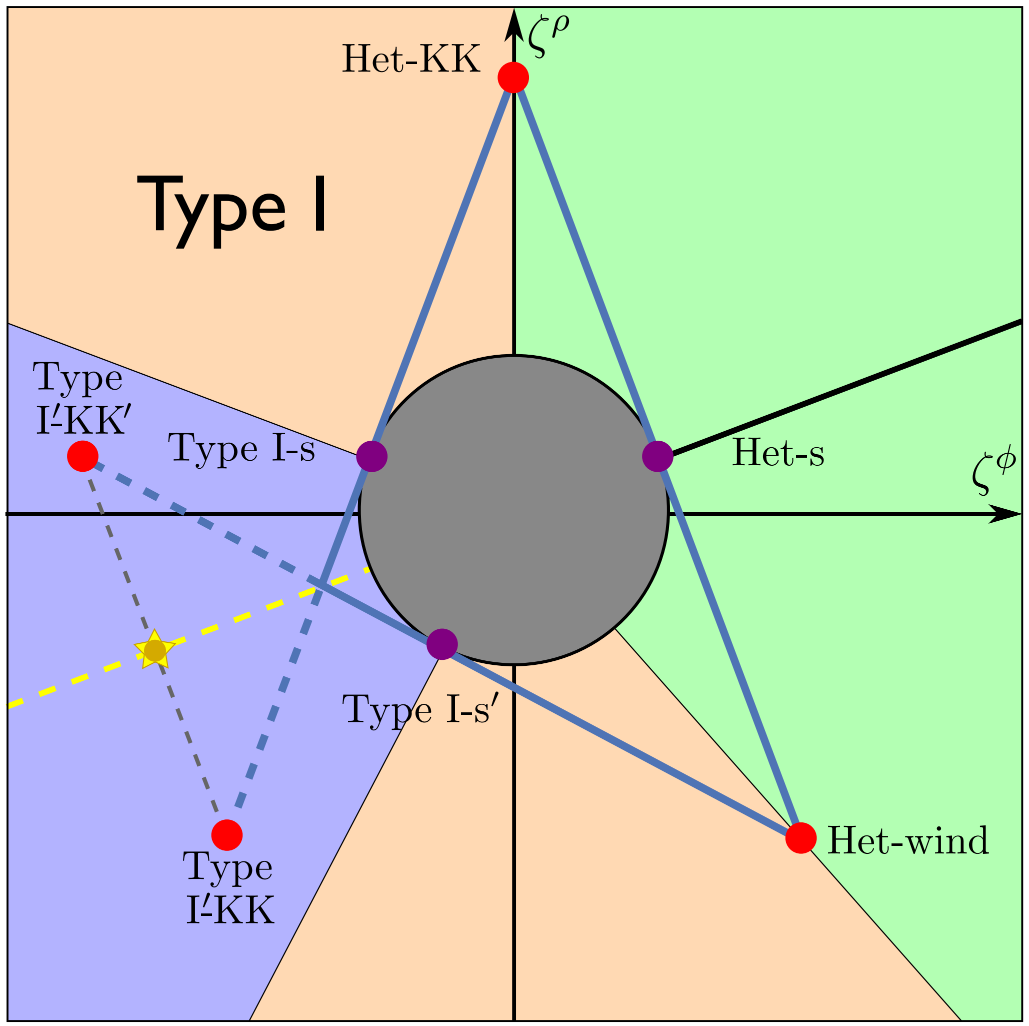
$\mathcal{N}=1$

G	Theory	c	α
$SU(N)$	Table 2, #1	$\frac{1}{24}(7N^2-5)$	$\sqrt{\frac{7}{12}}$
$SU(N)$	Table 2, #5	$\frac{1}{24}(6N^2+3N-5)$	$\frac{1}{\sqrt{2}}$
$SU(N)$	Table 3, #4	$\frac{1}{24}(7N^2-4)$	$\sqrt{\frac{7}{12}}$
$SU(N)$	Table 5, #4	$\frac{1}{24}(8N^2-3)$	$\sqrt{\frac{2}{3}}$
$USp(2N)$	Table 12, #1	$\frac{1}{24}(14N^2+15N-1)$	$\sqrt{\frac{7}{12}}$
$USp(2N)$	Table 13, #9	$\frac{1}{8}(4N^2+8N-1)$	$\frac{1}{\sqrt{2}}$
$USp(2N)$	Table 13, #10	$\frac{1}{24}(14N^2+21N-2)$	$\sqrt{\frac{7}{12}}$
$SO(N)$	Table 18, #1	$\frac{1}{48}(7N^2-21N-4)$	$\sqrt{\frac{7}{12}}$
$SO(N)$	Table 18, #2	$\frac{1}{48}(7N^2-15N-2)$	$\sqrt{\frac{7}{12}}$
$SO(N)$	Table 18, #3	$\frac{1}{24}(4N^2-9N-1)$	$\sqrt{\frac{2}{3}}$

Example of running decompactification



SO(32) slice



E8xE8 slice