A Minimal Weak Gravity Conjecture

work with Cesar Cota, Alessandro Mininno, Max Wiesner

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see also talk by
A. Mininno on Thursday

Timo Weigand, String Phenomenology 2024, Padova

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The problem in a nutshell

Tower Weak Gravity Conjecture (WGC):

Existence of tower of super-extremal states in every direction of charge lattice.

☑ Observed to hold in many frameworks of string/M-theory.

☑ Motivated by consistency of ordinary WGC under KK reduction.
[Heidenreich,Reece,Rudelius'15/16]

Problem: In various instances, no obvious / known / controlled WGC tower is in sight.

Key example: Conifold in M-theory on CY_3:

Direction in charge lattice with only a single BPS state ⊗

$\Longrightarrow$ at least no (super-)extremal BPS tower available

Two types of super-extremal towers:

particle towers

Necessary for existence:

Asymptotically weak coupling

- ☑ super-extremal (non-)BPS towers (essentially) proven in all known frameworks of string/M-theory
- ☑ guaranteed by Emergent String Conjecture

Asymptotically strong coupling

- ☑ super-extremal BPS towers can be proven to exist in suitable frameworks -

but not generally!

black hole towers

- ☑ observed/confirmed in M-theory on CY3 for directions where BPS = extremality

but not generally!

Two types of super-extremal towers:

particle towers | **black hole towers**

not generally!

Criterion
for existence:

Asymptotically weak coupling

- ☑ super-extremal (non-)BPS towers (more or less) proven in all known frameworks
- ☑ guaranteed by Emergent String Conjecture

Asymptotically strong coupling

not generally!

When are towers needed by consistency with KK reduction?

What happens in cases where no tower is known (yet)?

Main results

[Cota,Mininno,TW,Wiesner'23]

1. Consistency of WGC under KK reduction does not generically require a super-extremal tower.

2. Super-extremal towers are required by KK consistency only

   - for asymptotically weakly coupled gauge groups coupled to gravity - i.e. for KK U(1)s or perturbative heterotic gauge groups (up to duality)

   - for directions where BPS = extremality. *cf [Alim,Heidenreich,Rudelius'21]*

   Uses: Properties of species scale and Emergent String Conjecture

3. In all known cases where no super-extremal tower is known (yet), this is not in contradiction with KK reduction: **WGC well and alive!**

(Super)-extremal towers are required by KK consistency only

- for asymptotically weakly coupled gauge groups coupled to gravity.
- for directions where BPS = extremality.

motivates

conjectural part

Minimal Tower Weak Gravity Conjecture

Super-extremal towers of particle states are present if and only if they are required by consistency of the WGC under KK reduction.

Tower Weak Gravity Conjecture

[Arkani-Hamed,Motl,Nicolis,Vafa'06]
[Heidenreich,Reece,Rudelius'15]
[Montero,Shiu,Soler'16]
[Andriolo,Junghans,Noumi,Shiu'18]

A U(1) gauge theory coupled to quantum gravity possesses a tower of infinitely many super-extremal states of arbitrarily high charges:

$$\boxed{\frac{g_{\mathrm{U(1)}}^2 q^2}{m_D^2} \geq \gamma \frac{1}{M_{Pl,D}^{D-2}}}$$

γ : determined by extremality bound for black holes

i) tower of super-extremal particles:

In asymptotic region $\mathscr{M}$ of moduli space: $\exists$ super-extremal particle with $m_n \leq M_{\mathrm{BH,min}}$ and charge $n\,q$ $\forall n \in \mathscr{I}_q$ $\longleftarrow$ infinite set

ii) tower of super-extremal states at/above BH threshold

Tower Weak Gravity Conjecture: Particle version

Light super-extremal tower:

$$\boxed{\frac{g^2_{\mathrm{U}(1)}\, q^2}{m_D^2} \geq \gamma \frac{1}{M_{Pl,D}^{D-2}}}$$

i) $g_{\mathrm{U}(1)} \to 0$

ii) $\gamma \to \infty$

[Cota,Mininno,TW,Wiesner'23]

Two possibilities (necessary conditions):

i) Asymptotic weak coupling limit

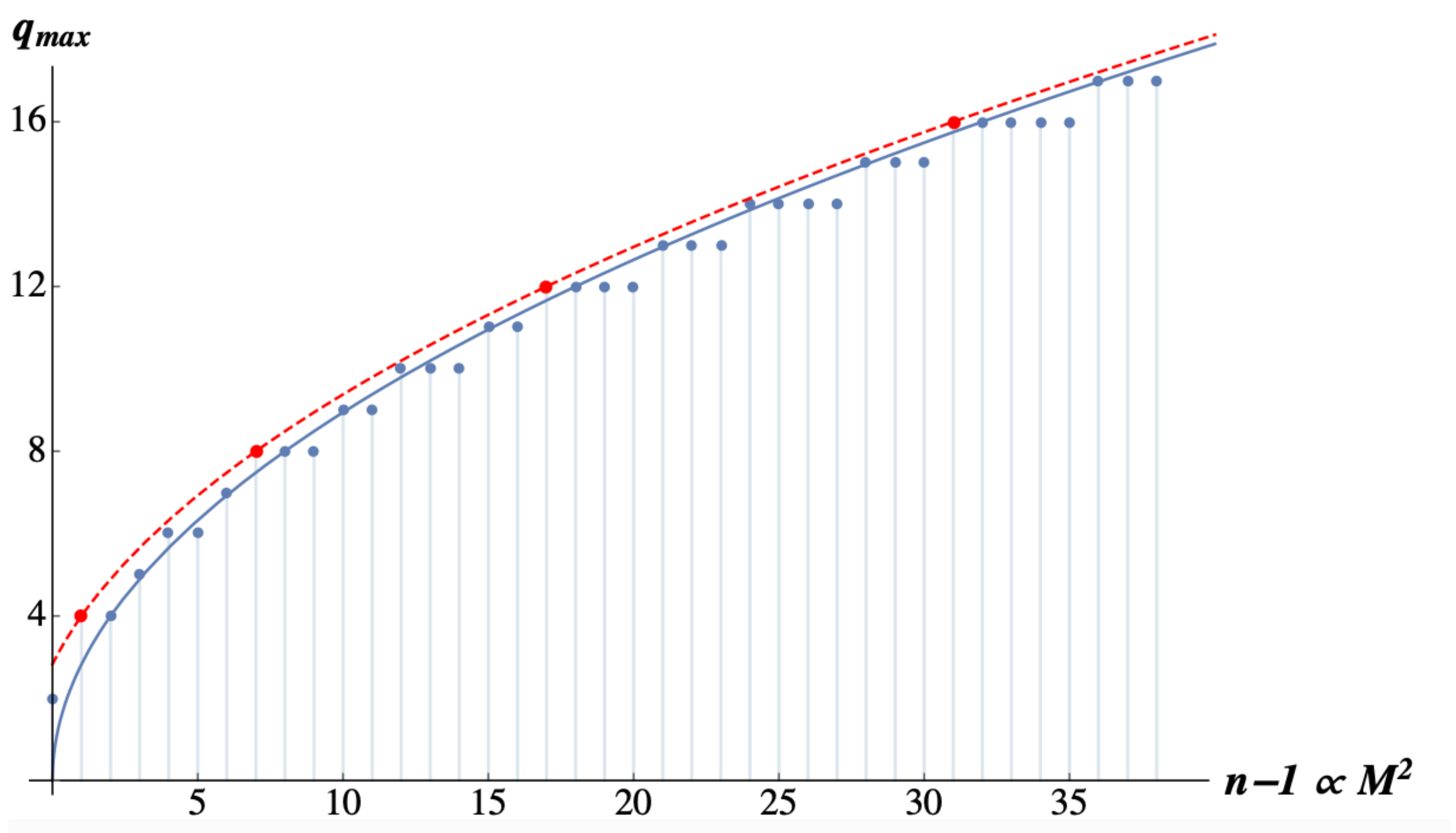
$$g^2_{\mathrm{U}(1)} M^{D-4}_{\mathrm{Pl,D}} \to 0 \quad \text{and} \quad \frac{g^2_{\mathrm{U}(1)} M^{D-2}_{\mathrm{Pl,D}}}{M^2_{\mathrm{Pl},\infty}} \to 0$$

Planck mass of asymptotic theory at infinite distance / asymptotic weak coupling

ii) Strong coupling limit at finite distance

$$g^2_{\mathrm{U}(1)} M^{D-4}_{\mathrm{Pl,D}} \to \infty \quad \text{such that} \quad \gamma \to \infty$$

Tower Weak Gravity Conjecture: Tests



1) Particle towers at asymptotic weak coupling

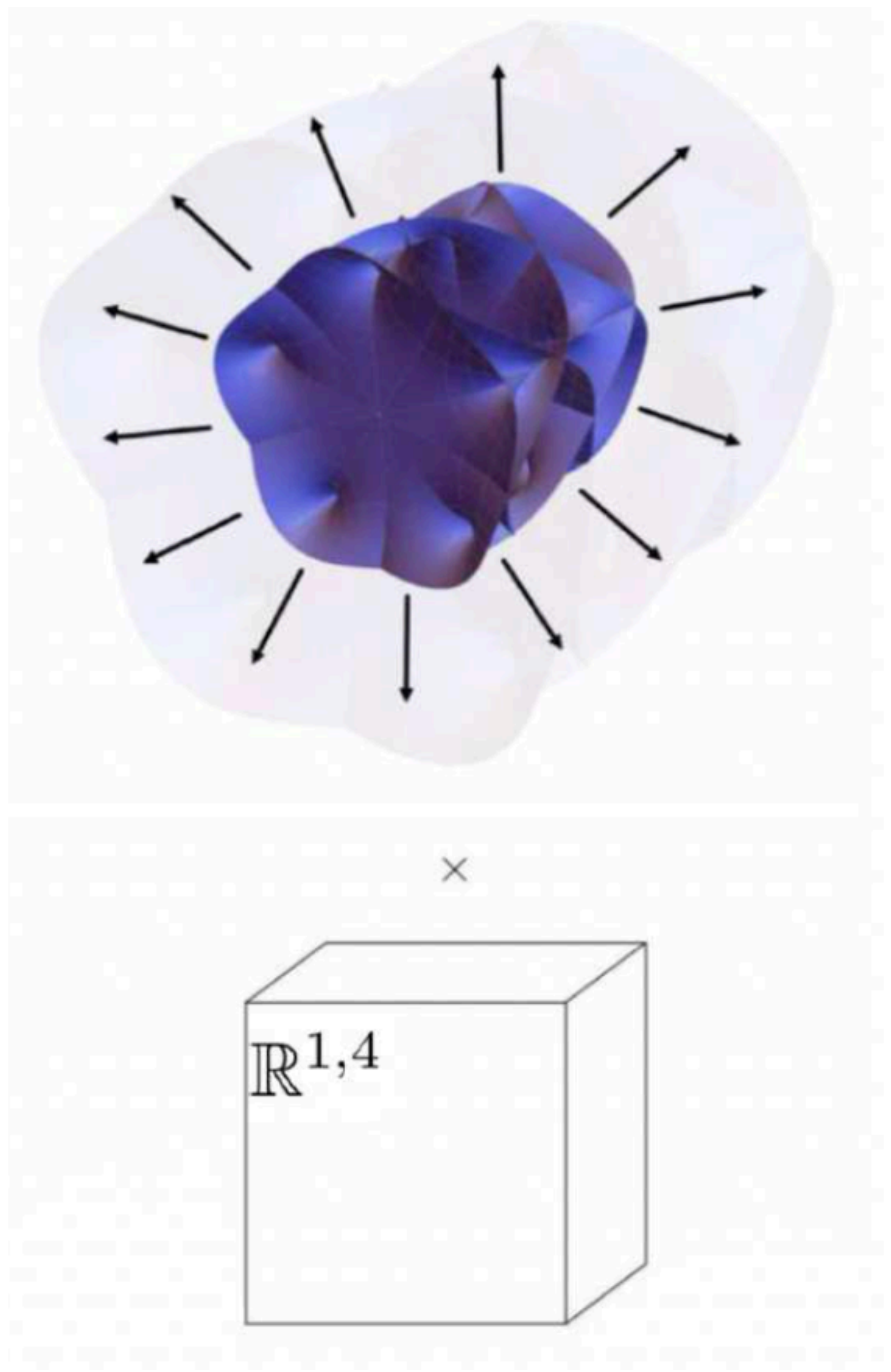
KK towers in (dual) decompactification limit:

- **KK U(1)s** *[Heidenreich,Reece,Rudelius'15]*
- **Type IIB on CY 3-fold in asymptotic complex structure regions** *[Grimm,Palti,Valenzuela'18] [Bastian,Grimm,Heisteeg'20] [Gendler,Valenzuela'21]*
- **M-theory on CY 3-fold in weakly coupled gauge sector** *[Lee,Lerche,TW'19] [Cota,Mininno,TW,Wiesner'22,23]*

String excitation towers:

- **Perturbative heterotic** *[AMNV'06] [Heidenreich,Reece,Rudelius'15]*
- **Closed perturbative bosonic** *[Heidenreich,Lotito'24]*
- **General F-theory in weakly coupled gauge sector** *[Lee,Lerche,TW'18,'19] [Kläwer,Lee,TW,Wiesner'20]*
- **M-theory on CY3 in weakly coupled gauge sector** *[Cota,Mininno,TW,Wiesner'22,23]*

Tower Weak Gravity Conjecture: Particle version



M-theory on CY X_3: $U(1)_\Sigma = \int_\Sigma C_3 \qquad \Sigma \in H_2(X_3)$

$$g_\Sigma^2 M_{\rm Pl} = \Sigma_\alpha f^{\alpha\beta}\Sigma_\beta\,, \qquad f_{\alpha\beta} = \int_{X_3} J_\alpha \wedge *J_\beta\,, \qquad \Sigma = \Sigma_\alpha C^\alpha \qquad \int_{C^\alpha} J_\beta = \delta^\alpha_\beta$$

Necessary for weak coupling: $g_\Sigma^2 M_{\rm Pl} \to 0 \quad \Longleftrightarrow$ Limit with infinite divisor volume at finite volume of X_3 *[Lee,Lerche,TW'19]*

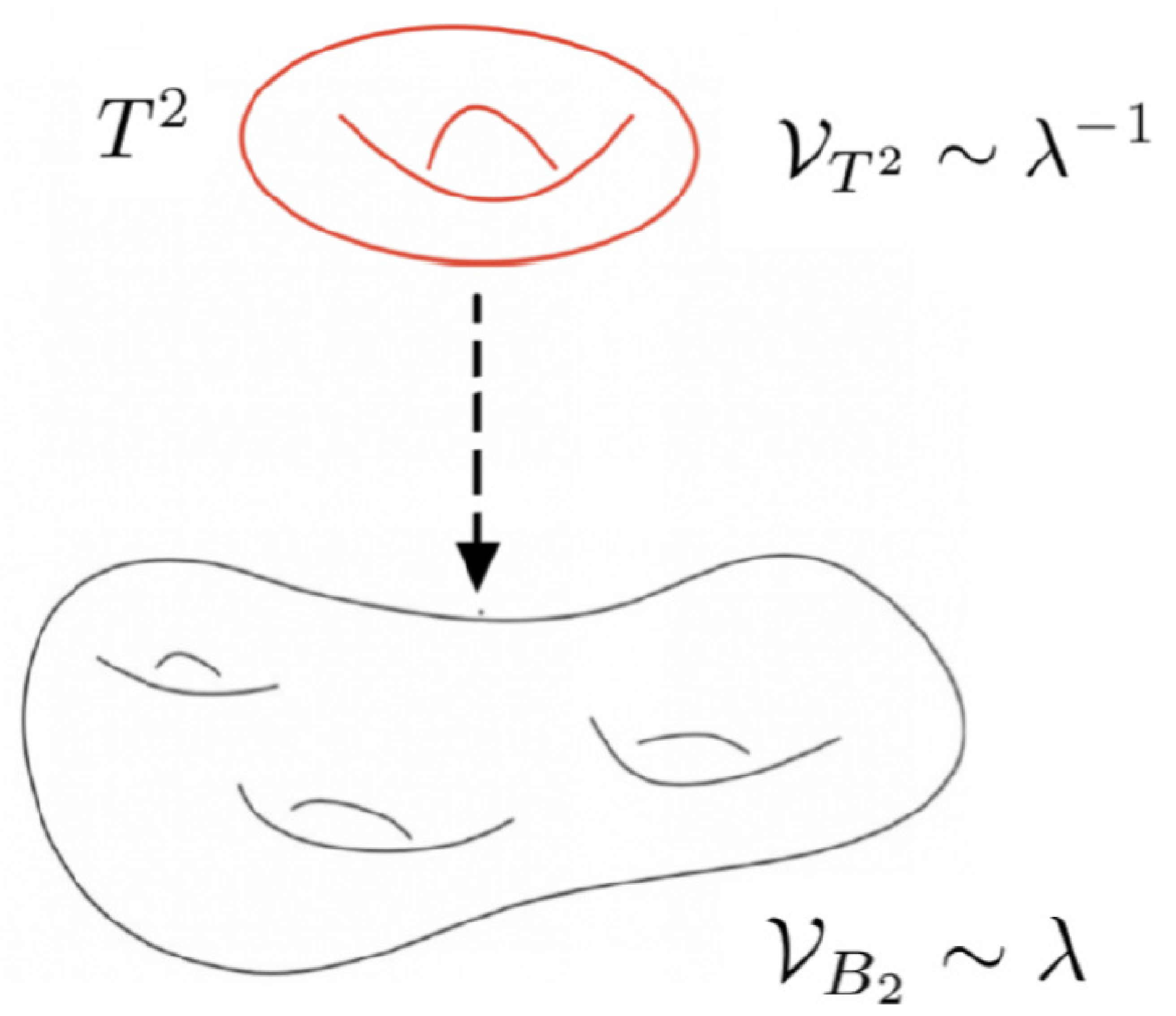
Additional criterion: $\dfrac{g_\Sigma^2 M_{\rm Pl}^3}{M_{{\rm Pl},\infty}^2} \to 0 \quad \Longleftrightarrow$ Careful analysis of asymptotic theories reveals 2 possibilities *[Cota,Mininno,TW,Wiesner'22,23]*

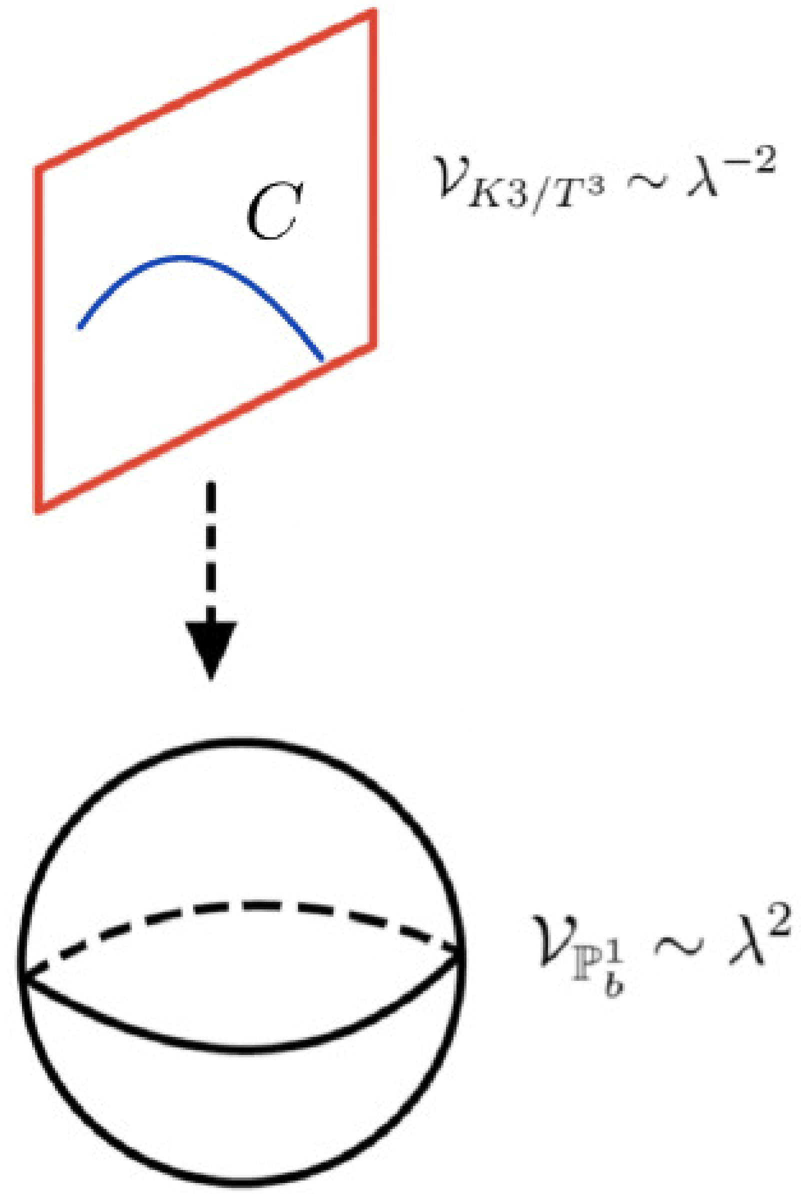
Tower Weak Gravity Conjecture: Particle version

$U(1)_\Sigma = \int_\Sigma C_3 \qquad \Sigma \in H_2(X_3)$

[Cota,Mininno,TW,Wiesner'23]

$\Sigma = T^2$





$\Sigma = C \subset K_3$

- $U(1)_\Sigma$: KK U(1) for 5d →6d
- $M_{\text{Pl},\infty} = M_{\text{Pl,6d}}$
- WGC tower = dual KK tower
  = BPS tower (M2-branes on Σ)

- $U(1)_\Sigma$: U(1) of dual weakly coupled 5d heterotic theory
- $M_{\text{Pl},\infty} = M_{\text{het,5d}}$
- if $\Sigma \cdot_{K3} \Sigma \geq 0$: WGC tower = BPS particle tower
- if $\Sigma \cdot_{K3} \Sigma < 0$: WGC tower = non-BPS het. string tower

Tower Weak Gravity Conjecture: Tests

1) Particle towers at weak coupling → **KK towers (up to duality)** / **String towers**

2) Towers away from weak coupling

- **BPS black hole towers in M-theory on CY 3-folds**

  in particular along directions where BPS = extremality

  [Alim,Heidenreich,Rudelius'21], [Gendler,Heidenreich,McAllister,Moritz,Rudelius'22]

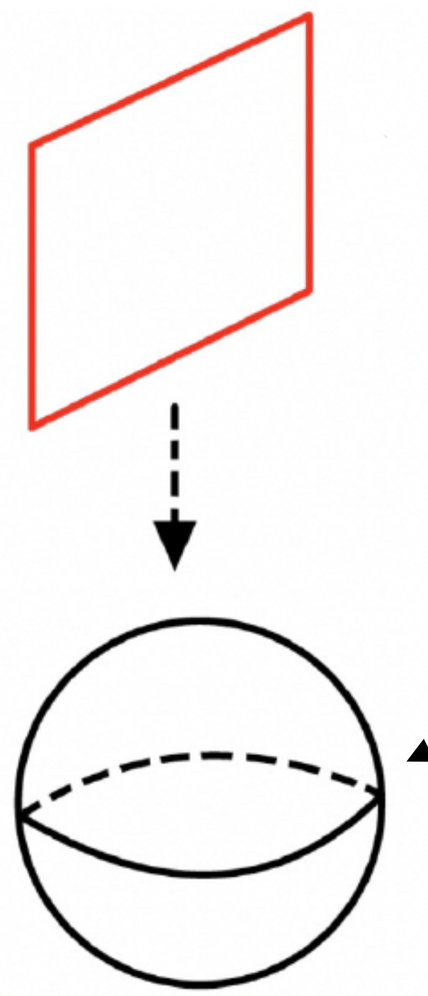
- **BPS SCFT sectors in M-theory on CY3 as particles**

Tower Weak Gravity Conjecture: Counter-examples?

Examples:

- U(1)s associated with conifold transitions in M-theory:
  cf. *[Alim,Heidenreich,Rudelius'21]*

  no *known* *tower* of charged particles or BHs -
  but maybe non-BPS tower of BHs unknown to us?



conifold curve $\mathbb{P}^1$ supports no tower of BPS states

- Open string U(1)s: *[Heidenreich,Reece,Rudelius'21][Cota,Mininno,TW,Wiesner'22]*

  no *known* *tower* of charged particles - non-pert. towers at best at BH level

Consistency under dimensional reduction

Is absence of a super-extremal tower consistent with dimensional reduction of the theory along a circle?

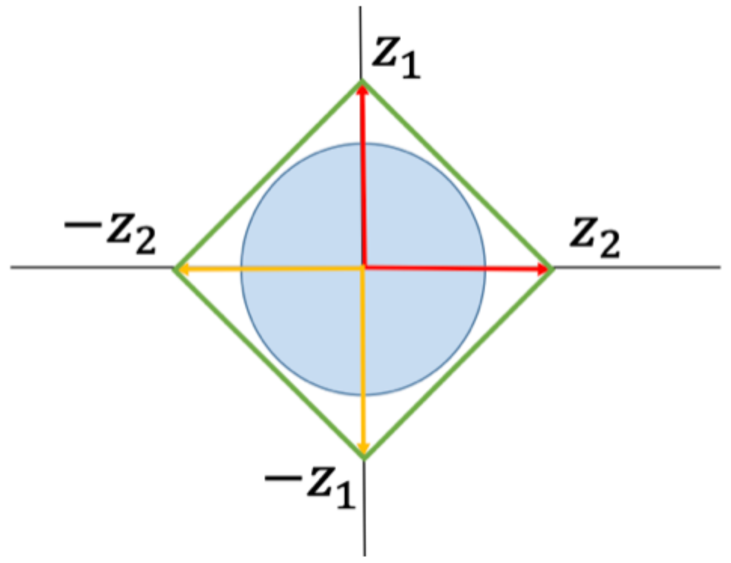
1) Review consistency under circle reduction
2) Loop hole: Minimal radius in generic circle reductions
3) Consequences for tower WGC

Reminder: WGC under dimensional reduction

U(1) Theory on $\mathbb{R}^{1,D-2} \times S^1$: $U(1)_D \longrightarrow U(1)_{D-1} \times U(1)_{\mathrm{KK}}$

$\Longrightarrow$ **Need to satisfy the CHC for** $U(1)_{D-1} \times U(1)_{KK}$

[Heidenreich,Reece,Rudelius'15]



CHC:

[Cheung,Remmen 14]

This requires existence of state in D dim. such that for all allowed values of r_{S^1} and θ:

$$(m_D r_{S^1})^2 \geq \frac{1}{4z_D^2(z_D^2 - 1)} + \frac{q\theta(1 - q\theta)}{z_D^2} \qquad z_D = g_D M_{Pl,D}^{\frac{D-2}{2}} \gamma^{1/2} \frac{|q|}{m_D}$$

θ : Wilson line

➡Problematic regime: i) $r_{S^1} \to 0$ or ii) $z_D = 1$ for all WGC states

(when BPS = extremality)

Reminder: WGC under dimensional reduction

Key Observation: *[Heidenreich,Reece,Rudelius'15/16]*

Super-extremal Tower in D dimensions $\Longrightarrow$ **CHC even for** $r_{S^1} \to 0$

Potential loopholes: *(cf. [Heidenreich,Reece,Rudelius'15])*

- The quantum gravity theory may not admit a limit $r_{S^1} \to 0$.
- Quantum corrections near $r_{S^1} \to 0$ may become relevant.

When is an EFT a KK reduction on S^1 ?

KK tower of mass $M_{\mathrm{KK}} \sim \frac{1}{2\pi r_{S^1}}$ detectable as particles (not black holes):

$$\boxed{\frac{1}{2\pi r_{S^1}} \sim M_{\mathrm{KK}} \leq M_{\mathrm{BH,min}}}$$

Minimal BH mass in D-1 dim:

$$\frac{M_{\mathrm{BH,min.}}}{M_{\mathrm{Pl,D-1}}} = \left(\frac{M_{\mathrm{Pl,D-1}}}{\Lambda_{\mathrm{QG}}}\right)^{D-4}$$

$$\Lambda_{\mathrm{QG}} \sim r_{\mathrm{BH,min}}^{-1}$$

Species Scale ≡ QG cutoff [*Dvali'07*]

In a **typical** theory in D-1 - away from boundaries of moduli space: $\Lambda_{\mathrm{QG}} \sim M_{\mathrm{Pl.,D-1}}$

In **this** case require: $2\pi r_{S^1} \geq M_{\mathrm{Pl,D-1}}^{-1}$

Minimal radius for *typical* D-1

EFT to be a KK reduction

When could the minimal radius argument fail?

Loophole: QG cutoff scale may drop below Planck scale parametrically:

$$\Lambda_{\mathrm{QG}} \ll M_{\mathrm{Pl.,D-1}}$$

due to tower of light weakly coupled states at infinite distance in moduli space:

Swampland Distance Conjecture [*Ooguri,Vafa`06*]

Emergent String Conjecture: [*Lee,Lerche,TW`19*]

Infinite distance physics is a decompactificaton limit or a weakly coupled string theory

Decompactification:

$$\Lambda_{\mathrm{QG}} = \text{higher-dim. } M_{\mathrm{Pl}}$$

[Long,Montero,Vafa,Valenzuela'21]
[Marchesano,Melotti'22]
[Castellano,Herraez,Ibanez'22]
[Heisteeg,Vafa,Wiesner,Wu'22]
[Cribiori,Lüst,Staudt'22] ….

Emergent string limit:

$$\Lambda_{\mathrm{QG}} \sim M_{\mathrm{str.}}$$

[*Dvali,Lüst'09*]
[*Dvali,Gomez'10*]

When could the minimal radius argument fail?

Case 1:

S^1 reduction of a D-dim theory

(a) in a decompactification limit

(b) in an emergent string limit

Case 2:

Limit $r_{S^1} \to 0$ itself corresponds to

(a) a (dual) decompactification limit

(b) an emergent string limit

When could the minimal radius argument fail?

Case 1:

S^1 reduction of a D-dim theory

(a) in a decompactification limit

$\Longrightarrow$ S^1 reduction of higher dim theory ✔

(b) in an emergent string limit

$\Longrightarrow$ S^1 reduction of a string theory

Case 2:

Limit $r_{S^1} \to 0$ itself corresponds to

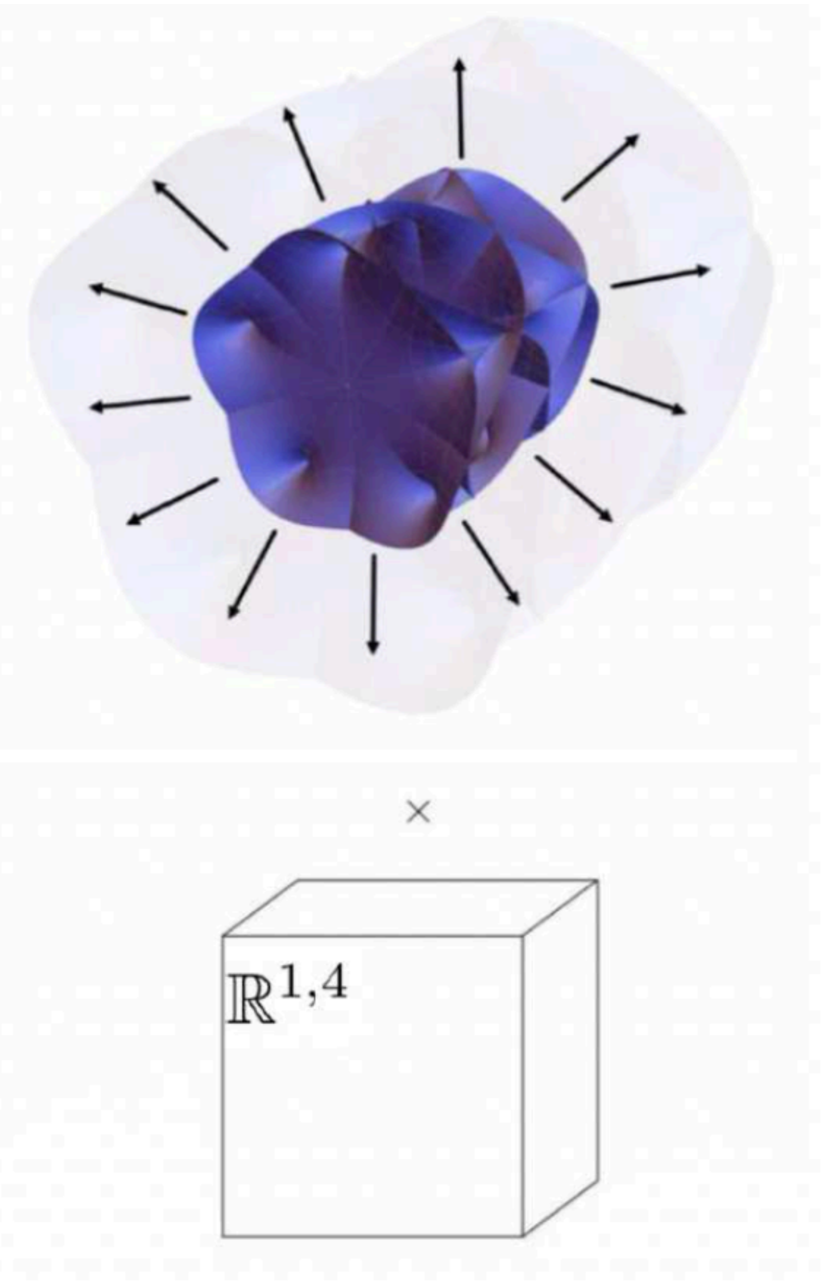
(a) a (dual) decompactification limit

$\Longrightarrow$ S^1 reduction of a string theory

(b) an emergent string limit

$\Longrightarrow$ S^1 reduction of M-theory

Circle reduction of M-theory on CY



- $M_{\mathrm{Pl},5}^3 = 4\pi M_{11\mathrm{d}}^3 \mathscr{V}_{X_3}$ $\qquad$ $\mathscr{V}_{X_3}$ volume in units of M_{11d}

- KK reduction on S^1: Can we take $r_{S^1} M_{\mathrm{Pl},5} \to 0$ at constant $M_{\mathrm{Pl},5}$

  i.e. $r_{S^1} M_{11\mathrm{d}} \to 0$ at $\mathscr{V}_{X_3}$ constant ?

M-theory-IIA duality:

Volume in string units

i) $g_{\mathrm{IIA}}^{2/3} = 2\pi M_{11\mathrm{d}} r_{S^1}$ and ii) $g_{\mathrm{IIA}}^{1/3} = \dfrac{M_s}{M_{11\mathrm{d}}}$ $\quad \Longrightarrow \quad$ $\dfrac{\mathscr{V}_{X_3,s}}{g_{\mathrm{IIA}}^2} = \mathscr{V}_{X_3}$: universal hyper

$\Longrightarrow \quad 2\pi r_{S^1} M_{11\mathrm{d}} = \left(\dfrac{\mathscr{V}_{X_3,s}}{\mathscr{V}_{X_3}}\right)^{1/3} \to 0$ at $\mathscr{V}_{X_3}$ constant requires co-scaling $\mathscr{V}_{X_3,s} \sim (r_{S^1} M_{11\mathrm{d}})^3 \to 0$

By mirror symmetry: $\quad \mathscr{V}_{X_3,s} \sim (r_{S^1} M_{11\mathrm{d}})^3 \to 0$ is not part of the quantum Kahler moduli space

Circle reduction of M-theory on CY

Interpretation:

$$2\pi\, r_{S^1}^{\text{min.}} M_{11d} = \left(\frac{\mathcal{V}_{X_3,\text{str.}}^{\text{min}}}{\mathcal{V}_{X_3}} \right)^{1/3}$$ is bound on theory to behave like a KK EFT

➡ This is not a bound on g_{IIA}, but below $g_{\text{IIA}}^{\text{min}} \sim \left(2\pi\, r_{S^1}^{\text{min.}} M_{11d}\right)^{3/2}$ KK reduction not a good description

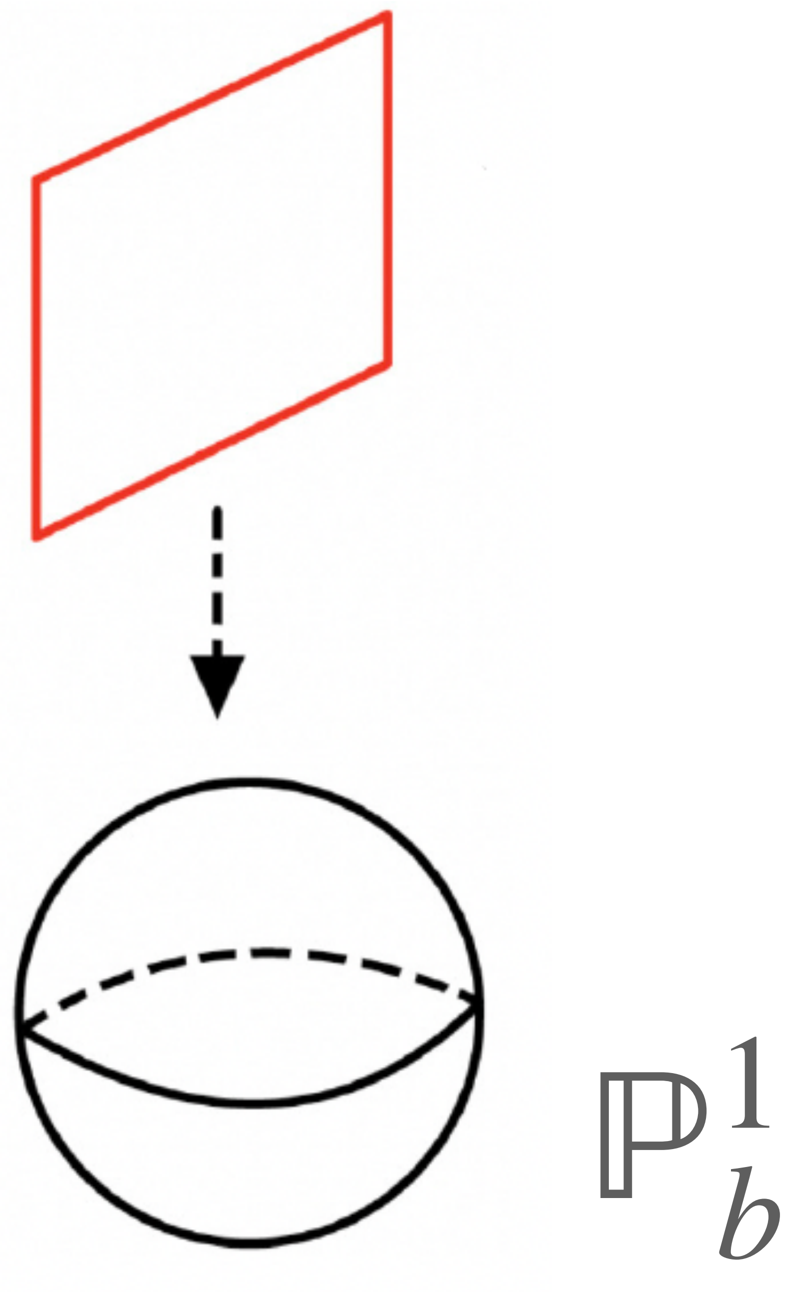
➡ This is a consequence of quantum geometry of compactification and does not occur for circle compactification of 11d M-theory

Conifold revisited

Check CHC bound explicitly in regime $r \geq r_{\min}$ for U(1) with

- no weak coupling limit
- without a tower of charged BPS or known tower of charged non-BPS states

Example: $A = \int_{\mathbb{P}^1_b} C_3$ $\qquad$ $\mathbb{P}^1_b$: base of K3-fibration



Concrete examples:

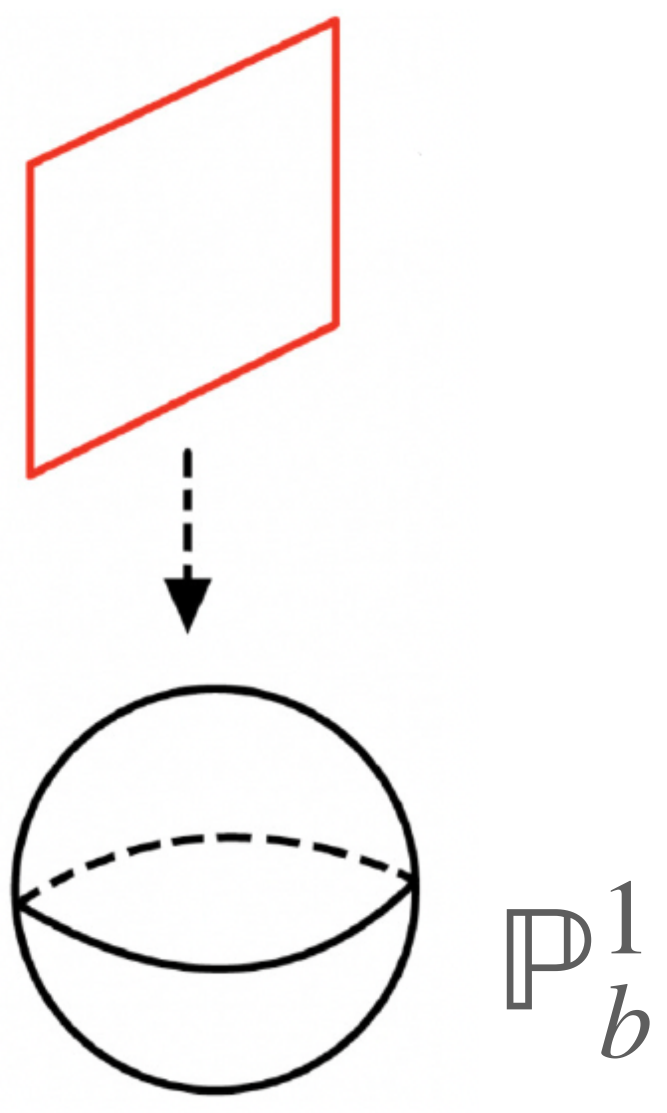
$$\mathbb{P}^4_{11222}[8] \qquad\qquad \mathbb{P}^4_{11226}[12]$$

$$\mathcal{I}(\mathbb{P}^4_{11222}[8]) = 8J_1^3 + 4J_1^2 J_2 \qquad \mathcal{I}(\mathbb{P}^4_{11226}[12]) = 4J_1^3 + 2J_1^2 J_2$$

Conifold revisited

Evaluate CHC bound explicitly for U(1) with

- no weak coupling limit
- without a tower of charged BPS or known non-BPS states



Numerically: *cf. [Candelas,Font,Katz,Morrison'94]* *[Blumenhagen,Kläwer,Schlechter,Wolf'18]*

$$\mathcal{V}^{\min}_{\mathbb{P}^4_{11222}[8],\mathrm{str}} \simeq 2.83 \qquad \mathcal{V}^{\min}_{\mathbb{P}^4_{11226}[12]} \simeq 6.00$$

at LG point of mirror!

$$\mathrm{RHS}_{\mathbb{P}^4_{11222}[8]} \lesssim 0.17 \qquad \mathrm{RHS}_{\mathbb{P}^4_{11226}[12]} \lesssim 0.10$$

CHC at $r \geq r_{\min}$ requires:

$$\boxed{(\mathcal{V}^{\min})^{\frac{2}{3}} \geq \frac{\gamma}{2Q_\alpha f^{\alpha\beta} Q_\beta}\left(\frac{\gamma \mathcal{V}_C}{|q|^2\left(2|q|^2 Q_\alpha f^{\alpha\beta} Q_\beta \mathcal{V}_{X_3}^{2/3} - \gamma \mathcal{V}_C\right)} + \frac{q\theta(1-\theta q)}{|q|^2}\right)}$$

✔

Circle reduction of closed string theory

Minimal radius criterion:

$$\frac{1}{2\pi r_{S^1}} = M_{\rm KK} \leq M_{\rm BH,min} = \frac{M_s}{g_s^2} \implies r_{S^1}^{\rm min} \geq g_s^2\sqrt{\alpha'} \to 0$$

as $g_s \to 0$: **No minimal radius!**

$$\frac{M_{\rm BH,min.}}{M_{\rm Pl,D}} = \left(\frac{M_{\rm Pl,D}}{\Lambda_{\rm QG}}\right)^{D-3}, \qquad \Lambda_{\rm QG} = M_s$$

Example: Apply to heterotic

1) **Perturbative sector:** $g^2_{U(1)_{\rm pert},D} M_{\rm het}^{D-4} \propto g_{\rm het}^2 \implies$ **Tower of super-extremal states required** ✓

2) **Non-perturbative sector:** E.g. from NS5-branes in comp. to 6d $g^2_{U(1)_{\rm n.p.},6} M_{\rm het}^2 \propto g_{\rm het}^{-2}$

E.g. for massless charged sector: $\frac{r_{S^1}^2}{\alpha'} \geq \frac{q\theta(1-q\theta)}{m_6^2 z_6^2} \propto g_{\rm het}^4 q\theta_6(1-q\theta_6)$ **no parametric clash**

no tower needed in agreement with absence of known candidates! ✓

Circle reduction of string theory: open

$g^2_{U(1)_{\text{pert}},D} M^{D-4}_{\text{het}} \propto g_s \implies$ **parametric clash for CHC and naively requires tower**

However:

No super-extremal particle tower in open pert. spectrum of increasing charge!

Solution:

$r \ll \sqrt{\alpha'}$ and CHC with $U(1)_{\text{KK}}$ $\xleftrightarrow{\text{T-duality}}$ $r \gg \sqrt{\alpha'}$ and CHC with $U(1)_{\text{winding}}$ for theory localised along S^1

Furthermore: In limit $g_s \to 0$ gauge theory on brane decouples from gravity! ✓

WGC under dimensional reduction

Consider a

- D-dimensional $U(1)_D$ gauge theory in a D-dimensional theory of quantum gravity such that
- the WGC is realized by a set of super-extremal particle-like states.

$\Longrightarrow$ In the (D-1)-dimensional theory after S^1 reduction, the CHC for $U(1)_D \times U(1)_{\mathrm{KK}}$ is satisfied

- by KK replicas of the D-dimensional super-extremal particle states
- for any value of the circle radius which allows for an interpretation as a circle reduction of the D-dimensional gauge theory coupled to gravity.

This holds **irrespective of whether the particles are part of a tower in the D-dimensional theory**.

Conclusions

✳ **WGC tower** of super-extremal particles **present and required** for consistency **for**:

i) perturbative heterotic string U(1)

ii) KK U(1)

Emergent String

⟷

Conjecture

all U(1) with a weak coupling limit

✳ All **known cases without** established super-extremal **tower** are **consistent**:

- conifold U(1) M-theory
- pert. open string U(1)
- non-pert. sector in 6d/4d heterotic
- generic F-theory away from emergent string limits

Conclusions

Open question:

Super-extremal particle tower present also for **strongly coupled BPS sectors** (5d SCFTs):

They would be required by circle reduction if these were strictly extremal.

Are they?

If so, this would motivate a

Minimal Tower Weak Gravity Conjecture:

Super-extremal particle towers are present ***if and only if*** *they are required by consistency of the WGC under circle reduction.*

Appendix

When could the minimal radius argument fail?

S^1 reduction of M-theory:

- No minimal radius from BH argument:

$$r_{S^1} \geq M_{\mathrm{Pl,D-1}}^{-1} \left(\frac{M_s}{M_{\mathrm{Pl,D-1}}} \right)^{D-4} \to 0$$

$M_s = \Lambda_{\mathrm{QG}}$ at small r_{S^1}

- A different argument does show **minimal radius for M-theory comp.** *generically*

✓ consistent with absence of known towers for <u>generic</u> theories

S^1 reduction of perturbative string theory:

- **No minimal radius** despite T-duality

➡ for **heterotic string**, this necessitates tower of WGC states in agreement with spectrum

➡ for **open string theory**, no tower required:

(see later)

✓ consistent with absence of established towers for open string

Weak Gravity Conjecture: Criterion for particles

Claim/Conjecture: *[Cota,Mininno,TW,Wiesner'23]*

The WGC must hold at the particle level for a genuine 0-form gauge theory coupled to gravity:

i) not a defect theory in a higher dimensional theory:

$\ell_{\text{perp.}}$: size of extra dimensions perpendicular to gauge brane

$\ell_{\text{min.}} = \dfrac{1}{\Lambda_{\text{QG}}}$: minimal length scale of QG $\Lambda_{\text{QG}} \sim r^{-1}_{\text{BH,min}}$: Species scale [Dvali,07]

hence require: $\ell_{\text{perp.}} \leq \ell_{\text{min}}$

ii) not secretly a higher-form symmetry:

$\ell_{\text{perp.}}$: size of cycle over which a higher-form was reduced : $\ell_{\text{perp.}} \leq \ell_{\text{min}}$

iii) gauge degrees of freedom not decoupled from gravitational sector