Quintessence, the cosmological constant, and the weak gravity conjecture

Francisco Gil Pedro

Based on:

W.I.P. with M. Cicoli, C. Cunillera and A. Padilla

and

arXiv: 2303.17723 & 2404.02961

w/ Y. Liu and A. Padilla

Outline

- Introduction
- Dynamical alternatives to the cosmological constant
    - Axionic quintessence
    - Embedding axionic quintessence
- Summary I
- The cosmological constant and the WGC
- Summary II

The CC problem, observationally

1998 SN Ia -> Universe in accelerated expansion

$$H_0 = 100\ h\ km/s/Mpc \approx 10^{-60} M_{pl}$$

$h = 0.675$ PLANCK 2018

$h = 0.73$ SH0ES

Explained by a fluid with $p = \omega\rho$ $\quad \omega \approx -0.99^{+0.15}_{-0.13}$ $\quad \Omega \sim 0.7$

Compatible with a cosmological constant $\Lambda \approx 10^{-120} M_P^4$

See however DESI: 2404.03002 $\quad \omega(a) = \omega_0 + (1-a)\omega_a$

$-0.55^{+0.39}_{-0.21}$ $\quad < -1.32$

The CC problem, theoretically

Why is $\Lambda \approx 10^{-120} M_P^4$?

How to embed into string theory?

Why is $\Lambda \neq M_{UV}^4$?

Alternatives to Λ ?

What is the role of anthropics?

"A decade ago the high energy physics community had a well-defined challenge: show why the dark energy density vanishes. Now there seems to be a new challenge and clue: show why the dark energy density is exceedingly small but not zero "

Peebles and Ratra, 2002

The CC problem, theoretically

What if $\omega \neq -1$?

(Too) Many alternatives:

Quintessence: Single field $\lambda = -\frac{V_\phi}{V}$

$\lambda < 0.6$ [Agrawal et al. '18] [Akrami et al. '18]

No dS conjecture? $\lambda \geq \mathcal{O}(1)$

How to explain the DE scale?

Radiative stability?

Fifth forces?

The CC problem, theoretically

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Alternatives:

-multifield dynamics [(Brinkman),Cicoli,Dibitetto,FGP '20-22]

-axionic quintessence [Cicoli, Cunillera,Padilla,FGP '21]

Axionic DE

[Kaloper,Sorbo '05]

Axionic DE: -Potential generated by non perturbative effects

-Derivative couplings to matter

-Radiative stability

scale suppression

fifth force suppression

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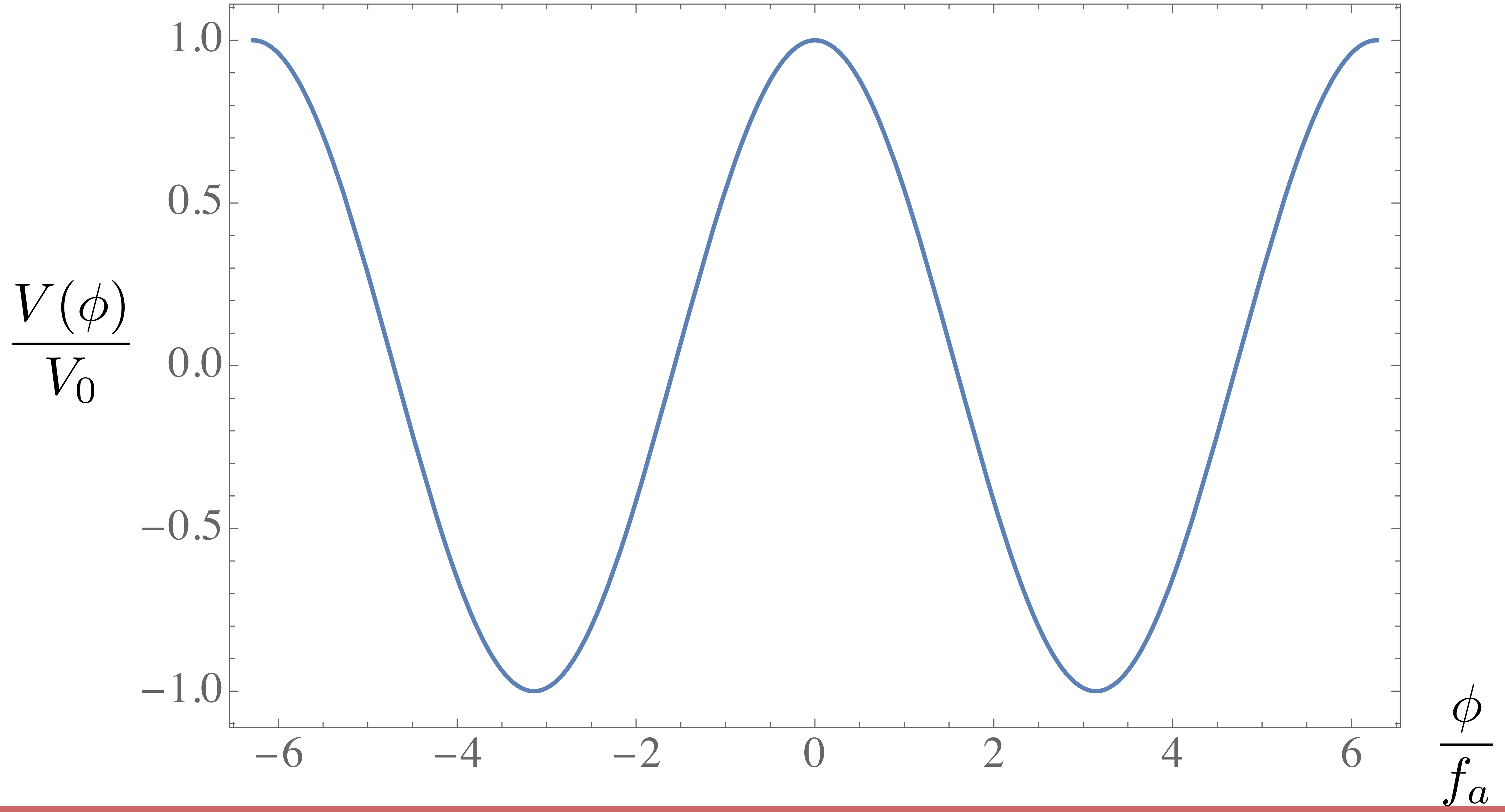
$$V = V_0\left(1 - \cos\frac{\phi}{f_a}\right)$$



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$f_a > M_P$

$\phi < \phi_{ip}$

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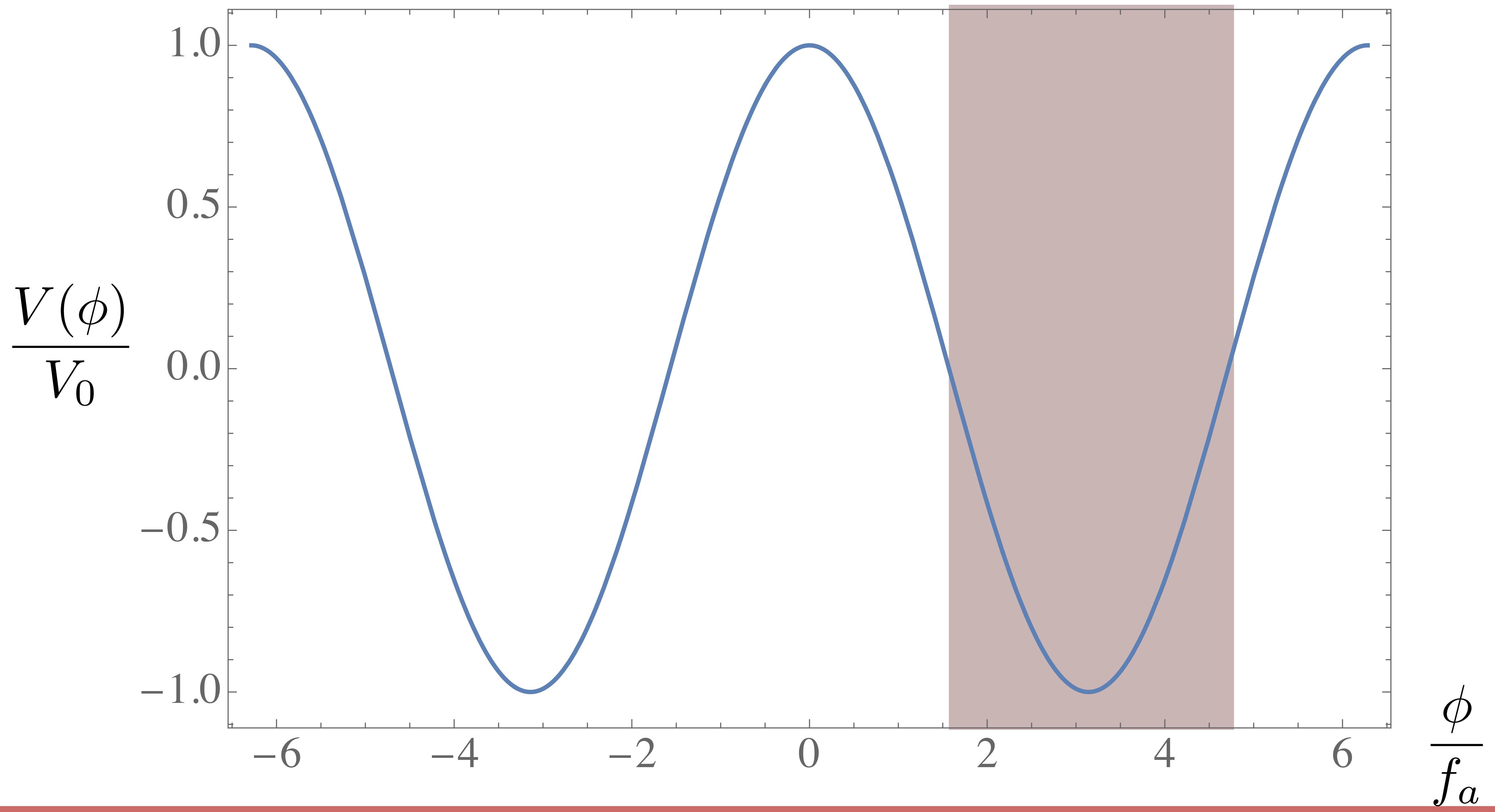

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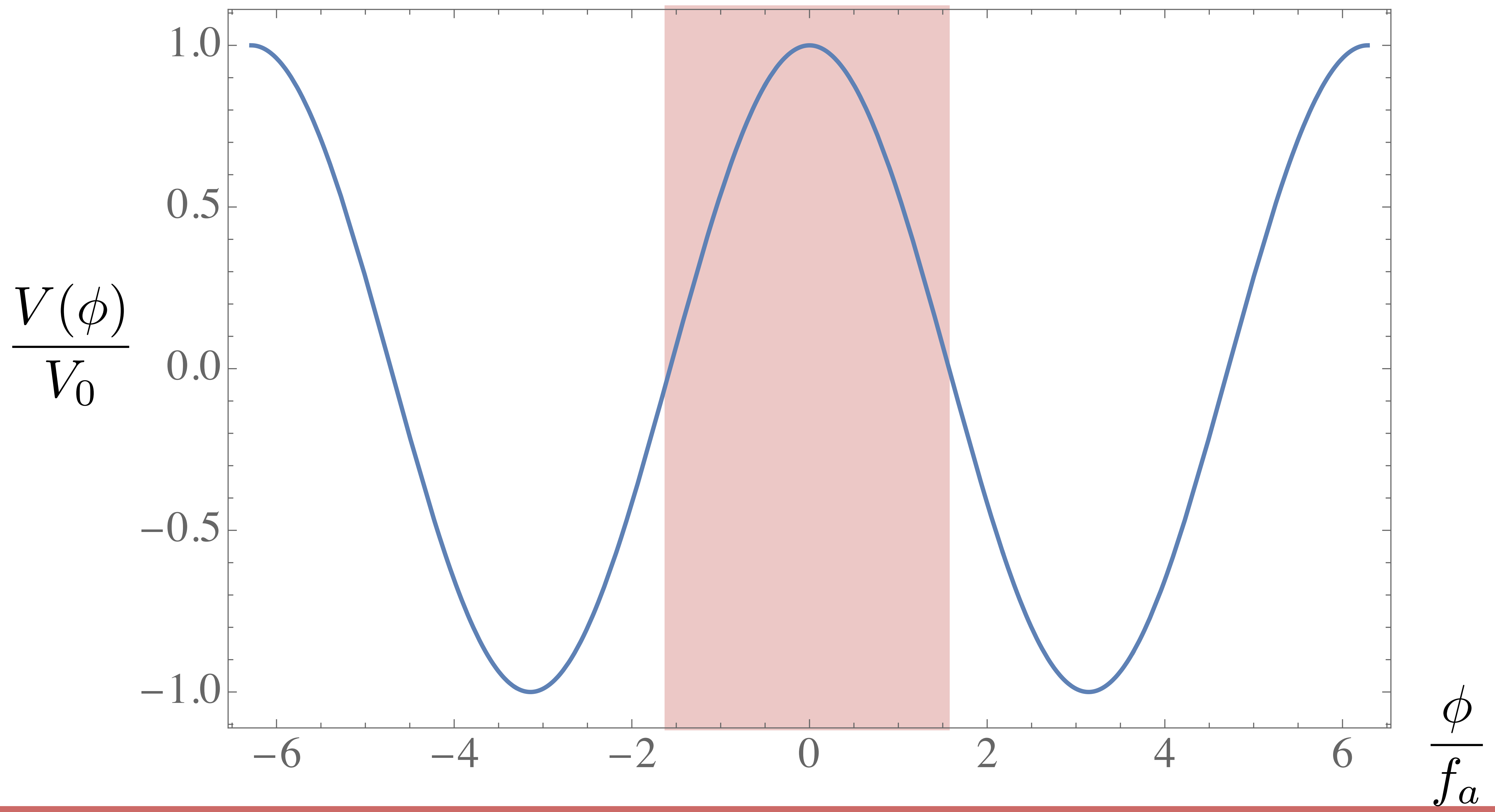

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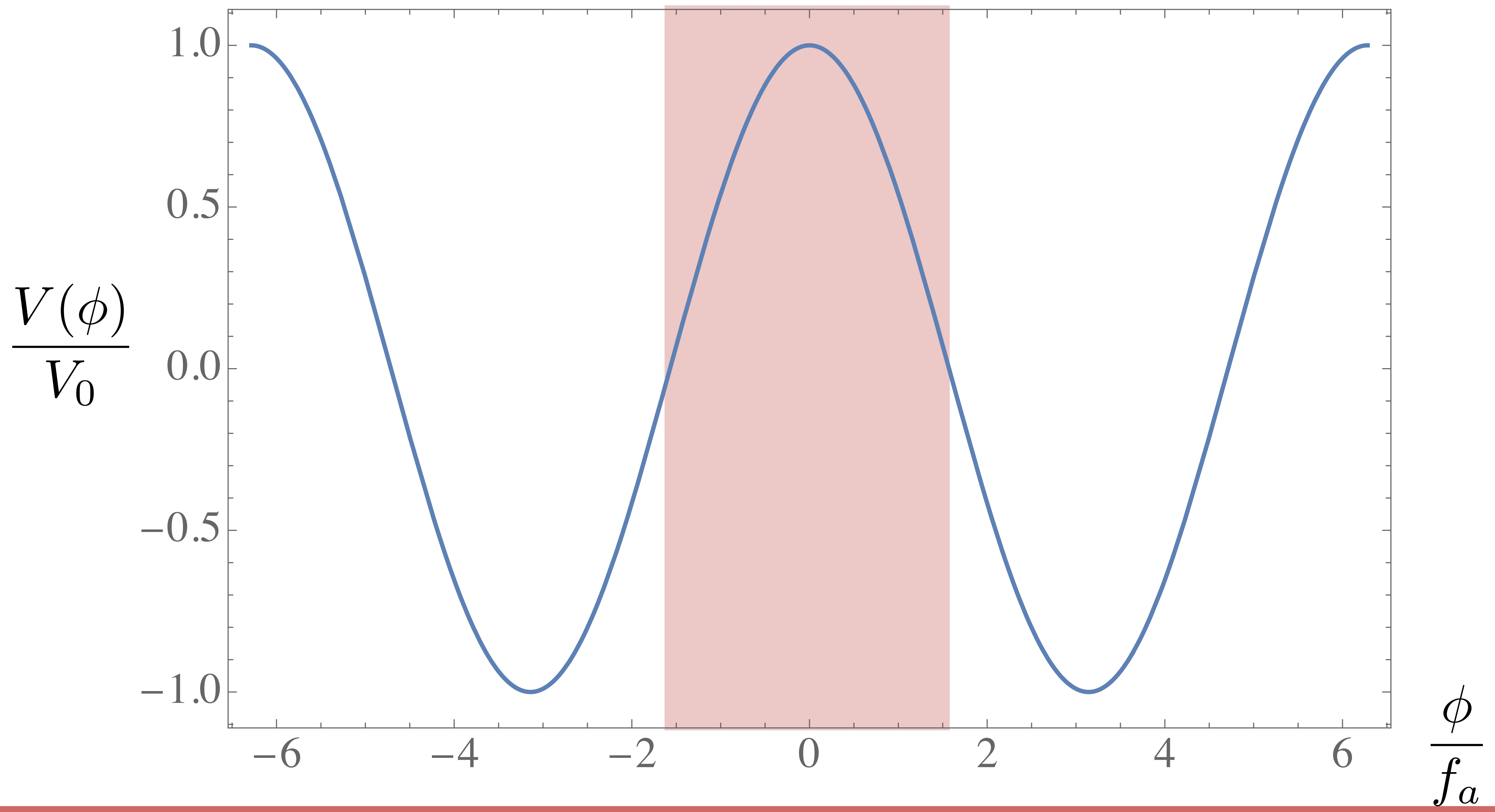
[Banks et al. '03] $\frac{f_a}{M_P} \sim \frac{(g_s)^p}{vol^q}$

axionic WGC: $S_a \ f_a < 1$

$f_a < M_P$

$\phi > \phi_{ip}$

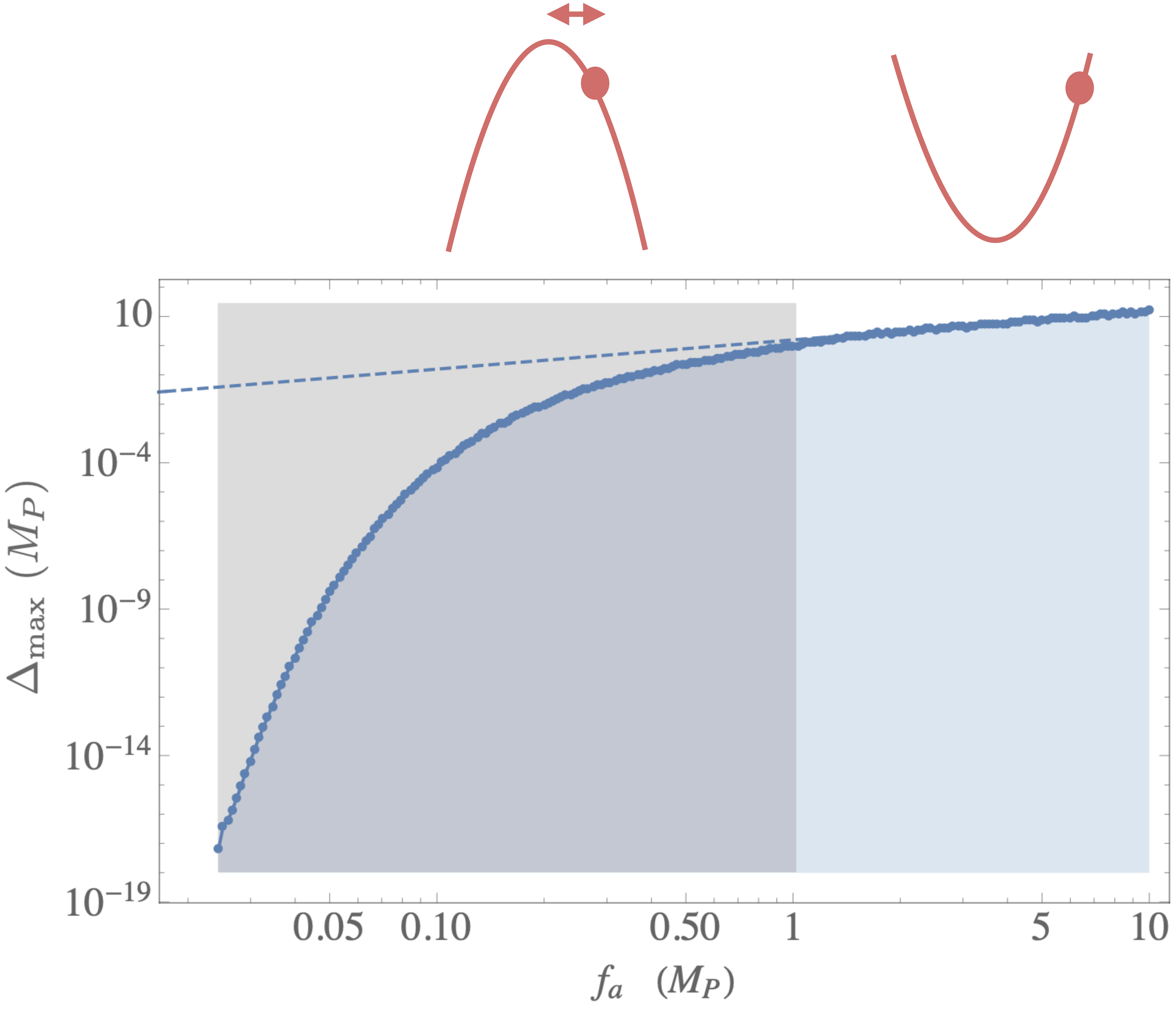


Axionic DE

$$\Delta_{max} = |\phi_i - \phi_{max}|$$

[Dutta and Scherrer '08]

[Cicoli, Cunillera,Padilla,FGP '21]



[Olguin-Trejo et al. '18]

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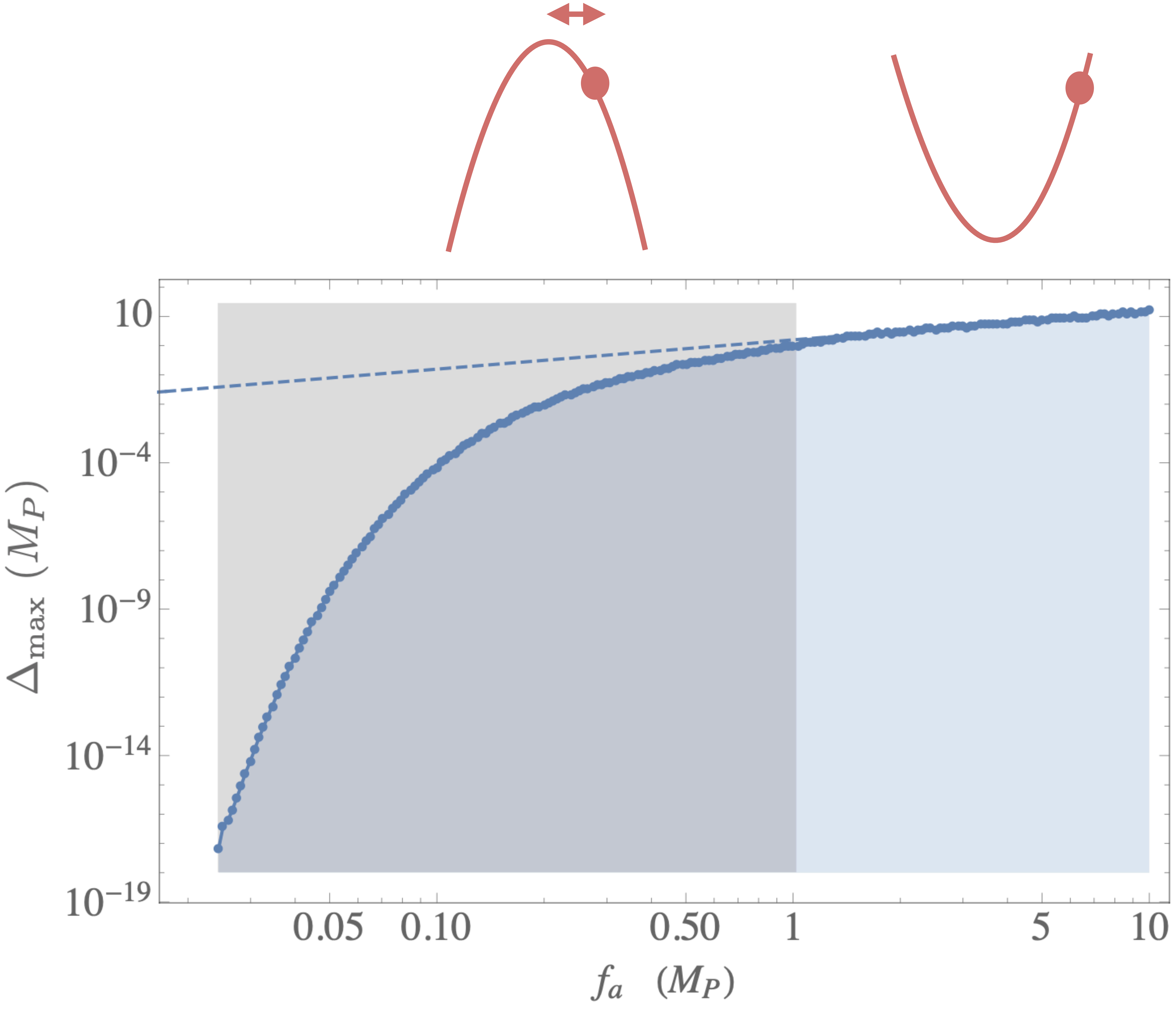

Small decay constant

Start very close to top of the hill

tuning of initial conditions

[Olguin-Trejo et al. '18]

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Axion is classically frozen until today

$$\frac{\partial \phi}{\partial N} = -\frac{V_\phi}{3H_{\rm inf}^2}$$

$$V_\phi \ll H_{inf}^2 \qquad \frac{\partial \phi}{\partial N} = 0$$

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Quantum diffusion during inflation

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Choice of ics gets blurred during inflation

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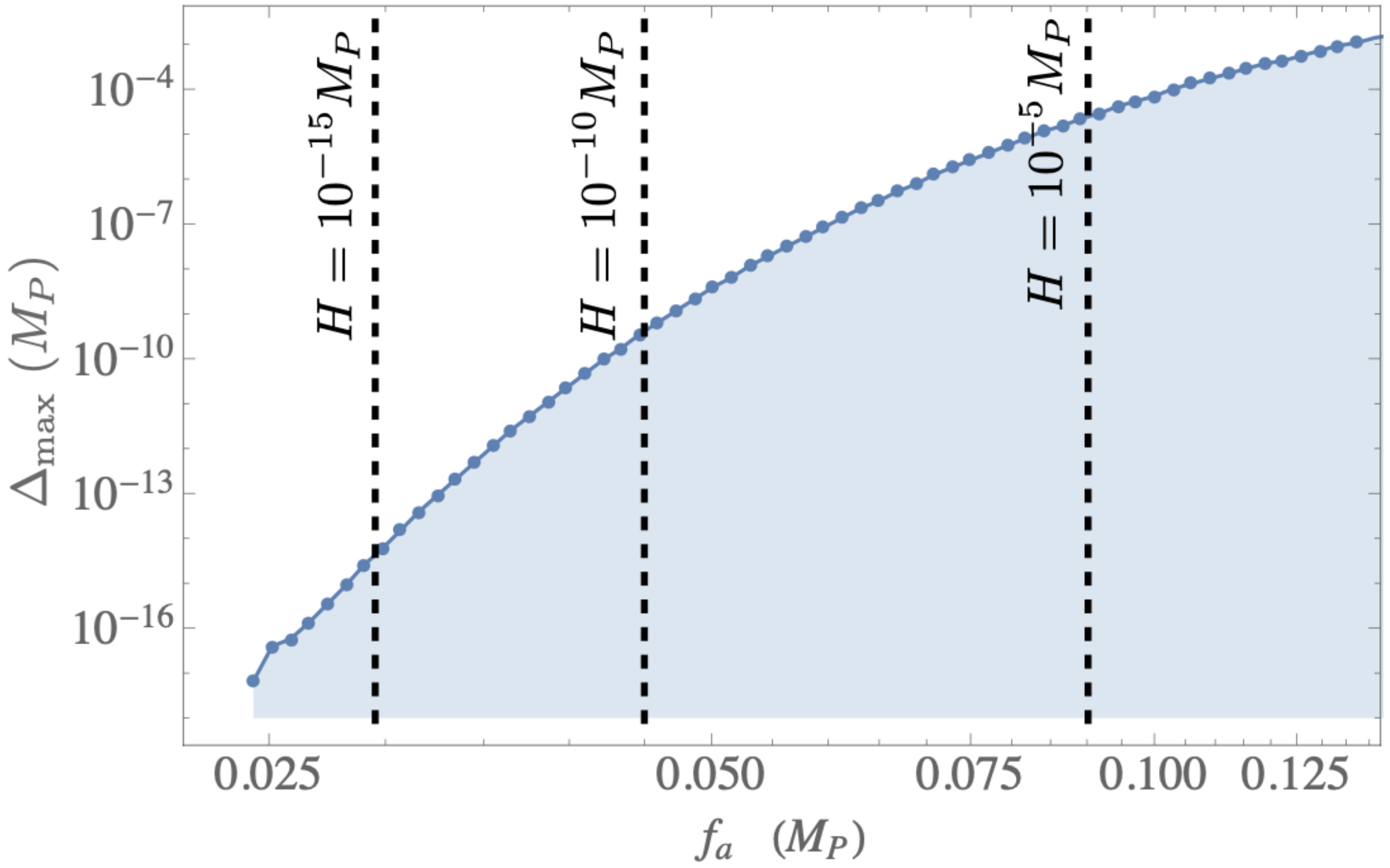

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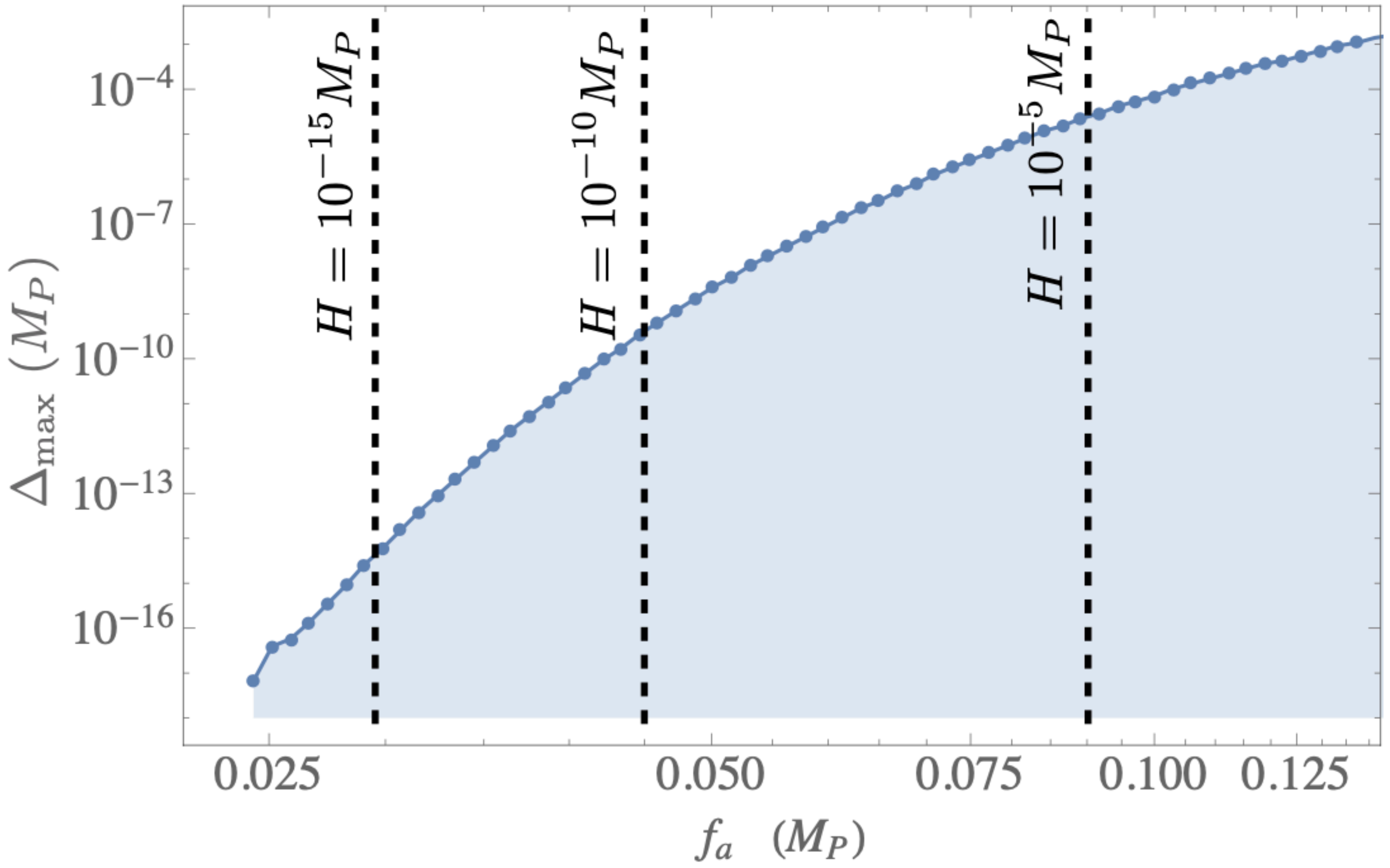

example: $H_{inf} \sim 10^{-5} M_P \rightarrow f_a > 0.08 M_P$

Axionic DE embedding

[Cicoli, Padilla, FGP, '24 WIP]

Embedding into type IIB compactifications

$$\mathcal{V} = t_1 t_2^2 - t_s^3 = \sqrt{\tau_1}\tau_2 - \tau_s^{3/2} \qquad T_j = \tau_j + i\theta_j$$

3 moduli + 3 axions

$$V = V_{LVS}(\mathcal{V}, \tau_s, \theta_s) + V_{inf}(\tau_1/\tau_2) + V_{late}(\theta_1, \theta_2)$$

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LVS stabilisation α'^3 corrections + n.p. effects

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Fibre inflation: string loops (W) + HD

[Cicoli et al. '08]

$$V_{\rm inf}(\sigma) = V_0\left[\left(1 - e^{-\frac{\sigma}{\sqrt{3}}}\right)^2 - 2\mathcal{R}\left(1 - \cosh\left(\frac{\sigma}{\sqrt{3}}\right)\right)\right].$$

$H_{inf} \sim 10^{-5} M_P$

$\mathcal{V} \sim \mathcal{O}(10^3)$

Axionic DE embedding

Late time potential generated by poly-instanton or or KNP alignment

$$W_{\rm np} = A_s\, e^{-a_s T_s} + A_2\, e^{-a_2 T_2 + A_1\, e^{-a_1 T_1}}$$
$$= A_s\, e^{-a_s T_s} + A_2\, e^{-a_2 T_2} + A_2 A_1\, e^{-(a_2 T_2 + a_1 T_1)} + \ldots$$

[Blumenhagen et al. '08-'12]
[Lüst, Zhang.'13]

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$$V_{\rm late} = \Lambda_2^4\,[1 - \cos{(a_2\theta_2)}] + \Lambda_1^4\,[1 - \cos{(a_2\theta_2 + a_1\theta_1)}]$$

$$\Lambda_2^4 = \frac{4W_0 A_2}{\mathcal{V}^2}(a_2\langle\tau_2\rangle)e^{-a_2\langle\tau_2\rangle} \qquad \Lambda_1^4 = \frac{4W_0 A_1 A_2}{\mathcal{V}^2}(a_1\langle\tau_1\rangle + a_2\langle\tau_2\rangle)e^{-a_1\langle\tau_1\rangle - a_2\langle\tau_2\rangle}$$

Hierarchy: $\Lambda_1^4 \ll \Lambda_2^4$

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Hierarchy: $\Lambda_1^4 \ll \Lambda_2^4$

$$f_1 = \frac{1}{\sqrt{2}a_1\langle\tau_1\rangle} \qquad f_2 = \frac{1}{a_2\langle\tau_2\rangle}$$

$$V_{DE} = \Lambda_1^4\left(1 - \cos\frac{\phi_1}{f_1}\right)$$

$$f_1, f_2 \ll 1$$

Axionic DE embedding

Example: <u>safe ics</u> ⟷ $f_1 = 0.085$ <u>DE scale</u> $\Lambda_1^4 \sim 10^{-120}$ ⟷ $f_2 = 0.0038$

↓

$\Lambda_2^4 \sim 10^{-117}$

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axionic mass hierarchy

$$m_2 \sim 10^{-56} = 10^{-29} eV$$

$$< H_{eq} \sim 10^{-27} eV$$

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$a_i = \frac{2\pi}{N_i}$

$N_1 = 1$ $\langle \tau_1 \rangle = 1.3$ $N_2 = 27$ $\langle \tau_2 \rangle \sim 10^3$

$N_1 = 5$ $\langle \tau_1 \rangle \sim 5$ $N_2 = 10$ $\langle \tau_2 \rangle \sim 500$

Summary I

- From a bottom up perspective dynamical DE is no easier than dS
- But if observationally $\omega \neq -1$ or theoretically no dS in QG
- An alternatives: axionic quintessence
  - radiative stability (shift symmetry)
  - scale suppression
  - evades fifth force
  - $f_a < M_P$ hilltop

A unicorn in a duck suit ?

As long as observationally $\omega = -1$, a (unnaturally small) CC fits the bill.

Occam's razor: assume it IS a CC

"If a poet sees something that walks like a duck and swims like a duck and quacks like a duck, we will forgive him for entertaining more fanciful possibilities. It could be a unicorn in a duck suit—who's to say! But we know that more likely, it's a duck."

[Bousso, TASI Lectures on the Cosmological Constant]

Explore string **<u>inspired</u>** models for CC.

Λ sourced by 4-forms in 4 dimensions

Bousso-Polchinski

[Bousso and Polchinski '00]

Toy landscape in 4D: J four-forms and J types of branes $J \gg 1$

$$S = \int d^4x \sqrt{|g|} \left[\frac{M_P^2}{2} R - \sum_i \frac{1}{2} F_i^2 \right] + S_{matter} + S_{bdy} + \sum_i S_{brane_i}$$

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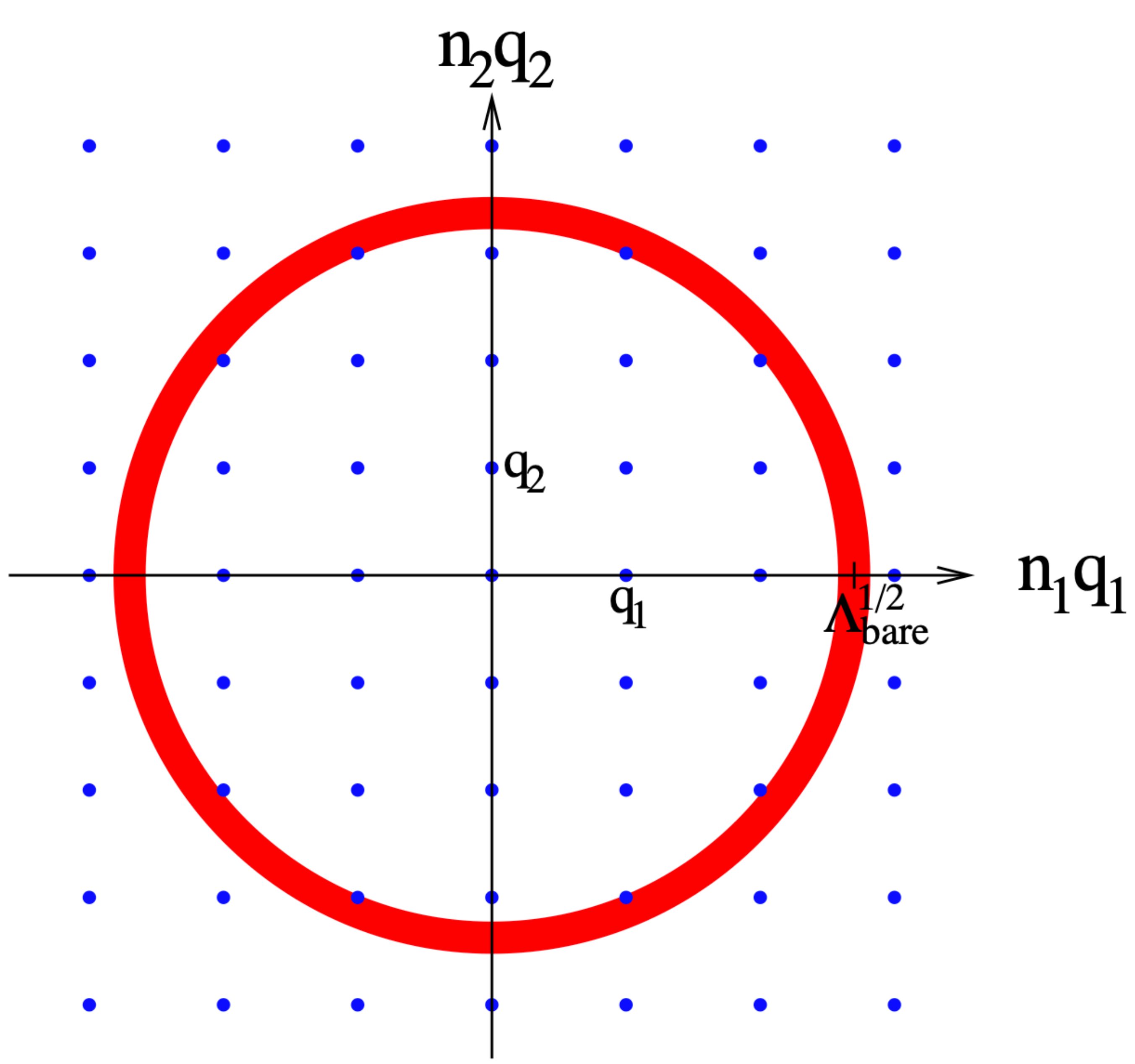
$$\frac{2|\Lambda_{bare}|}{3M_{pl}^2}\sqrt{\pi/J}\left(\frac{Jq^2}{e\pi|\Lambda_{bare}|}\right)^{J/2} \leq H_0^2 \qquad q \sim 0.01 M_P^2 \qquad J \sim 100$$



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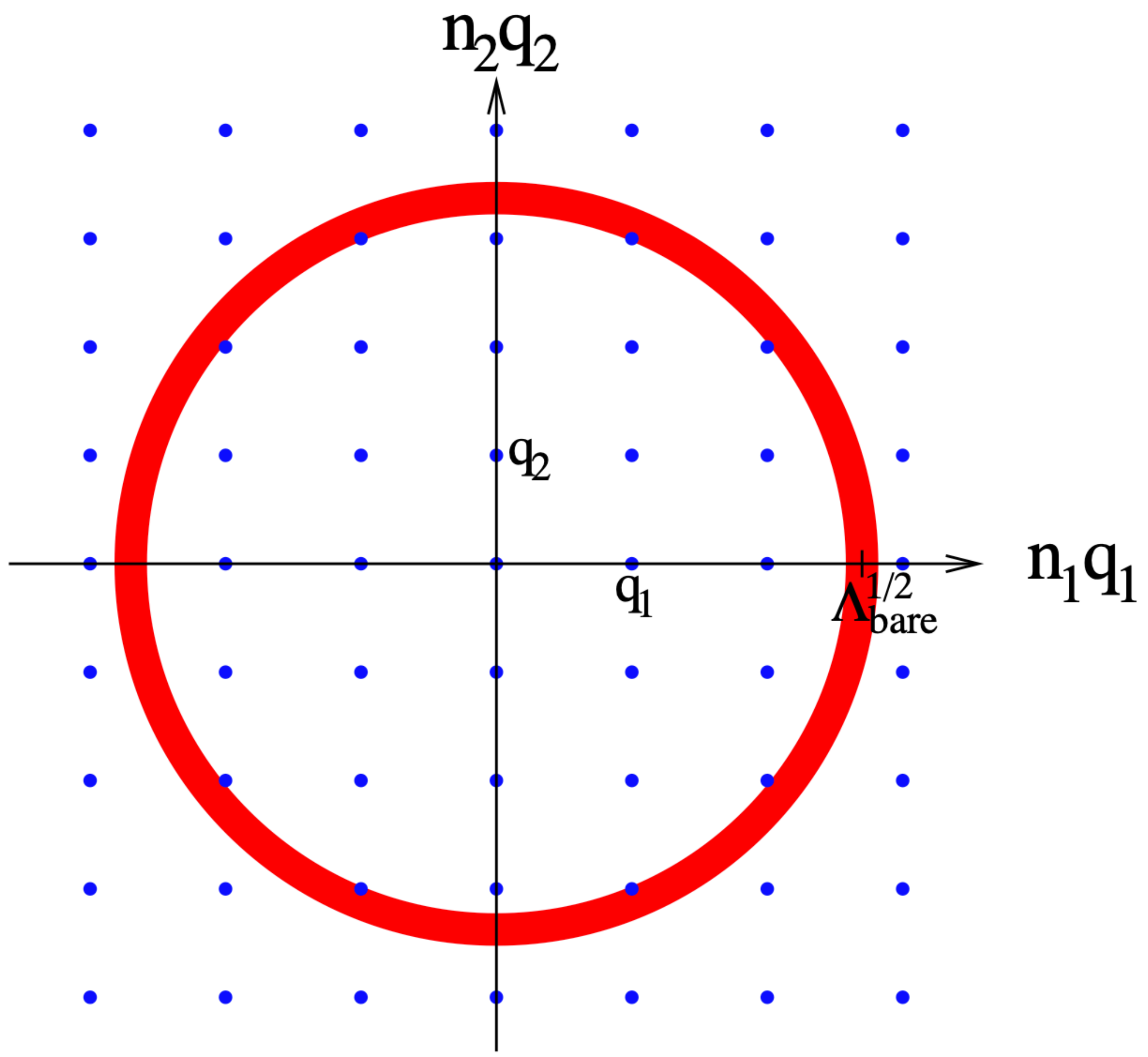

Anthropic solution

Kaloper-Westphal

Unimodular (x 2) with branes

[Kaloper&Westphal '22-23]

[Kaloper '22-23]

$$S = \int d^4x \Big\{ \sqrt{g}\Big(\frac{M_{\rm Pl}^2}{2}R - M_{\rm Pl}^2(\lambda + \hat{\lambda}) - \mathcal{L}_{\tt QFT}\Big) - \frac{\lambda}{3}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu \mathcal{A}_{\nu\lambda\sigma} - \frac{\hat{\lambda}}{3}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu \hat{\mathcal{A}}_{\nu\lambda\sigma} \Big\}$$

$$+ S_{\tt boundary} - \mathcal{T}_A \int d^3\xi \sqrt{\gamma}_A - \mathcal{Q}_A \int \mathcal{A} - \mathcal{T}_{\hat{A}} \int d^3\xi \sqrt{\gamma}_{\hat{A}} - \mathcal{Q}_{\hat{A}} \int \hat{\mathcal{A}} \,.$$

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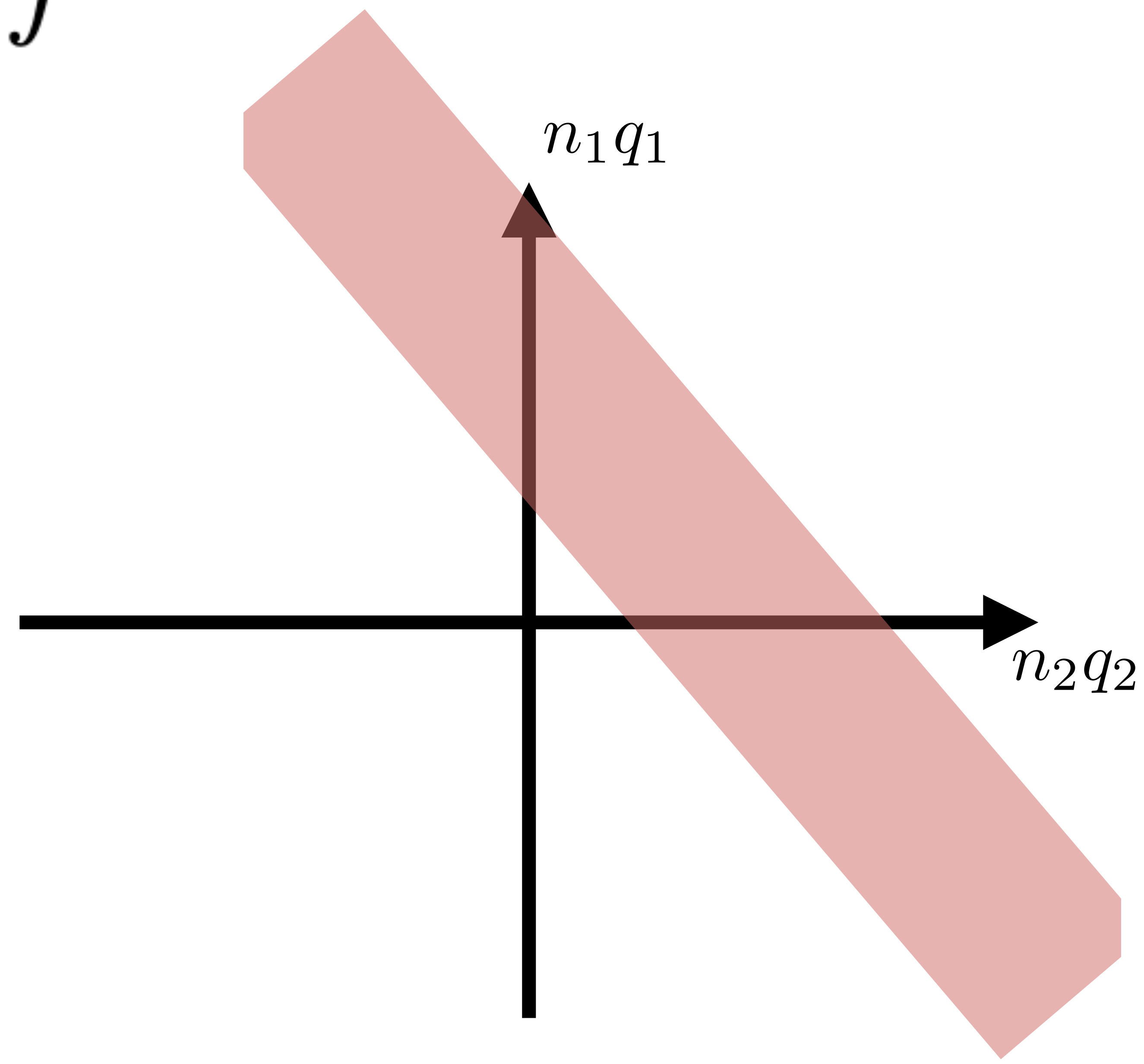
$$\Lambda = \Lambda_{bare} + (n_1q_1 + n_2q_2)M_{uv}^2$$



"Unimodular landscape"

Kaloper-Westphal

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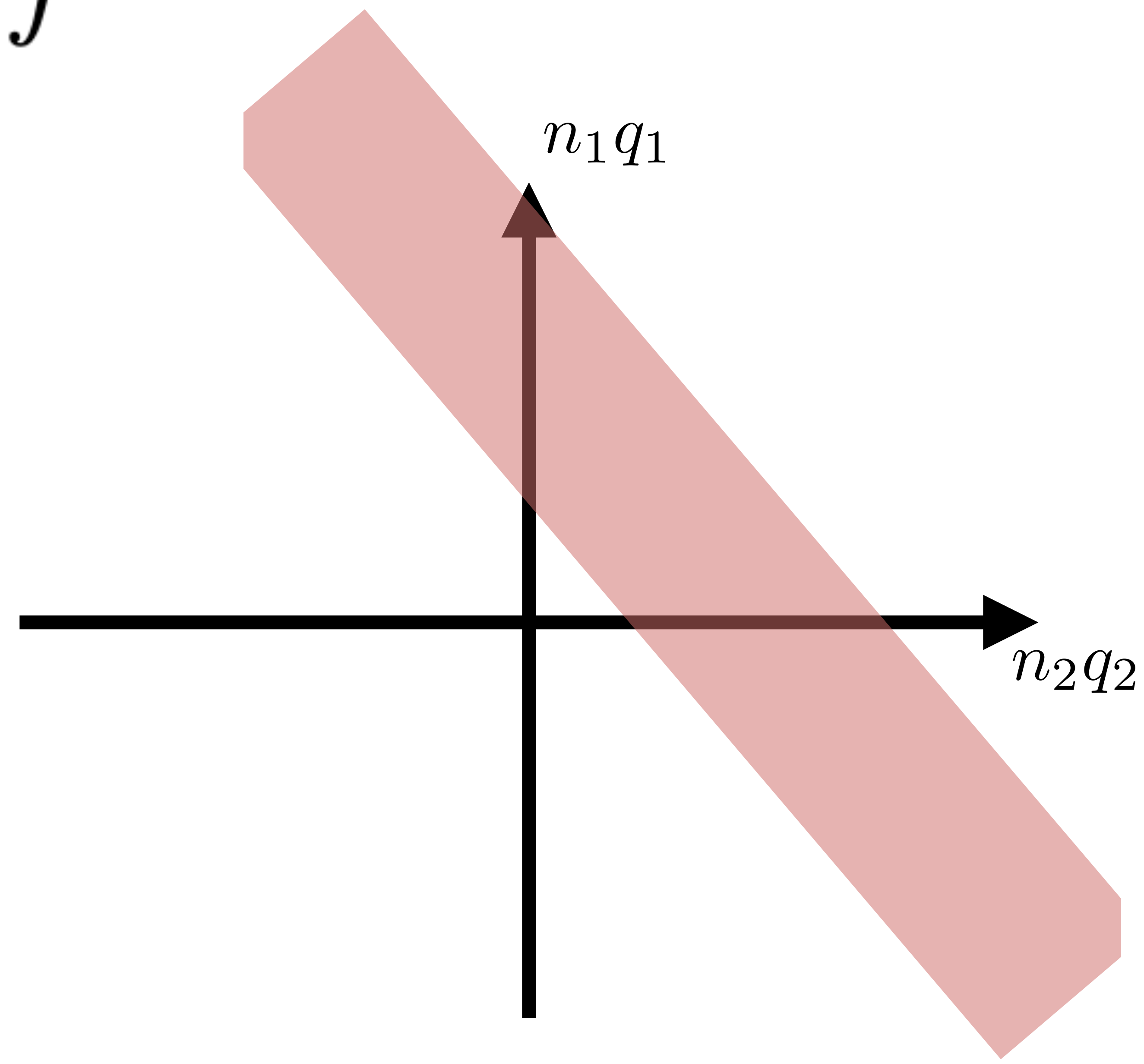

"Unimodular landscape"

For $\dfrac{q_1}{q_2}$ irrational $\quad \Lambda \approx H_0^2 \quad q_1, q_2 \gg H_0^2$

Thou shall not decay !

!! Minkowski can be stable !!

$$\Gamma \underset{\Lambda \to 0^+}{\longrightarrow} 0 \quad \text{if} \quad \frac{M_P q_i}{T_i} \frac{M_P M_{uv}^2}{T_i} < 1$$

[Kaloper&Westphal ’22]

“Since the instability dynamically stops at $\Lambda = 0$, the evolution favors the terminal Minkowski space without a need for anthropics.”

Thou shall not decay !

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$$\Gamma \xrightarrow[\Lambda \to 0^+]{} 0 \qquad \text{if} \qquad \frac{M_P q_i}{T_i}\frac{M_P M_{uv}^2}{T_i} < 1$$

[Kaloper&Westphal ’22]

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?? Is this specific to the linear model ??

The weak gravity conjecture

Existence of vacua with the observed CC >> conditions on brane charges

Stability of low scale de Sitter >> condition on $\frac{qM_{pl}}{\tau}$

[hep-th/0601001]

Einstein-Maxwell theory: decay of charged BH

$$q \gtrsim m$$

"gravity is the weakest force"

Brane WGC:

tension τ

charge q

$$qM_p \geq \tau$$

Instantons

[Liu,Padilla,FGP 23/24]

$$S = \int_{\mathcal{M}} d^4x \sqrt{|g|} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} \omega^{ij}(\phi) \nabla_\mu \phi_i \nabla^\mu \phi_j - V(\phi) \right] + \int_{\mathcal{M}} \left[-\frac{1}{2} Z_{ij}(\phi) F^i \wedge \star F^j + \sigma_i(\phi) F^i \right] + S_{\text{boundary}} + S_{\text{membranes}}$$

Instantons

[Liu,Padilla,FGP 23/24]

$$S = \int_{\mathcal{M}} d^4x\sqrt{|g|}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}\omega^{ij}(\phi)\nabla_\mu\phi_i\nabla^\mu\phi_j - V(\phi)\right] + \int_{\mathcal{M}}\left[-\frac{1}{2}Z_{ij}(\phi)F^i\wedge\star F^j + \sigma_i(\phi)F^i\right] + S_{\text{boundary}} + S_{\text{membranes}}$$

Wick rotate $t \to -it_E, \qquad S \to iS_E$

Spherical symmetry $d_s^2 = dr^2 + \rho(r)^2 d\Omega_3$

$$3M_{pl}^2\left[\frac{1}{\rho^2} - \left(\frac{\rho'}{\rho}\right)^2\right] = V - \frac{1}{2}\omega^{ij}\phi_i'\phi_j' + \frac{1}{2}Z_{ij}\frac{A^{i\prime}A^{j\prime}}{\rho^6}$$

$$3M_{pl}^2k^2 \equiv \Lambda_{eff}$$

$$M_{pl}^2\left[\frac{1}{\rho^2} - \left(\frac{\rho'}{\rho}\right)^2 - 2\frac{\rho''}{\rho}\right] = V + \frac{1}{2}\omega^{ij}\phi_i'\phi_j' + \frac{1}{2}Z_{ij}\frac{A^{i\prime}A^{j\prime}}{\rho^6}$$

[Coleman/de Lucia ’80]

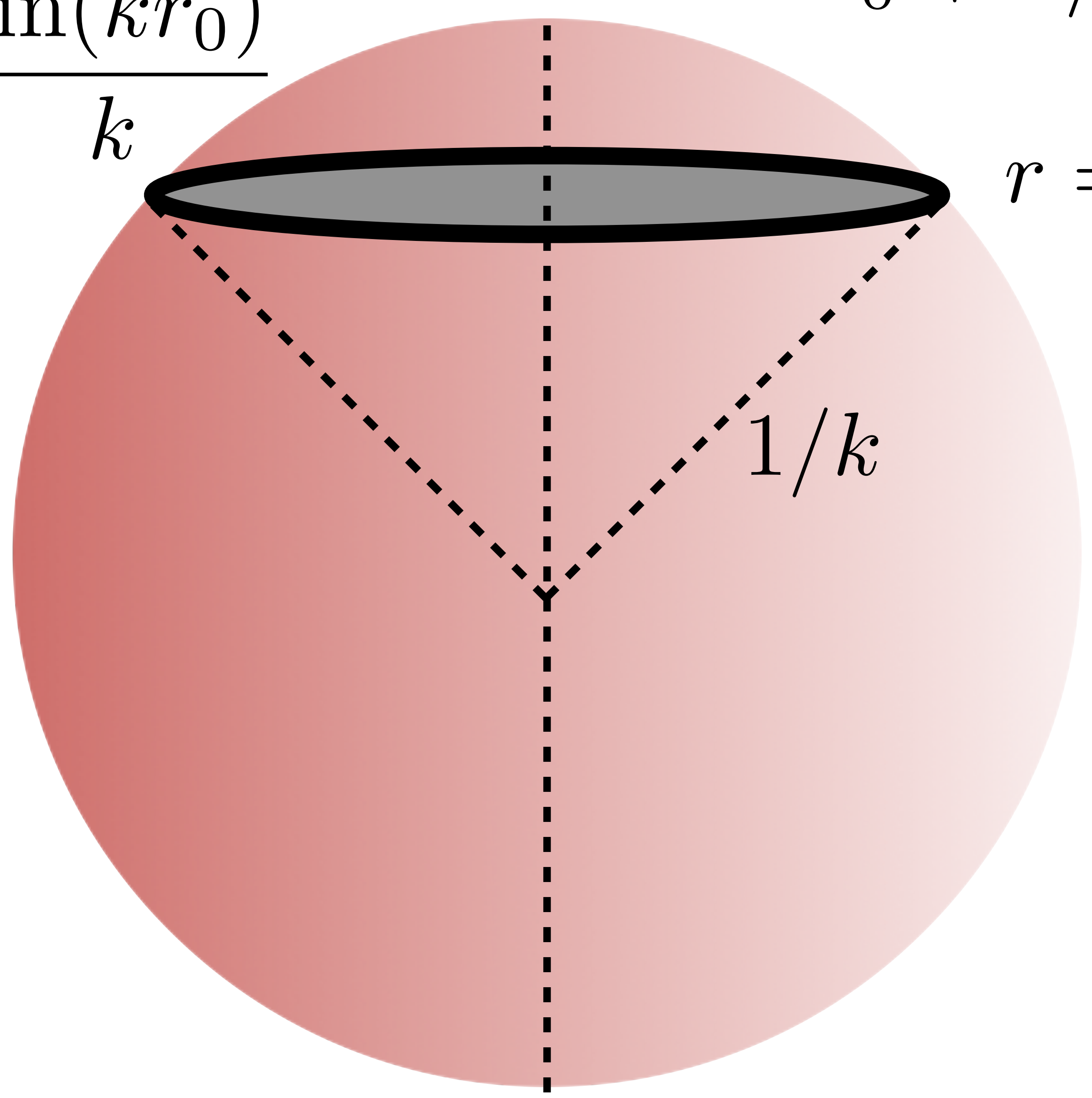
Instantons

$$3M_{pl}^2 k^2 \equiv \Lambda_{eff}$$

$k^2 > 0$

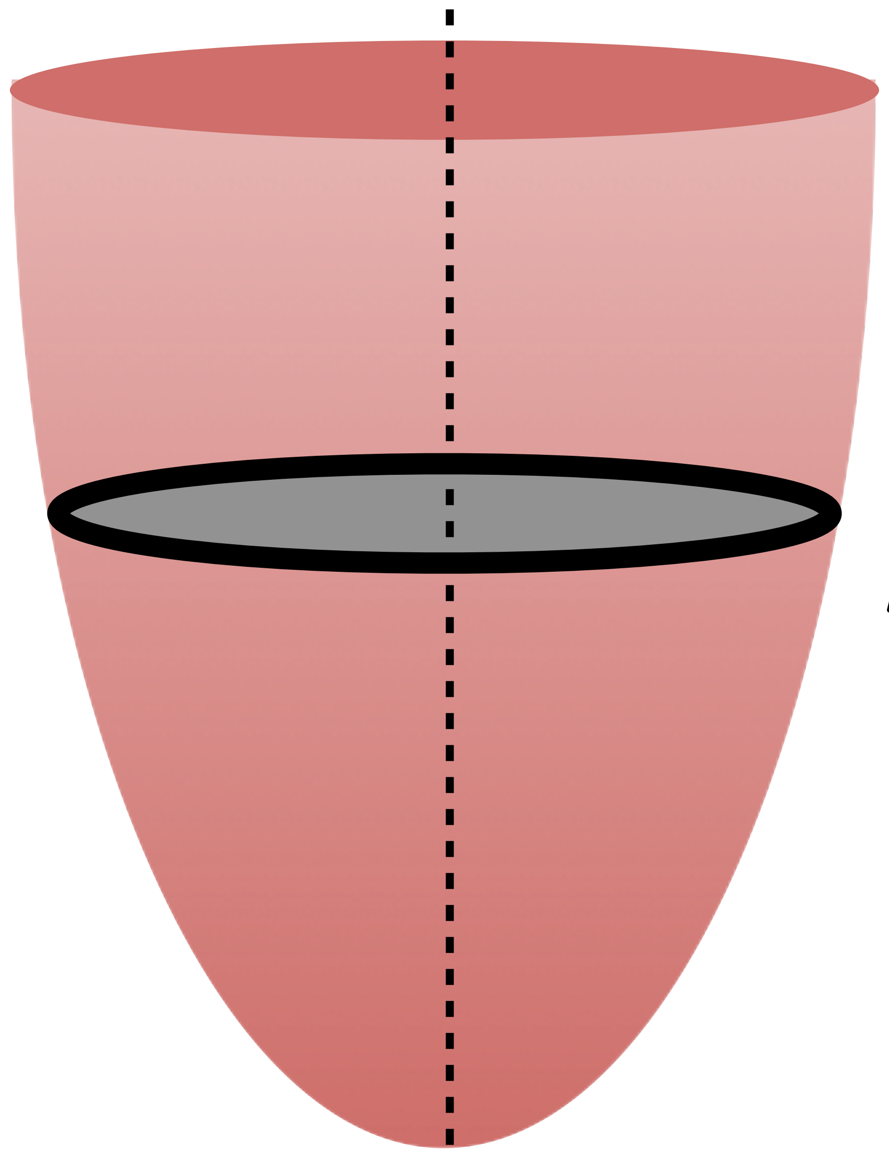
$$\rho(k) = \frac{\sin[k(r+r_0)]}{k}$$

$$\rho(k) = \frac{\sin(kr_0)}{k}$$



$k^2 < 0$

$$\rho(k) = \frac{\sinh[|k|(\epsilon r + r_0)]}{|k|}$$



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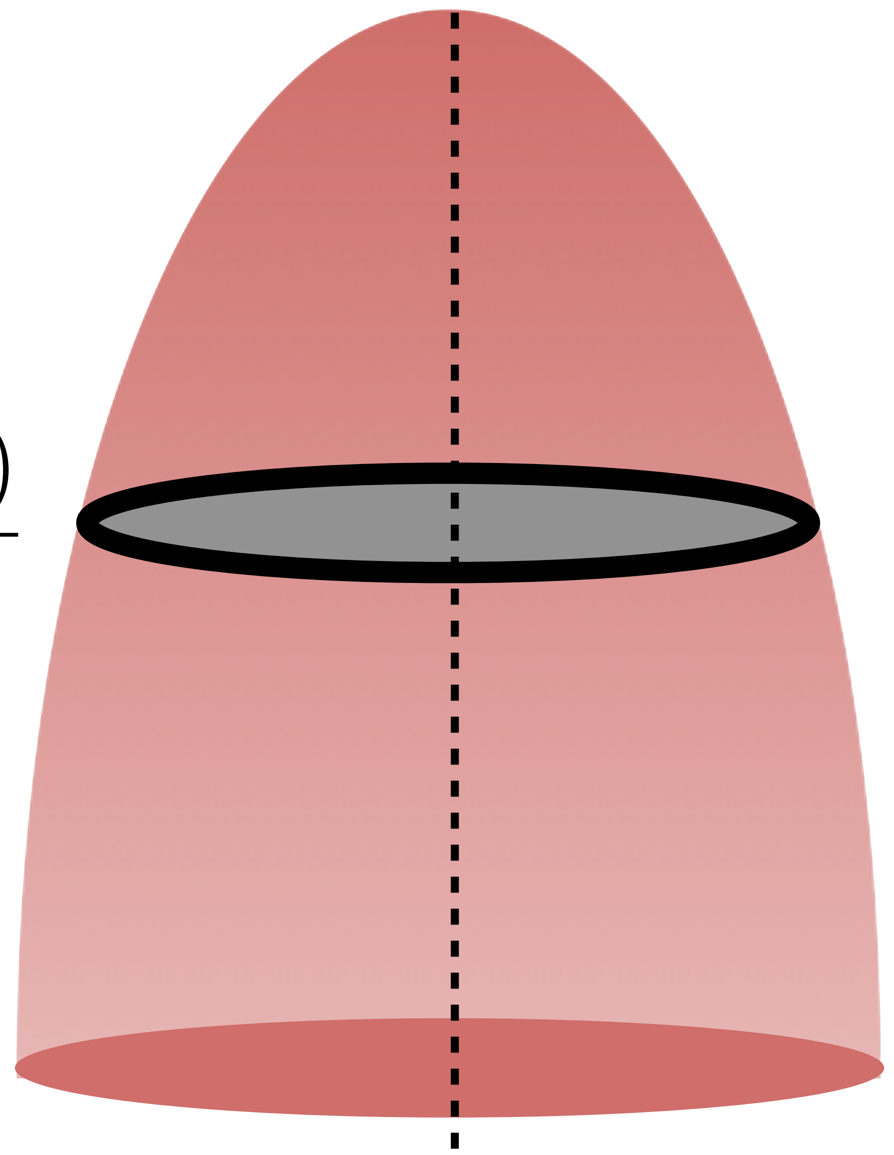
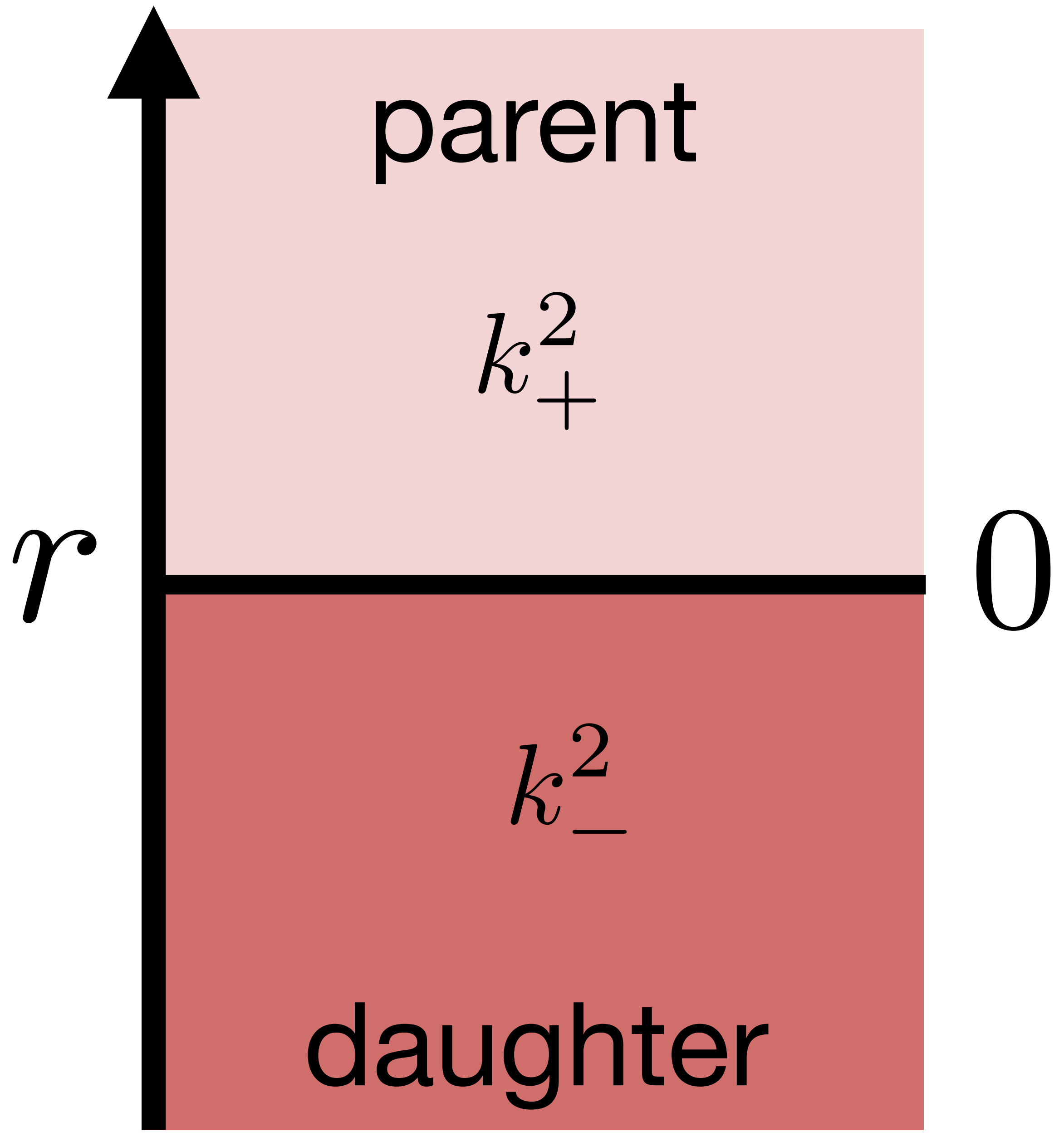


Instantons

Transition probability: $\frac{\Gamma}{\text{Vol}} \sim e^{-B/\hbar}$ $B = S_E(\text{instanton}) - S_E(\text{parent}).$



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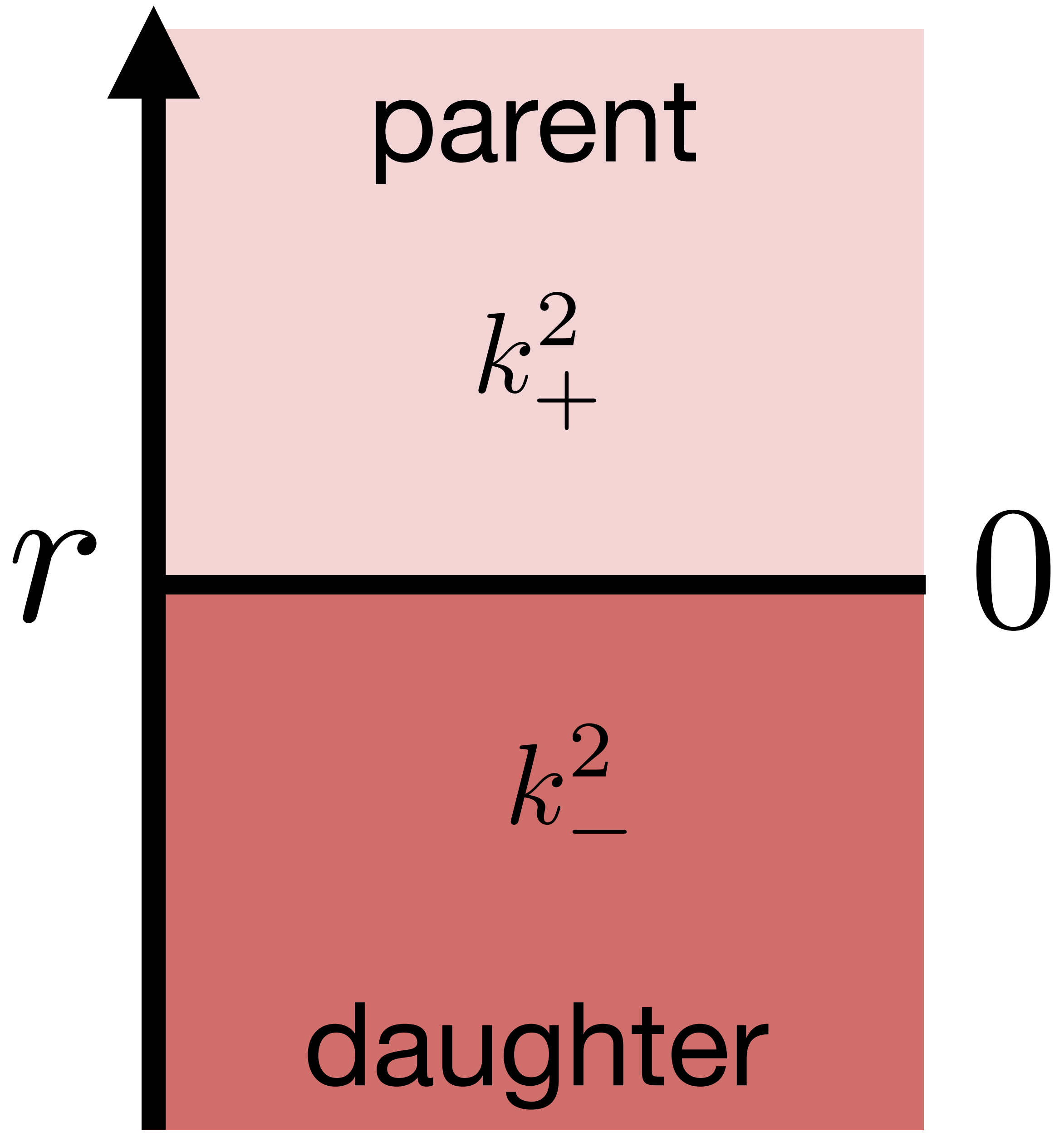

Junction conditions:

$$\Delta[\rho(0)] = 0 \qquad \Delta[\rho'(0)] = -\frac{T}{2M_{pl}^2}\rho(0)$$

$$\epsilon_+ \cos(kr_0)_+ = \rho'(0^+) < \rho'(0_-) = \epsilon_- \cos(kr_0)_-$$

Instantons

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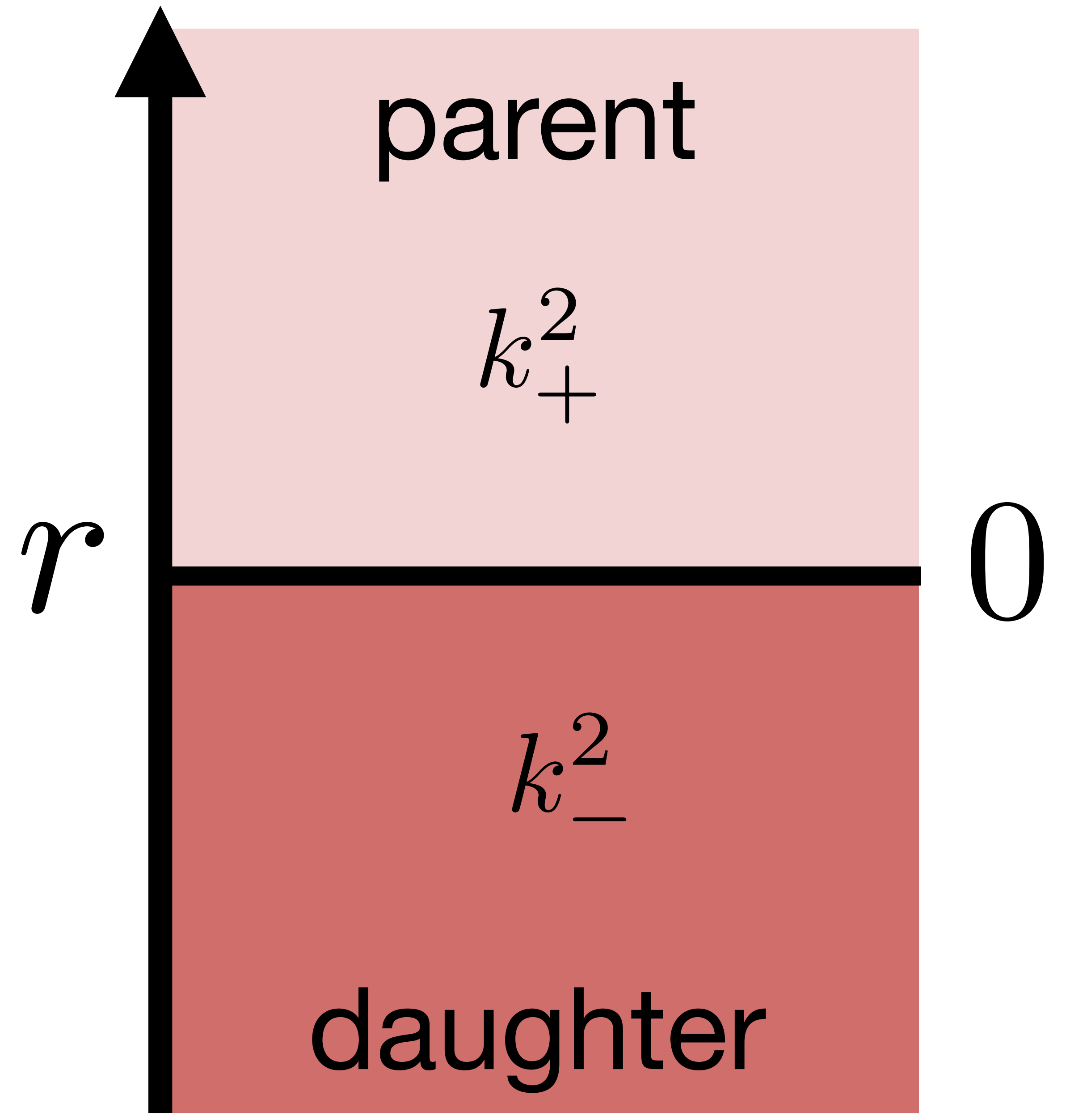

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kinematic constraint on the possible instanton configurations $\quad X \equiv \frac{4\Delta k^2 M_{pl}^4}{T^2}$

Instantons

<table>
<tr><th></th><th colspan="2">dS$_+$
$\epsilon_+ = +1$</th><th colspan="2">Minkowski/AdS$_+$
$\epsilon_+ = +1$</th><th colspan="2">Minkowski/AdS$_+$
$\epsilon_+ = -1$</th></tr>
<tr><td rowspan="2">dS$_-$
$\epsilon_- = +1$</td><td>$(kr_0)_+ \geq \frac{\pi}{2} \geq (kr_0)_-$
allowed for $|X| \leq 1$</td><td>$(kr_0)_+ \geq (kr_0)_- \geq \frac{\pi}{2}$
allowed for $X \leq -1$</td><td colspan="2" rowspan="2">negative tension</td><td rowspan="2">$(kr_0)_- \geq \frac{\pi}{2}$
kinematically
allowed for $X \leq -1$,
infinitely suppressed</td><td rowspan="2">$\frac{\pi}{2} \geq (kr_0)_-$
kinematically
allowed for $-1 \leq X \leq 0$,
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<tr><td>$\frac{\pi}{2} \geq (kr_0)_+ \geq (kr_0)_-$
allowed for $X \geq 1$</td><td>$(kr_0)_+ < (kr_0)_-$
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Instantons

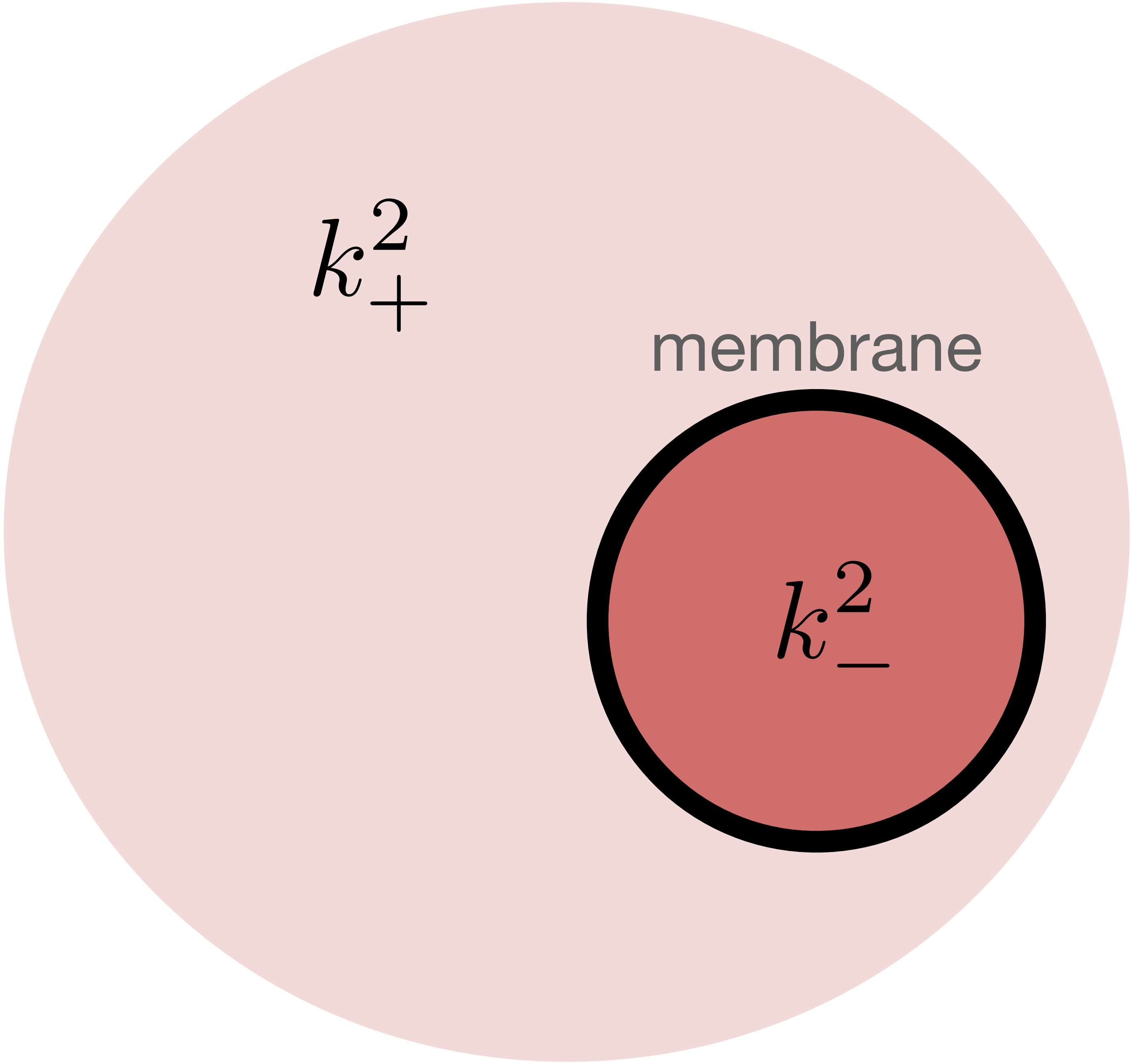
dS decays

	dS_+, $\epsilon_+ = +1$		Minkowski/AdS_+, $\epsilon_+ = +1$		Minkowski/AdS_+, $\epsilon_+ = -1$	
dS_- $\epsilon_- = +1$	$(kr_0)_+ \geq \frac{\pi}{2} \geq (kr_0)_-$ allowed for $\lvert X \rvert \leq 1$ $\frac{\pi}{2} \geq (kr_0)_+ \geq (kr_0)_-$ allowed for $X \geq 1$	$(kr_0)_+ \geq (kr_0)_- \geq \frac{\pi}{2}$ allowed for $X \leq -1$ $(kr_0)_+ < (kr_0)_-$ negative tension	negative tension		$(kr_0)_- \geq \frac{\pi}{2}$ kinematically allowed for $X \leq -1$, infinitely suppressed	$\frac{\pi}{2} \geq (kr_0)_-$ kinematically allowed for $-1 \leq X \leq 0$, infinitely suppressed
Minkowski/AdS_- $\epsilon_- = +1$	$(kr_0)_+ \geq \frac{\pi}{2}$ allowed for $0 \leq X \leq 1$	$\frac{\pi}{2} \geq (kr_0)_+$ allowed for $X \geq 1$	$\lvert k_- \rvert \geq \lvert k_+ \rvert$ allowed for $X \geq 1$	$\lvert k_- \rvert < \lvert k_+ \rvert$ negative tension	kinematically allowed for $\lvert X \rvert \leq 1$, infinitely suppressed	
Minkowski/AdS_- $\epsilon_- = -1$	negative tension		negative tension		$\lvert k_- \rvert > \lvert k_+ \rvert$ negative tension	$\lvert k_- \rvert \leq \lvert k_+ \rvert$ kinematically allowed for $X \leq -1$, infinitely suppressed

Instantons

Transition probability: $$\frac{\Gamma}{\text{Vol}} \sim e^{-B/\hbar}$$ $$B = S_E(\text{instanton}) - S_E(\text{parent}).$$

For dS to AdS/Mink transitions: $$B = \frac{4M_{pl}^2\Omega_3}{k_+^2}\left[\frac{1+Y-X}{Y(1+Y+X)}\right]$$



$$Y(X) = \sqrt{(X-1)^2 + 16k_+^2 M_{pl}^4/T^2}$$

$$X \equiv \frac{4\Delta k^2 M_{pl}^4}{T^2}$$

Vacuum Stability

Is our Universe safe?

$$B_{M_+\to AdS_-} \sim \frac{2M_{pl}^2\Omega_3}{k_+^2}(1-S(X)) + \frac{8M_{pl}^6\Omega_3}{T^2X(X-1)^2}\left[(X-1)^2(1-S(X))+2S(X)\right]$$

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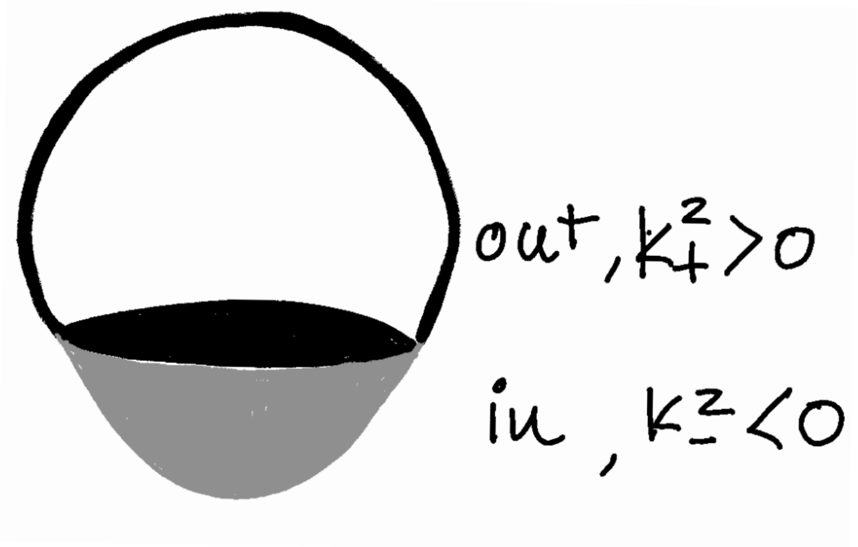

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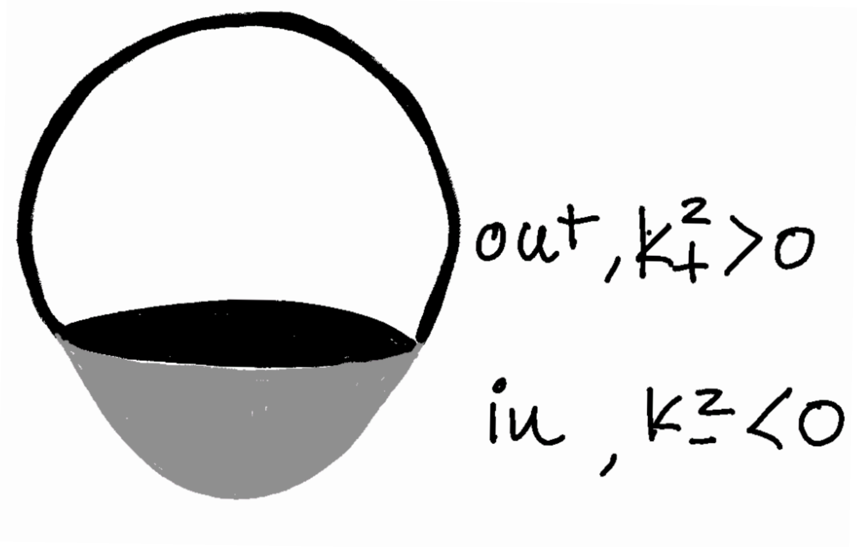

$$\frac{4M_{pl}^2\Omega_3}{k_+^2}, \qquad X<1.$$

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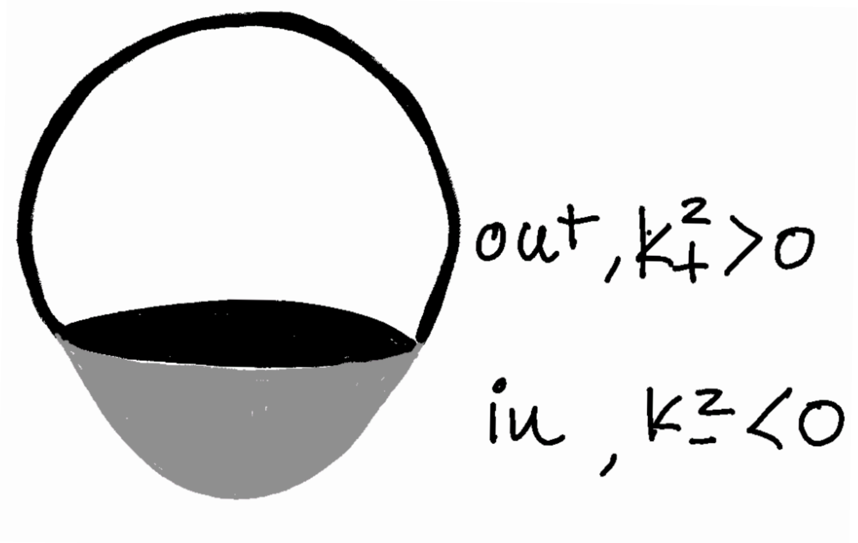

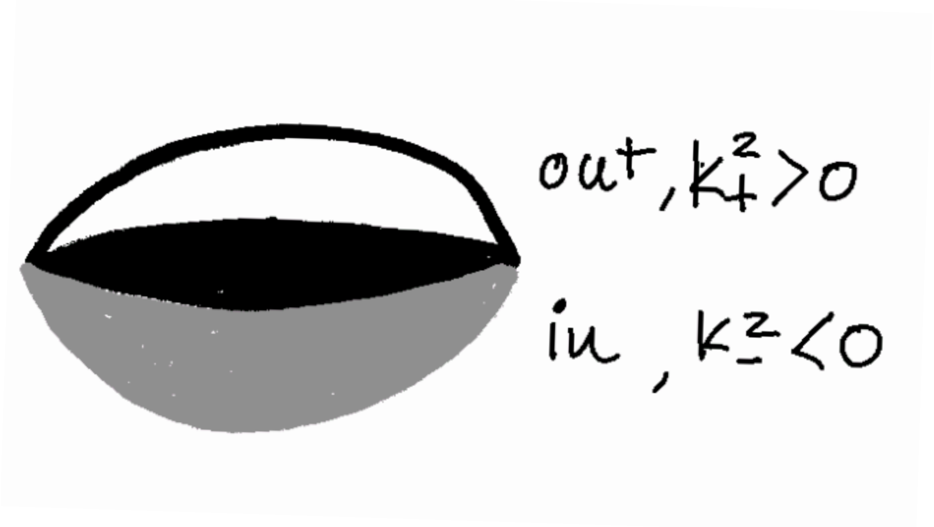

What is the price of stability?

$$X \equiv \frac{4M_{pl}^2\Delta k^2}{T^2}$$

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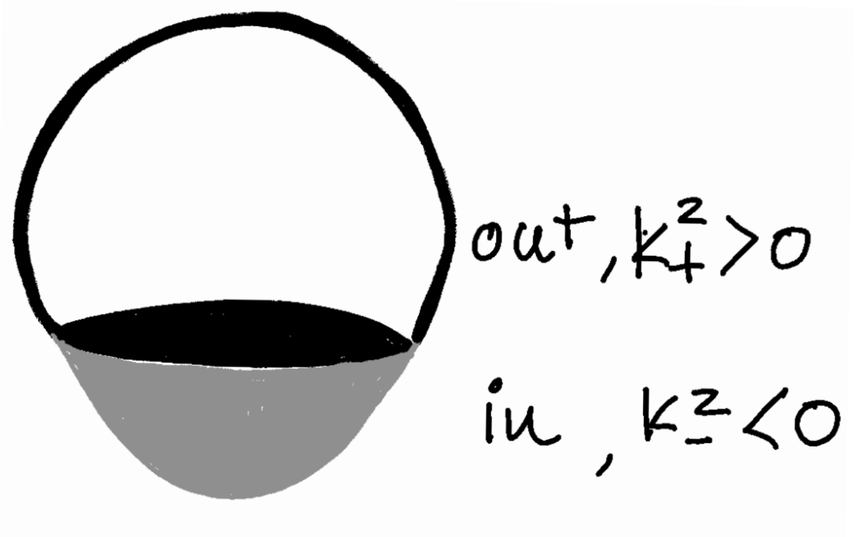

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linear model: $\Delta k^2 = q$ → $\frac{qM_{pl}}{T}\frac{M_{pl}M_{uv}^2}{T}<1$ → $\frac{qM_{pl}}{T}<1$ ~~WGC~~

Vacuum Stability

$$N = N_* = \sqrt{2|\Lambda_{bare}|}/q$$

quadratic model: $\Delta k^2(N) = (N + \frac{1}{2})\frac{q^2}{M_{pl}^2}$ → $X_* \propto \frac{qM_{pl}}{T}\frac{M_{uv}^2 M_{pl}}{T}\frac{\sqrt{|\Lambda_{bare}|}}{M_{uv}^2}$

↓

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Conclusion holds for BT and BP and for general F^n

"A (sufficient) requirement for membrane creation to stop is […]" $\frac{|e|}{m^2} \lesssim 6\pi G\sqrt{\frac{4\pi G}{-\lambda}}$.

[Brown Teitelboim, '88]

Vacuum Stability

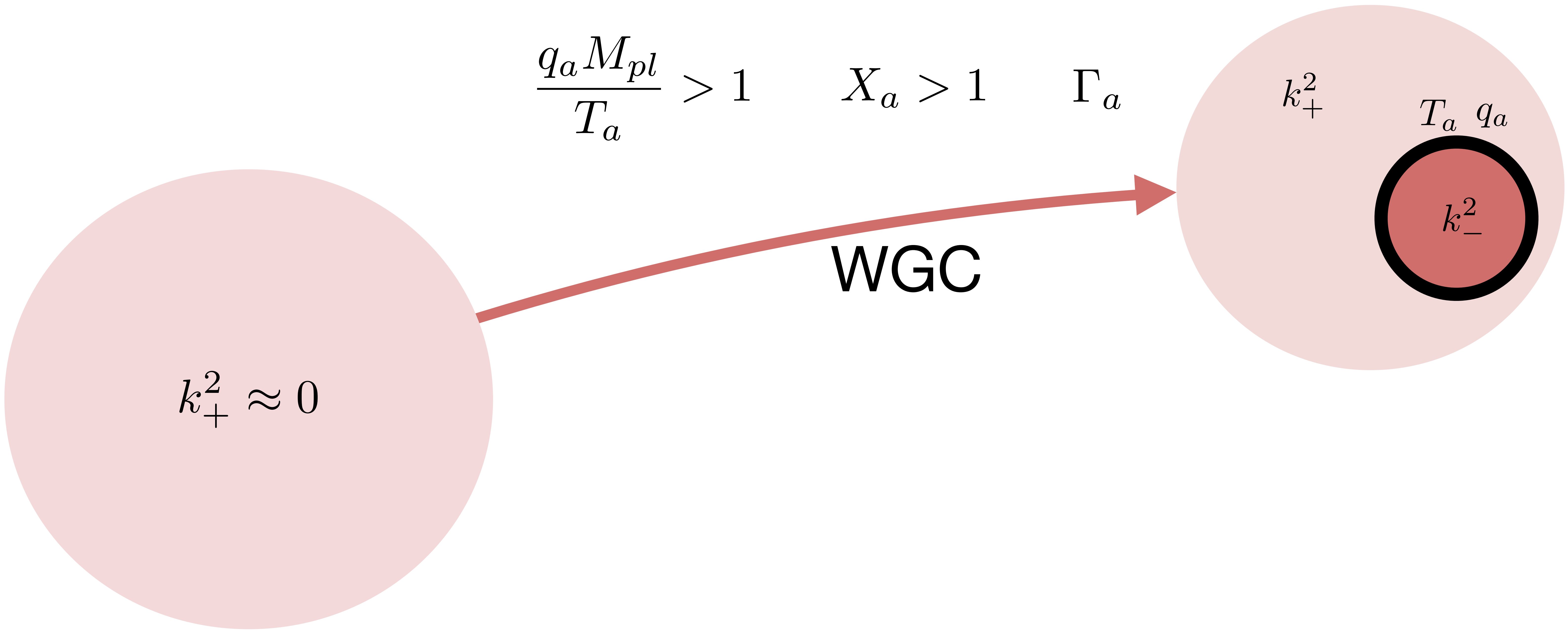
[Liu,Padilla,FGP '24]

$k_+^2 \approx 0$

Vacuum Stability

[Liu,Padilla,FGP '24]

$$\frac{q_a M_{pl}}{T_a} > 1 \qquad X_a > 1 \qquad \Gamma_a$$



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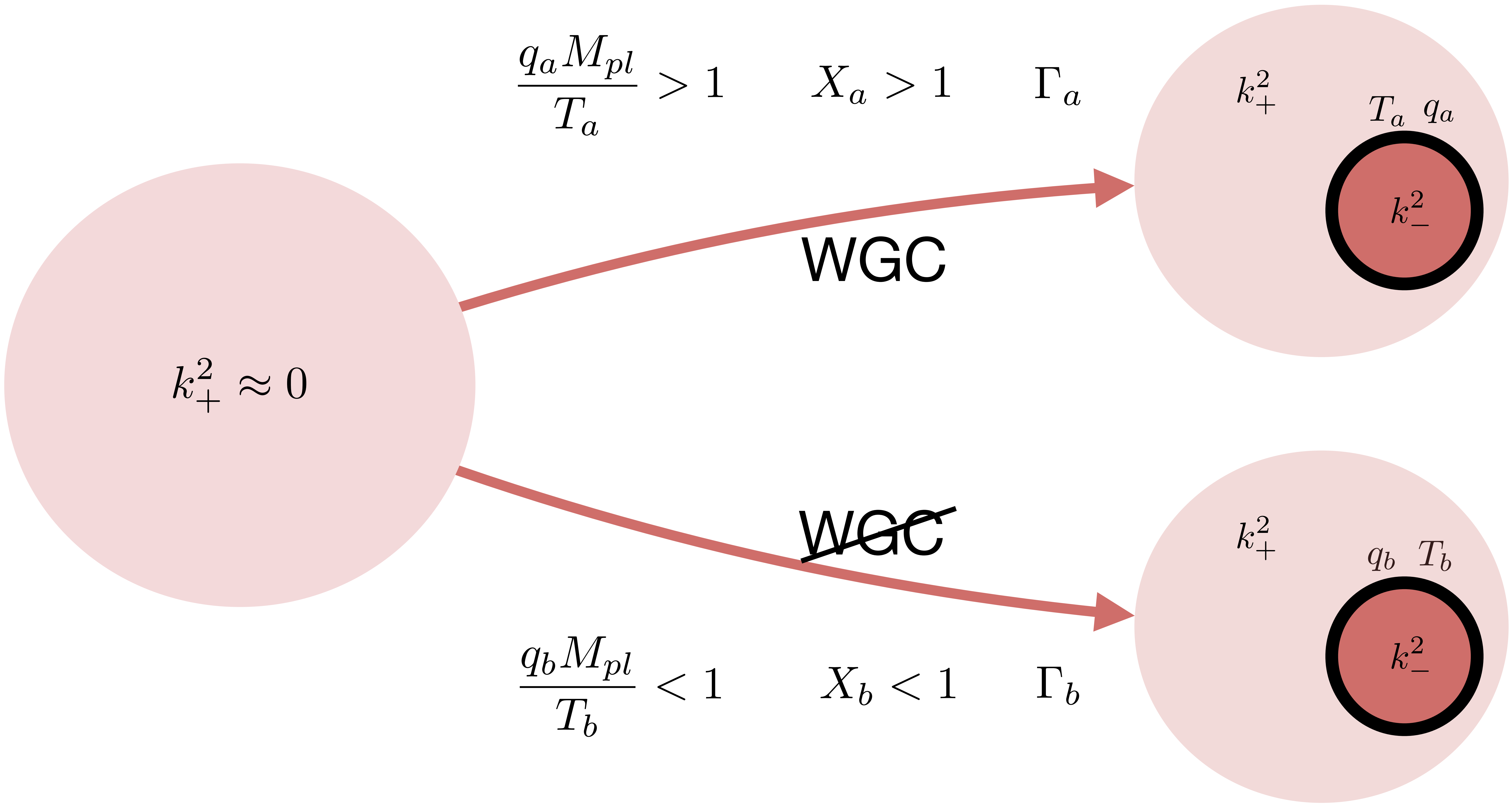
[Liu,Padilla,FGP '24]



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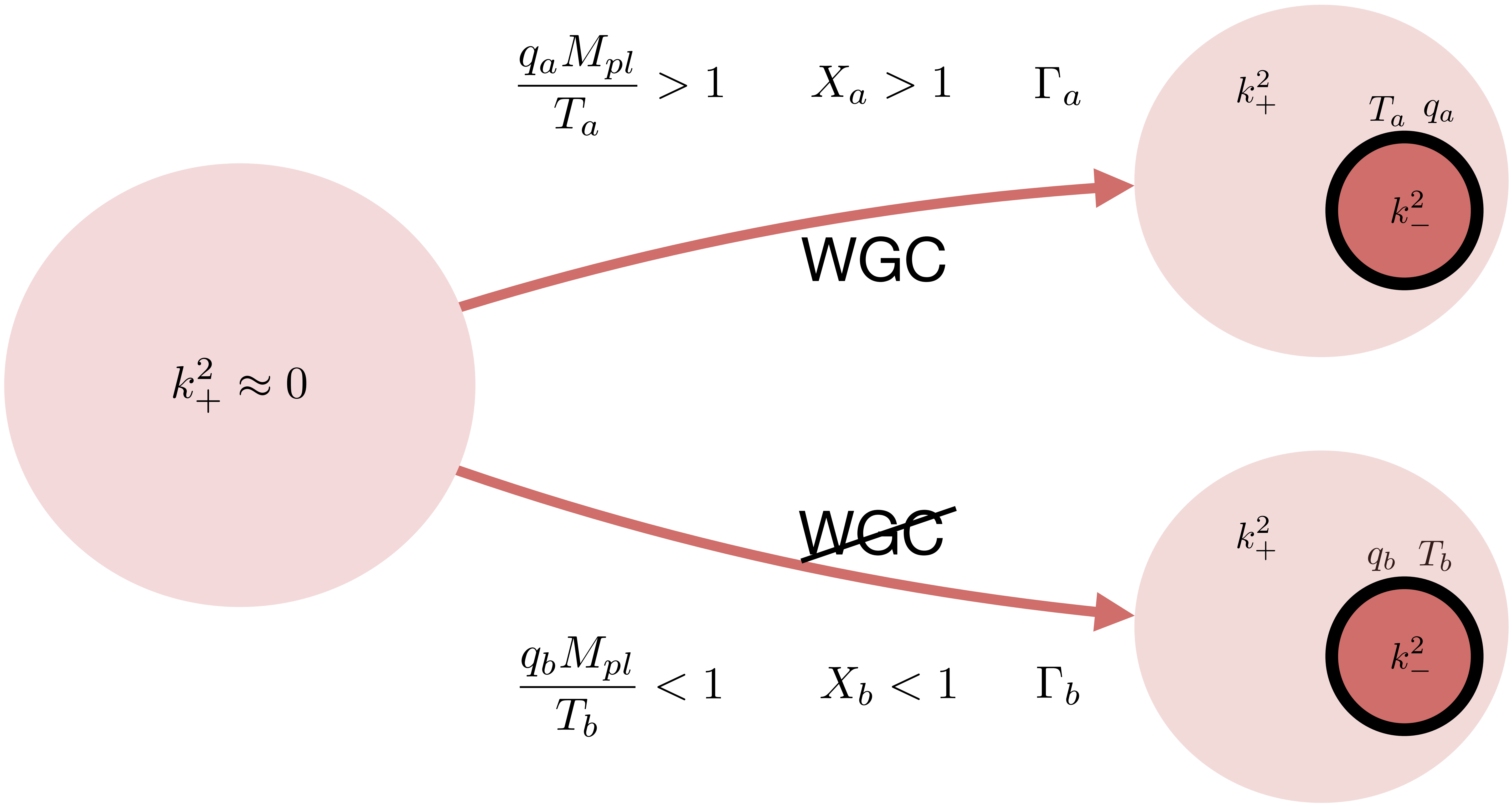


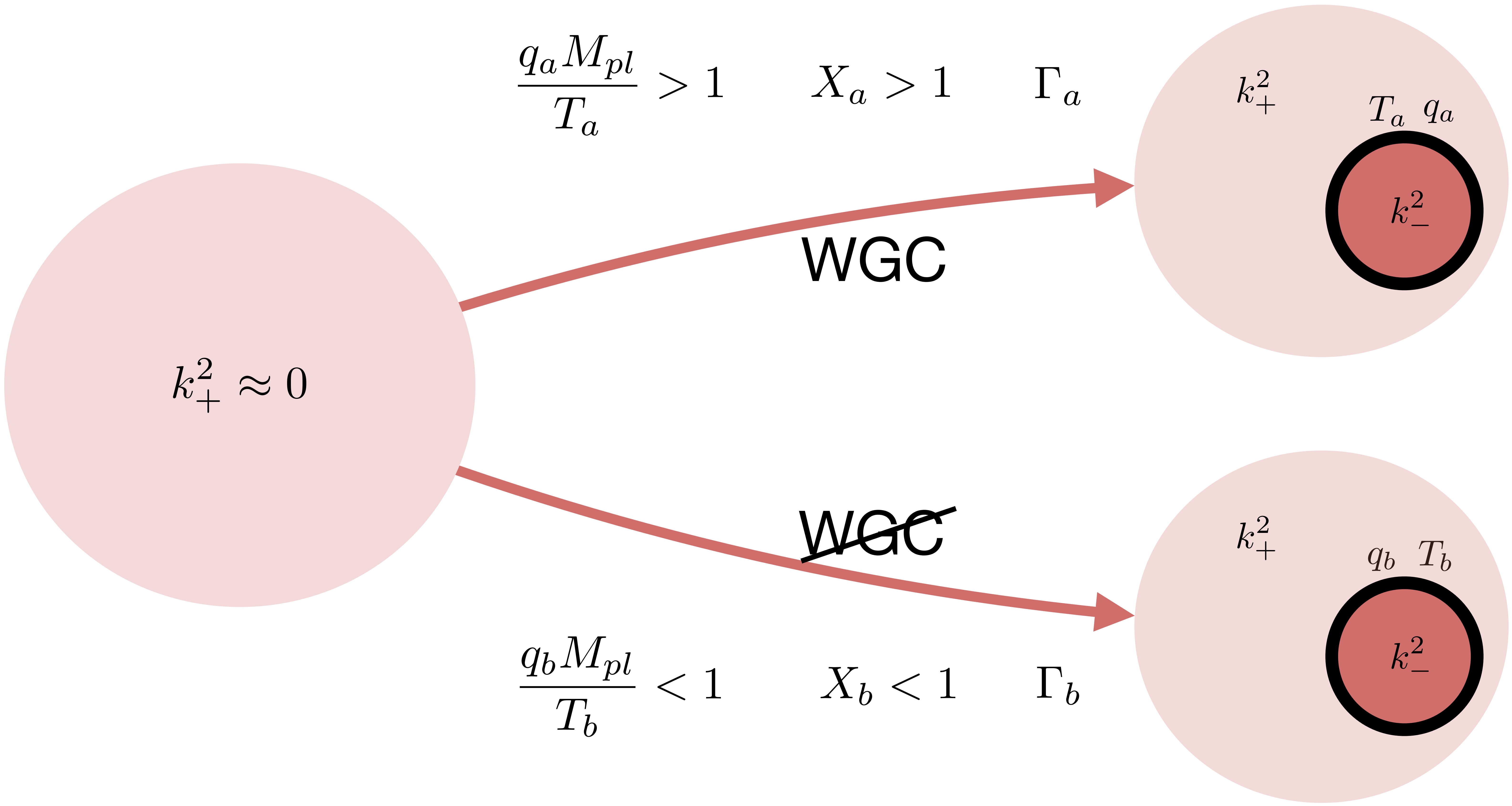
$$\frac{dB}{dX} < 0 \qquad B_a < B_b$$

$$\Gamma_a \gg \Gamma_b \approx 0$$

WGC respecting decays dominate decay dynamics

Vacuum Stability

[Liu,Padilla,FGP '24]



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WGC respecting decays dominate decay dynamics

Stability of Minkowski preserved only if all brane species violate WGC

Similar to charged BH argument

Summary II

- Link between stability of our vacuum and the (violation of) WGC
- Link holds for generic 4 form theories
- Stability removed if $\exists$ WGC respecting branes

• • • •

- Very special landscapes, can similar arguments hold in general?
- How to go beyond this 4D EFT picture?
- Why $H_0 \sim 10^{-60} M_{pl}$?
- Does longevity = likelihood in the landscape ?

Thank you

arXiv: 2303.17723 / 2404.02961

Extras

Multifield Quintessence

[Spintessence, Boyle et al.'01]
[Sonner and Townsend '06]
[Achucarro and Palma '18]

Multifield modulus+axion

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\gamma_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi) \ ,$$

[(Brinkman),Cicoli,Dibitetto,FGP '20-22]

$$\gamma_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & f^2(\phi_1) \end{pmatrix} \qquad f(\phi_1) = e^{-k_1\phi_1} \qquad V = V_0\, e^{-k_2\phi_1}$$

$$R_{fs} = -2k_1^2$$

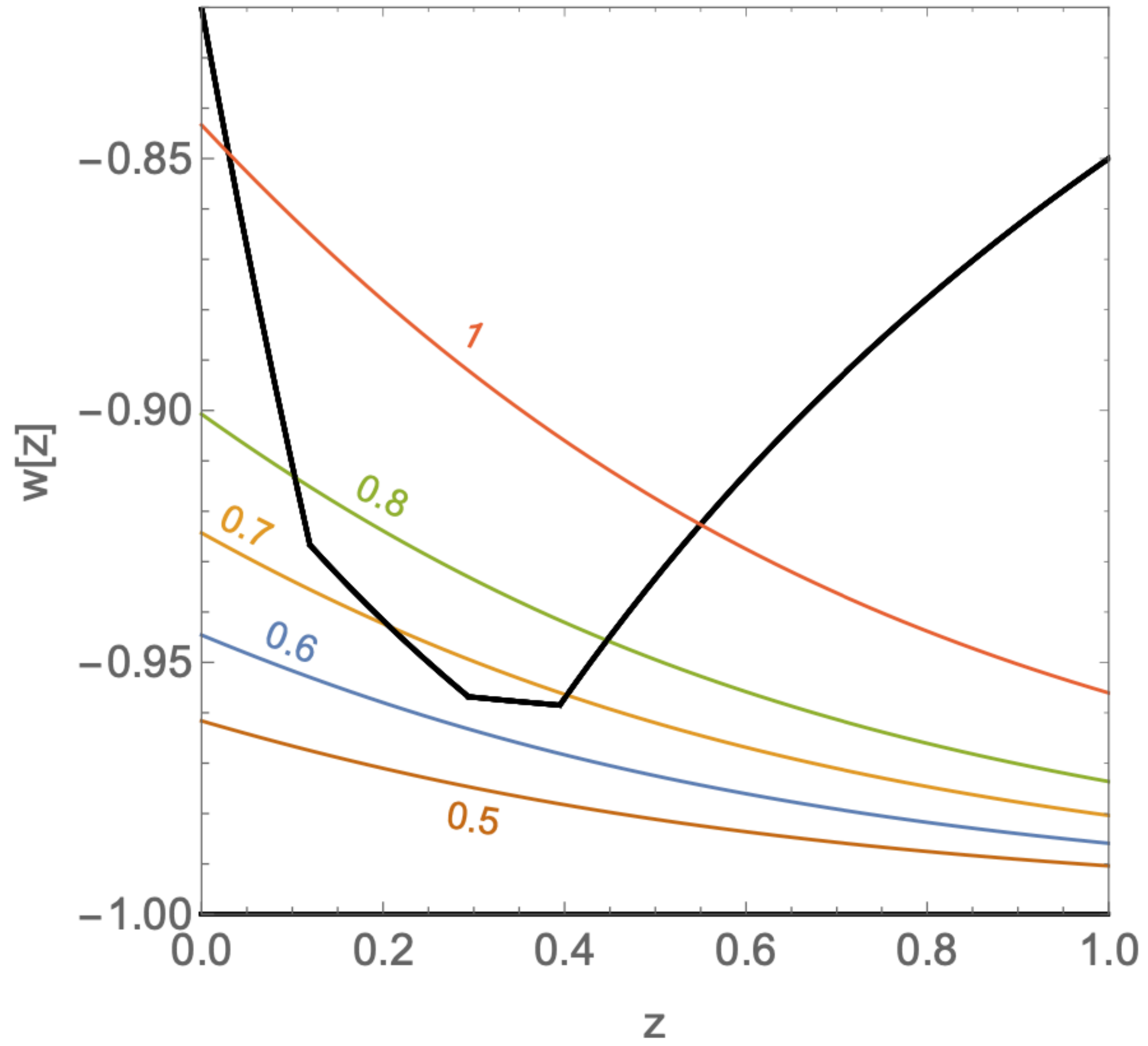
Exploit difference between $\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2$ and $\epsilon_H = -\frac{\dot{H}}{H^2}$

accelerated expansion in steep potentials: $\frac{k_2^2}{2} \gtrsim \mathcal{O}(1)$

Multifield Quintessence

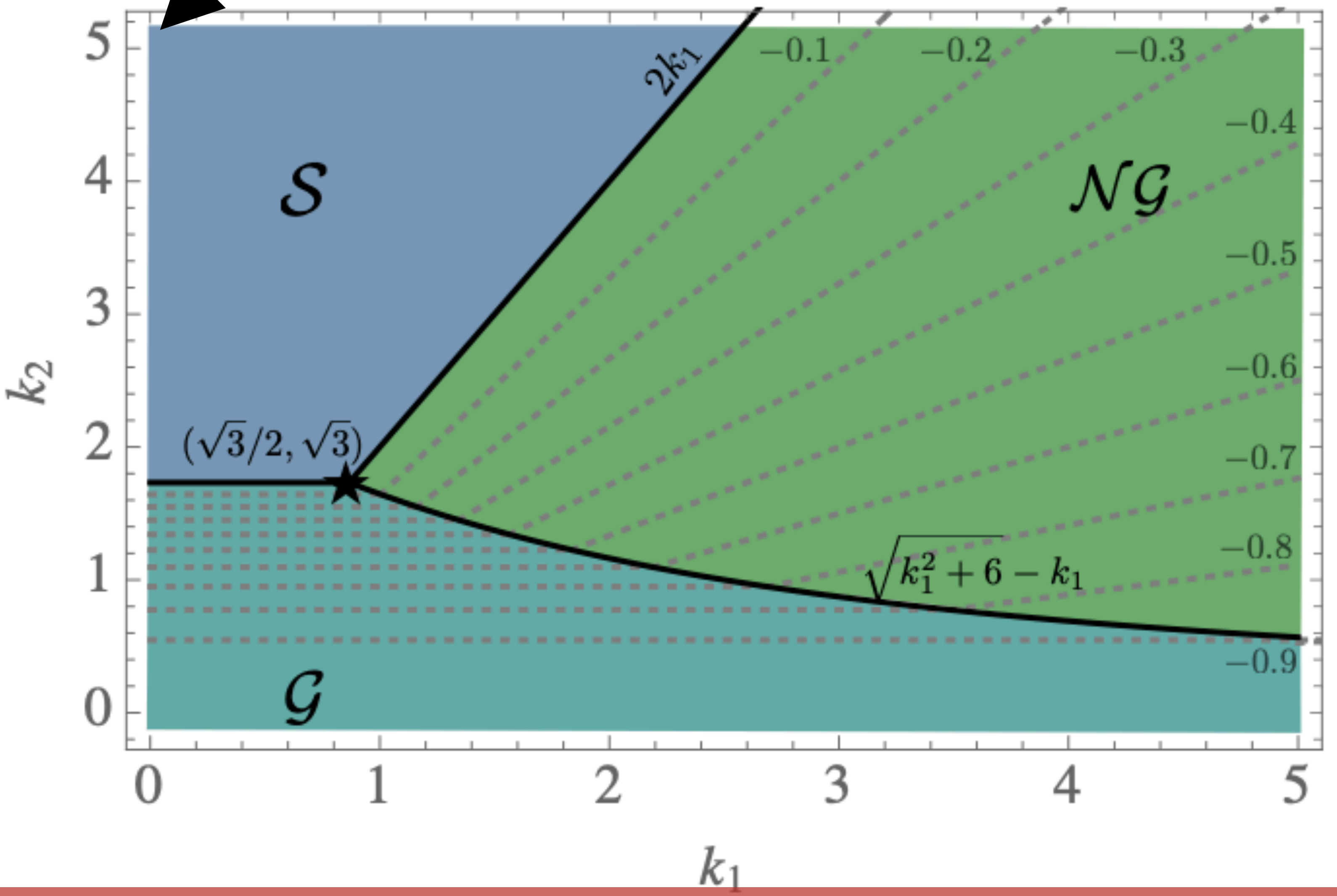
Look for observationally viable quintessence

$$\omega_{DE} \sim -1$$
$$\Omega_{DE} = 0.7$$



[Agrawal et al. '18]

Single field regime $k_1 = 0$



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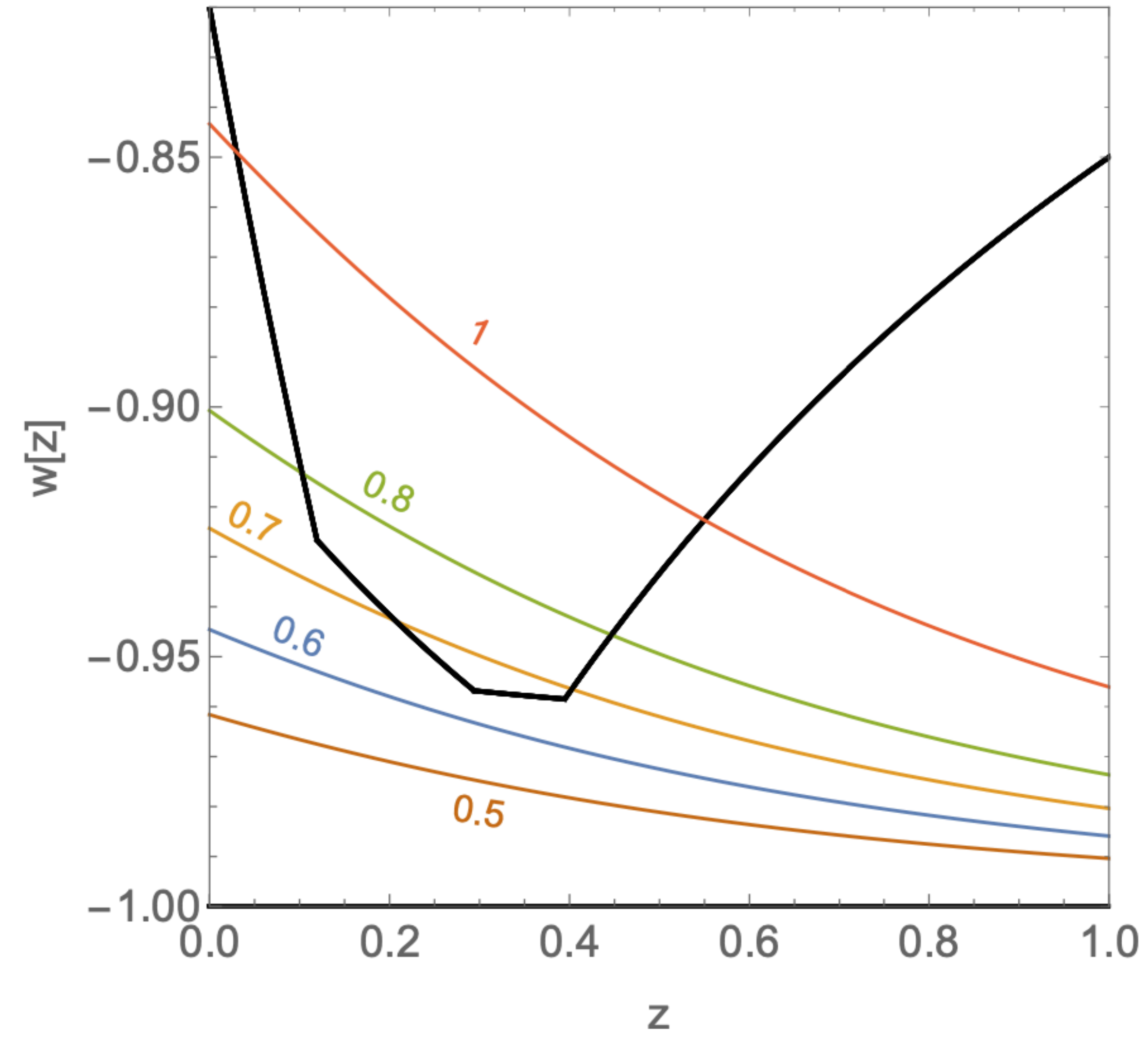

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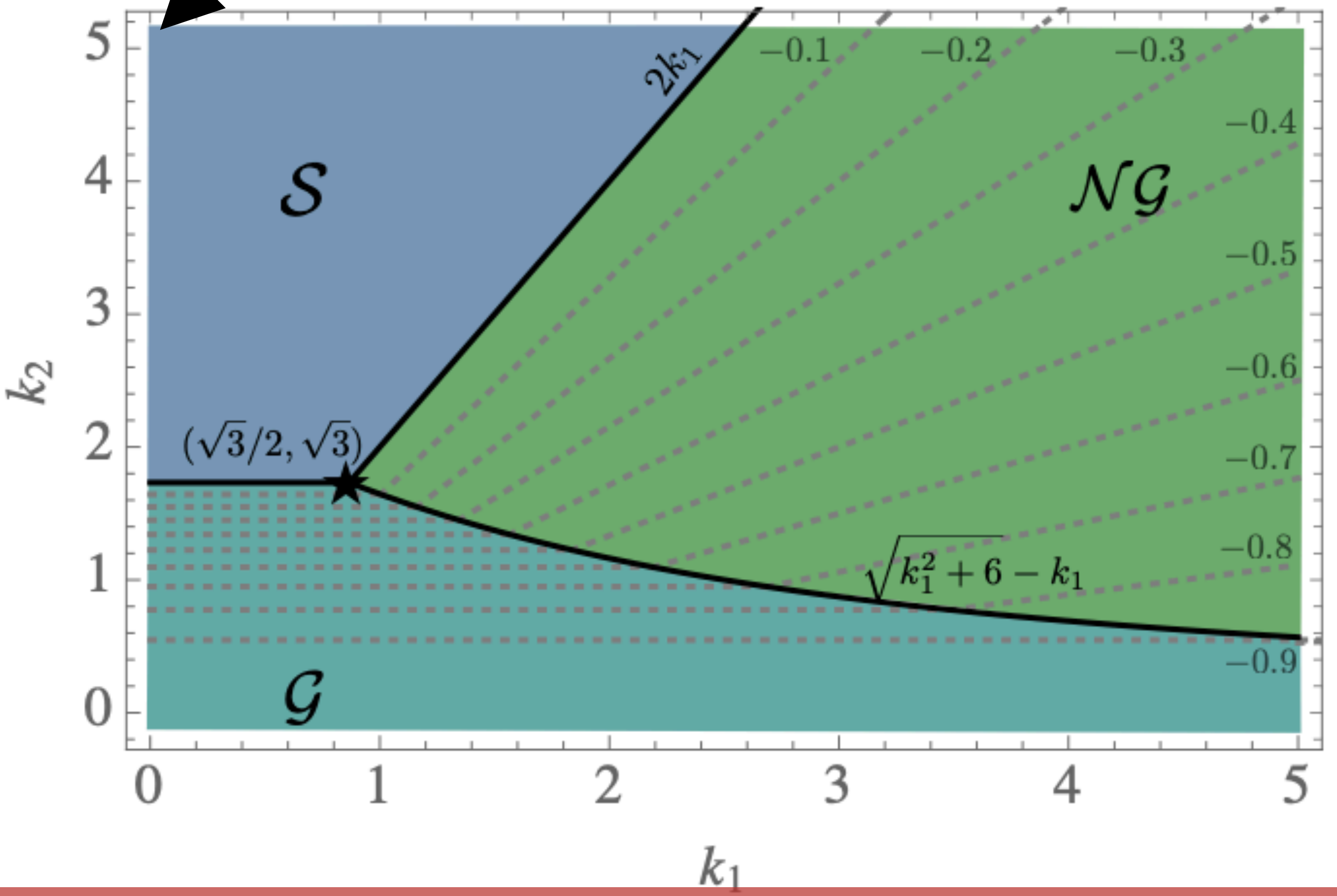

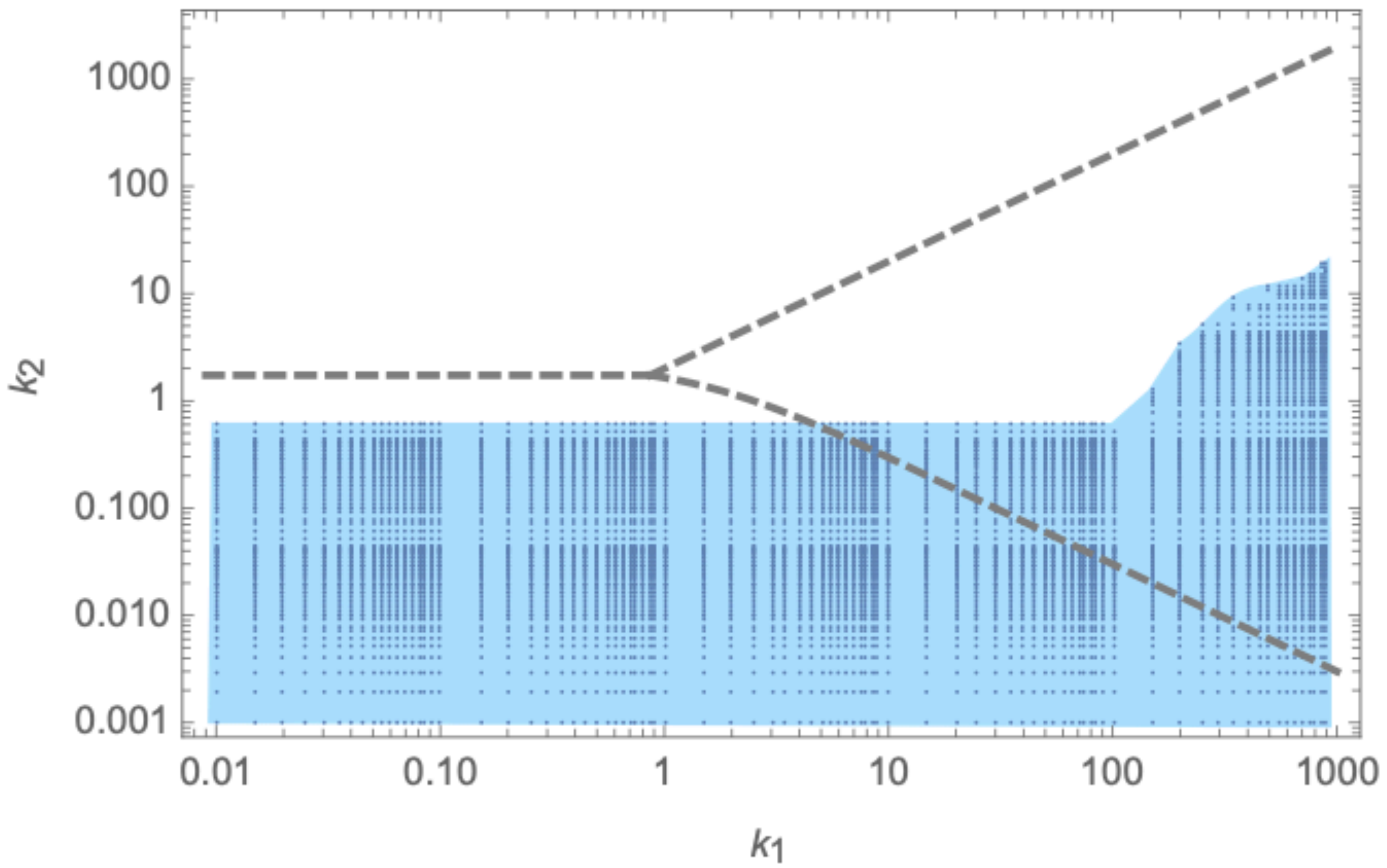

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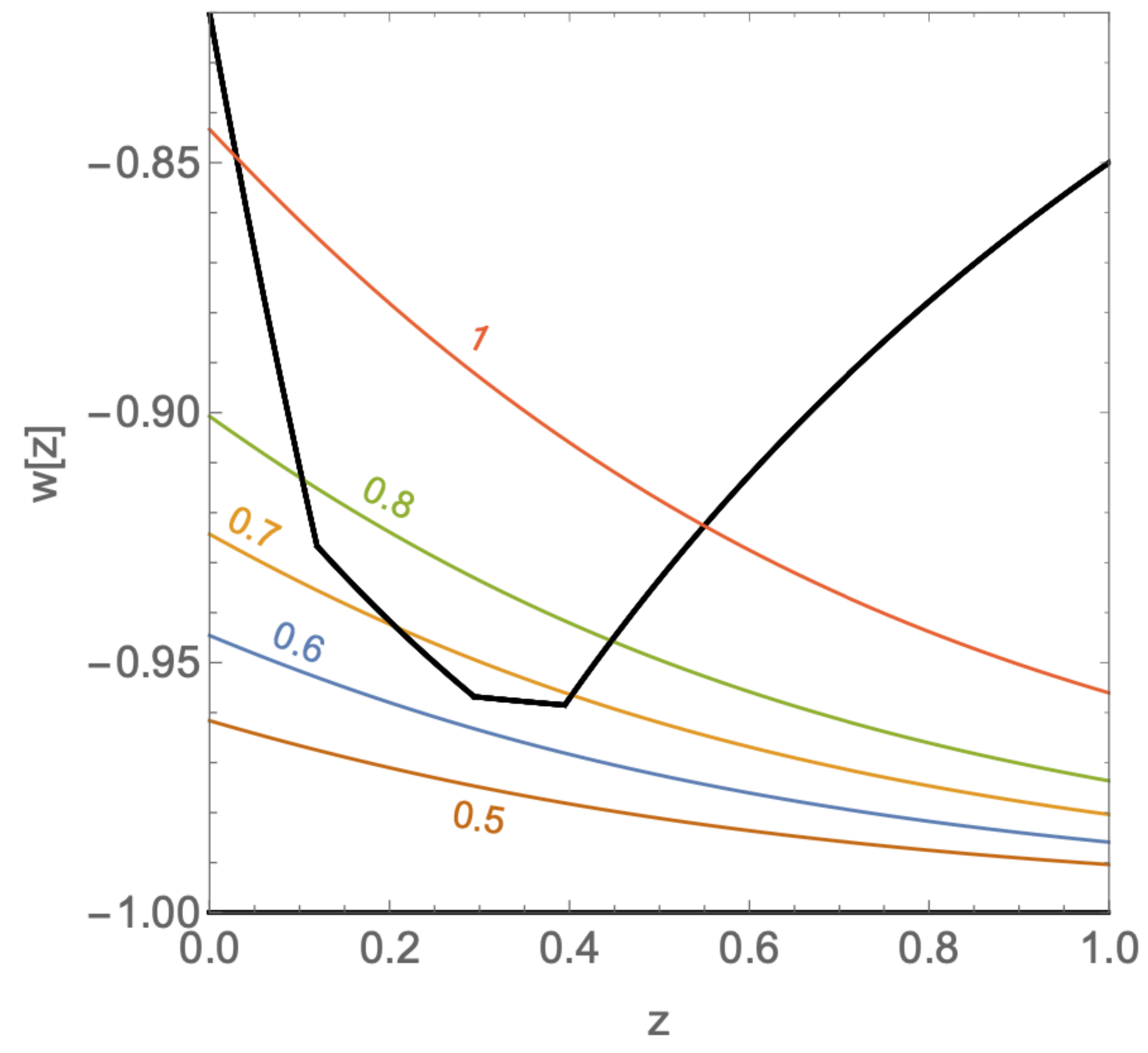

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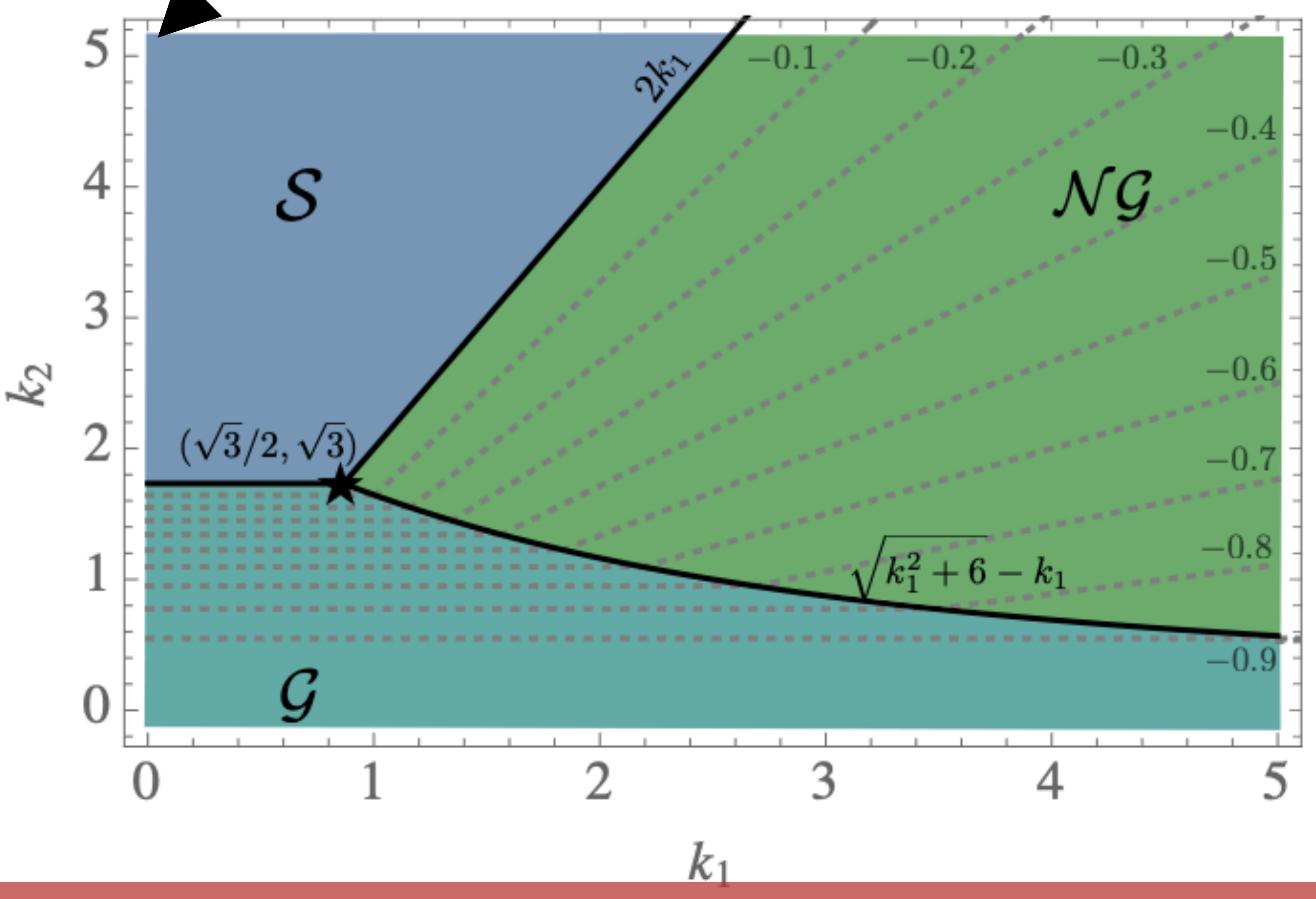

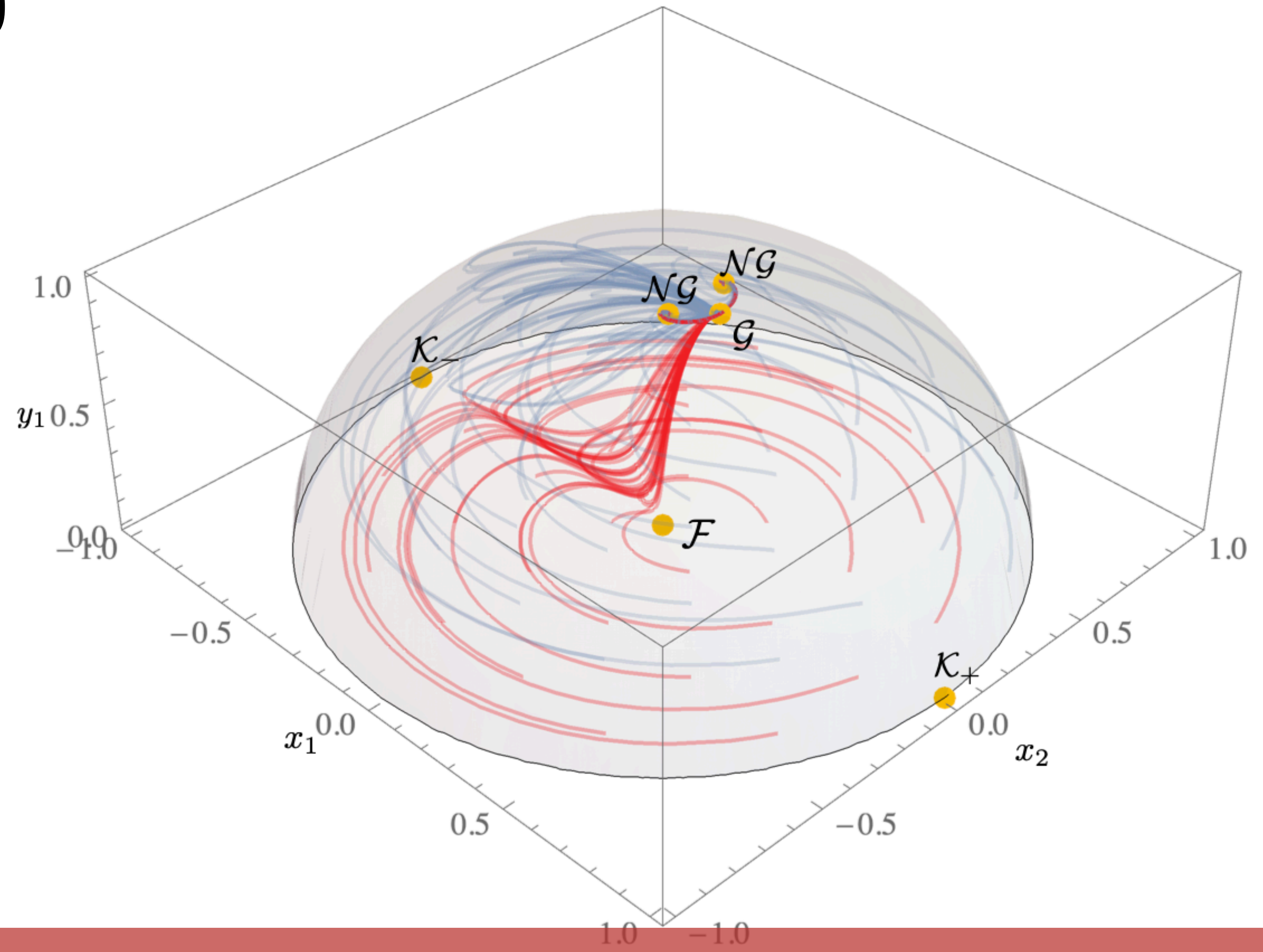

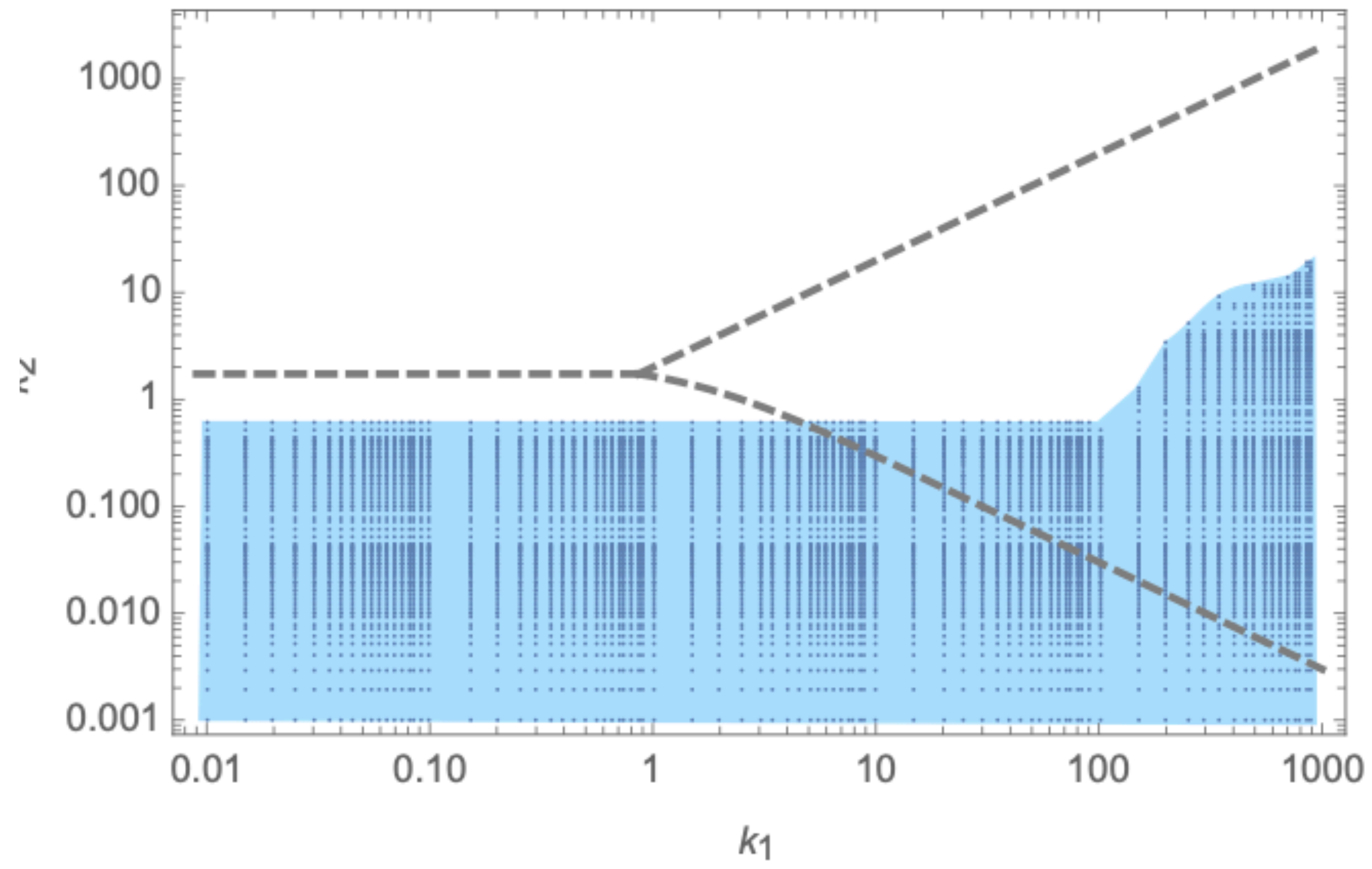

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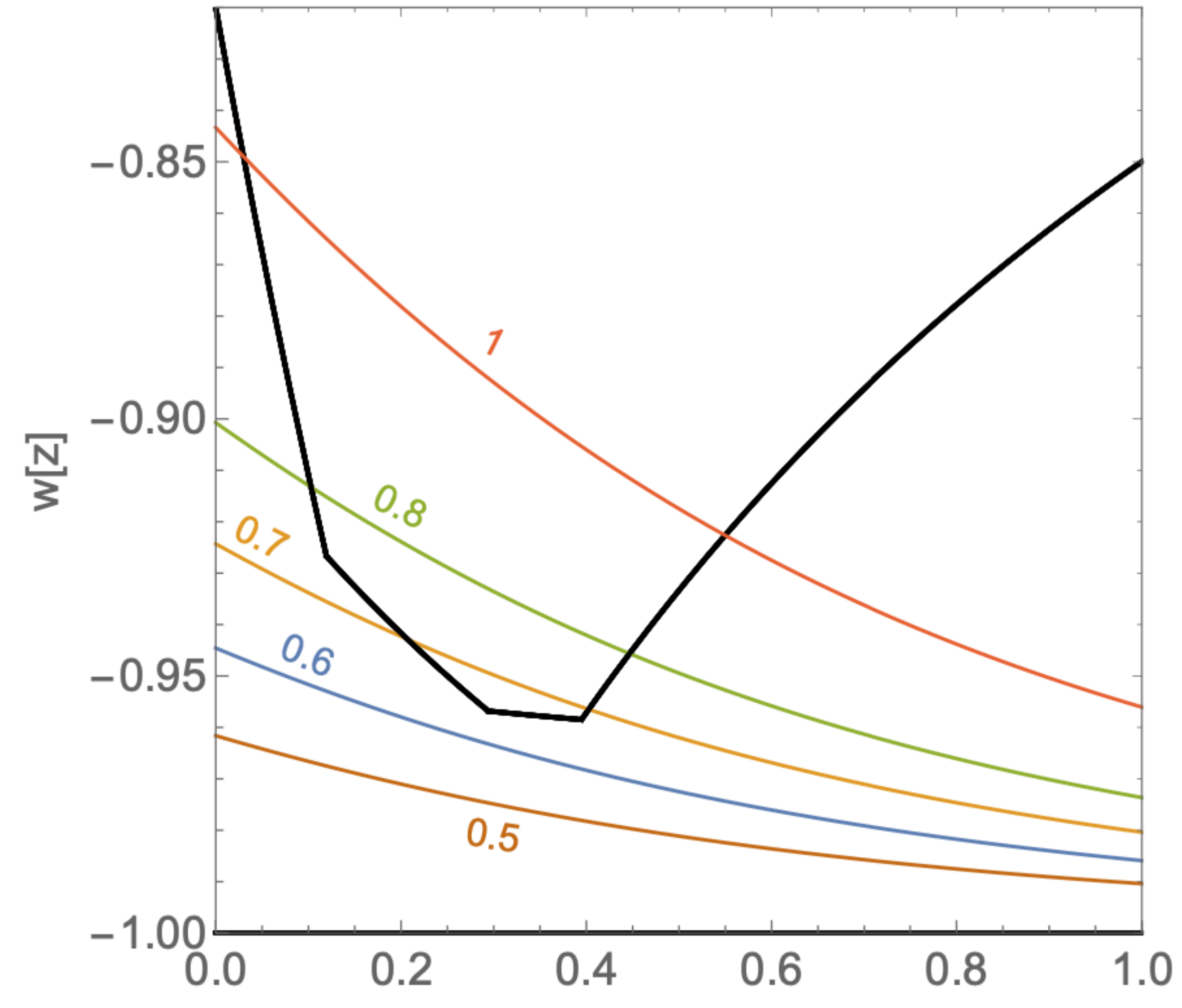

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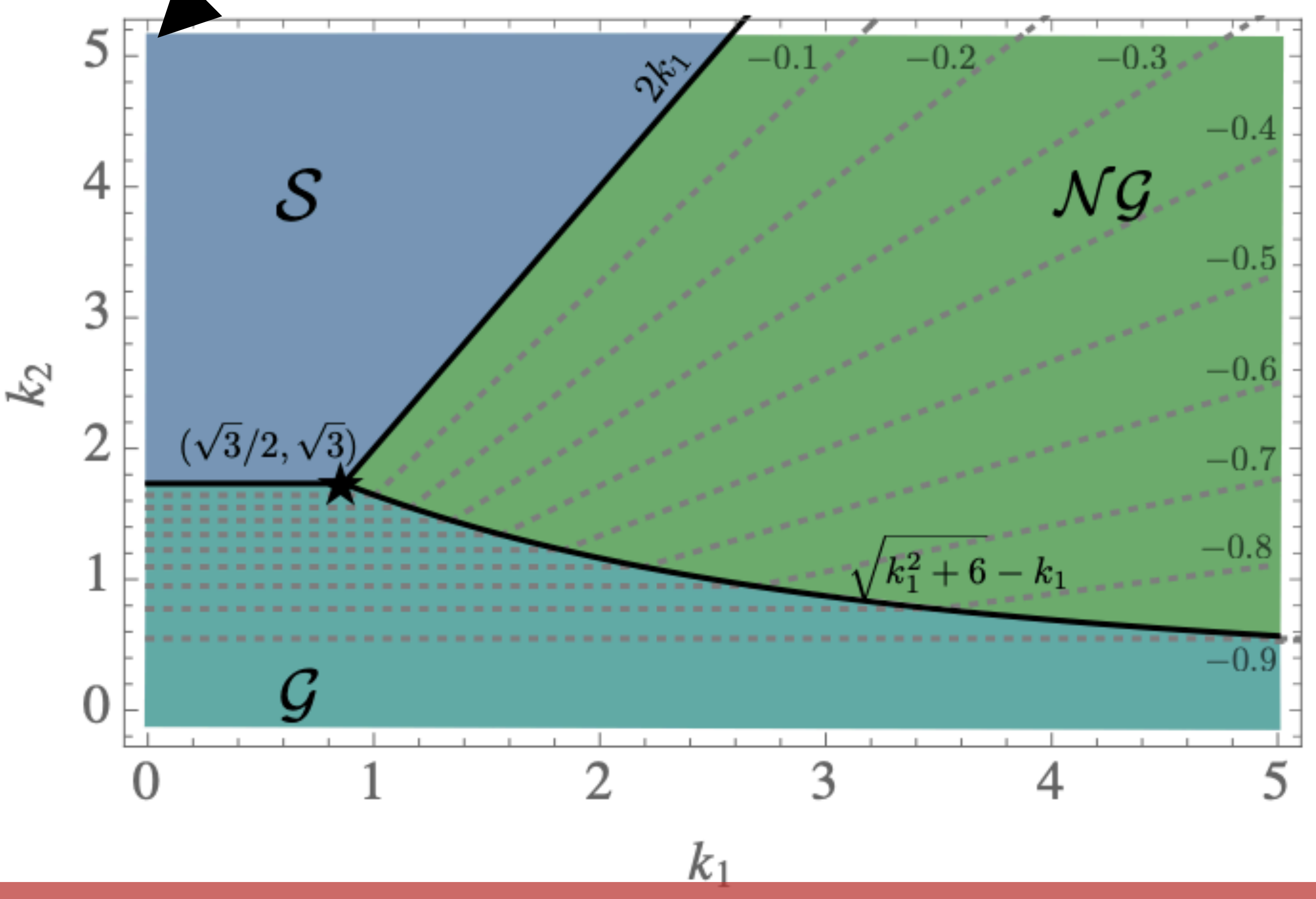
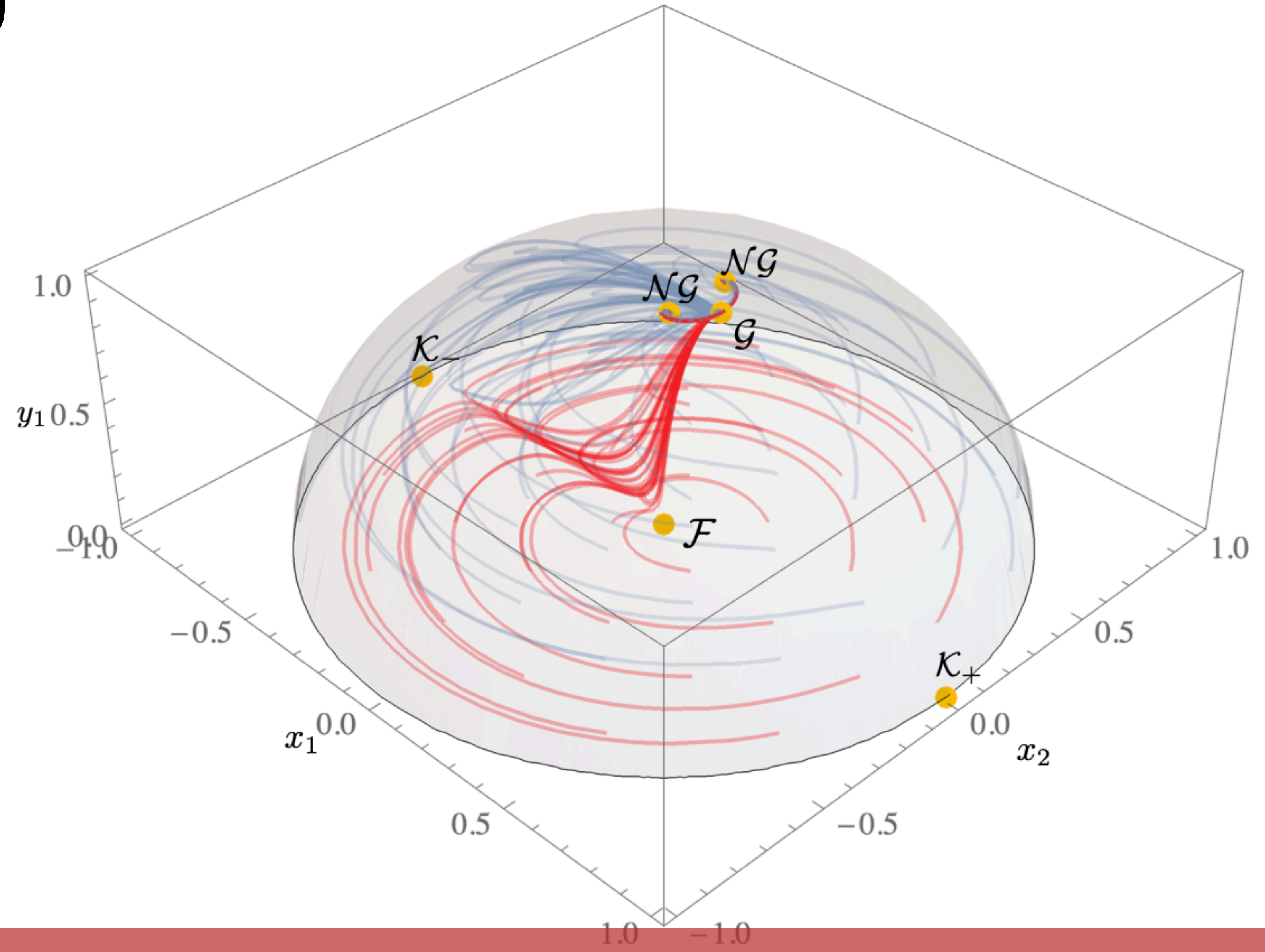
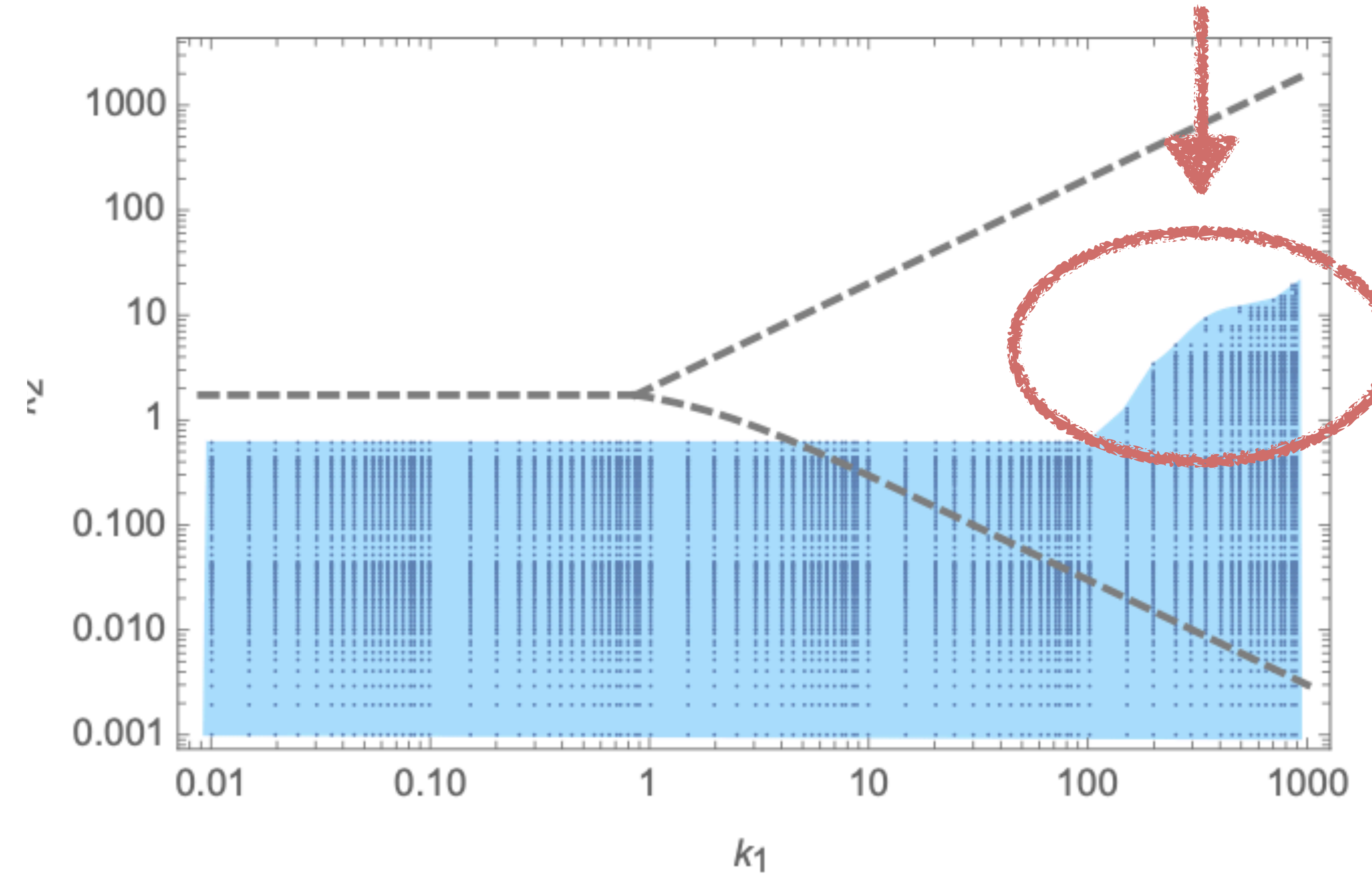


Steep potentials require LARGE field space curvatures





Multifield Quintessence

[Saltman,Silverstein, ’04]
[(Brinkman),Cicoli,Dibitetto,FGP 2020-22]

$$K = -p \ln[X + \bar{X}] \qquad V_K \propto e^K = (X + \bar{X})^{-p}$$

$$k_1 = \sqrt{2/p} \qquad \text{and} \qquad k_2 = \sqrt{2p}\,.$$

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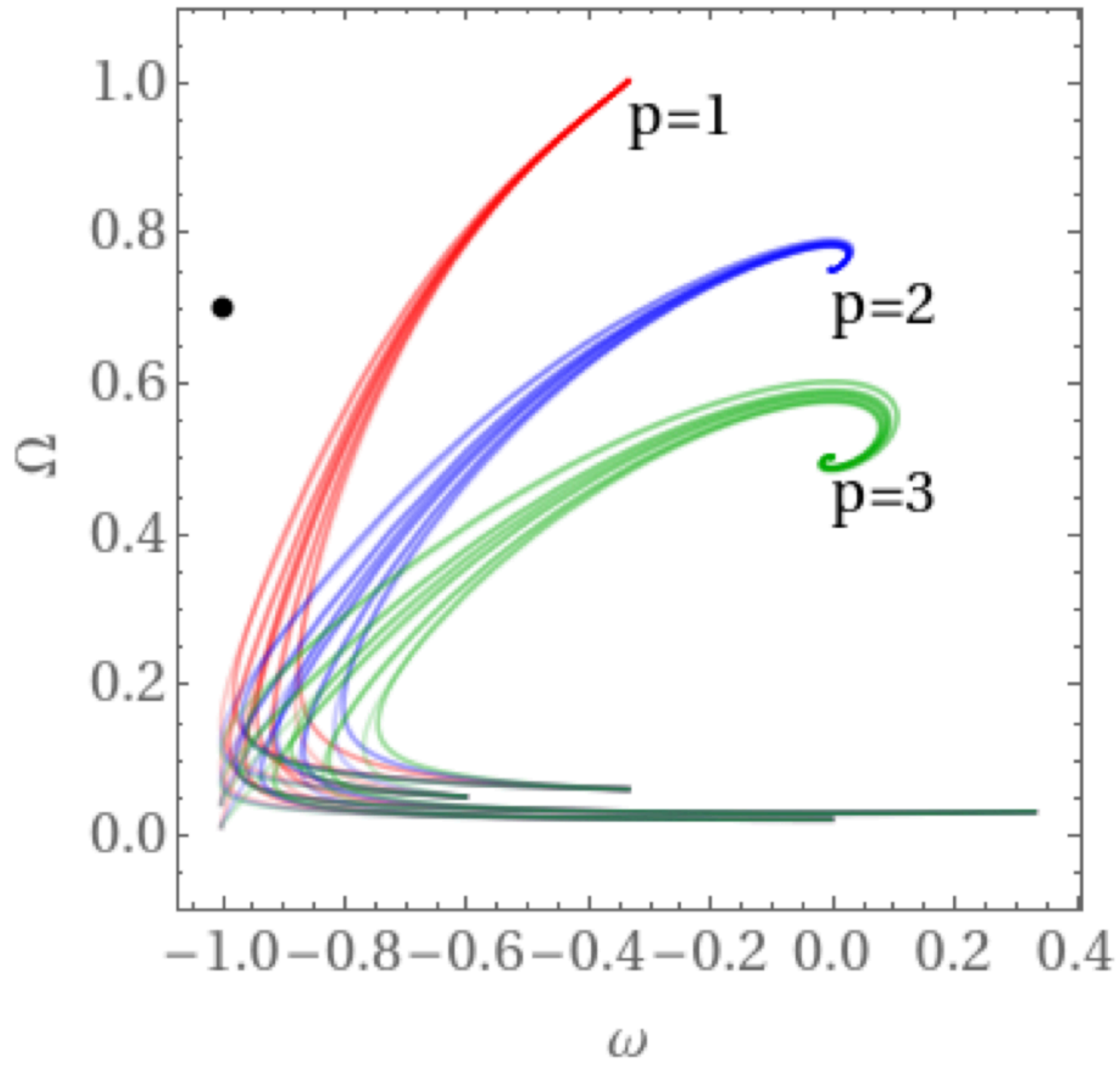

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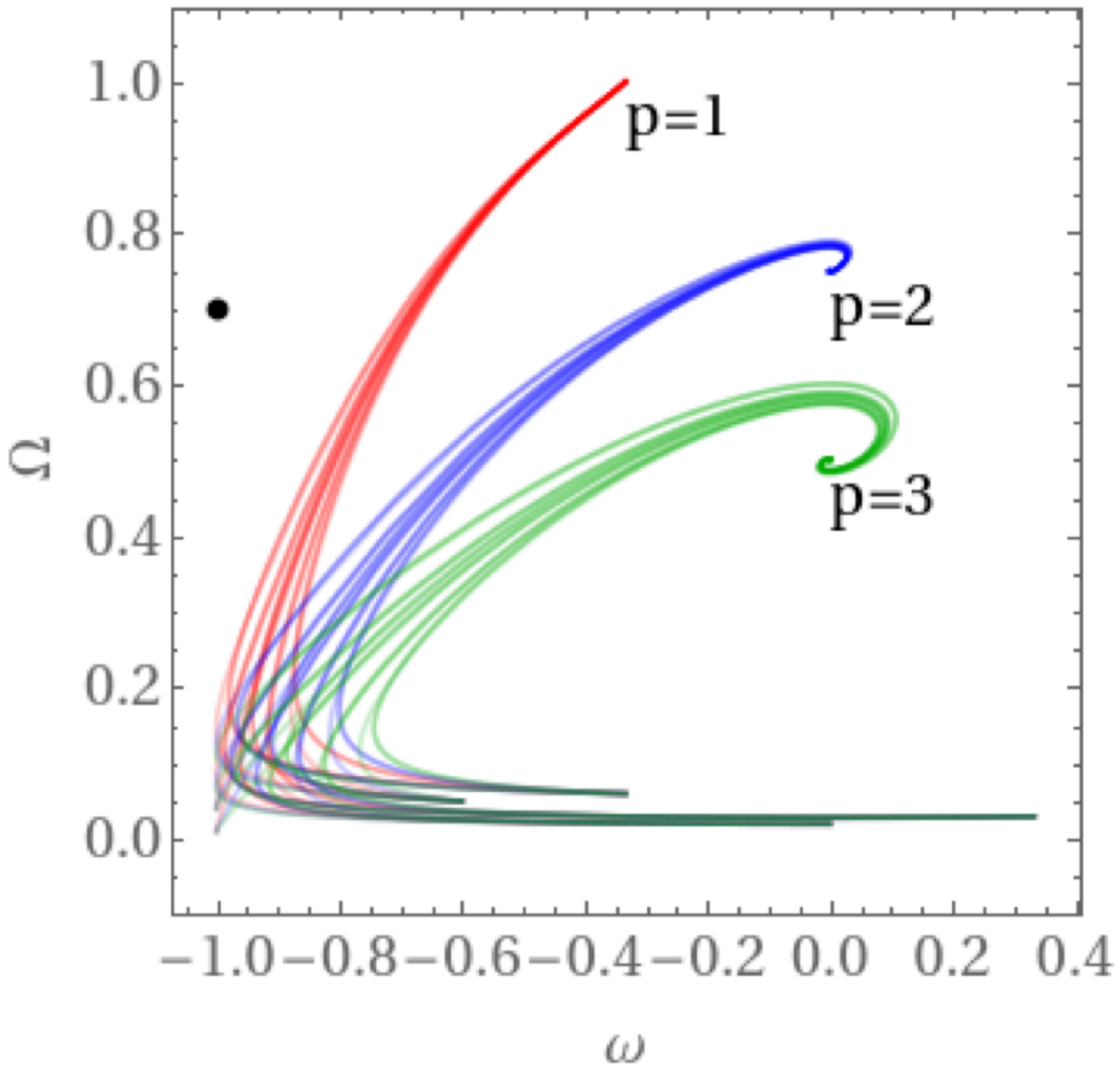
?Embeddable into string theory?



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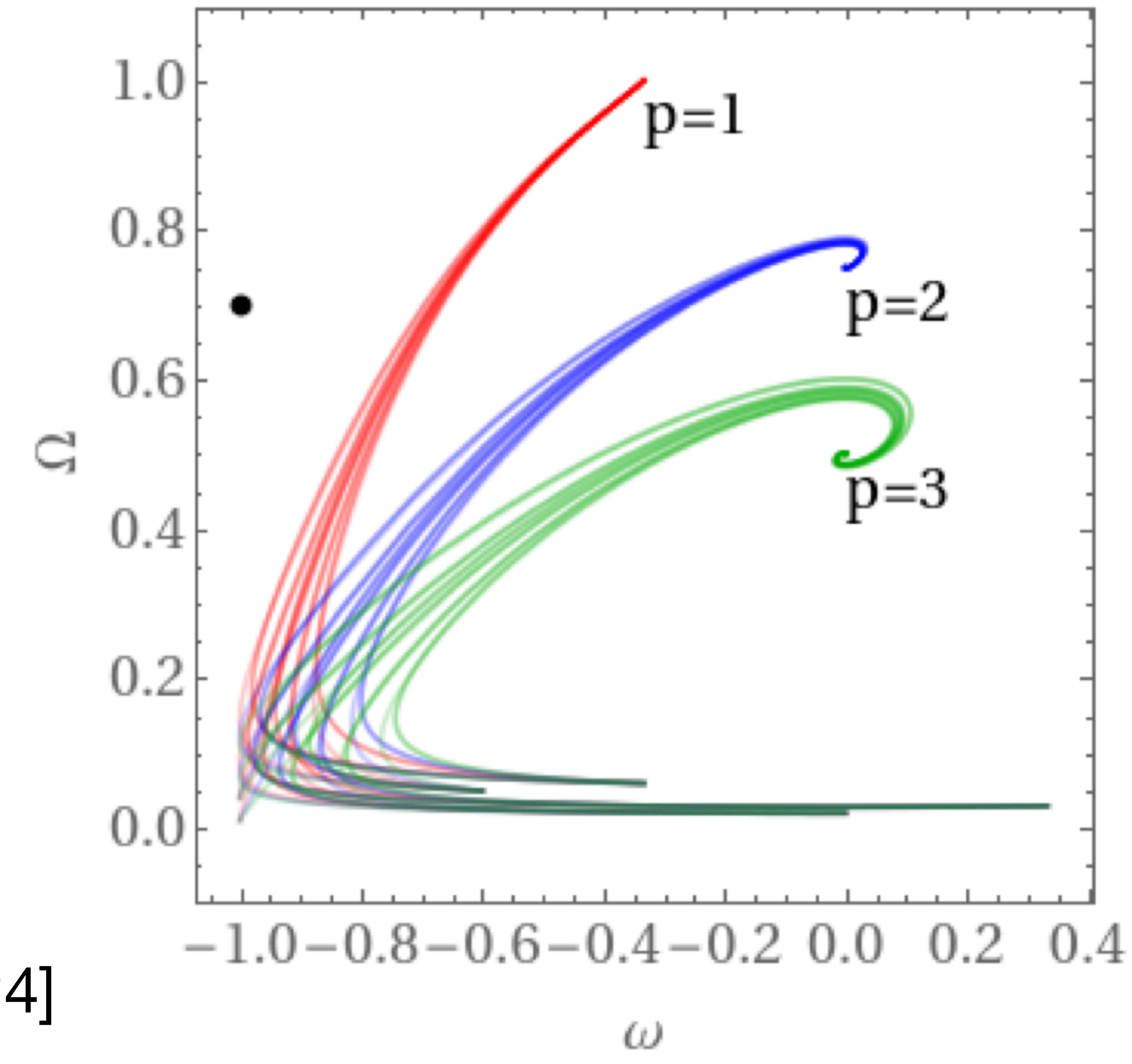
$$K = -p\ln[X + \bar{X}] \qquad V_K \propto e^K = (X + \bar{X})^{-p}$$

$$k_1 = \sqrt{2/p} \qquad \text{and} \qquad k_2 = \sqrt{2p}\,.$$



?Embeddable into string theory?

Recent analysis with SDC&TCC [Payeur et al. ’24]

Axionic DE

[Cicoli, Cunillera,Padilla,FGP '21]

Axion is classically frozen until today

$$\frac{\partial \phi}{\partial N} = -\frac{V_\phi}{3H_{\text{inf}}^2}$$

$$V_\phi \ll H_{inf}^2 \qquad \frac{\partial \phi}{\partial N} = 0$$

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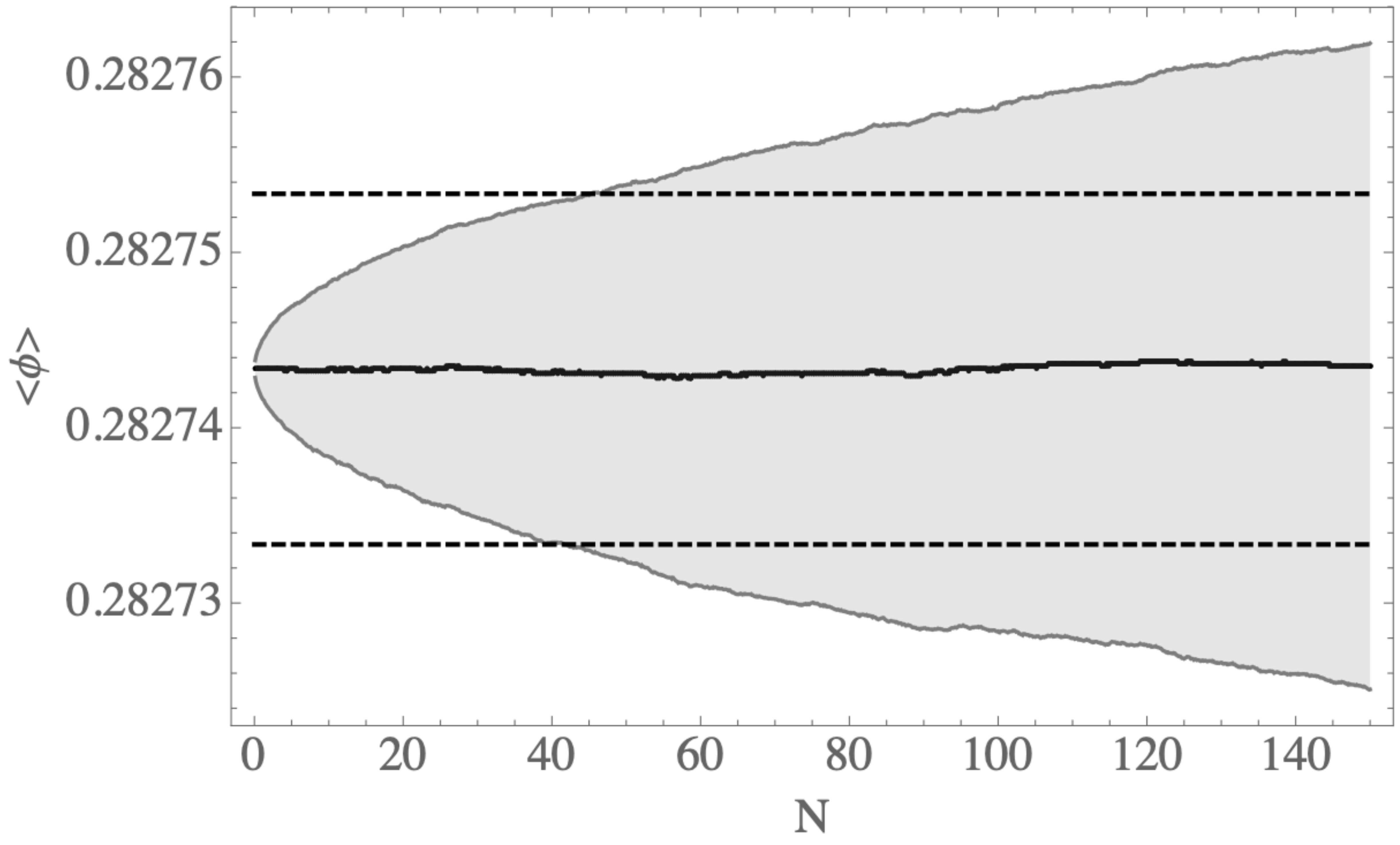

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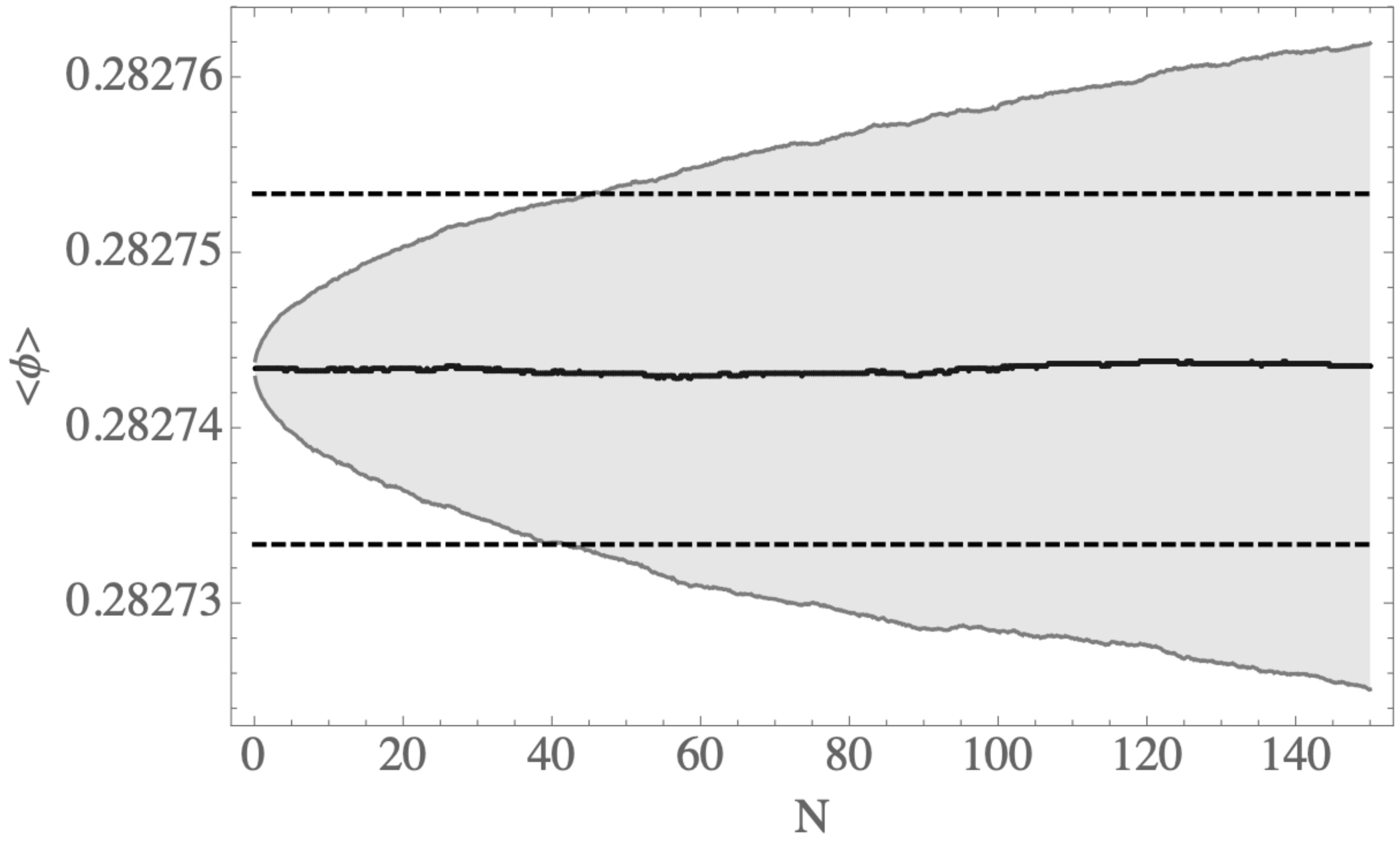


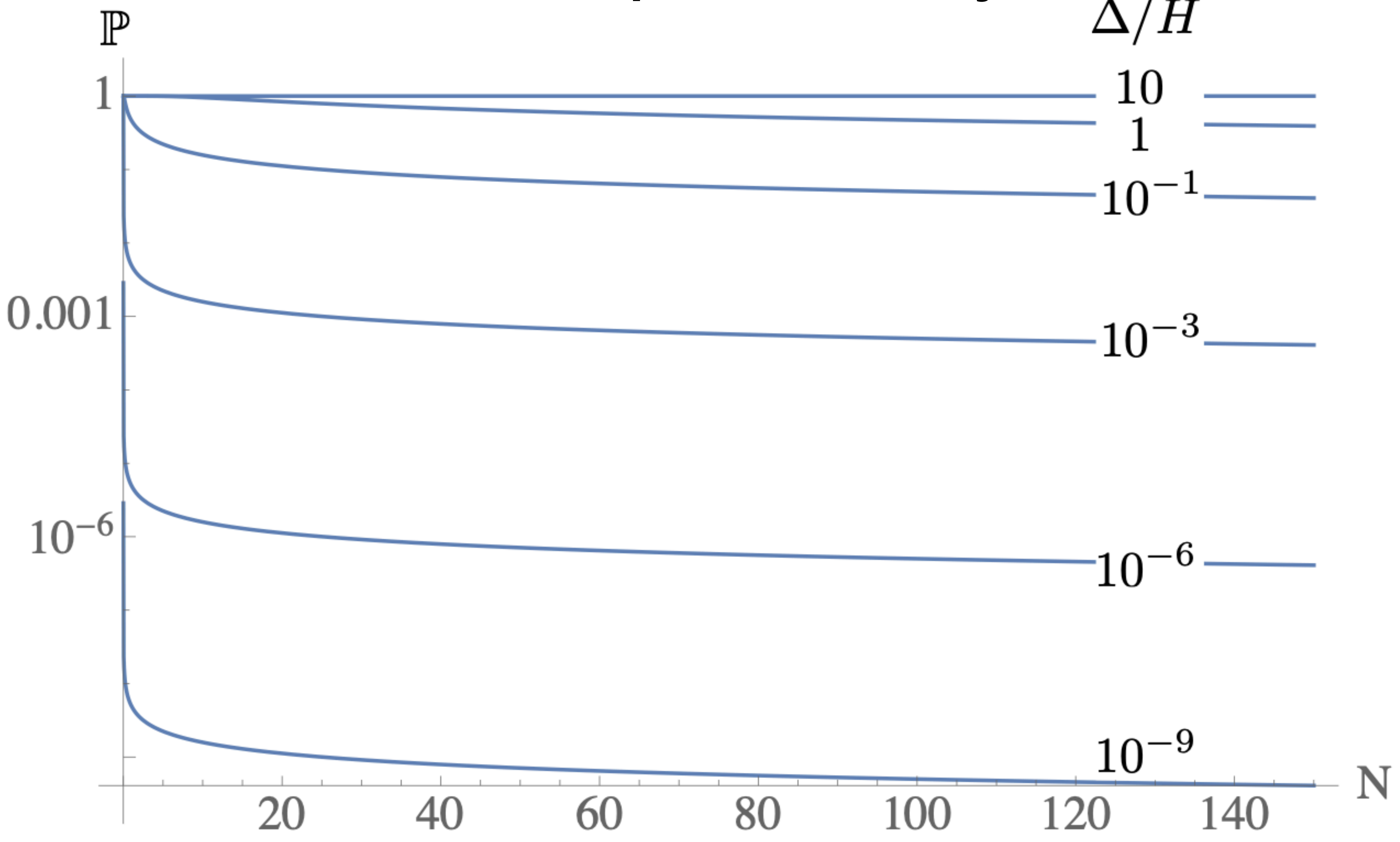
Quantum diffusion during inflation

Choice of ics gets blurred during inflation

Axionic DE

[Cicoli, Cunillera,Padilla,FGP ’21]

Survival probability



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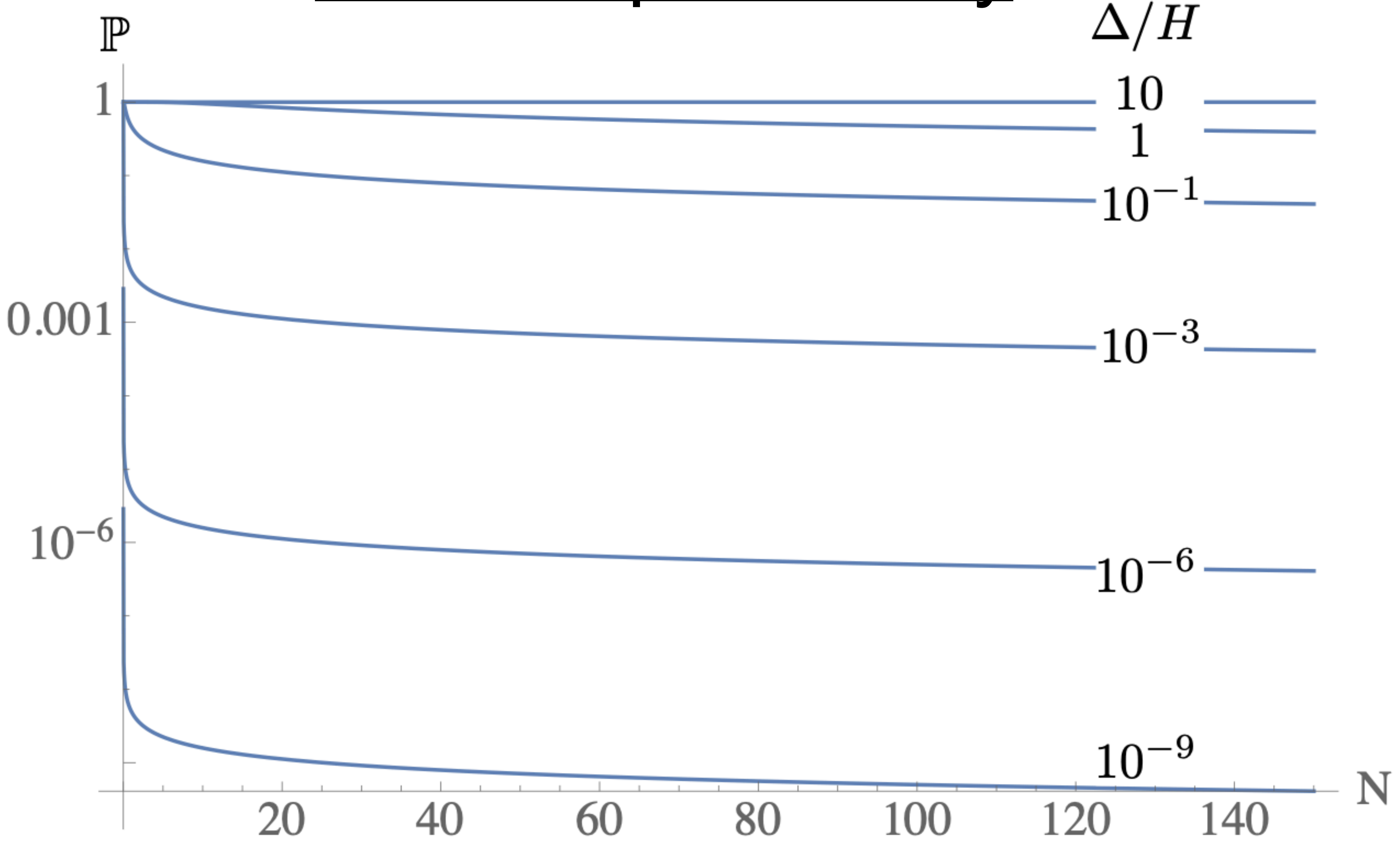

Safe from diffusion if $\Delta_{max} > H_{inf}$

Axionic DE

[Cicoli, Cunillera,Padilla,FGP '21]

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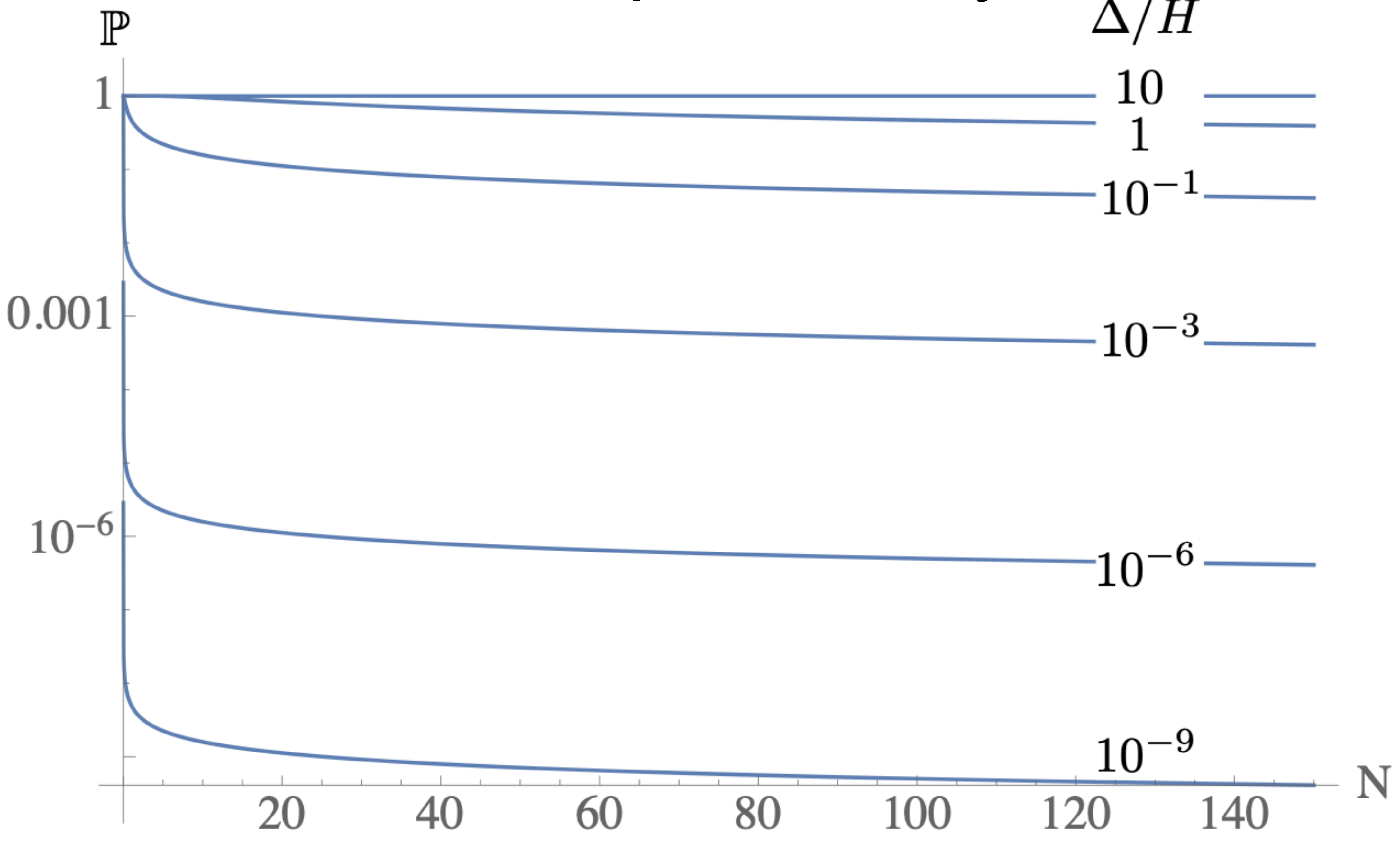

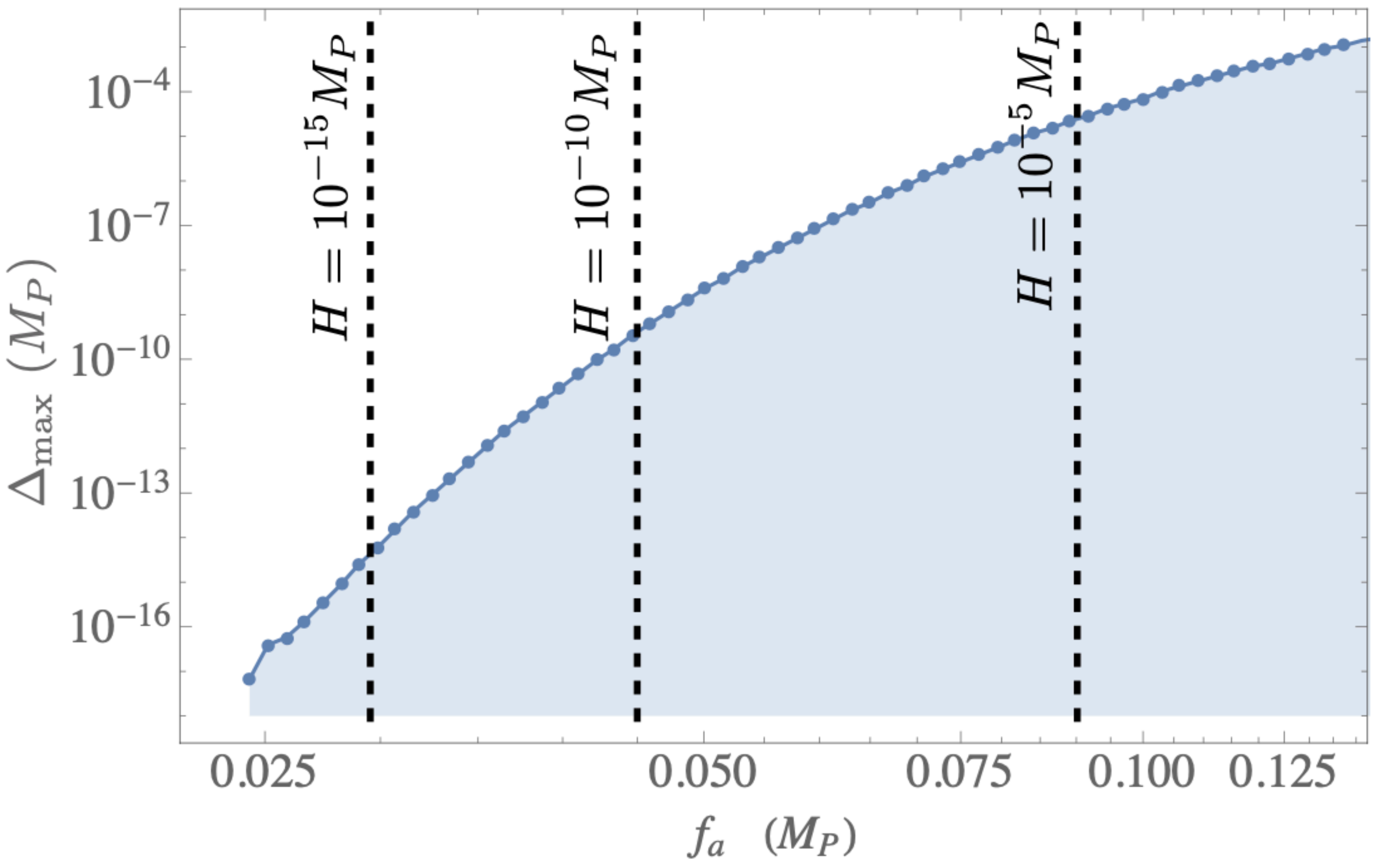

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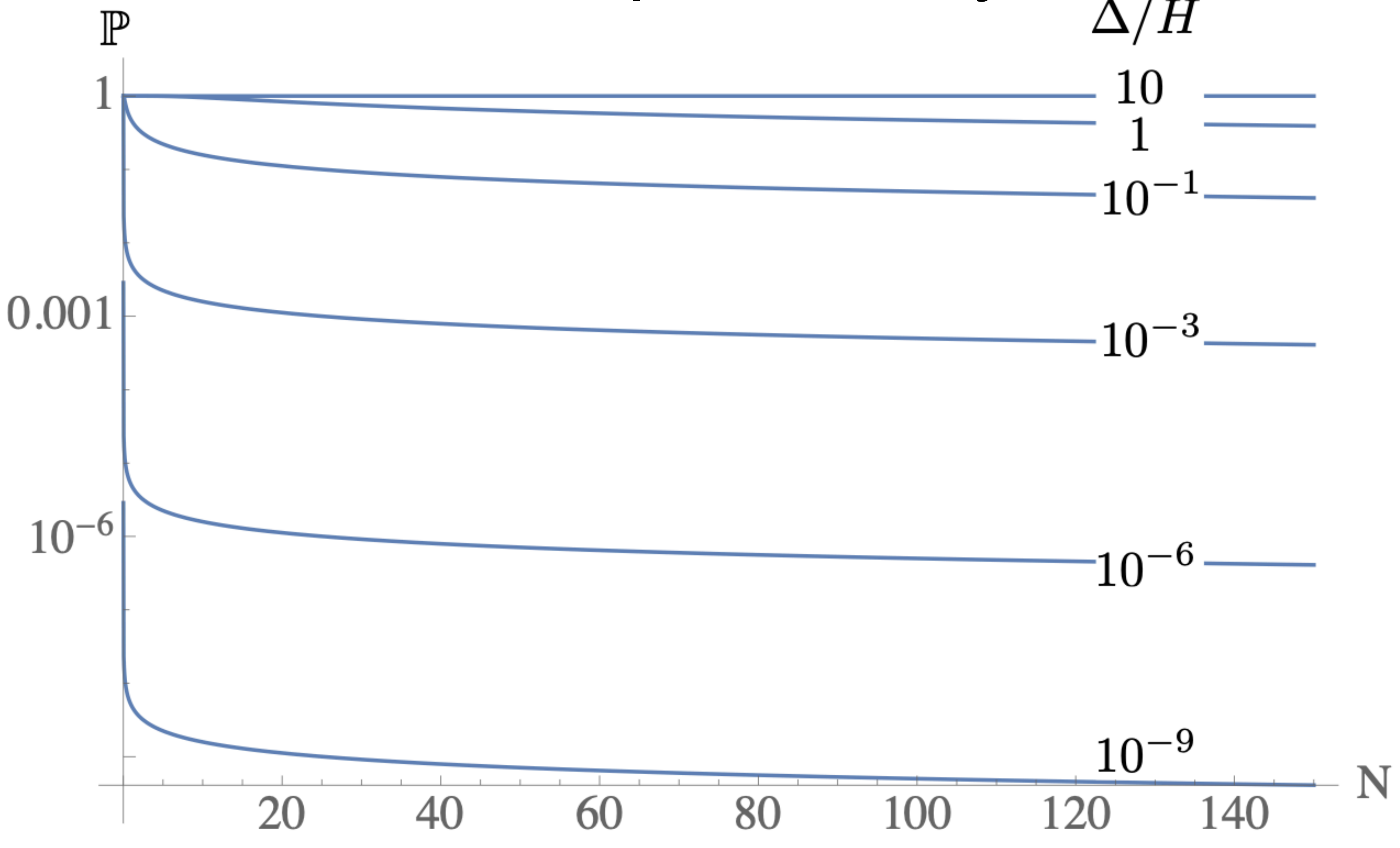
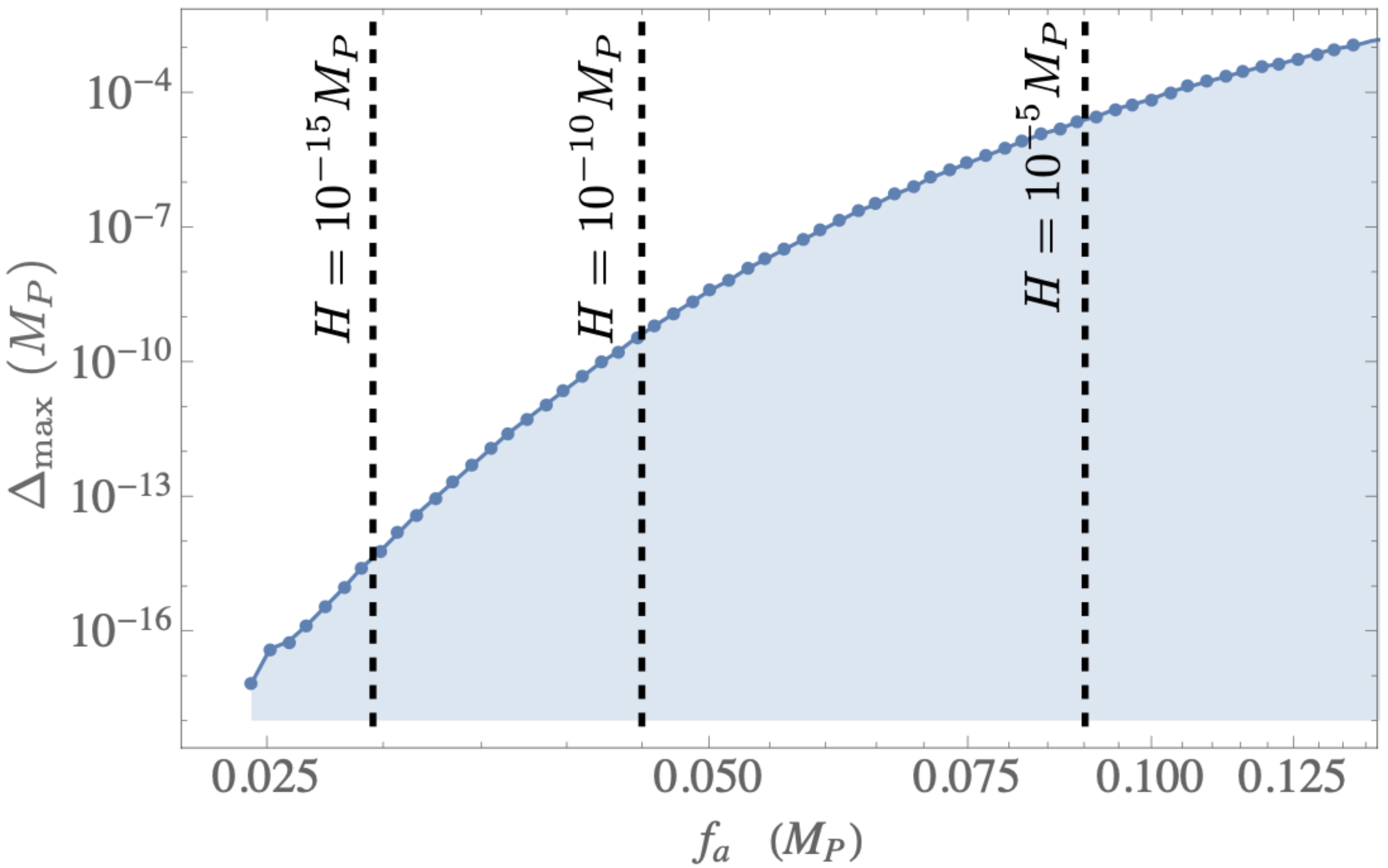




Safe from diffusion if $\Delta_{max} > H_{inf}$

example: $H_{inf} \sim 10^{-5} M_P \rightarrow f_a > 0.08 M_P$

Axionic DE embedding

Decay constants: $\mathcal{L}_{\rm kin} \supset -\frac{1}{4\langle\tau_1\rangle^2}(\partial\theta_1)^2 - \frac{1}{2\langle\tau_2\rangle^2}(\partial\theta_2)^2\,.$ $f_1 = \frac{1}{\sqrt{2}a_1\langle\tau_1\rangle}$ $f_2 = \frac{1}{a_2\langle\tau_2\rangle}$

$f_1, f_2 \ll 1$

$$\Lambda_2^4 \approx \frac{1}{\mathcal{V}^2}\frac{1}{f_2}e^{-\frac{1}{f_2}} \qquad \Lambda_1^4 \approx \frac{1}{\mathcal{V}^2}\left(\frac{1}{f_2}+\frac{1}{\sqrt{2}f_1}\right)e^{-\frac{1}{f_2}-\frac{1}{\sqrt{2}f_1}}$$

Since $\Lambda_1^4 \ll \Lambda_2^4$, in vacuum $\langle\theta_2\rangle \approx 0$

$$V_{DE} = \Lambda_1^4\left(1-\cos\frac{\phi_1}{f_1}\right)$$

Fibre inflation: $H_{inf} \approx 10^{-5}$ $\mathcal{V} \approx 10^3$

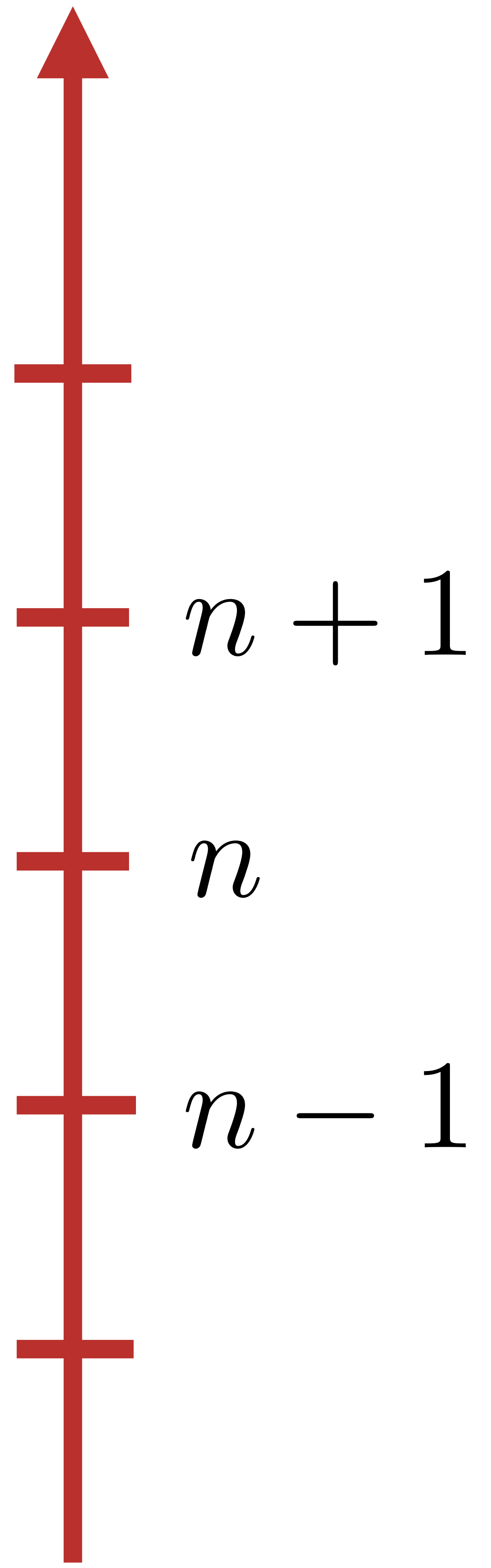
axionic ics: $\Delta_{max} > H_{inf}$ $f_1 > 0.08$

Brown-Teitelboim

[Brown and Teitelboim ’88]

Single gravitationally coupled 4 form+branes

$$S = \int d^4x \sqrt{|g|} \left[\frac{M_P^2}{2} R - \frac{1}{2} F^2 \right] + S_{matter} + S_{bdy} + S_{brane}$$



$F = nq$ quantised

Landscape of vacua: $\Lambda(n) = \Lambda_{bare} + \frac{n^2 q^2}{2}$

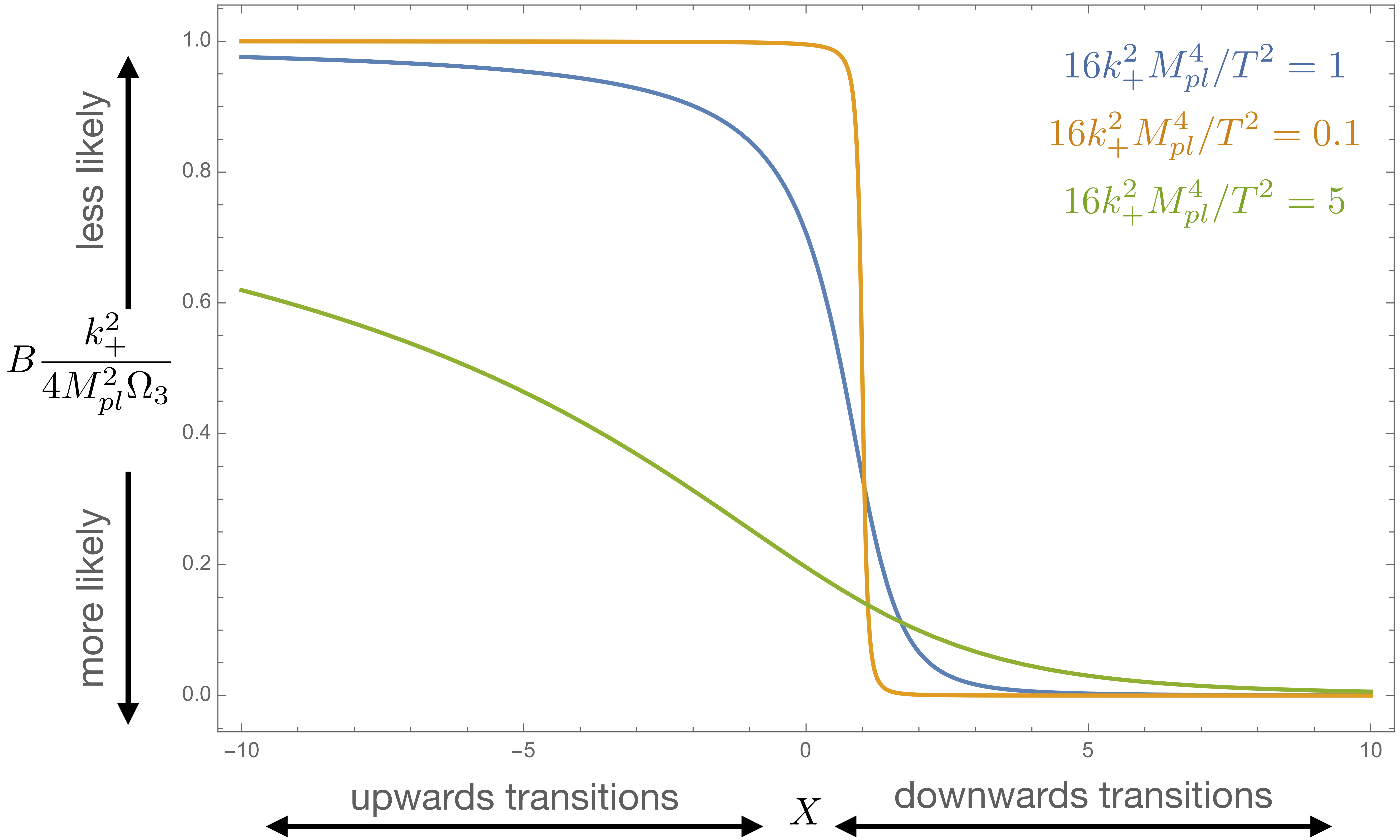
No fine tuning: $\Delta\Lambda(n) = q^2(n - 1/2)$ $\quad\Delta\Lambda \sim q^2 \sim H_0^2 \ll M_{uv}^2$

Empty universe problem

A workaround: nucleate brane stacks

[Feng et al. 00’]

Instantons



$16k_+^2 M_{pl}^4/T^2 = 1$
$16k_+^2 M_{pl}^4/T^2 = 0.1$
$16k_+^2 M_{pl}^4/T^2 = 5$
less likely
more likely
$B\frac{k_+^2}{4M_{pl}^2\Omega_3}$
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10
upwards transitions
X
downwards transitions

Instantons

Let $M_\pm$ be our vacuum with $k^2 = \Lambda \approx H_0^2 \approx 10^{-120}$

Large jump:

$$B_{dS_+ \to M_-} \sim \frac{4M_{pl}^2 \Omega_3}{k_+^2 \left(1 + \frac{4M_{pl}^4 k_+^2}{T^2}\right)^2}.$$

How did we get here?

Multiple small jumps:

$$B_{dS_+ \to dS_- \approx dS_+} \sim \frac{4M_{pl}^2 \Omega_3}{k_+^2 \sqrt{1 + \frac{16M_{pl}^4 k_+^2}{T^2}}}.$$

Dependent on brane charges

Unimodular Gravity

Vary EH action wrt volume preserving diffs: $\det(-g) = 1$

$$\mathcal{L} \supset \lambda(\det(-g) - 1)$$

CC as an integration constant

Covariant formulation in terms of four-forms

[Henneaux and Teitelboim ’89]

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - M_P^2\lambda + \mathcal{L}_{QFT}\right) - \frac{\lambda}{3}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}$$

$$T_{\mu\nu} \supset \lambda g_{\mu\nu} \qquad \partial_\mu\lambda = 0$$

$$\Lambda = \Lambda_{bare} + \lambda$$

Radiatively unstable

[Padilla ’15]

Vacuum Stability

[Liu,Padilla,FGP '24]

More generally

$$\mathcal{L} \supset \sqrt{|g|} \sum_i \frac{c_n^i}{n!} \frac{(F_i)^n}{M_{uv}^{2n-4}}$$

For very low scale dS/Minkowski: $$X_* \sim \frac{qM_{pl}}{T} \frac{M_{pl} M_{uv}^2}{T} \left(\frac{|\Lambda_{bare}|}{M_{uv}^4} \right)^{1-1/n} \left(\frac{n^n}{n!} \right)^{1/n} c_n^{1/n}$$

Stability requires $X_* < 1$ which implies:

- ~~WGC~~ $$\frac{qM_{pl}}{T} < 1$$
- heavy branes $$\frac{M_{pl} M_{uv}^2}{T} < 1$$
- suppressed bare CC $$\left(\frac{|\Lambda_{bare}|}{M_{uv}^4} \right)^{1-1/n} < 1$$
- suppressed couplings $$c_n^{1/n} < 1$$