# **Euclidean Wormholes and String Phenomenology**

**Gary Shiu University of Wisconsin-Madison**



## Analytic bounds on late-time cosmology



Flavio Tonioni **Hung V. Tran** 

- `Accelerating universe at the end of time," GS, Tonioni, Tran, *PRD* [\[arXiv:2303.03418\].](https://arxiv.org/abs/2303.03418)
- 
- ``Collapsing universe before time," GS, Tonioni, Tran, *JCAP* [\[arXiv: 2312.06772\]](https://arxiv.org/abs/2312.06772).
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[See Flavio Tonioni's parallel talk on Thursday, 17:00, Session B4]



• "Late-time attractors and cosmic acceleration," GS, Tonioni, Tran, *PRD* [\[arXiv:2306.07327\].](https://arxiv.org/abs/2306.07327) • CAnalytic bounds on late-time axion-scalar cosmologies," GS, Tonioni, Tran, [\[arXiv:2406.17030\]](https://arxiv.org/abs/2406.17030).

• These findings involving Euclidean wormholes were obtained in 2D gravity in which explicit calculations are under control. Do they play similar roles in  $D\geq 4$  with dynamical gravity?

• Are they genuine saddle points? Perturbatively stable? **Loges, GS, Sudhir, '22**]

• Can we construct Euclidean wormholes from compactifications of string theory? **Loges**, [GS, Van Riet, '23\]](https://arxiv.org/abs/2302.03688). Explicit AdS wormhole solutions made precision holography possible.

## Why Euclidean Wormholes?

- - Maldacena, Shaghoulian, Tajdini]; …
	-
- 
- Do they arise as Euclidean saddles of UV complete theories such as string theory?
	-
	-

### Recent prominent roles in string theory, especially from the quantum information perspective:

• Page-curve for black holes [Penington];[Almheiri, Engelhardt, Marolf, Maxfield];[Almheiri, Mahajan, Maldacena, Zhao]; [Almheiri, Hartman,

• AdS wormholes jeopardize factorization in AdS/CFT [Maldacena, Maoz, '04] (ensemble average?)

## Why Euclidean Wormholes at String Pheno?

Wormholes break global symmetries.

- Cottrell, GS, Soler, '15];[Montero, Valenzuela, Uranga, '15]; [Heidenreich, Reece, Rudelius, '15]; [Hebecker,Mangat-Theissen-Witkowski, '16]; …
	-

which was used to constraint and some large feld and inflation to delete a wormhole on a single univers Figure 4: Wofrnholes: A Memiwormhole (left) <sub>standriolo, Huang, Noumi, Ooguri, GS, '20];<br>Figure 4: Wofrnholes: A Memiwormhole (left) sandrio<del>wer menoles.con</del>necting tw</sub> evidence that WGC bound is set by wormholes: [Andriolov GSI Soler, Nahl Riet, (22)

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- 
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· Gauss-Bonnet term Wh New Was argued to be positive using Swampland criterial steams as <sub>22</sub> he trace (WGC); [Martucci, Riss@QdiJationeFT string probes) leads to an additional exponential suppression of wormhole effects & provides an alternativ $\bm{\mathit{e}}$  definition of species scal $\bm{\mathit{e}}$  Martucci, Riggg, Valenti, Vecchi, ' $\bm{\mathit{q}}$ 4] • Other phenomenological implications for wormholes reviewed fil rebecker, Mikhail, Solet, '14]  $S_{w_i} =$  $\mathbb{F}$  $\frac{1}{2}$ V  $H \wedge *H =$ <br>rmholes revi  $\partial \! \widehat{\kappa}$  $\frac{1}{2}$  $\frac{2}{2}$  $\exists \ket{\boldsymbol{\phi}}$   $\mathcal{R}$ *r*0 *dr rai*  $\sqrt{5q}$ <del>er,</del> '1<del>8</del> C =  $\sqrt[3]{4}\sqrt{6}$ 4 Notice the factor 2 appearing because a wormhole consists of two solutions of two solutions of the factor 2 appearing because a wormhole consists of two solutions. of  $(11)$ , each restricted to  $r > r$ 





Play a key role in the axionic Weak Gravity Conjecture [Arkani-Hamed, Motl, Nicolis, Vafa, '05];[Rudelius, '14, '15]; [Brown,

• Coleman's α-parameters [Coleman, '89] lead a -1-form global symmetries [McNamara, Vafa, '20]

[Giddings, Strominger, '88]; [Abbott, Wise, '89];[Coleman, Lee, '90]; [Kallosh, Linde, Linde, Susskind, '95]; …

# Giddings-Strominger Wormhole

role of affine parameter along the geodesic. pe or alline parameter along role of affine parameter along the geodesic.

 $\cdot$   $h(r)$  is a harmonic function, normalized to  $h' = f/a^{d-1}$  so that  $\;\star\; h = \mathrm{vol}_{d-1}$ ; plays the (*r*) is a harmonic function, normalized to  $h' - f/a^{d-1}$  so that  $\star h - \mathrm{vol}$ , the target space the constant constant constant constant constant  $\alpha$  be and  $\alpha$  *n* and  $\alpha$  *n* and  $\alpha$  plays the angle of the angle  $\frac{1}{\sqrt{2}}$  is a harmonic function, normalized to  $\frac{1}{\sqrt{2}}$ , and  $\frac{1}{\sqrt{2}}$ , and  $\frac{1}{\sqrt{2}}$ •  $h(r)$  is a harmonic function, normalized to  $h' = f/a^{a-1}$  so that  $\star h = \mathrm{vol}_{d-1}$ ; plays t

#### Giddings-Strominger Solutions Giddings-Strominger Solutio 2 Regular Giddings-Strominger wormholes

· Consider the following Euclidean action in d ≥ 3 dimensions: dimensions:  $G$  anoider the following  $\Gamma$ uelideen estien in de  $\Omega$  dimensioner regularity condition provided in action in deep annononol.<br>Consider the following Euclidean action in deep annononol.

• A simple set of solutions with O(d) symmetry take the form [Giddings, Strominger, '88]: A simple set of solutions with O(d) symmetry take the form *d* A simple set of solutions with O(*d*) symmetry take the form

$$
S = \frac{1}{2\kappa_d^2} \int \left( \star (\mathcal{R} - 2\Lambda) - \frac{1}{2} G_{ij}(\varphi) \mathrm{d}\varphi^i \wedge \star \mathrm{d}\varphi^j \right)
$$

$$
ds^{2} = f(r)^{2} dr^{2} + a(r)^{2} d\Omega_{d-1}^{2},
$$

$$
\left(\frac{a'}{f}\right)^{2} = 1 + \frac{a^{2}}{\ell^{2}} + \frac{c}{2(d-1)(d-2)a^{2d-4}},
$$

$$
c = G_{ij}(\varphi) \frac{d\varphi^{i}}{dh} \frac{d\varphi^{j}}{dh} = constant
$$

### Three Classes of Euclidean Geometries



### $c > 0$  c  $< 0$ space-like geodesic null geodesic time-like geodesic

⇥

only time-like geodesics

Core instanton Extremal instanton, e.g. D-instanton Wormhole  $\frac{1}{2}$  **2** Extremal instanton e a D-instanton Marmh space-like geodesic null geodesic time-like geodesic

 $G_{ij}(\varphi) d\varphi^{i} d\varphi^{j} = -d\chi^{2}$ 



 $\mathsf{c}=0$ 

⇥

 $c \geq 0$  all possible, but longest time-like geodesic has length  $\frac{2\pi}{|A|}$  $|\beta|$ 

### Three Classes of Euclidean Geometries



### $c > 0$  c  $< 0$ space-like geodesic null geodesic time-like geodesic c *>* 0 c = 0 c *<* 0

Core instanton Extremal instanton, e.g. D-instanton Wormhole <u>space-listantum controllitum controllitum controlle</u><br>Cole instantum controller controller and controller and controller and controller and controller and controlle





 $\mathsf{c}=0$ 

#### • Required geodesic length for wormholes only depends on the wormhole size in AdS units: Required geodesic length for wormholes only depends on the wormhole size in AdS unit red geodesic length for wormholes only depends on the wormhole size in AdS units:

 $=$  "length of geodesic required by geometry"

 $\bm{\cdot}\,\,\,$  There must exist a time-like geodesic longer than  $D_d(q_0)$  [Arkani-Hamed, Orgera, Polchinski, '07] . There must exist a time-like geodesic longer than  $D_d(q_0)$  [Arkani-Hamed, Orgera, Polchinski, '07]

There must exist a time-like geodesic *longer* than *Dd*(*q*0) [Arkani-Hamed, Orgera, Polchinski – '07]

#### Wormhole Regularity GS wormhole regularity smaller when the wormhole size is large in AdS units, *a*<sup>0</sup> `: wormnoie Regu *Dd*(1) = ⇡

$$
D_d\left(\frac{a_0}{\ell}\right)
$$
 = "length of geodesic required by geometry"

•  $D_d(q_0)$  is monotonic in  $q_0 \equiv a_0/\ell$ :  $D$   $(a)$  is monotonic in  $q - q$  $\cdot$   $D_d(q_0)$  is monotonic in  $q_0$  $\sim$   $\sim$  0 $\sim$ 

$$
\sum_i \frac{1}{\beta_i^2} > \frac{d-1}{2(d-2)}
$$

$$
2\pi \sqrt{\frac{d-1}{2(d-2)}} = D_d(0) \geq D_d(q_0) \geq D_d(\infty) = 2\pi \sqrt{\frac{d-2}{2(d-1)}}
$$
 distance conjecture?  

## Extremal Instantons and Wormholes

- 
- For a given axion charge:
	- Extremal instanton has the minimal action if there are no regular wormholes;
	-
- no naked singularity to warn us about possible sickness.
- - Negative modes that lower the Euclidean action [Hertog, Truijen, Van Riet, '19]
	- $\cdot$   $\,$  Violate  $\, {\rm Tr} (F \pm \star F)^2 \geq 0$  in the dual CFT  $_{\rm [Bergshoeff,~Collinucci,~Ploegh,~Vandoren,~Van Riet,~'05]}$
- Our works [Loges, GS, Sudhir, '22]; [Loges, GS, Van Riet, '23] Settled these puzzles.

• The Euclidean action of core instanton, extremal instanton and wormhole depends on  $\beta_i$ .

• Regular wormhole has the minimal action if the dilaton coupling allows it to exist.

• Euclidean axion wormholes are over-extremal. Unlike over-extremal black holes, there is

• This has led some to speculate that Euclidean axion wormholes are in the swampland:



# Wormhole Stability

• Previous works (25+ years) on perturbative stability of axion wormholes have led to contradictory claims, casting doubts on their contributions to the Euclidean path integral.

[Loges, GS, Sudhir, '22]





#### **Gary Shiu - Amplitudes meet the** Swampland

101 views · 1 year ago

## Wormhole Stability



• My talk at [String Pheno 2022](https://youtu.be/3gRpiBS0-NE?si=3kwNC1pwB_zkjoxG) in Liverpool touched upon this issue.

• Under diffeomorphism, metric and 3-form perturbations are mixed:

- Gauge invariant perturbations satisfy Dirichlet boundary conditions in the 3-form picture.
- Conformal mode problem is absent:  $j = 0$  mode is not physical (wormhole size is set by axion charge).
- The conformal model problem is conceivably only be a feature of pure gravity. Could it be evaded for the wavefunction of the universe in the context of inflation? Loges, GS, work in progress].
- $-(1 + 2\phi) d\eta^2 + 2\partial_i B d\eta dx^i + ((1 2\psi)\gamma_{ij} + 2\nabla_i \partial_j E)$  $\int dx^{i}dx^{j}$  $\overline{1}$ 
	- $\sqrt{\gamma} \epsilon_{ijk} \partial^i w \, dx^j \wedge dx^k$

 $\Rightarrow$   $\delta H \rightarrow 0$  asymptotically

• Metric perturbations vanish far from the wormhole neck so are the 3-form perturbations:

### Gauge Invariance and Boundary Conditions Scalar perturbations

$$
ds^{2} = a(\eta)^{2} \left[ -(1+2\phi) d\eta^{2} + 2\partial_{i}L \right]
$$

$$
H = \frac{n}{2\pi^{2}} \left[ (1+s)\text{vol}_{3} + d\eta \wedge \left(\frac{1}{2}\sqrt{1+s}\right) \right]
$$

$$
n = \int_{S^3} H \in \mathbb{Z} \qquad \Rightarrow \delta H \to 0 \text{ asymptot}
$$

# String Theory Embeddings

[Loges, GS, Van Riet, '23]

d theory contains 11 scala ~

$$
ds_{10}^2 = e^{-6b\varphi} ds_4^2 + e^{2b\varphi} R^2 \mathcal{M}_{ij} d\theta^i d\theta^j
$$
  

$$
\mathcal{M}_{ij} = diag(e^{\vec{\beta}_1 \cdot \vec{\Phi}}, e^{\vec{\beta}_2 \cdot \vec{\Phi}}, \dots, e^{\vec{\beta}_6 \cdot \vec{\Phi}})
$$
  

$$
C_3 = \chi_1 d\theta^{123} + \chi_2 d\theta^{145} + \chi_3 d\theta^{256} + \chi_4 d\theta^{346}
$$



 $1 \times 10^{34}$ 

<sup>1</sup> Ad theory contains 11 scalars:<br> **i** orentzian "overextremal" branes *j*  $\overline{L}$   $\overline{U}$   $\overline{C}$   $\overline{I}$  $\sim$  $\frac{1}{2}$ <sup>6</sup>)*·* Lorentzian "overextremal" branes.

$$
S_4 = \frac{1}{2\kappa_4^2} \int \left[ -\mathcal{R} + \frac{1}{2} \sum_{i=1}^4 \left[ (\partial s_i)^2 + e^{2s_i} (\partial \chi_i)^2 \right] + \frac{1}{2} \sum_{i=5}^7 (\partial s_i)^2 \right]
$$

There are four decoupled axio-dilaton pairs with  $\beta = 2$ :

$$
\sum_{i=1}^{4} \frac{1}{\beta_i^2} = 1 \quad > \quad \frac{3}{4} \quad \checkmark
$$



$$
\sum_{i=1}^{4} \frac{1}{\beta_i^2}
$$

### Euclidean Axion Wormholes in Flat Space reduction of the contract of t

• Reduction ansatz motivated by the extremal solution:

### Euclidean Axion Wormholes in AdS Space [Loges, GS, Van Riet, '23] *T*<sup>1</sup>*,*<sup>1</sup> ansatz

### $T^{1,1} = [SU(2) \times SU(2)] / U(1) \sim$

Background solution: Reduction ansatz;  $ds_{10}^2 = \ell^2 ds_5^2 + \ell^2 (ds_{KE}^2 + \eta^2)$   $ds_1^2$  $e^{\Phi} = g_s$  *e*<sup> $\Phi$ </sup>  $B_2 = 0$  $C_0 = 0$  $C_2 = 0$  $F_5 = 4\ell^2(1 - i\star)\text{vol}_{T^{1,1}}$ [Klebanov, Witten]



$$
ds_{10}^{2} = \ell^{2} e^{-\frac{2}{3}(4u+v)} ds_{5}^{2} + \ell^{2} (e^{2u} ds_{KE}^{2} + e^{2v} \eta^{2})
$$
  
\n
$$
e^{\Phi} = g_{s} e^{\phi}
$$
  
\n
$$
B_{2} = \ell^{2} g_{s}^{1/2} b \Phi_{2}
$$
  
\n
$$
C_{0} = ig_{s}^{-1} \chi
$$
  
\n
$$
C_{2} = i\ell^{2} g_{s}^{-1/2} c \Phi_{2}
$$
  
\n
$$
F_{5} = 4\ell^{2} (1 - i\star) \text{vol}_{T^{1,1}}
$$

#### Consistent Reduction to 5D Consistent reduction to 5D

#### [Loges, GS, Van Riet, '23]



$$
+\frac{28}{3}(\partial u)^2 + \frac{8}{3}\partial u \partial v + \frac{4}{3}(
$$

$$
e^{2u} ds_{\text{KE}}^2 + e^{2v} \eta^2
$$

**Not Giddings-Strominger wormhole!**

#### Consistent Reduction to 5D Consistent reduction to 5D

#### [Loges, GS, Van Riet, '23]



$$
+\frac{28}{3}(\partial u)^2 + \frac{8}{3}\partial u \partial v + \frac{4}{3}(
$$

$$
e^{2u} ds_{\text{KE}}^2 + e^{2v} \eta^2
$$

**Not Giddings-Strominger wormhole!**

#### [Loges, GS, Van Riet, '23]

### **Symmetries** 4.2.2 Symmetries and axion charges

 $\cdot$  The gauge symmetries associated with  $C_0, C_2, B_2$  lead to three shift symmetries:

 $\cdot$  Exploit these symmetries, which in the SUGRA approximation, are unbroken to discrete subgroups. xploit these symmetries, which in the SUGRA approximation, are unbroken to discrete subgrou broken to discrete subgroups, we can use the above perturbative symmetries to simplify the

$$
\begin{aligned}\n\delta_1 \chi &= \lambda_1 \,, & \delta_2 \chi &= 0 \,, & \delta_3 \chi &= 0 \,, \\
\delta_1 b &= 0 \,, & \delta_2 b &= \lambda_2 \,, & \delta_3 b &= 0 \,, \\
\delta_1 c &= \lambda_1 b \,, & \delta_2 c &= 0 \,, & \delta_3 c &= \lambda_3 \,,\n\end{aligned}
$$

 $\cdot$  The  $\delta_1$  transformation is an element of  $SL(2,\mathbb{R})$  under which: (*F*<sup>3</sup> = d*C*<sup>2</sup> *C*<sup>0</sup> ^ *H*3) is gauge-invariant. The <sup>1</sup> transformation is an element of SL(2*,* R), ne  $o_1$  transformation is an element of he  $\delta_1$  transformation is an element o

$$
e^{-\phi} \mapsto e^{-\phi} \,, \qquad \chi \mapsto \chi + \lambda_1 \,, \qquad \begin{bmatrix} b \\ c \end{bmatrix} \mapsto \begin{bmatrix} 1 & 0 \\ \lambda_1 & 1 \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix} \,.
$$

**• There is also the rescaling symmetry of**  $SL(2,\mathbb{R})$  **for which:** nere is also the rescaling symmetry of  $SL(2,\mathbb{R})$  for which: here is also the rescaling symmetry of  $SL(2,\mathbb{R})$  for which:

*.* (4.11)

$$
e^{-\phi} \mapsto a^2 e^{-\phi}, \qquad \chi \mapsto a^2 \chi, \qquad \begin{bmatrix} b \\ c \end{bmatrix} \mapsto \begin{bmatrix} a^{-1} & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix}
$$

1*b* = 0 *,* 2*b* = <sup>2</sup> *,* 3*b* = 0 *,*

- $\delta_1 \chi = \lambda_1 \,, \qquad \qquad \delta_2 \chi = 0 \,, \qquad \qquad \delta_3 \chi = 0 \,,$  $\delta_{21} = 0$  ,  $\delta_{22} = 0$  ,  $\delta_{22} = 0$ 1*c* = 1*b ,* 2*c* = 0 *,* 3*c* = <sup>3</sup> *,*  $\delta_1 h = 0$   $\delta_2 h = \lambda_2$   $\delta_3 h = 0$
- $\delta_1 c = \lambda_1 b$ ,  $\delta_2 c = 0$ ,  $\delta_3 c = \lambda_3$ ,  $\sigma_1 c - \lambda_1 \sigma, \quad \sigma_2 c - \sigma, \quad \sigma_3 c - \lambda_3,$

 $\overline{\phantom{a}}$ 

*.* (4.12)

#### [Loges, GS, Van Riet, '23]

 $\,\cdot\,$  The axion EOMs can be solved in terms of the constant axion charges (assoc. with  $\delta_2,\delta_3$ ):  $\sum_{i=1}^{n}$ 

### **Since we want in the supergravity and supervisors are not in the supergravity and supervisors are not in the supergravity and symmetries are not in the supergravity and symmetric symmetries are not in the supergravity are** broken to discrete subgroups, we can use the above perturbative symmetries to simplify the

and the rescaling symmetry: and under the "rescaling symmetry,"

•  $\mathbf{q}_1$  and  $\mathbf{q}_2$  are both conserved and quantized;  $\mathbf{q}_1$ is gauge-dependent (Page charge),  $\mathbf{q}_2$  is  $\cdot$   $\mathfrak{q}_1$  and  $\mathfrak{q}_2$  are both conserved and quantized;  $\mathfrak{q}_1$  is gauge-dependent (Page charge),  $\mathfrak{q}_2$  is integer-quantized in units of (`s*/*`)<sup>6</sup> and ` `s, we are justified in taking a continuous-charge

 $\cdot$   $\cdot$   $\mathfrak{q}_{1,2}$  quantized in units of  $(\ell_s/\ell)^6$  and  $\ell \gg \ell_s$ : justifying continuous charge approximation. *e* of  $(\ell_s / \ell)^6$  and  $\ell \gg \ell_s$ : justifying continuous  $\sigma_{S}$ . Jaomynig commacac charge approximation.  $\iota_{\mathcal{S}}(\iota_{\mathcal{S}})$  and  $\iota_{\mathcal{S}} \gg \iota_{\mathcal{S}}$ ; justifying continuous charge

 $q_1 \mapsto a q_1, \qquad q_2 \mapsto a^{-1} q_2.$ 

$$
\star [e^{-4u-\phi} \, \mathrm{d}b + e^{-4u+\phi} \chi(\mathrm{d}c - \chi \, \mathrm{d}b)] \equiv \mathfrak{q}_1 \operatorname{vol}_4,
$$
  

$$
\star [e^{-4u+\phi}(\mathrm{d}c - \chi \, \mathrm{d}b)] \equiv \mathfrak{q}_2 \operatorname{vol}_4.
$$

- gauge-invariant (Maxwell charge).  $1$  and  $42$  are both conserved and quantized,  $41$ s gauge-dependent (Fage charge),  $42$ integral in units of the state in taking a complete invariant (Maxwell charge).  $\cdot$   $\mathsf{q}_1$  and  $\mathsf{q}_2$  are both conserved and quantized;  $\mathsf{q}_1$ is gauge-dependent (Page
- approximation and will not be both  $\epsilon$ ized in units of  $({\mathscr E}_s/{\mathscr E})^{\mathrm{o}}$  and  ${\mathscr E} \gg {\mathscr E}_s$ : justifying co
- Under the shift symmetries: *eF*<sup>3</sup>  $\delta_1\mathfrak{q}_1 = \lambda_1\mathfrak{q}_2 \,, \qquad \quad \delta_2\mathfrak{q}_1 = 0 \,, \qquad \quad \delta_3\mathfrak{q}_1 = 0 \,,$  $\delta_1 q_2 = 0$ ,  $\delta_2 q_2 = 0$ ,  $\delta_3 q_2 = 0$ ,  $\delta_1$ g<sub>1</sub> =  $\lambda_1$ g<sub>2</sub>,  $\delta_2$ g<sub>1</sub> = 0,  $\delta_3$ g<sub>1</sub>  $o_1q_2 =$

[Loges, GS, Van Riet, '23] *p*  $$ 

 $\cdot$  Solutions have definite parity when  $\mathfrak{q}_1 = 0$ ; generate solutions with  $\mathfrak{q}_1 \neq 0$  by shift  $\delta_1.$ Here *h*(*r*) is harmonic and normalized to *h*0 = *f /q*<sup>4</sup> so that ?d*h* = vol4. Solutions have definite pris have definite parity when  $\mathsf{q}_1 = \mathsf{U},$  generate solutions with  $\mathsf{q}_1 \neq \mathsf{U}$  by shilt  $o_1.$ 

 $\alpha$  be undone using the 1 transformation above as long as  $\alpha$ 

#### Equations of Motion and under the "rescaling symmetry," and "rescaling symmetry,"  $r_{\rm eff}$

• In terms of the constant axion charges, the remaining 5d EOMs: IS OF the constant axion charges, the remaining 5d EQIVIS.  $\blacksquare$ 

$$
\Box \phi = -e^{2\phi} (\partial \chi)^2 - \frac{1}{2} e^{4u} \left[ e^{\phi} (\mathfrak{q}_1 - \mathfrak{q}_2 \chi)^2 + e^{-\phi} \mathfrak{q}_2^2 \right] (\partial h)^2 ,
$$
  
\n
$$
d \star (e^{2\phi} d\chi) = -e^{4u + \phi} \mathfrak{q}_2 (\mathfrak{q}_1 - \mathfrak{q}_2 \chi) (\partial h)^2 ,
$$
  
\n
$$
\Box (7u + v) = \frac{3}{8} \partial_u \mathcal{V} - \frac{3}{4} e^{4u} \left[ e^{\phi} (\mathfrak{q}_1 - \mathfrak{q}_2 \chi)^2 - e^{-\phi} \mathfrak{q}_2^2 \right] (\partial h)^2 ,
$$
  
\n
$$
\Box (u + v) = \frac{3}{8} \partial_v \mathcal{V} ,
$$
  
\n
$$
2R_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - e^{2\phi} \partial_\mu \chi \partial_\nu \chi + e^{4u} \left[ e^{\phi} (\mathfrak{q}_1 - \mathfrak{q}_2 \chi)^2 - e^{-\phi} \mathfrak{q}_2^2 \right] \partial_\mu h \partial_\nu h
$$
  
\n
$$
+ \frac{56}{3} \partial_\mu u \partial_\nu u + \frac{8}{3} (\partial_\mu u \partial_\nu v + \partial_\mu v \partial_\nu u) + \frac{8}{3} \partial_\mu v \partial_\nu v - \frac{2}{3} g_{\mu\nu} \mathcal{V} .
$$

- 
- The  $B_2, C_2$  axions source the saxions  $u, v$ : cannot simply set  $u = v = 0$ .  $b_2$ ,  ${\mathsf C}_2$  axions source the saxions  ${\mathcal U},$   ${\mathcal V}$ : cannot sin

#### Supersymmetric Instanton cation, axion charges and especially whether or not our wormholes violate positivity bounds, supersymmetric inst Sunersymmetric Instanton

- Setting  $u = v = 0$  at the potential minimum requires:  $\mathfrak{a}_1 - \mathfrak{a}_2 \mathfrak{v} = + \mathfrak{a}_2 e^{-\phi}$
- **Picking the bottom sign:** *v v* transforma under constant shifts of . Picking the bottom sign:

[Loges, GS, Van Riet, '23] Ellie (5d equations of Manuel Street, 1931 that the stress tensor vanishes tensor vanishes tensor vanishes te<br>In the matter stress tensor vanishes tensor vanishes tensor vanishes tensor vanishes tensor van in tensor van<br>In [Loges, GS, Van Riet, 23]<br>**the contract of the contract of** 

- This is the supersymmetric D-instanton. pupersymmer<br>Pupersymmer<br>Pupersymmer  $\mathsf{C}$   $\mathsf{C}$
- IIB axio-dilaton which has  $\beta=2$  which does not allow for regular wormholes. der
- $\cdot$  Non-singular wormhole solutions require non-zero  $u, v$ .

$$
\mathfrak{q}_1 - \mathfrak{q}_2 \chi = \pm \mathfrak{q}_2 \, e^{-\phi}
$$



• Another option is to consider  $b, c$  = constant (hence  $\mathfrak{q}_1 = \mathfrak{q}_2 = 0$ ), leaving only the type = const. =) *<sup>F</sup>*<sup>3</sup> <sup>=</sup> *ieH*<sup>3</sup> *.* (4.19) ||
|
|-

$$
\chi - e^{-\phi} = \frac{\mathfrak{q}_1}{\mathfrak{q}_2} = \text{const.} \qquad \Longrightarrow \qquad F_3 = -ie^{-\Phi}H_3 \,.
$$

#### [Loges, GS, Van Riet, '23]

#### Boundary Conditions In constructing numerical solutions have the freedom to choose coordinates such that 4.2.5 Numerical solutions

• Metric Ansatz:  $ds^2 = f(r)^2 dr^2 + (q_0^2 + r^2)d\Omega^2$  where  $q_0$ =wormhole size (in AdS units).  $r^2 + r^2)d\Omega_4^2$  $\frac{2}{4}$  where  $q_0$ *q*2 <sup>0</sup> + *r*<sup>2</sup> *,* (4.27)  $\frac{a}{0} + r^2$ )*d*s2<sup>2</sup><sub>4</sub> whe *,* (4.27)

• The boundary conditions for the saxions  $u, v$  as  $r \to \infty$  can be found by first identifying the mass eigenstates: expanding around the potential minimum and diagonalizing: **u** *diamate in the potential minimum* of the set of the The boundary conditions for the saxions  $u, v$  as  $r \to \infty$  can be found by first identifying the pouradity conditions for the same for, a dependence our pollowing mother in the following way. By the following

 $\frac{2}{2}$ / $r^8$  as  $r \rightarrow \infty$ they are both subiect to the axion source term which goes as  $a_2^2/r^8$  as  $r \to \infty$ . They are both subject to the axion source term which goes as  $a^2/r^8$  as  $r\rightarrow \infty$ *f f* 20 *for large <i>r*  $\alpha$  *f*  $\alpha$ 

 $u-v \sim 1/r^6$  and  $4u + v \sim \log r/r^8$ to the asymptotic AdS region; "initial" conditions at *r* = 0, For fixed *q*<sup>0</sup> we use a shooting method to construct solutions which connect the neck region <sup>2</sup>(@*h*)<sup>2</sup> ⇠  $u-v \sim 1/r^6$  and  $4u+v \sim \log r/r^8$ 

. Metric Ansatz: 
$$
ds_5^2 = f(r)^2 dr^2 + (q_0^2 + r^2) d\Omega_4^2
$$
 where  $q_0$ =wormhole size (in AdS units).

- AdS solutions as  $r \to \infty$ , i.e., interested in AdS solutions as  $r \to \infty$ , i.e.,  $f \to 1/q \sim 1/r$  as  $r \to \infty$ . AUD SUIUIUIIS  $dS' \rightarrow \infty$ , i.e.,  $J \rightarrow 1/4 \sim 1/7$  as  $r \rightarrow \infty$ *<sup>q</sup>*(*r*) = <sup>q</sup> *q*2 <sup>0</sup> + *r*<sup>2</sup> *,* (4.27)
- the boundary conditions for the saxions  $u, v$  as  $r \rightarrow \infty$  can be to  $v$  the mass eigenstates: oxpanding around the petertial minimum of the minimum of *V* and diagonalizing, one finds two mass eigenstates:  $\mathcal{I}$  The houndary conditions for the saxions  $\mathcal{I}$ ,  $\mathcal{I}$  as  $\mathcal{I} \rightarrow \infty$  can be interpoundary conditions for the saxions  $u, v$  as  $r \rightarrow \infty$  can be  $r$ .<br>The mass eigenstates: expanding around the notential minimum are mass cigaristates. Capariumy around the potential minimum

$$
u - v: \t m_1^2 = 12, \t \Delta_1 = 6, 4u + v: \t m_2^2 = 32, \t \Delta_2 = 8.
$$

they are both subject to the axion source term which goes as  $\mathfrak{q}_2^2/r^{\mathfrak{0}}$  as  $r\to\infty$ .  $\mathbf{r}$  indy are both subject to the axion source term will goes as  $\mathbf{q}_2$ ,  $\mathbf{r}_4$  as  $\frac{1}{2}$   $\$ These are both subject to a source term from the axions in (4.17) which goes as  $\Delta$   $\sim$ 

$$
u-v \sim 1/r^6 \; \varepsilon
$$

 $\cdot$  Massless scalars  $\phi, \chi, b, c$  fall off as  $1/r^4$ . Massless scalars  $\phi$ ,  $\gamma$ ,  $b$ ,  $c$  fall off as  $1/r<sup>4</sup>$ . <sup>6</sup> The scalars *, , b, c* are all massless and will fall o↵ as 1*/r*<sup>4</sup>  $\blacksquare$  Massless scalars  $\phi, \chi, b, c$  tall off as  $1/r$ .

$$
u - v
$$
:  $m_1^2 = 12$ ,  $\Delta_1 = 6$ ,  
 $u + v$ :  $m_2^2 = 32$ ,  $\Delta_2 = 8$ .

## Constructing Wormhole Solutions

- For fixed wormhole size  $q_0$  and  $(e^{\varphi}\chi_0)'$ , use shooting method to construct solutions which connect the neck region to the asymptotic AdS region.  $q_0$  and  $(e^{\phi}\chi_0)'$
- Adjust  $u_0$ ,  $v_0$  to match BCs.
- Two-parameter family of wormhole solutions.
- For fixed  $q_0 = a_0 / \ell$ , can arrange for all  $r$  :
	- Weak coupling:  $e^{\Phi} = g_s e^{\phi}$
	- Small curvatures:  $\mathscr{R}[g_5], \mathscr{R}[g_{10}] \sim \ell^{-2}$

#### [Loges, GS, Van Riet, '23]





## Dual CFT and Operators Positivity



$$
C_0 \longleftrightarrow \theta_1 + \theta_2
$$
  

$$
\int_{S^2} \tilde{C}_2 \longleftrightarrow \theta_1 - \theta_2
$$
  

$$
(\mathrm{d}\tilde{C}_2 = \mathrm{d}C_2 - C_0 \,\mathrm{d}B_2)
$$

$$
\mathcal{O}_{C_0} = \text{Tr}(F_1 \wedge F_1 + F_2 \wedge F_2)
$$

$$
\mathcal{O}_{\tilde{C}_2} = \text{Tr}(F_1 \wedge F_1 - F_2 \wedge F_2)
$$

Dual operators:

 $\mathcal{O}_{\Phi} = \text{Tr}(F_1 \wedge \star F_1 + F_2 \wedge \star F_2)$  $\mathcal{O}_{B_2} = \text{Tr}(F_1 \wedge *F_1 - F_2 \wedge *F_2)$ 

Operator positivity:

$$
\langle \text{Tr}[(F_i \pm \star F_i)^2] \rangle \ge 0
$$

#### [Loges, GS, Van Riet, '23]

Type IIB on  $T^{1,1}$  is dual to an  $\mathcal{N}=1$  quiver CFT with two nodes [Klebanov, Witten – '98]

$$
^{2}]\rangle \geq 0 \qquad \Longrightarrow \qquad \langle \mathcal{O}_{\Phi} \rangle \pm \langle \mathcal{O}_{B_{2}} \rangle \geq \langle \mathcal{O}_{C_{0}} \rangle \pm \langle \mathcal{O}_{\tilde{C}_{2}} \rangle
$$

#### Violation of Positivity Bounds  $\frac{f}{f} = \frac{f}{f}$ *B*2  $\overline{C}$  *F*2  $\overline{C}$  *F2*  $\overline{C}$



#### [Loges, GS, Van Riet, '23]

With the fully explicit 10d gravity solution, we can check whether  $\langle O_\Phi \rangle \pm \langle O_{B_2} \rangle \geq \langle O_{C_0} \rangle \pm \langle O_{\tilde{C}_2} \rangle$ 

This is **always violated** (for all  $q_0$  and  $\chi_{\infty}$ )! G. J. Loges 10D Axion Wormholes 28 / 34

#### [Loges, GS, Van Riet, '23]

$$
\begin{array}{c}\n\mathcal{D}_{C_0}\rangle = 0 \\
\mathcal{D}_{\tilde{C}_2}\rangle = 0\n\end{array}\n\right\} \qquad \Longrightarrow \qquad \phi_4 \ge \pm \frac{\mathfrak{q}_2}{4} \chi_{\infty} \quad \checkmark
$$

### One boundary vs two?



## Positivity bounds?

[Loges, GS, Van Riet, '23]

- For classical field configurations,  $\mathrm{Tr}(F\pm\star F)^2\geq 0$  is a complete square of Hermitian operators and should be positive.
- In QFT, we subtract an infinite constant when we normal order. The normal ordered operator is not a complete square.
- Evaluating on the quantum state,  $\langle \text{Tr}(F \pm \star F)^2 \rangle$  can be negative (similar argument for stress tensor in [Hofman, Maldacena, '08]).
- Wormhole solutions connecting two asymptotic regions may correspond to such a quantum state in the dual CFT.
- Constructing such quantum state in the dual CFT is an interesting question for the future.



• Establish that GS wormhole is perturbatively stable. The 3-form picture makes gauge invariance and proper

• Conclusion of stability may carry over to AdS space and with additional dilatons: (physical) perturbations

- boundary boundary conditions transparent.
- are localized to the wormhole throat.
- Construct explicit Euclidean axion wormholes in flat and AdS space from string theory:
	- $\bm{\cdot}\;$  Flat space wormholes from type IIA on  $T^6$ : cannot Wick rotate to only Lorentzian branes.
	- $\cdot$  AdS space wormholes from type IIB on  $T^{1,1}$ 
		- Not Giddings-Strominger type: saxions have a potential and are sourced by the axions.
		-
		- $\circ$
- where one might integrate out wormhole effects a la Coleman, the solutions break down.

### **Summary**

Known CFT dual: violation of positivity bounds in the CFT state for two-boundary solutions.

Massive scalars  $u, v$  dual to irrelevant operators may play a crucial role in identifying such CFT state.

• Other conceptual issues remain, e.g.,  $\alpha$ -parameters? Baby universes? For small wormholes (in AdS units)