Euclidean Wormholes and String Phenomenology

Gary Shiu

University of Wisconsin-Madison

Analytic bounds on late-time cosmology

Flavio Tonioni

Hung V. Tran

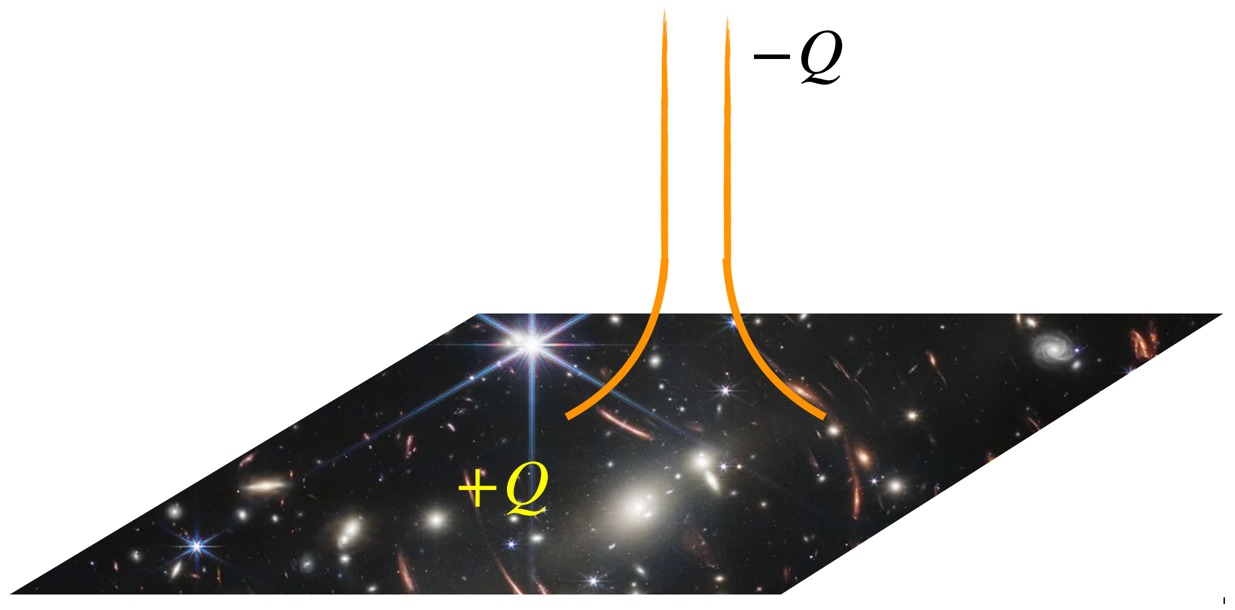
- ``Accelerating universe at the end of time,” GS, Tonioni, Tran, *PRD* [arXiv:2303.03418].
- ``Late-time attractors and cosmic acceleration,” GS, Tonioni, Tran, *PRD* [arXiv:2306.07327].
- ``Collapsing universe before time,” GS, Tonioni, Tran, *JCAP* [arXiv: 2312.06772].
- ``Analytic bounds on late-time axion-scalar cosmologies,” GS, Tonioni, Tran, [arXiv:2406.17030].

[See Flavio Tonioni’s parallel talk on Thursday, 17:00, Session B4]

Why Euclidean Wormholes?

- Recent prominent roles in string theory, especially from the quantum information perspective:
  - Page-curve for black holes [Penington];[Almheiri, Engelhardt, Marolf, Maxfield];[Almheiri, Mahajan, Maldacena, Zhao]; [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini]; …
  - AdS wormholes jeopardize factorization in AdS/CFT [Maldacena, Maoz, '04] (ensemble average?)
- These findings involving Euclidean wormholes were obtained in 2D gravity in which explicit calculations are under control. Do they play similar roles in $D \geq 4$ with dynamical gravity?
- Do they arise as Euclidean saddles of UV complete theories such as string theory?
  - Are they genuine saddle points? Perturbatively stable? [Loges, GS, Sudhir, '22]
  - Can we construct Euclidean wormholes from compactifications of string theory? [Loges, GS, Van Riet, '23]. Explicit AdS wormhole solutions made precision holography possible.

Why Euclidean Wormholes at String Pheno?



- Wormholes break global symmetries.

  [Giddings, Strominger, '88]; [Abbott, Wise, '89];[Coleman, Lee, '90]; [Kallosh, Linde, Linde, Susskind, '95]; …

- Play a key role in the axionic Weak Gravity Conjecture [Arkani-Hamed, Motl, Nicolis, Vafa, '05];[Rudelius, '14, '15]; [Brown, Cottrell, GS, Soler, '15];[Montero, Valenzuela, Uranga, '15]; [Heidenreich, Reece, Rudelius, '15]; [Hebecker,Mangat-Theissen-Witkowski, '16]; …

$$f \cdot S_{instanton} \lesssim M_P$$

evidence that WGC bound is set by wormholes:
[Andriolo, Huang, Noumi, Ooguri, GS, '20];
[Andriolo, GS, Soler, Van Riet, '22]

which was used to constrain some large field inflation models.

- Coleman's α-parameters [Coleman, '89] lead a -1-form global symmetries [McNamara, Vafa, '20]
- Gauss-Bonnet term which was argued to be positive using Swampland criteria: [Aalsma, GS, '22] (WGC); [Martucci, Risso, Weigand, '22] (EFT string probes) leads to an additional exponential suppression of wormhole effects & provides an alternative definition of species scale [Martucci, Risso, Valenti, Vecchi, '24]
- Other phenomenological implications of wormholes reviewed in [Hebecker, Mikhail, Soler, '18]

Giddings-Strominger Wormhole

Giddings-Strominger Solutions

- Consider the following Euclidean action in d ≥ 3 dimensions:

$$S = \frac{1}{2\kappa_d^2} \int \left(\star(\mathcal{R} - 2\Lambda) - \frac{1}{2} G_{ij}(\varphi) \mathrm{d}\varphi^i \wedge \star \mathrm{d}\varphi^j \right)$$

- A simple set of solutions with O(d) symmetry take the form [Giddings, Strominger, '88]:

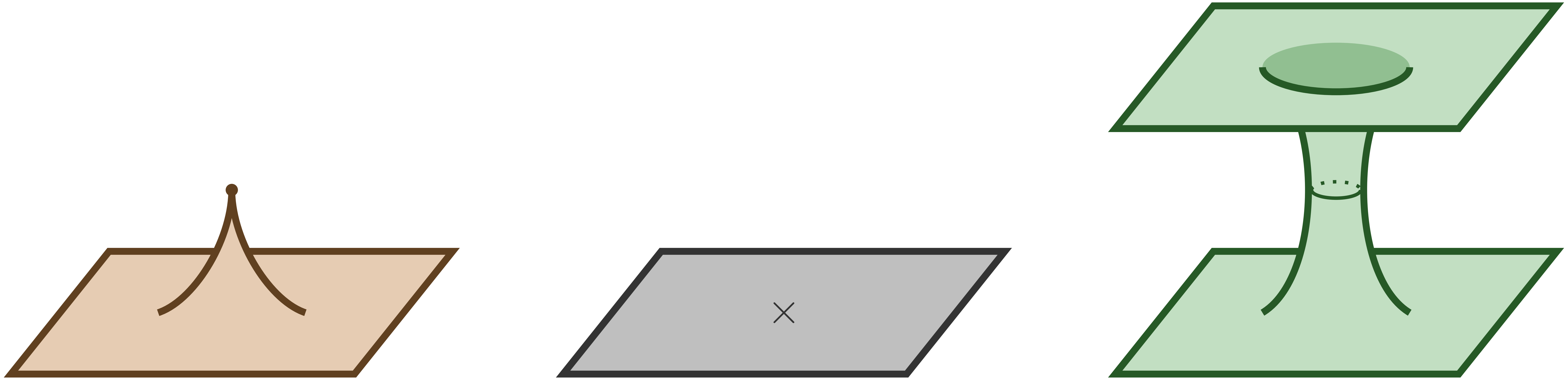
$$\mathrm{d}s^2 = f(r)^2\, \mathrm{d}r^2 + a(r)^2\, \mathrm{d}\Omega_{d-1}^2 \,,$$

$$\left(\frac{a'}{f}\right)^2 = 1 + \frac{a^2}{\ell^2} + \frac{\mathsf{c}}{2(d-1)(d-2)a^{2d-4}} \,,$$

$$\mathsf{c} = G_{ij}(\varphi) \frac{\mathrm{d}\varphi^i}{\mathrm{d}h} \frac{\mathrm{d}\varphi^j}{\mathrm{d}h} \qquad = \text{constant}$$

- $h(r)$ is a harmonic function, normalized to $h' = f/a^{d-1}$ so that $\star\, h = \mathrm{vol}_{d-1}$; plays the role of affine parameter along the geodesic.

Three Classes of Euclidean Geometries

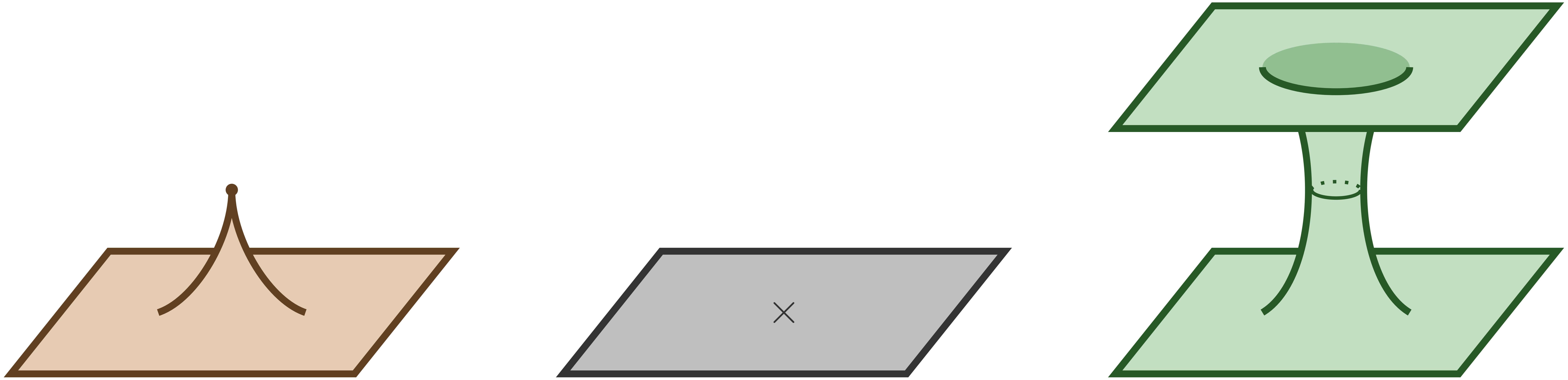


$c > 0$	$c = 0$	$c < 0$
space-like geodesic	null geodesic	time-like geodesic
Core instanton	Extremal instanton, e.g. D-instanton	Wormhole

$$G_{ij}(\varphi)\,\mathrm{d}\varphi^i \mathrm{d}\varphi^j = -\mathrm{d}\chi^2$$

only time-like geodesics

Three Classes of Euclidean Geometries



c > 0
space-like geodesic
Core instanton

c $= 0$
null geodesic
Extremal instanton, e.g. D-instanton

c < 0
time-like geodesic
Wormhole

$$G_{ij}(\varphi)\,\mathrm{d}\varphi^i \mathrm{d}\varphi^j = \mathrm{d}\phi^2 - e^{\beta\phi}\,\mathrm{d}\chi^2$$

c $\gtrless 0$ all possible, but longest time-like geodesic has length $\frac{2\pi}{|\beta|}$

Wormhole Regularity

- Required geodesic length for wormholes only depends on the wormhole size in AdS units:

$$D_d\left(\frac{a_0}{\ell}\right) = \text{“length of geodesic required by geometry”}$$

- $D_d(q_0)$ is monotonic in $q_0 \equiv a_0/\ell$:

$$2\pi\sqrt{\frac{d-1}{2(d-2)}} = D_d(0) \quad \geq \quad D_d(q_0) \quad \geq \quad D_d(\infty) = 2\pi\sqrt{\frac{d-2}{2(d-1)}}$$

distance conjecture?
but
metric with indefinite sign

- There must exist a time-like geodesic longer than $D_d(q_0)$ [Arkani-Hamed, Orgera, Polchinski, ‘07]

$$\sum_i \frac{1}{\beta_i^2} > \frac{d-1}{2(d-2)}$$

Extremal Instantons and Wormholes

- The Euclidean action of core instanton, extremal instanton and wormhole depends on β_i.
- For a given axion charge:
  - Extremal instanton has the minimal action if there are no regular wormholes;
  - Regular wormhole has the minimal action if the dilaton coupling allows it to exist.
- Euclidean axion wormholes are over-extremal. Unlike over-extremal black holes, there is no naked singularity to warn us about possible sickness.
- This has led some to speculate that Euclidean axion wormholes are in the swampland:
  - Negative modes that lower the Euclidean action [Hertog, Truijen, Van Riet, '19]
  - Violate $\mathrm{Tr}(F \pm \star F)^2 \geq 0$ in the dual CFT [Bergshoeff, Collinucci, Ploegh, Vandoren, Van Riet, '05]
- Our works [Loges, GS, Sudhir, '22]; [Loges, GS, Van Riet, '23] settled these puzzles.

Wormhole Stability

Wormhole Stability

[Loges, GS, Sudhir, ’22]

- Previous works (25+ years) on perturbative stability of axion wormholes have led to contradictory claims, casting doubts on their contributions to the Euclidean path integral.

	Frame	Stable	Gauge-inv	j=0,1	B.C.
Rubakov, Shvedov, ‘96	axion	No	No	physical	✗
Alonso, Urbano, ‘17	axion	Yes	Yes	physical	✗
Hertog, Truijen, Van Riet, ‘18	axion	No	Yes	pure gauge	✗
Loges, GS, Sudhir, ‘22	3-form	Yes	Yes	pure gauge	✓
Hertog, Meanaut, Missoni, Tielemans, Van Riet, ‘24	axion	Yes	Yes	pure gauge	✓

- My talk at String Pheno 2022 in Liverpool touched upon this issue.

Gauge Invariance and Boundary Conditions

- Under diffeomorphism, metric and 3-form perturbations are mixed:

$$\mathrm{d}s^2 = a(\eta)^2\left[-(1+2\phi)\,\mathrm{d}\eta^2 + 2\partial_i B\,\mathrm{d}\eta\mathrm{d}x^i + \big((1-2\psi)\gamma_{ij} + 2\nabla_i\partial_j E\big)\,\mathrm{d}x^i\mathrm{d}x^j\right]$$

$$H = \frac{n}{2\pi^2}\left[(1+s)\mathrm{vol}_3 + \mathrm{d}\eta\wedge\left(\tfrac{1}{2}\sqrt{\gamma}\epsilon_{ijk}\partial^i w\,\mathrm{d}x^j\wedge\mathrm{d}x^k\right)\right]$$

- Metric perturbations vanish far from the wormhole neck so are the 3-form perturbations:

$$n = \int_{S^3} H \in \mathbb{Z} \qquad\qquad \Rightarrow \delta H \to 0 \text{ asymptotically}$$

- Gauge invariant perturbations satisfy Dirichlet boundary conditions in the 3-form picture.

- Conformal mode problem is absent: $j=0$ mode is not physical (wormhole size is set by axion charge).

- The conformal model problem is conceivably only be a feature of pure gravity. Could it be evaded for the wavefunction of the universe in the context of inflation? [Loges, GS, work in progress].

String Theory Embeddings

[Loges, GS, Van Riet, ’23]

Euclidean Axion Wormholes in Flat Space

[Loges, GS, Van Riet, '23]

- Reduction ansatz motivated by the extremal solution:

$$\mathrm{d}s_{10}^2 = e^{-6b\varphi}\,\mathrm{d}s_4^2 + e^{2b\varphi}R^2\mathcal{M}_{ij}\,\mathrm{d}\theta^i\mathrm{d}\theta^j$$

$$\mathcal{M}_{ij} = \mathrm{diag}\big(e^{\vec{\beta}_1\cdot\vec{\Phi}}, e^{\vec{\beta}_2\cdot\vec{\Phi}}, \ldots, e^{\vec{\beta}_6\cdot\vec{\Phi}}\big)$$

$$C_3 = \chi_1\,\mathrm{d}\theta^{123} + \chi_2\,\mathrm{d}\theta^{145} + \chi_3\,\mathrm{d}\theta^{256} + \chi_4\,\mathrm{d}\theta^{346}$$

	1	2	3	4	5	6
$D2_1$	$\times$	$\times$	$\times$			
$D2_2$	$\times$			$\times$	$\times$	
$D2_3$		$\times$			$\times$	$\times$
$D2_4$			$\times$	$\times$		$\times$

- 4d theory contains 11 scalars:

No Wick rotation that turns them into Lorentzian "overextremal" branes.

$$S_4 = \frac{1}{2\kappa_4^2}\int\left[-\mathcal{R} + \frac{1}{2}\sum_{i=1}^{4}\left[(\partial s_i)^2 + e^{2s_i}(\partial\chi_i)^2\right] + \frac{1}{2}\sum_{i=5}^{7}(\partial s_i)^2\right]$$

There are four decoupled axio-dilaton pairs with $\beta = 2$:

$$\sum_{i=1}^{4}\frac{1}{\beta_i^2} = 1 \quad > \quad \frac{3}{4} \qquad \checkmark$$

Euclidean Axion Wormholes in AdS Space

[Loges, GS, Van Riet, '23]

$$T^{1,1} = [\mathrm{SU}(2)\times\mathrm{SU}(2)]/\,\mathrm{U}(1) \quad \sim \quad \overbrace{S^2}^{\int_{S^2}(B_2,C_2)\text{ axions}} \times S^3$$

$S^2 \times S^3$ → F_5 flux

$\int_{S^2}(B_2, C_2)$ axions

<u>Background solution:</u>

$$\begin{aligned} \mathrm{d}s_{10}^2 &= \ell^2\,\mathrm{d}s_5^2 + \ell^2\big(\mathrm{d}s_{\mathrm{KE}}^2 + \eta^2\big) \\ e^{\Phi} &= g_{\mathrm{s}} \\ B_2 &= 0 \\ C_0 &= 0 \\ C_2 &= 0 \\ F_5 &= 4\ell^2(1 - i\star)\mathrm{vol}_{T^{1,1}} \end{aligned}$$

[Klebanov, Witten]

<u>Reduction ansatz</u>;

$$\begin{aligned} \mathrm{d}s_{10}^2 &= \ell^2 e^{-\frac{2}{3}(4u+v)}\,\mathrm{d}s_5^2 + \ell^2\big(e^{2u}\,\mathrm{d}s_{\mathrm{KE}}^2 + e^{2v}\eta^2\big) \\ e^{\Phi} &= g_{\mathrm{s}}e^{\phi} \\ B_2 &= \ell^2 g_{\mathrm{s}}^{1/2} b\,\Phi_2 \\ C_0 &= i g_{\mathrm{s}}^{-1}\chi \\ C_2 &= i\ell^2 g_{\mathrm{s}}^{-1/2} c\,\Phi_2 \\ F_5 &= 4\ell^2(1 - i\star)\mathrm{vol}_{T^{1,1}} \end{aligned}$$

Consistent Reduction to 5D

[Loges, GS, Van Riet, '23]

string dilaton

$H_3 \sim \mathrm{d}b \wedge \Phi_2$

$F_1 \sim \mathrm{d}\chi$

$F_3 \sim (\mathrm{d}c - \chi\,\mathrm{d}b) \wedge \Phi_2$
$(\mathrm{d}F_3 = H_3 \wedge F_1)$

$$S_5 = \frac{1}{2\kappa_5^2} \int \mathrm{d}^5x \sqrt{|g|} \Big[-\mathcal{R} + \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}e^{-4u-\phi}(\partial b)^2 + \tfrac{1}{2}e^{2\phi}(\partial\chi)^2 + \tfrac{1}{2}e^{-4u+\phi}(\partial c - \chi\,\partial b)^2$$

$$+ \tfrac{28}{3}(\partial u)^2 + \tfrac{8}{3}\partial u \partial v + \tfrac{4}{3}(\partial v)^2 + \underbrace{2e^{-\frac{8}{3}(4u+v)}\big(2e^{4u+4v} - 12e^{6u+2v} + 4\big)}_{\mathcal{V}} \Big]$$

$e^{2u}\,\mathrm{d}s^2_{\mathrm{KE}} + e^{2v}\,\eta^2$

[Cassani, Dall'Agata, Faedo – '10]

[Cassani, Faedo – '11]

Not Giddings-Strominger wormhole!

Consistent Reduction to 5D

[Loges, GS, Van Riet, '23]

string dilaton

$H_3 \sim \mathrm{d}b \wedge \Phi_2$

$F_1 \sim \mathrm{d}\chi$

$F_3 \sim (\mathrm{d}c - \chi\, \mathrm{d}b) \wedge \Phi_2$
$(\mathrm{d}F_3 = H_3 \wedge F_1)$

$$S_5 = \frac{1}{2\kappa_5^2} \int \mathrm{d}^5x \sqrt{|g|} \Big[-\mathcal{R} + \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2} e^{-4u-\phi} (\partial b)^2 + \tfrac{1}{2} e^{2\phi} (\partial\chi)^2 + \tfrac{1}{2} e^{-4u+\phi} (\partial c - \chi\, \partial b)^2$$

$$+ \tfrac{28}{3}(\partial u)^2 + \tfrac{8}{3}\partial u \partial v + \tfrac{4}{3}(\partial v)^2 + \underbrace{2e^{-\frac{8}{3}(4u+v)}\left(2e^{4u+4v} - 12 e^{6u+2v} + 4\right)}_{\mathcal{V}} \Big]$$

$e^{2u}\, \mathrm{d}s^2_{\mathrm{KE}} + e^{2v}\, \eta^2$

[Cassani, Dall'Agata, Faedo – '10]

[Cassani, Faedo – '11]

Not Giddings-Strominger wormhole!

Symmetries

[Loges, GS, Van Riet, '23]

- The gauge symmetries associated with C_0, C_2, B_2 lead to three shift symmetries:

$$\begin{aligned} \delta_1\chi &= \lambda_1\,, & \delta_2\chi &= 0\,, & \delta_3\chi &= 0\,, \\ \delta_1 b &= 0\,, & \delta_2 b &= \lambda_2\,, & \delta_3 b &= 0\,, \\ \delta_1 c &= \lambda_1 b\,, & \delta_2 c &= 0\,, & \delta_3 c &= \lambda_3\,, \end{aligned}$$

- The δ_1 transformation is an element of $SL(2,\mathbb{R})$ under which:

$$e^{-\phi} \mapsto e^{-\phi}\,, \qquad \chi \mapsto \chi + \lambda_1\,, \qquad \begin{bmatrix} b \\ c \end{bmatrix} \mapsto \begin{bmatrix} 1 & 0 \\ \lambda_1 & 1 \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix}\,.$$

- There is also the rescaling symmetry of $SL(2,\mathbb{R})$ for which:

$$e^{-\phi} \mapsto a^2 e^{-\phi}\,, \qquad \chi \mapsto a^2\chi\,, \qquad \begin{bmatrix} b \\ c \end{bmatrix} \mapsto \begin{bmatrix} a^{-1} & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix}\,.$$

- Exploit these symmetries, which in the SUGRA approximation, are unbroken to discrete subgroups.

Axion Charges

[Loges, GS, Van Riet, '23]

- The axion EOMs can be solved in terms of the constant axion charges (assoc. with δ_2, δ_3):

$$\begin{aligned} \star\left[e^{-4u-\phi}\,\mathrm{d}b + e^{-4u+\phi}\chi(\mathrm{d}c - \chi\,\mathrm{d}b)\right] &\equiv \mathfrak{q}_1\,\mathrm{vol}_4\,, \\ \star\left[e^{-4u+\phi}(\mathrm{d}c - \chi\,\mathrm{d}b)\right] &\equiv \mathfrak{q}_2\,\mathrm{vol}_4\,. \end{aligned}$$

- $\mathfrak{q}_1$ and $\mathfrak{q}_2$ are both conserved and quantized; $\mathfrak{q}_1$ is gauge-dependent (Page charge), $\mathfrak{q}_2$ is gauge-invariant (Maxwell charge).

- $\mathfrak{q}_{1,2}$ quantized in units of $(\ell_s/\ell)^6$ and $\ell \gg \ell_s$: justifying continuous charge approximation.

- Under the shift symmetries:

$$\begin{aligned} \delta_1\mathfrak{q}_1 &= \lambda_1\mathfrak{q}_2\,, \qquad & \delta_2\mathfrak{q}_1 &= 0\,, \qquad & \delta_3\mathfrak{q}_1 &= 0\,, \\ \delta_1\mathfrak{q}_2 &= 0\,, & \delta_2\mathfrak{q}_2 &= 0\,, & \delta_3\mathfrak{q}_2 &= 0\,, \end{aligned}$$

and the rescaling symmetry: $\mathfrak{q}_1 \mapsto a\,\mathfrak{q}_1\,, \qquad \mathfrak{q}_2 \mapsto a^{-1}\mathfrak{q}_2\,.$

Equations of Motion

[Loges, GS, Van Riet, '23]

- In terms of the constant axion charges, the remaining 5d EOMs:

$$\begin{aligned}
\Box\phi &= -e^{2\phi}(\partial\chi)^2 - \frac{1}{2}e^{4u}\big[e^{\phi}(\mathfrak{q}_1 - \mathfrak{q}_2\chi)^2 + e^{-\phi}\mathfrak{q}_2^2\big](\partial h)^2\,,\\
\mathrm{d}\star(e^{2\phi}\mathrm{d}\chi) &= -e^{4u+\phi}\mathfrak{q}_2(\mathfrak{q}_1 - \mathfrak{q}_2\chi)(\partial h)^2\,,\\
\Box(7u+v) &= \frac{3}{8}\partial_u\mathcal{V} - \frac{3}{4}e^{4u}\big[e^{\phi}(\mathfrak{q}_1 - \mathfrak{q}_2\chi)^2 - e^{-\phi}\mathfrak{q}_2^2\big](\partial h)^2\,,\\
\Box(u+v) &= \frac{3}{8}\partial_v\mathcal{V}\,,\\
2R_{\mu\nu} &= \partial_\mu\phi\partial_\nu\phi - e^{2\phi}\partial_\mu\chi\partial_\nu\chi + e^{4u}\big[e^{\phi}(\mathfrak{q}_1 - \mathfrak{q}_2\chi)^2 - e^{-\phi}\mathfrak{q}_2^2\big]\partial_\mu h\partial_\nu h\\
&\qquad + \frac{56}{3}\partial_\mu u\partial_\nu u + \frac{8}{3}(\partial_\mu u\partial_\nu v + \partial_\mu v\partial_\nu u) + \frac{8}{3}\partial_\mu v\partial_\nu v - \frac{2}{3}g_{\mu\nu}\mathcal{V}\,.
\end{aligned}$$

- Solutions have definite parity when $\mathfrak{q}_1 = 0$; generate solutions with $\mathfrak{q}_1 \neq 0$ by shift δ_1.
- The B_2, C_2 axions source the saxions u, v: cannot simply set $u = v = 0$.

Supersymmetric Instanton

[Loges, GS, Van Riet, '23]

- Setting $u = v = 0$ at the potential minimum requires:

$$\mathfrak{q}_1 - \mathfrak{q}_2 \chi = \pm \mathfrak{q}_2 \, e^{-\phi}$$

- Picking the bottom sign:

$$\chi - e^{-\phi} = \frac{\mathfrak{q}_1}{\mathfrak{q}_2} = \text{const.} \qquad \Longrightarrow \qquad F_3 = -ie^{-\Phi} H_3 \, .$$

- This is the supersymmetric D-instanton.

- Another option is to consider b, c = constant (hence $\mathfrak{q}_1 = \mathfrak{q}_2 = 0$), leaving only the type IIB axio-dilaton which has $\beta = 2$ which does not allow for regular wormholes.

- Non-singular wormhole solutions require non-zero u, v.

Boundary Conditions

[Loges, GS, Van Riet, '23]

- Metric Ansatz: $ds_5^2 = f(r)^2 dr^2 + (q_0^2 + r^2) d\Omega_4^2$ where q_0=wormhole size (in AdS units).
- AdS solutions as $r \to \infty$, i.e., $f \to 1/q \sim 1/r$ as $r \to \infty$
- The boundary conditions for the saxions u, v as $r \to \infty$ can be found by first identifying the mass eigenstates: expanding around the potential minimum and diagonalizing:

$$\begin{aligned} u - v: \quad & m_1^2 = 12\,, \quad & \Delta_1 = 6\,, \\ 4u + v: \quad & m_2^2 = 32\,, \quad & \Delta_2 = 8\,. \end{aligned}$$

they are both subject to the axion source term which goes as $\mathbf{q}_2^2/r^8$ as $r \to \infty$.

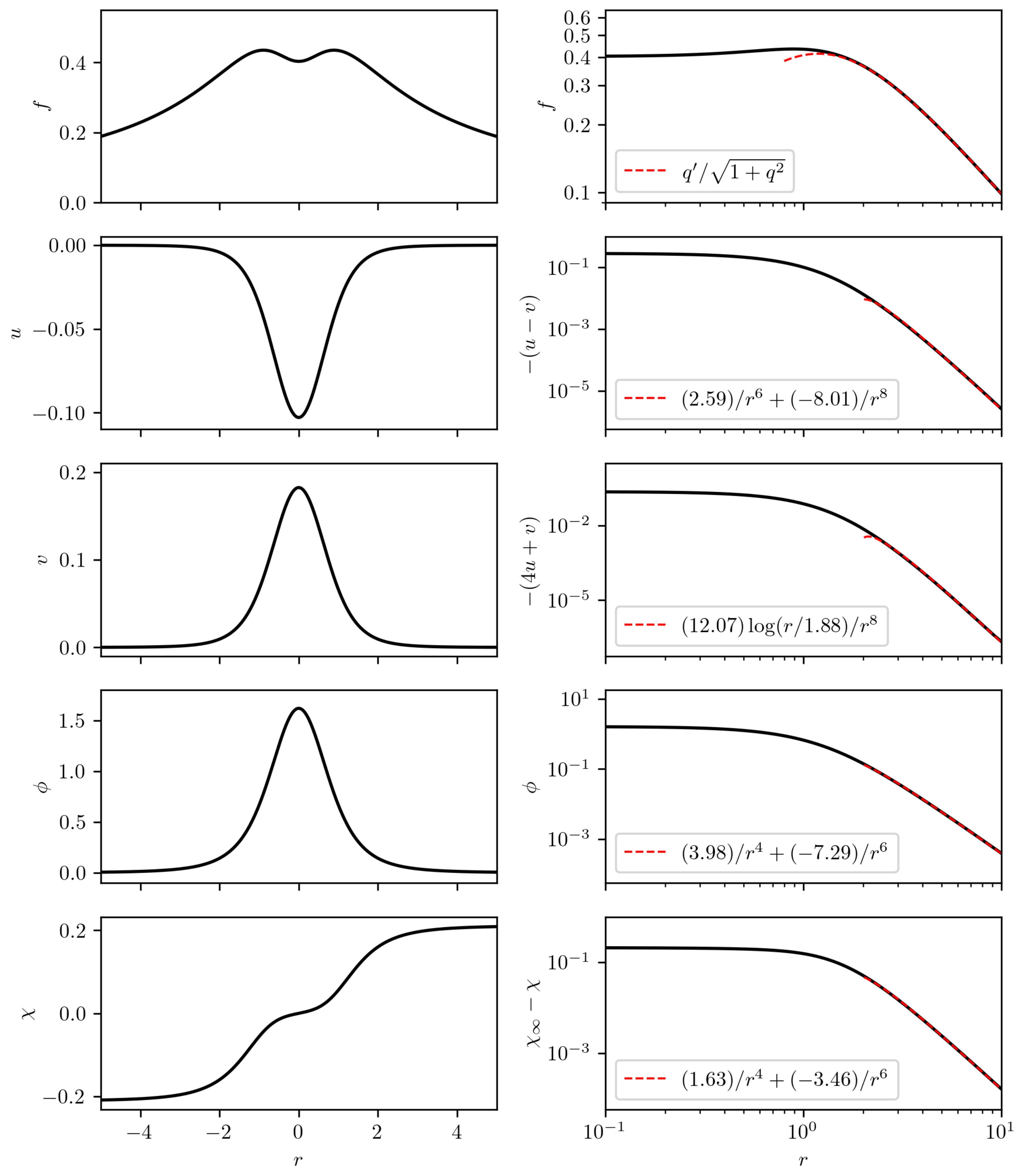
$$u - v \sim 1/r^6 \text{ and } 4u + v \sim \log r / r^8$$

- Massless scalars ϕ, χ, b, c fall off as $1/r^4$.

Constructing Wormhole Solutions

[Loges, GS, Van Riet, '23]

- For fixed wormhole size q_0 and $(e^{\phi}\chi_0)'$, use shooting method to construct solutions which connect the neck region to the asymptotic AdS region.
- Adjust u_0, v_0 to match BCs.
- Two-parameter family of wormhole solutions.
- For fixed $q_0 = a_0/\ell$, can arrange for all r :
  - Weak coupling: $e^{\Phi} = g_s e^{\phi}$
  - Small curvatures: $\mathscr{R}[g_5], \mathscr{R}[g_{10}] \sim \ell^{-2}$



Dual CFT and Operators Positivity

[Loges, GS, Van Riet, '23]

Type IIB on $T^{1,1}$ is dual to an $\mathcal{N}=1$ quiver CFT with two nodes [Klebanov, Witten – '98]

$$e^{-\Phi} \longleftrightarrow \frac{1}{g_1^2}+\frac{1}{g_2^2} \qquad\qquad C_0 \longleftrightarrow \theta_1+\theta_2$$

$$\int_{S^2} B_2 \longleftrightarrow \frac{1}{g_1^2}-\frac{1}{g_2^2} \qquad\qquad \int_{S^2} \tilde{C}_2 \longleftrightarrow \theta_1-\theta_2$$

$$(\mathrm{d}\tilde{C}_2=\mathrm{d}C_2-C_0\,\mathrm{d}B_2)$$

Dual operators:

$$\mathcal{O}_\Phi=\mathrm{Tr}(F_1\wedge\star F_1+F_2\wedge\star F_2) \qquad \mathcal{O}_{C_0}=\mathrm{Tr}(F_1\wedge F_1+F_2\wedge F_2)$$

$$\mathcal{O}_{B_2}=\mathrm{Tr}(F_1\wedge\star F_1-F_2\wedge\star F_2) \qquad \mathcal{O}_{\tilde{C}_2}=\mathrm{Tr}(F_1\wedge F_1-F_2\wedge F_2)$$

Operator positivity:

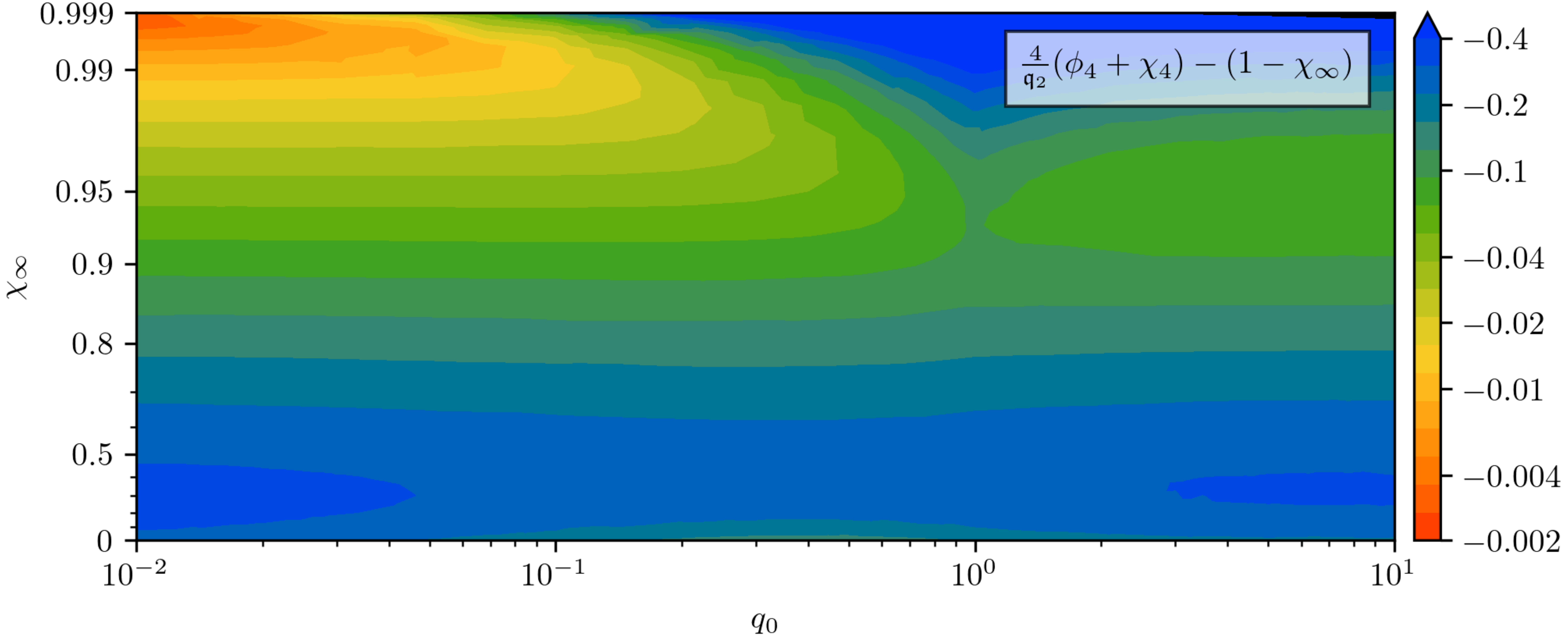
$$\langle\mathrm{Tr}[(F_i\pm\star F_i)^2]\rangle\geq 0 \qquad\Longrightarrow\qquad \langle\mathcal{O}_\Phi\rangle\pm\langle\mathcal{O}_{B_2}\rangle\geq\langle\mathcal{O}_{C_0}\rangle\pm\langle\mathcal{O}_{\tilde{C}_2}\rangle$$

Violation of Positivity Bounds

[Loges, GS, Van Riet, '23]

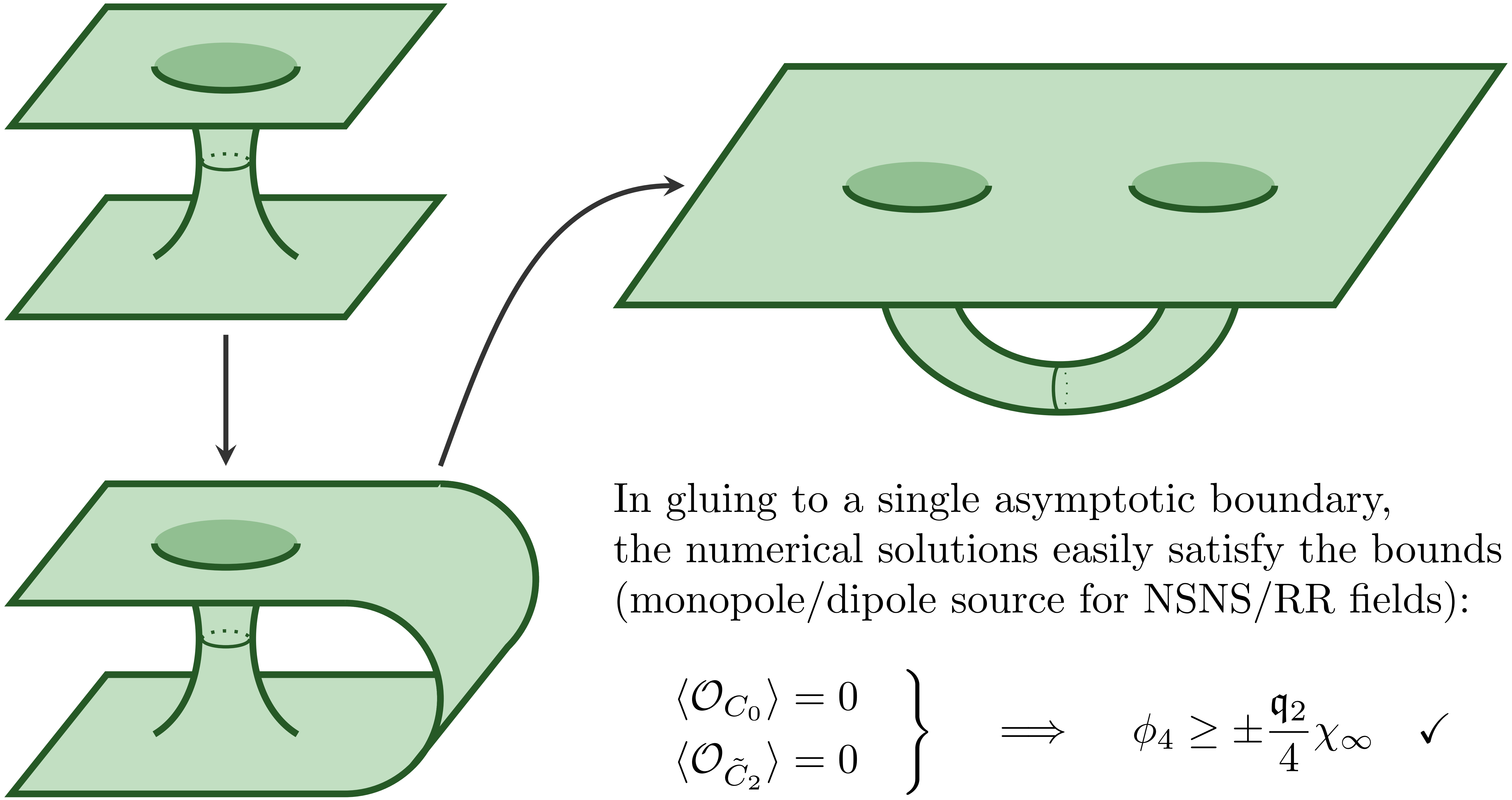
With the fully explicit 10d gravity solution, we can check whether $\langle \mathcal{O}_\Phi \rangle \pm \langle \mathcal{O}_{B_2} \rangle \geq \langle \mathcal{O}_{C_0} \rangle \pm \langle \mathcal{O}_{\tilde{C}_2} \rangle$

This is **always violated** (for all q_0 and χ_∞)!



One boundary vs two?

[Loges, GS, Van Riet, '23]



In gluing to a single asymptotic boundary, the numerical solutions easily satisfy the bounds (monopole/dipole source for NSNS/RR fields):

$$\left.\begin{array}{l}\langle \mathcal{O}_{C_0}\rangle = 0 \\ \langle \mathcal{O}_{\tilde{C}_2}\rangle = 0\end{array}\right\} \implies \phi_4 \geq \pm\frac{\mathfrak{q}_2}{4}\chi_\infty \quad \checkmark$$

Positivity bounds?

[Loges, GS, Van Riet, '23]

- For classical field configurations, $\mathrm{Tr}(F \pm \star F)^2 \geq 0$ is a complete square of Hermitian operators and should be positive.
- In QFT, we subtract an infinite constant when we normal order. The normal ordered operator is not a complete square.
- Evaluating on the quantum state, $< \mathrm{Tr}(F \pm \star F)^2 >$ can be negative (similar argument for stress tensor in [Hofman, Maldacena, '08]).
- Wormhole solutions connecting two asymptotic regions may correspond to such a quantum state in the dual CFT.
- Constructing such quantum state in the dual CFT is an interesting question for the future.

Summary

Summary

- Establish that GS wormhole is perturbatively stable. The 3-form picture makes gauge invariance and proper boundary boundary conditions transparent.
- Conclusion of stability may carry over to AdS space and with additional dilatons: (physical) perturbations are localized to the wormhole throat.
- Construct explicit Euclidean axion wormholes in flat and AdS space from string theory:
  - Flat space wormholes from type IIA on T^6: cannot Wick rotate to only Lorentzian branes.
  - AdS space wormholes from type IIB on $T^{1,1}$
    - Not Giddings-Strominger type: saxions have a potential and are sourced by the axions.
    - Known CFT dual: violation of positivity bounds in the CFT state for two-boundary solutions.
    - Massive scalars u, v dual to irrelevant operators may play a crucial role in identifying such CFT state.
- Other conceptual issues remain, e.g., α-parameters? Baby universes? For small wormholes (in AdS units) where one might integrate out wormhole effects a la Coleman, the solutions break down.