The Structure of the Flux Landscape

- Unlimited Edition -

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Utrecht University

Based on:

2404.12422 with Damian van de Heisteeg

2311.09295 with Jeroen Monnee

work in progress with Damian van de Heisteeg, David Prieto

Introduction

Effective actions from string theory

- Common properties of effective theory when compactifying on Y

$$S^{(4)} = \int d^4x\sqrt{G}\Big(R - \frac{1}{g^2(\phi)}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \theta(\phi)\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu} - K(\phi)(\partial\phi)^2 - V(\phi)\Big)$$

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Properties of functions $g^2(\phi),\ \theta(\phi),\ K(\phi),\ V(\phi)$ in terms Y-deformations

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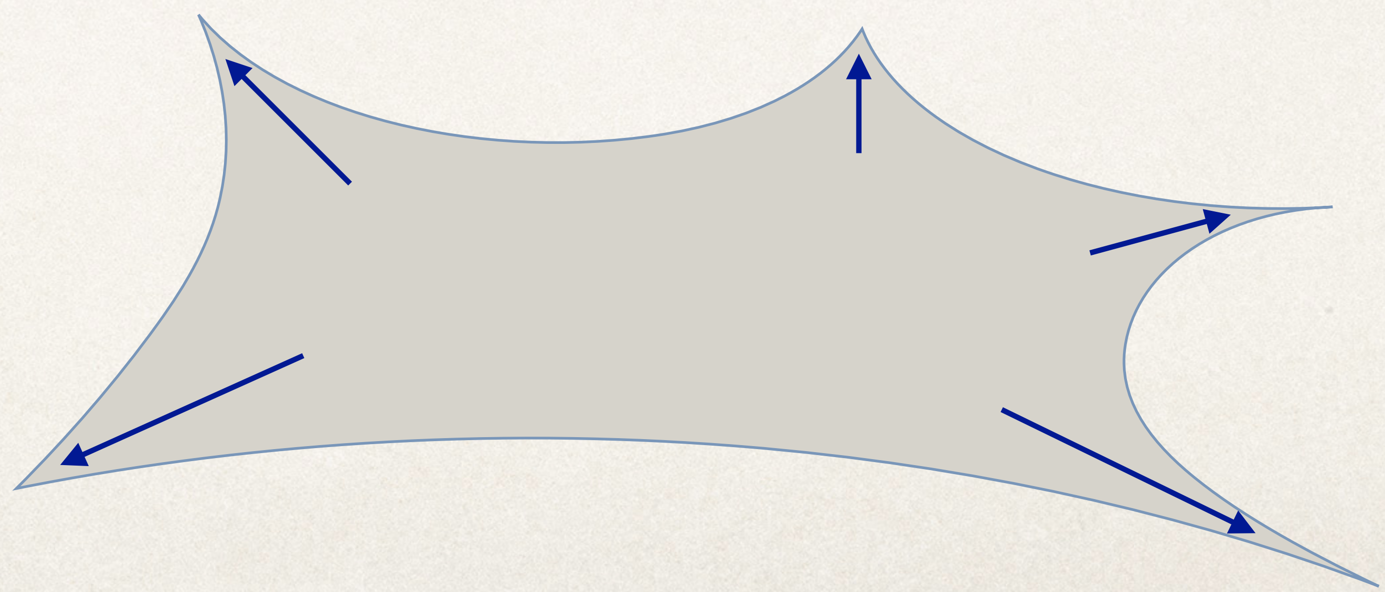
- Much recent progress: limits towards the boundaries $\phi \to \infty$ and $V(\phi) \cong 0$



Structure away from the boundaries

- In this talk: use properties of scalar potential to uncover universal structure of the vacuum landscape

  $\rightarrow$ patterns away from boundaries

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A key insight:

functions are algebraic (polynomial) vs. functions are transcendental (no polynomial relations, 'instantons')

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$\updownarrow$

Symmetry

susy? [Palti,Weigand,Vafa '22]

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Very different behavior when solving problems with integer parameters
$\rightarrow$ change the distribution of flux vacua

F-theory flux scalar potentials

- Concretely: flux compactifications of Type IIB and F-theory

reviews [Grana][Douglas,Kachru][Denef]

- F-theory on compact Calabi-Yau fourfold Y
  - 4-form flux: $G_4 \in H^4(Y,\mathbb{Z})$ $\qquad \int_Y G_4 \wedge G_4 = \ell$ $\qquad$ 'tadpole condition'

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$$\Pi_i(\phi) = \int_{C^i} \Omega$$ periods of (4,0) form

Focusing on a landscape of vacua

- Example landscapes:
  - W=0 landscape: $\partial_{\phi^i} W = 0 \ \text{ and } \ W = 0$
  - Minkowski landscape: $\partial_{\phi^i} V = 0 \ \text{ and } \ V = 0$
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- generalization to Calabi-Yau fourfolds and essentially complete picture

  [TG,van de Heisteeg '24]

How to find vacua?

A benchmark example

F-theory on Hulek-Verrill fourfold

- Hulek-Verrill fourfold: $(X^1, \dots, X^6) \in \quad \mathbb{T}^5 = \mathbb{P}^5 \backslash \{X_1 \cdots X_6 = 0\}$

$$(X^1 + X^2 + X^3 + X^4 + X^5 + X^6)\left(\frac{\phi^1}{X^1} + \frac{\phi^2}{X^2} + \frac{\phi^3}{X^3} + \frac{\phi^4}{X^4} + \frac{\phi^5}{X^5} + \frac{\phi^6}{X^6}\right) = 1$$

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- Periods: expanded around large complex structure [Jockers,Kotlewski,Kuusela '23]

$$\mathbf{\Pi} = \begin{pmatrix} \Pi^0 \\ \Pi^I \\ \Pi_{IJ} \\ \Pi_I \\ \Pi_0 \end{pmatrix}$$

e.g.

$$\Pi^0 = \sum_{n_1,\dots,n_6=0}^{\infty} \left(\frac{(n_1 + \ldots + n_6)!}{n_1! \cdots n_6!}\right)^2 (\phi^1)^{n_1} \cdots (\phi^6)^{n_6}$$

$$\Pi^I = \Pi^0 \frac{\log \phi^I}{2\pi i} + 2 \sum_{n_1,\dots,n_6} (H_{n_1+\ldots+n_6} - H_{n_I})(\phi^1)^{n_1} \cdots (\phi^6)^{n_6}$$

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- W=0 flux vacua: $\mathbf{G_4}^T \Sigma \mathbf{\Pi} = \mathbf{G_4}^T \Sigma \partial_{\phi^I} \mathbf{\Pi} = 0$

very transcendental $\rightarrow$ solve exactly?

- Numerical approach: start near a known limit - large complex structure - and add corrections

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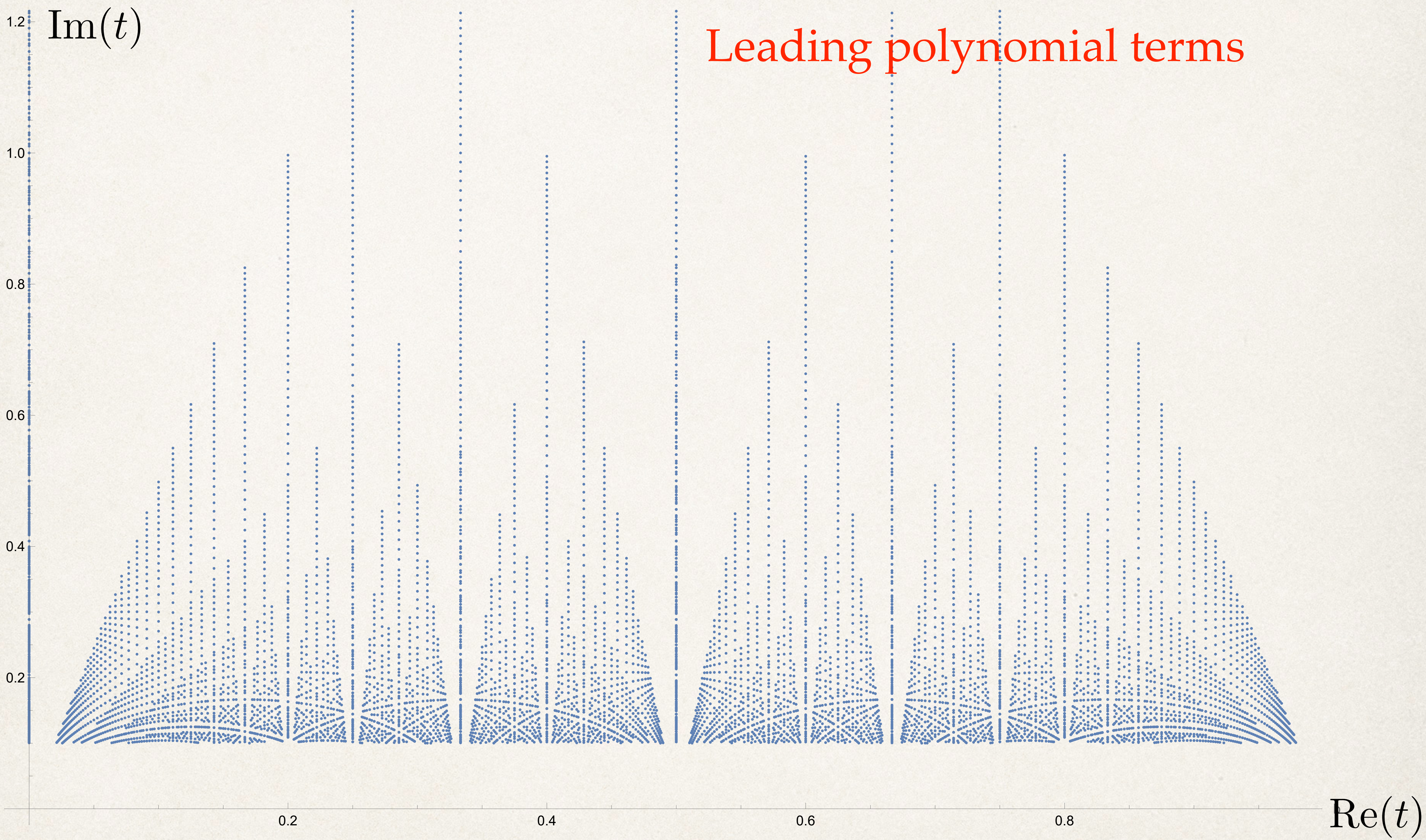

- along slice $\phi_1 = \ldots = \phi_6$ in standard "Kähler coordinate" t

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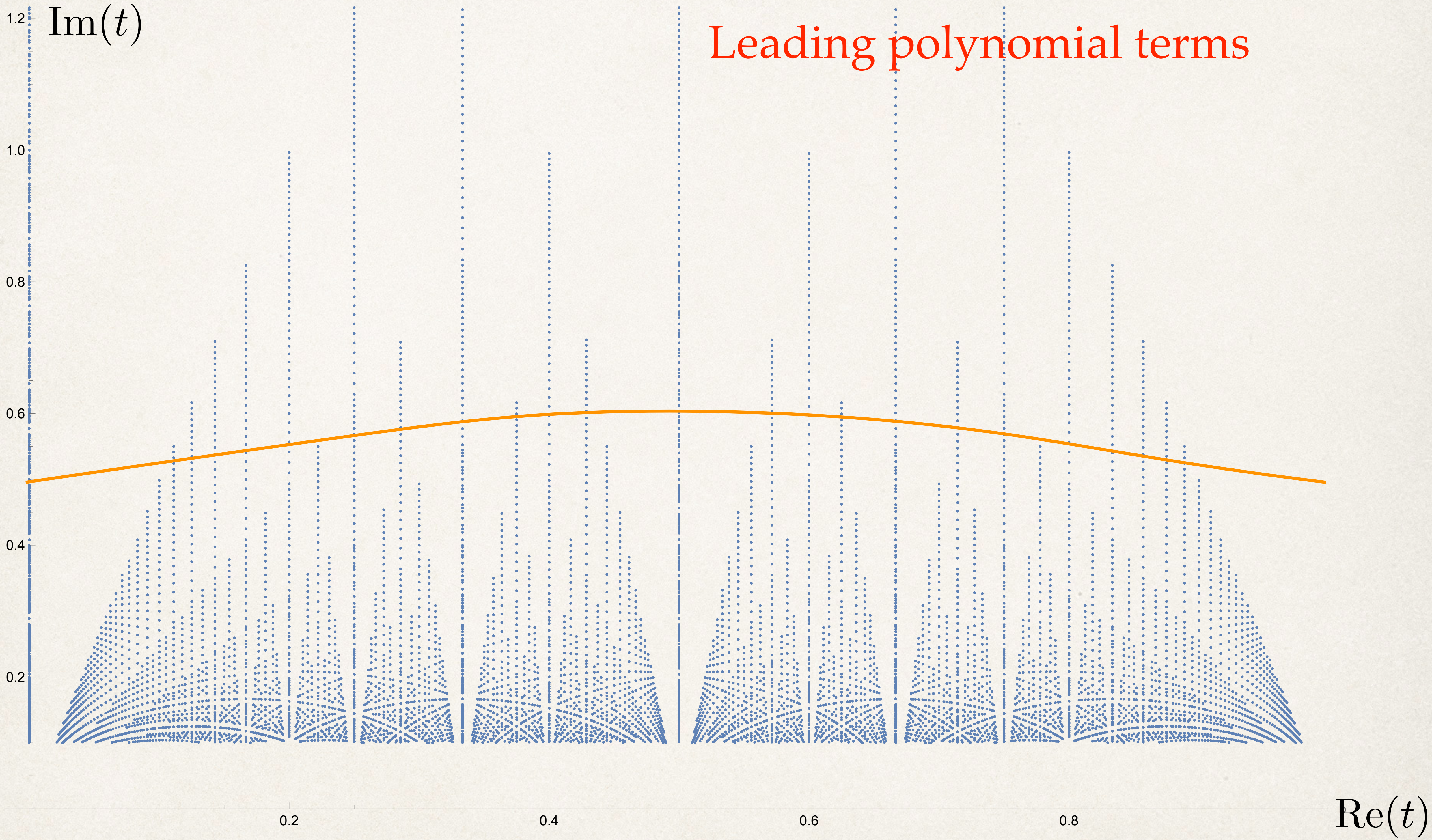

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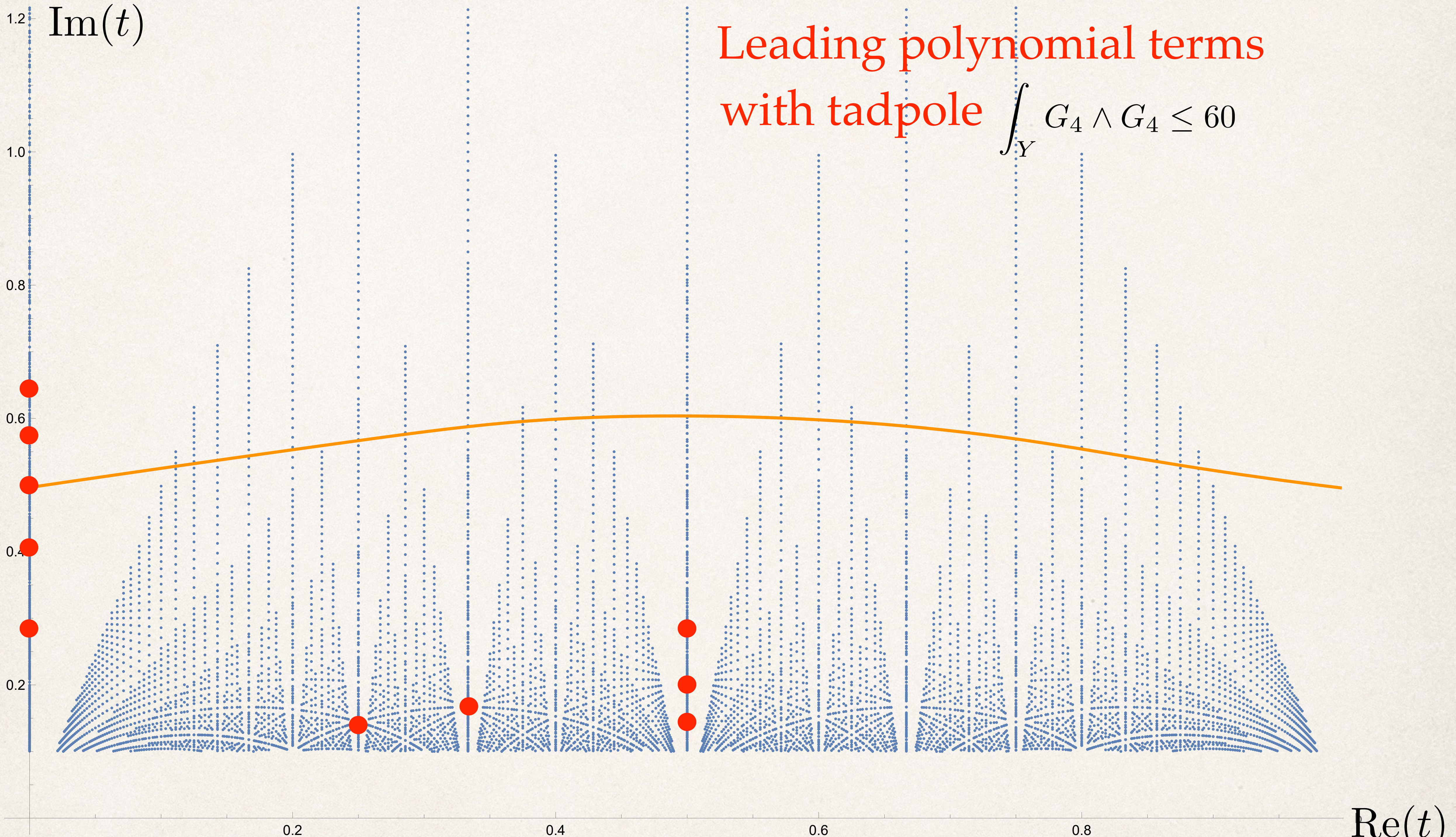

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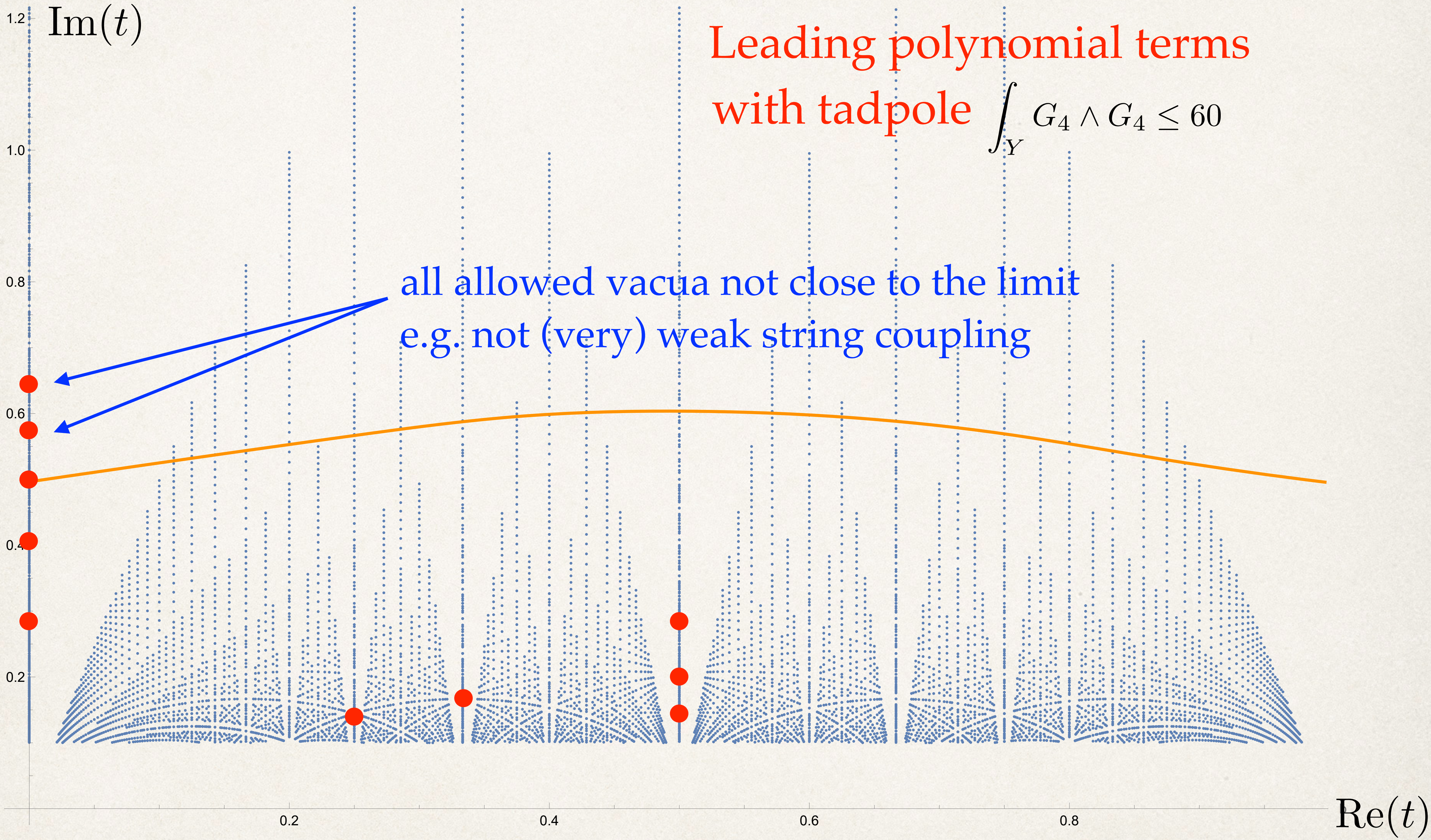

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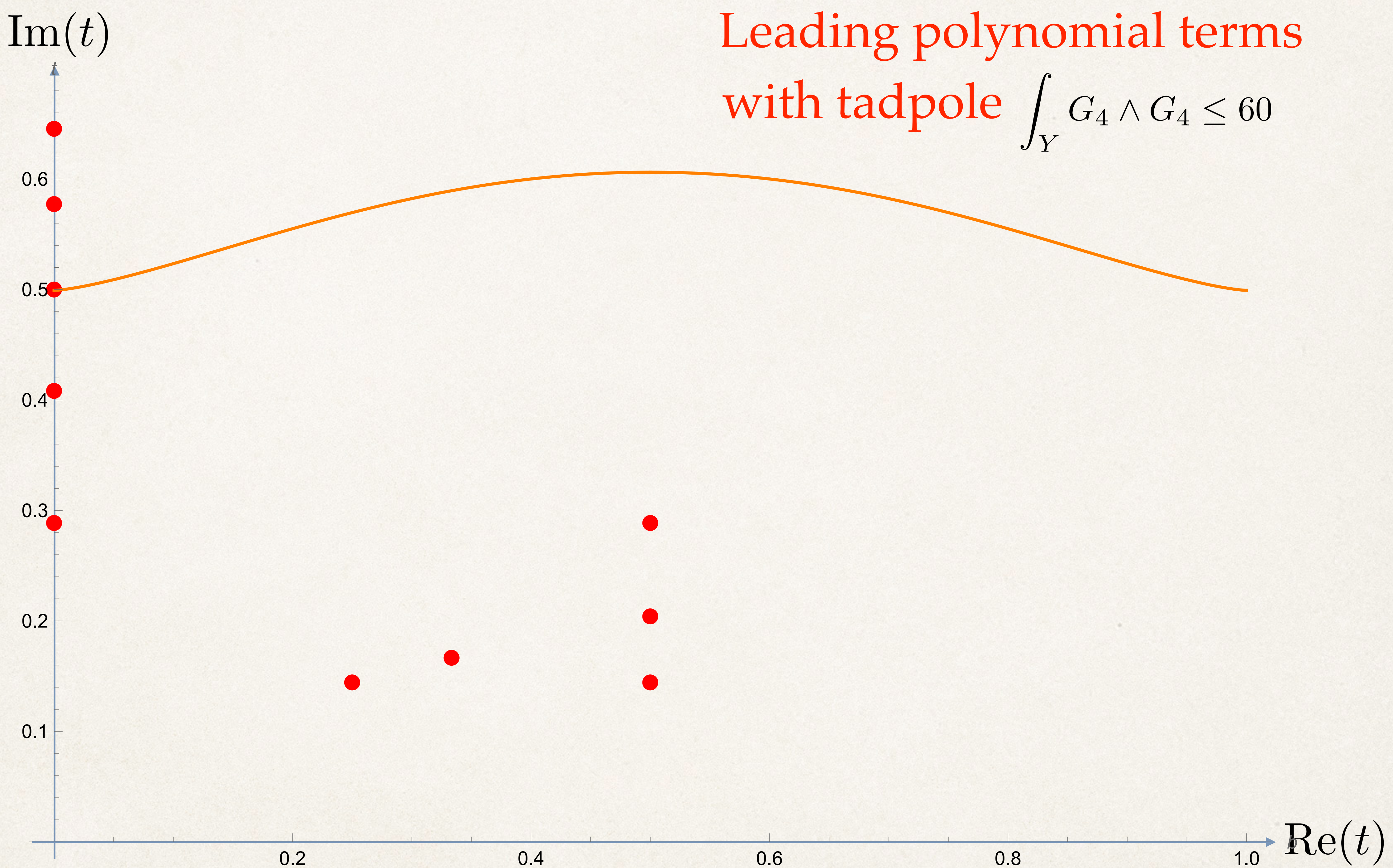

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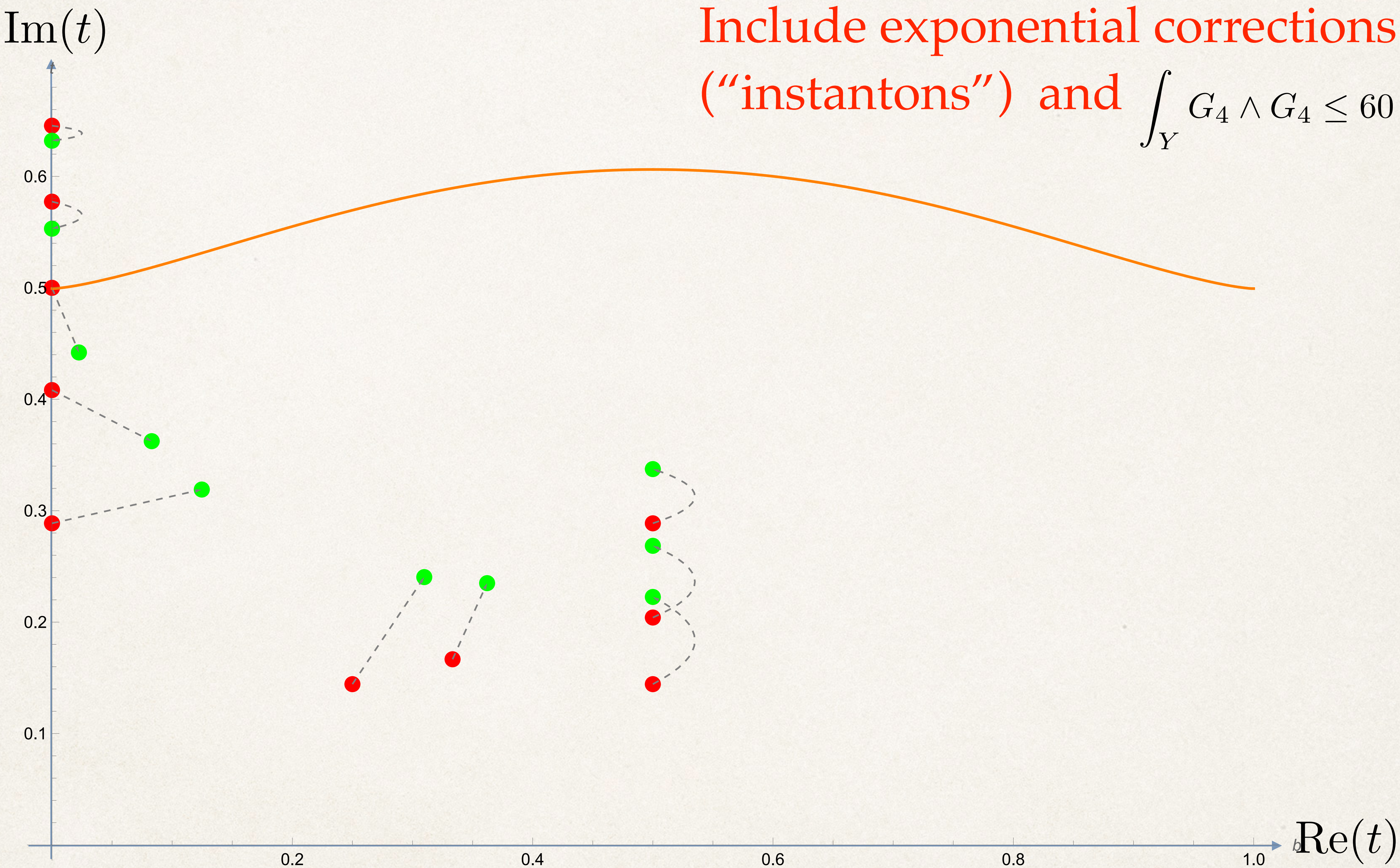
Include exponential corrections ("instantons") and $\int_Y G_4 \wedge G_4 \leq 60$



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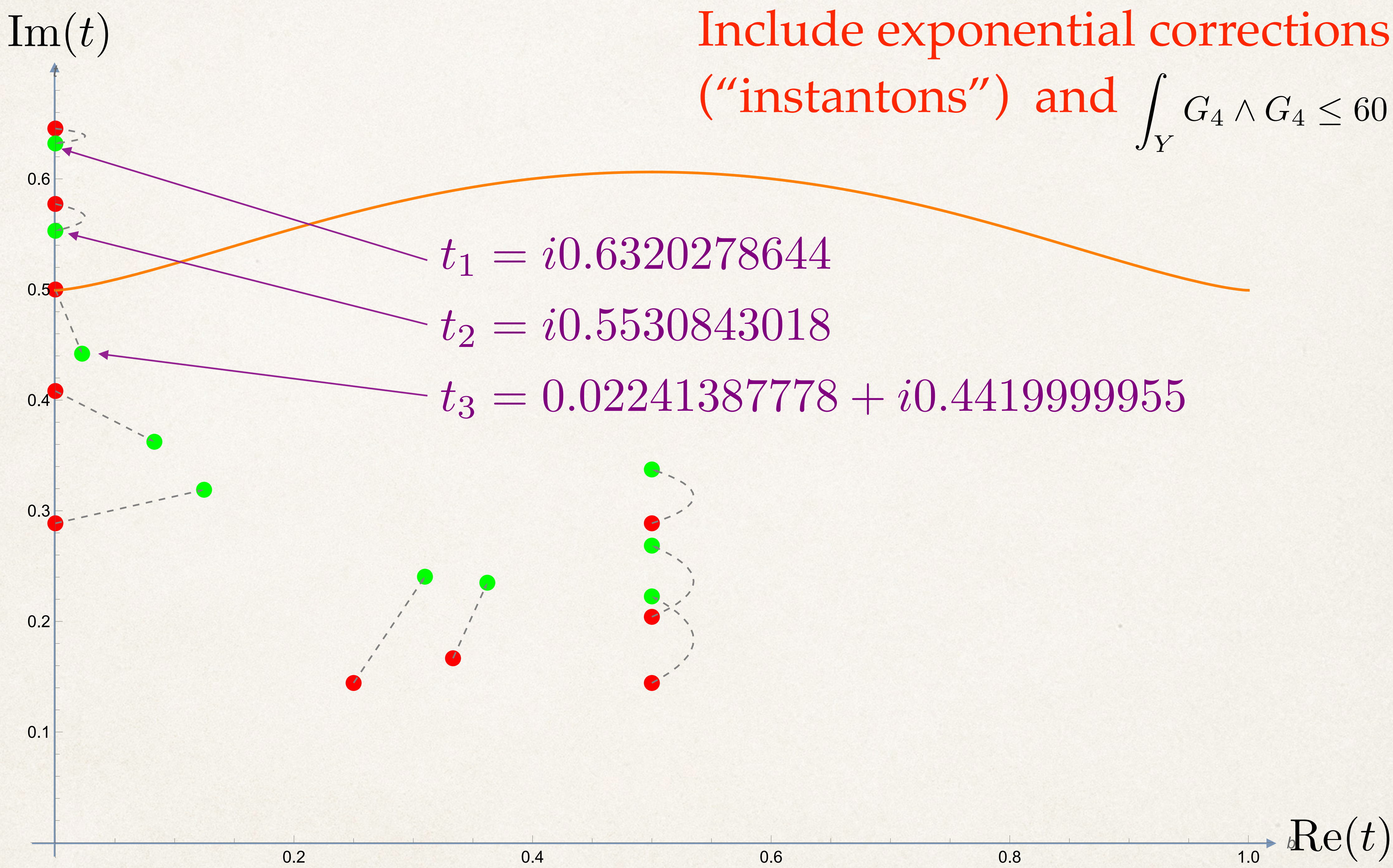

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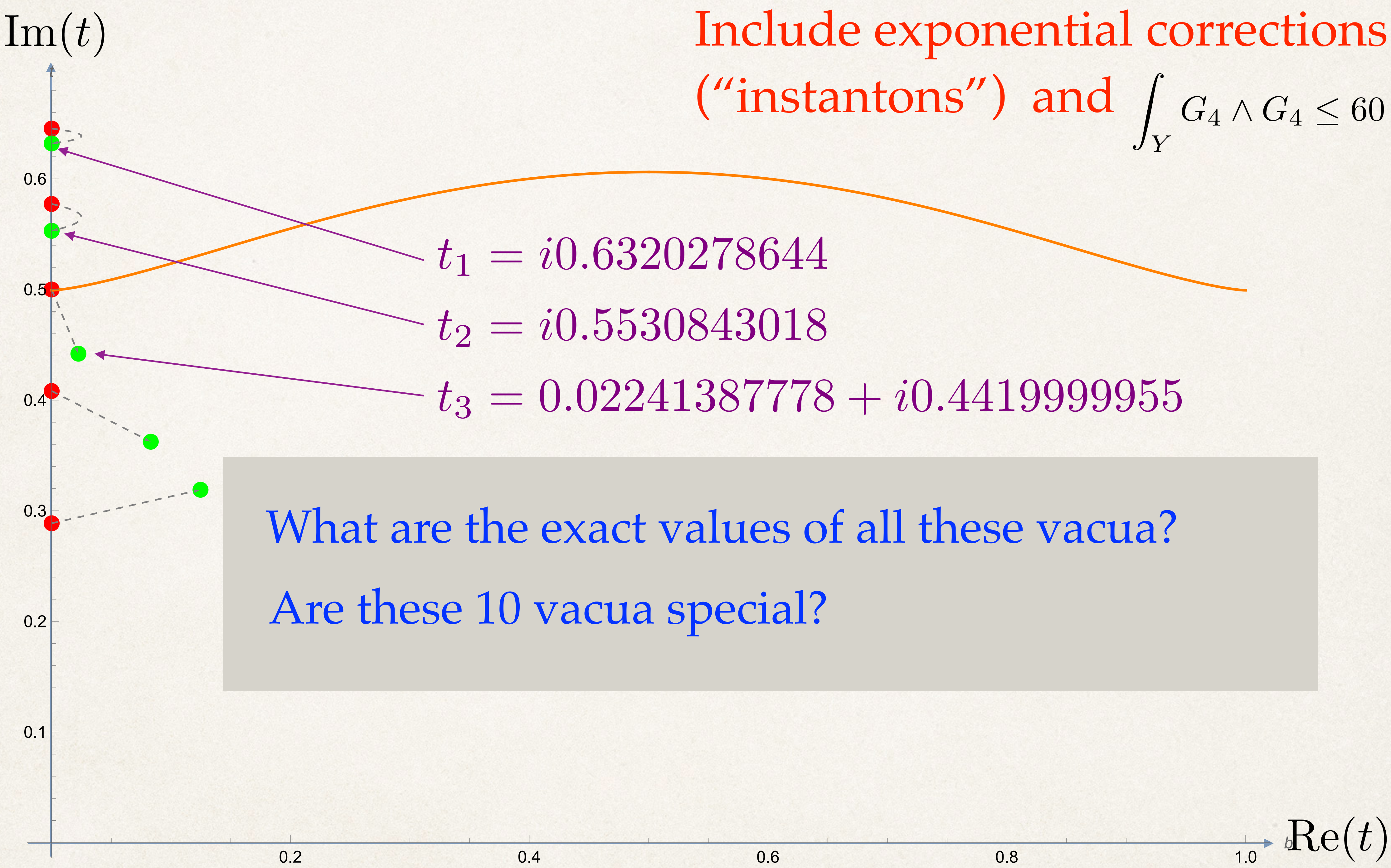

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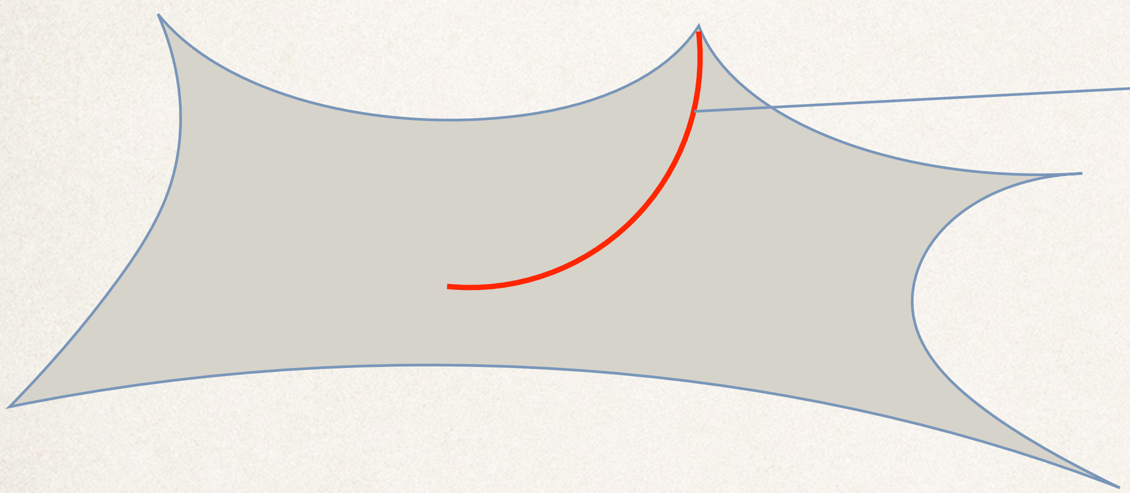

symmetry locus: $\phi^1 = \phi^2 \equiv \phi$

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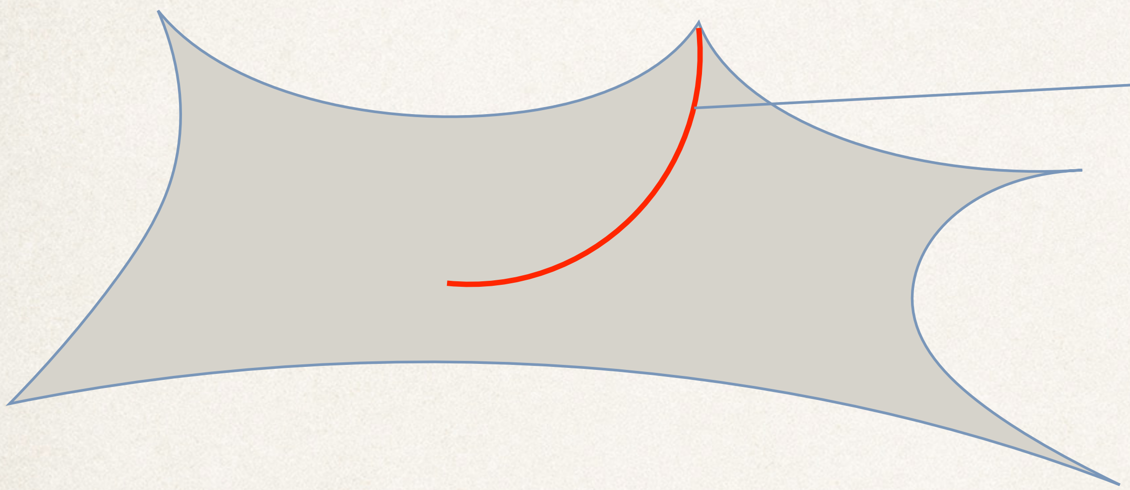

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Observe: Periods split off a piece that are K3 periods

$\rightarrow$ K3 parts of periods are algebraic in coords $\mathfrak{t}^i(\phi)$

(after using transcendental K3 mirror map)

Flux vacua from symmetries

- $M_{\rm s}$- odd flux: $\mathbf{G}_4 = g^0 \mathbf{v}^- + g^i \mathbf{v}_i + g_0 \mathbf{v}_-$ → breaks symmetry
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all exponentials absent → polynomial, not transcendental

→ exact flux vacua in the middle of the moduli space
→ Type IIB: stabilize dilaton-axion, complex structure,
7-brane moduli exactly [TG,van Heisteeg]

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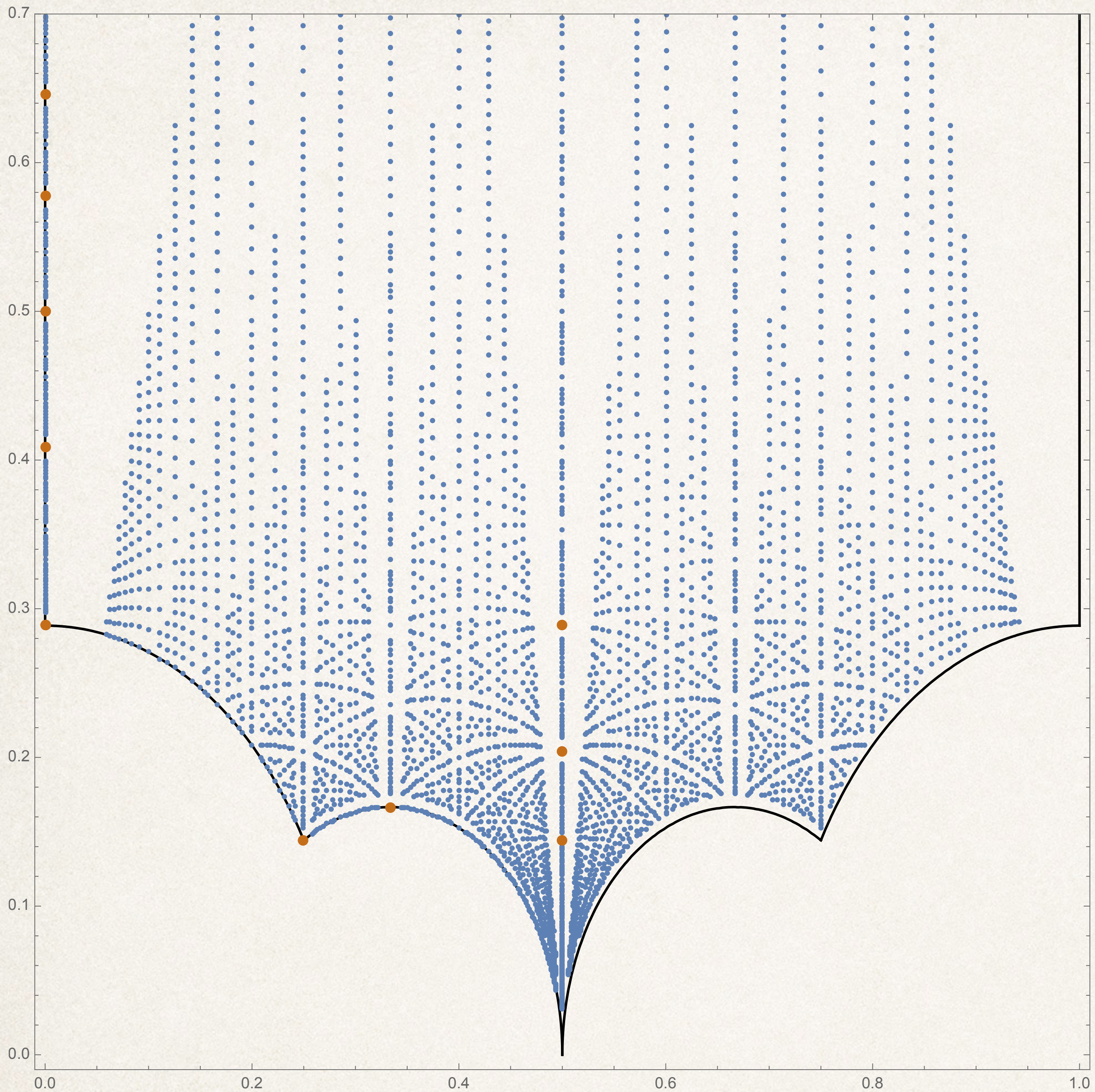

CY fourfolds containing
'attractive K3 surfaces'

[Moore '98]

$\mathfrak{t} \in \mathbb{Q}(i\sqrt{D}) \qquad D > 0$

given by flux

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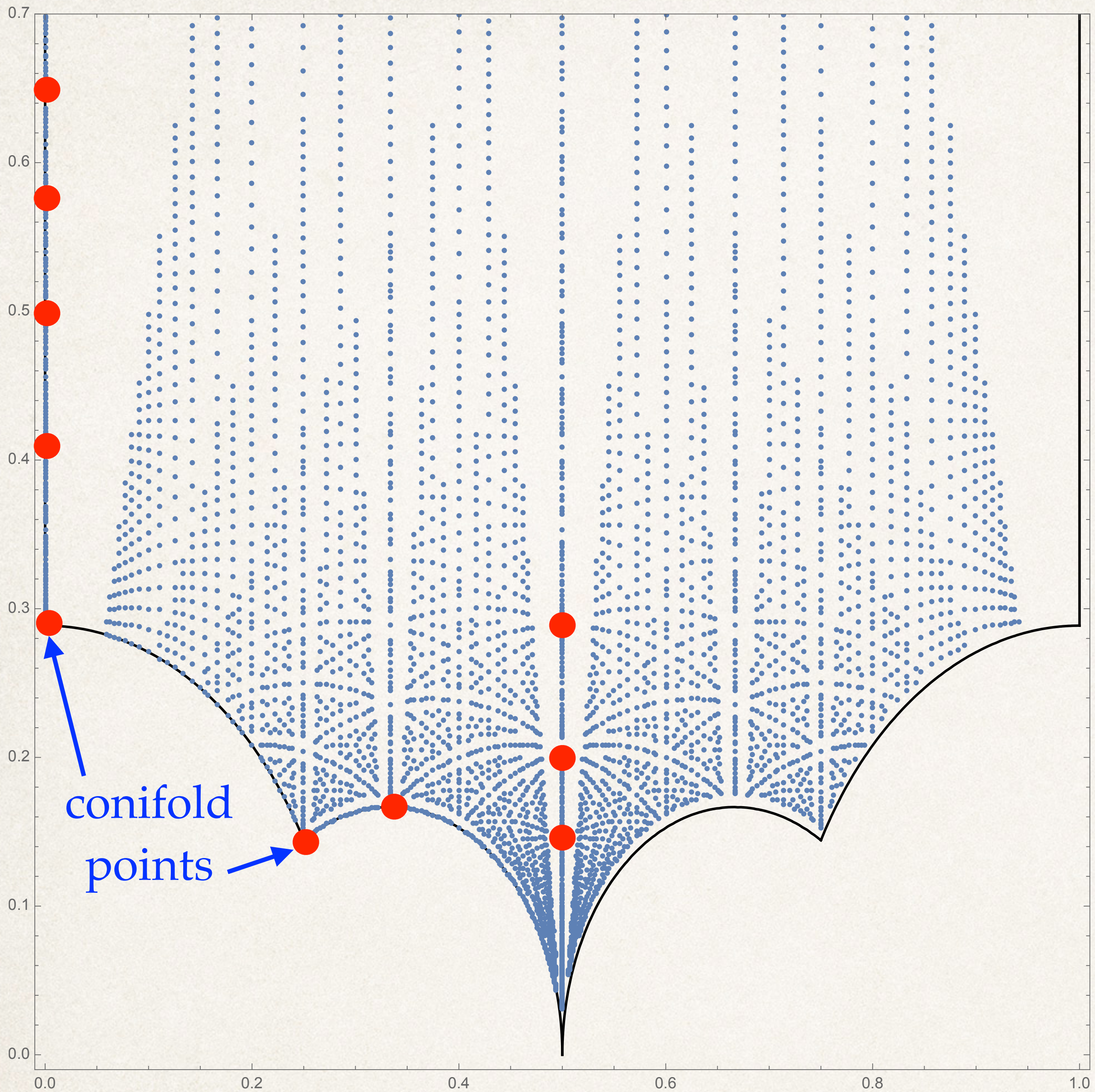

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10 vacua satisfying the tadpole bound $\chi = 720$: red dots

$\mathfrak{t}$	$\frac{i}{2\sqrt{3}}^{*}$	$\frac{i}{\sqrt{6}}$	$\frac{1}{2}+\frac{i}{2\sqrt{6}}$	$\frac{i}{2}$	$\frac{1}{3}+\frac{i}{6}$	$\frac{i}{\sqrt{3}}$	$\frac{1}{4}+\frac{i}{4\sqrt{3}}^{*}$	$\frac{1}{2}+\frac{i}{4\sqrt{3}}$	$\frac{1}{2}+\frac{i}{2\sqrt{3}}$	$\frac{i\sqrt{15}}{6}$
ϕ	$\frac{1}{16}$	$\frac{3\sqrt{3}-5}{4}$	$-\frac{5+3\sqrt{3}}{4}$	$\frac{1}{4}-\frac{\sqrt{3}}{8}$	$\frac{1}{4}+\frac{\sqrt{3}}{8}$	$\frac{1}{22+9\sqrt{6}}$	$\frac{1}{4}$	$-\frac{22+9\sqrt{6}}{2}$	$-\frac{1}{2}$	$\frac{1}{64}$

Exact vacua are very special !

- Gauge coupling function for RR U(1)s in IIB orientifolds and F-theory

  $\rightarrow$ holomorphic function $f(\phi)$ of complex structure of Y (includes τ)

  N=1 mirror symmetry: $f(t) = it^m \mathcal{K}_m + \sum_{\kappa_m} c_{\kappa_m} e^{2\pi i t^m \kappa_m}$ [TG '10]

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$f(t_*)$ special? Potential answer to theta-problem $\theta = \mathrm{Im} f(t_*)$?

[Cecotti,Vafa '18]

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  (2) Tadpole bound is key:

  e.g. in example $\mathrm{Re}\, \mathfrak{t} = 0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2} \quad \rightarrow \quad \phi = \mathrm{Re}\phi$

  generalizing [Cecotti,Vafa '18] [Bönisch,Elmi,Kashani-Poor,Klemm '22]

Structure of
$W=0$ Flux Landscape

Intuition behind general structure

[TG,Monnee]

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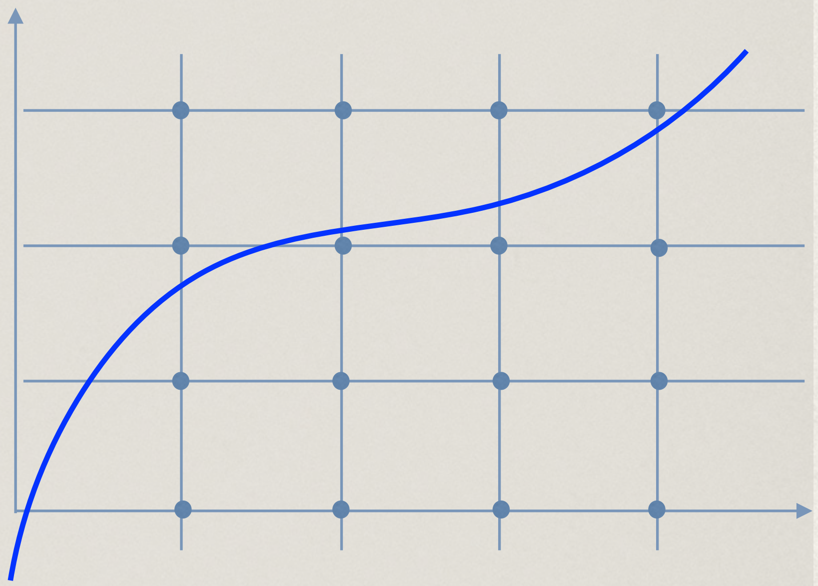

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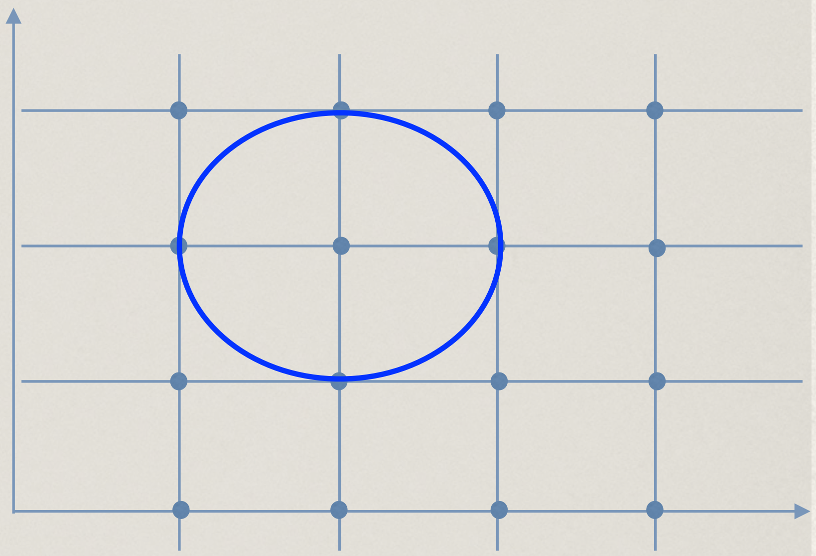

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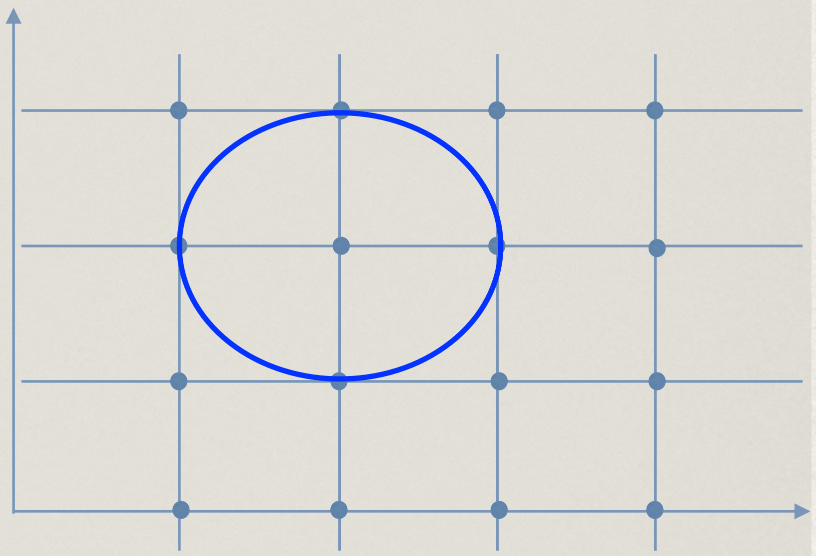

- made precise in theory of unlikely intersections (part of 'tameness revolution')

  $\rightarrow$ Pila-Wilkie and Ax-Schanuel theorems [Bakker,Tsimerman '17],[Gao,Klingler '21],[Chiu '21]…

  - Zilber–Pink conjecture for the Hodge locus (partial proof)

    [Baldi,Klingler,Ullmo '21]

A picture of the landscape

- Key lies in understanding the transcendentality of periods (without computing them)
  - $\rightarrow$ measure of transcendentality [Baldi,Klingler,Ullmo '21]

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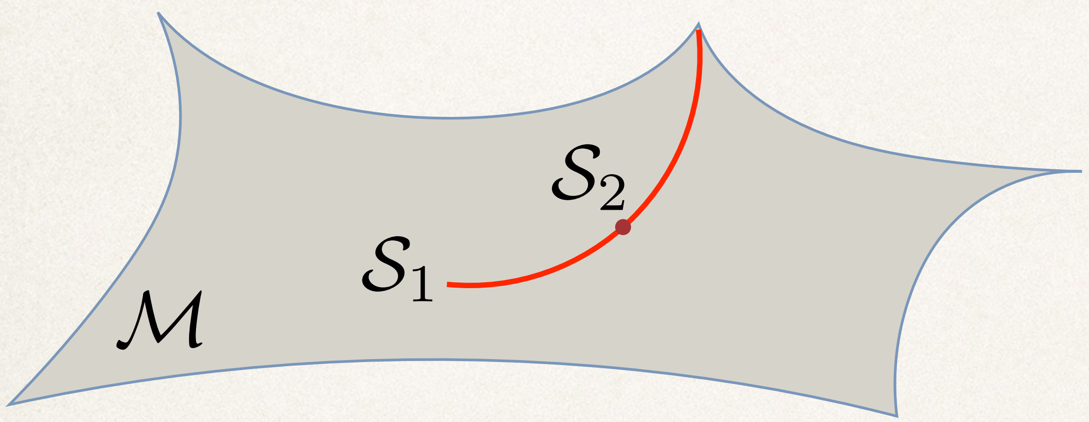

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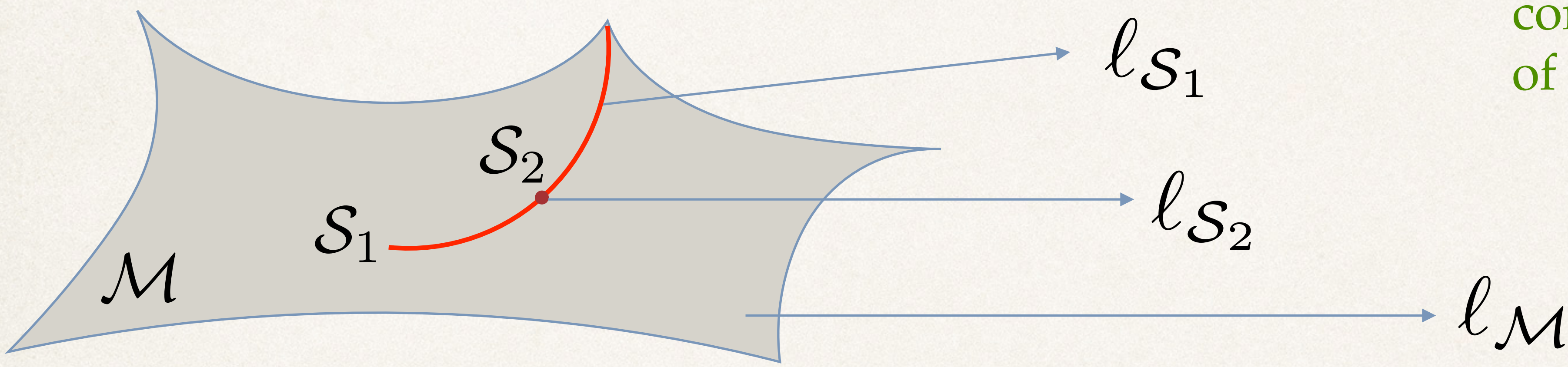

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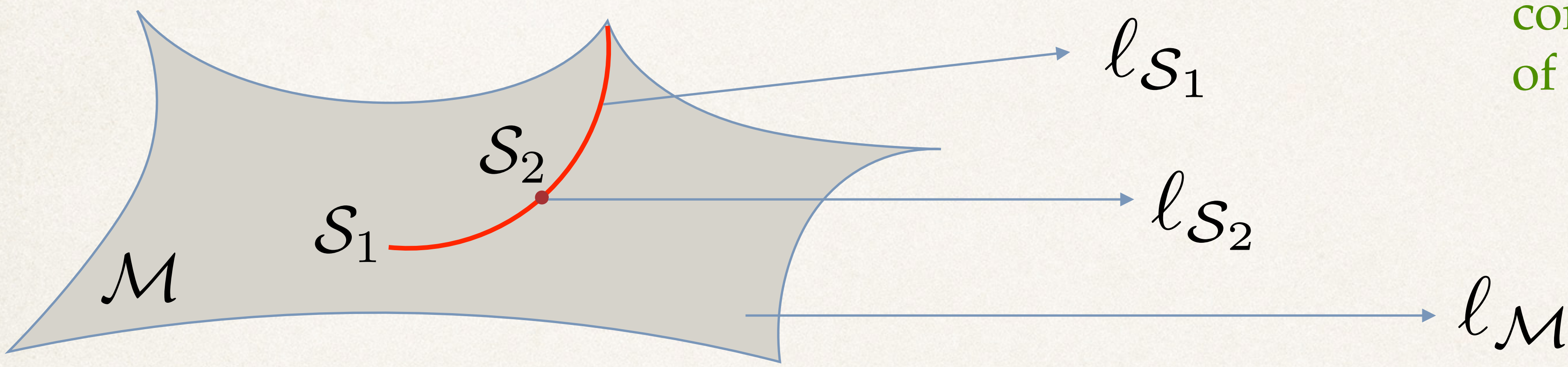

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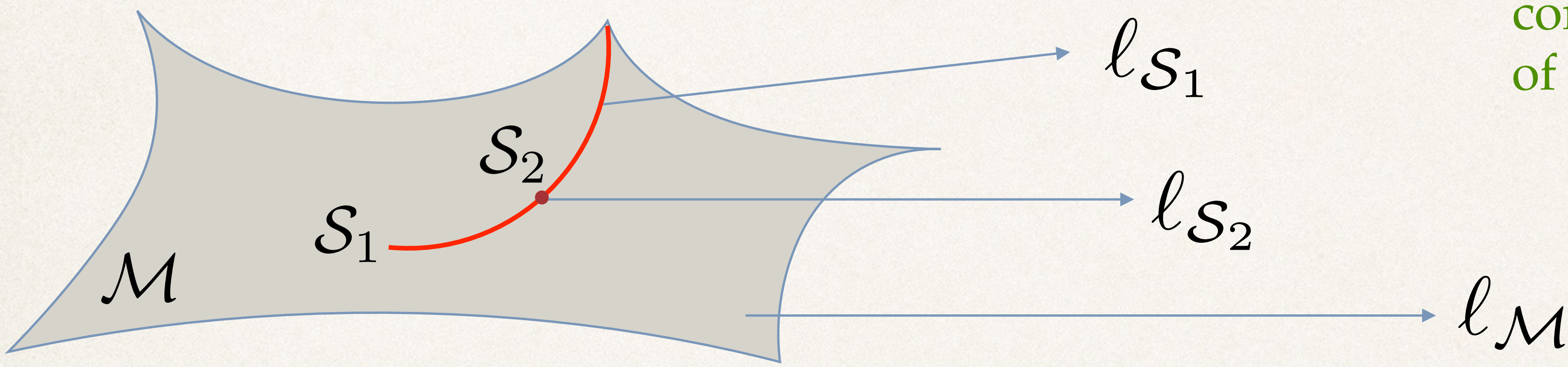

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CY_3, CY_4

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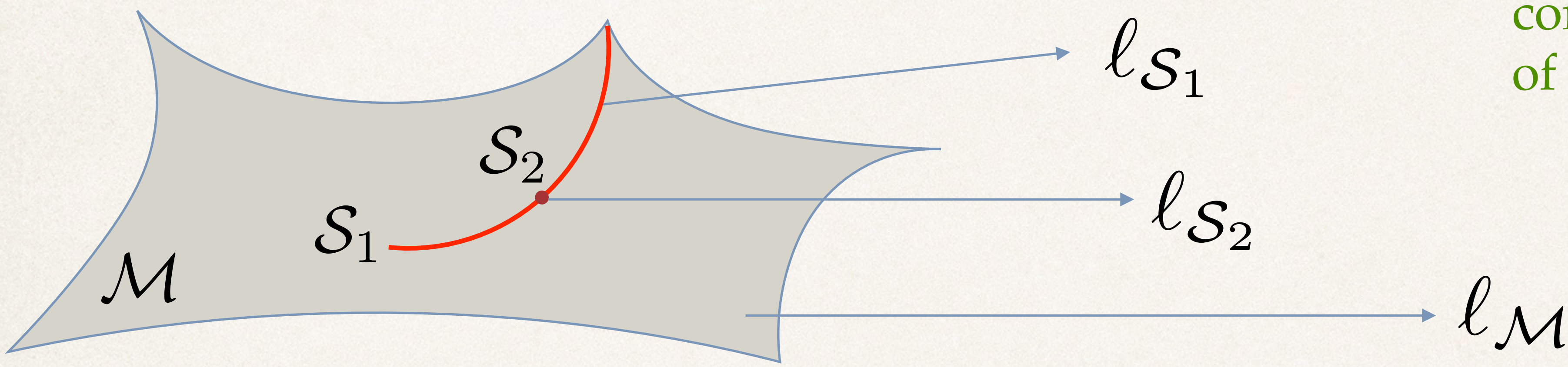

compare with classification of boundaries using HT

$\ell_{\mathcal{M}} = 1$ elliptic curve, K3 $\mathrm{K3} \times \mathrm{K3}$, $\mathrm{CY}_3 \times T^2$	$\ell_{\mathcal{M}} = 3$ CY_3, CY_4	$\ell_{\mathcal{M}} > 3$ CY_n, $n > 4$

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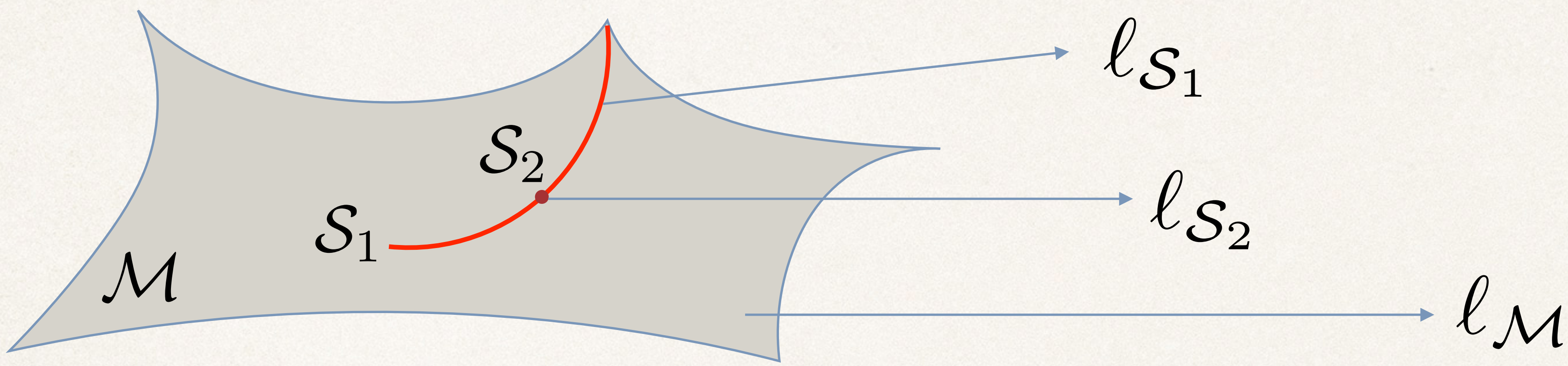
[TG,van de Heisteeg '24]



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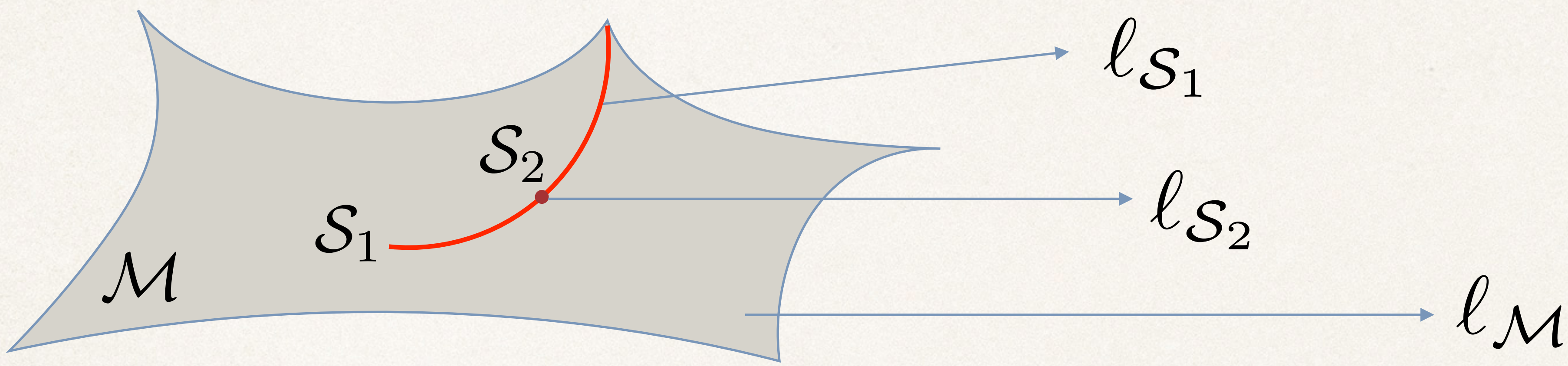

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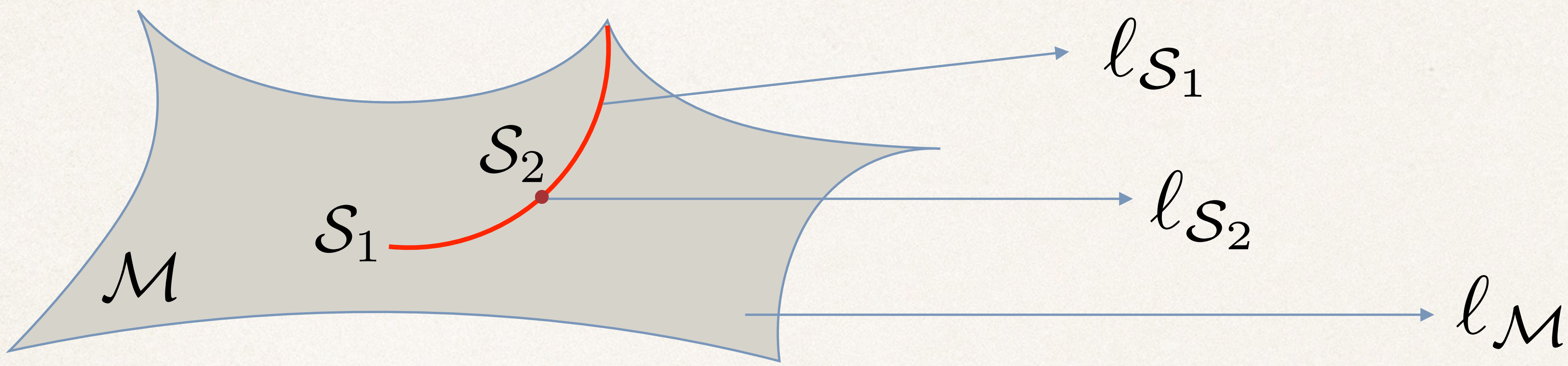

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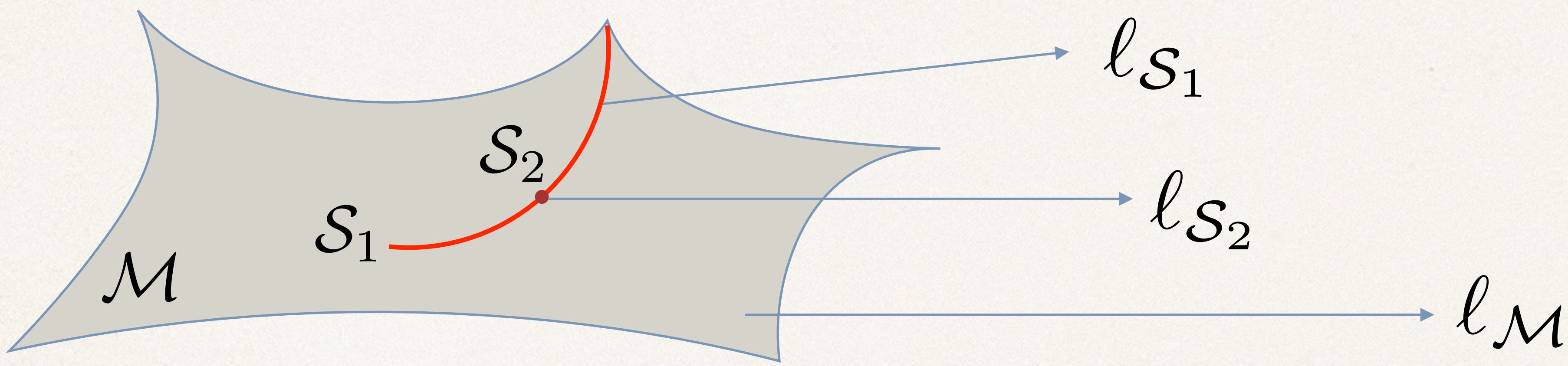

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  - G_4 with $W = 0$ on vacuum locus $\qquad G_4 \in H^{2,2}$

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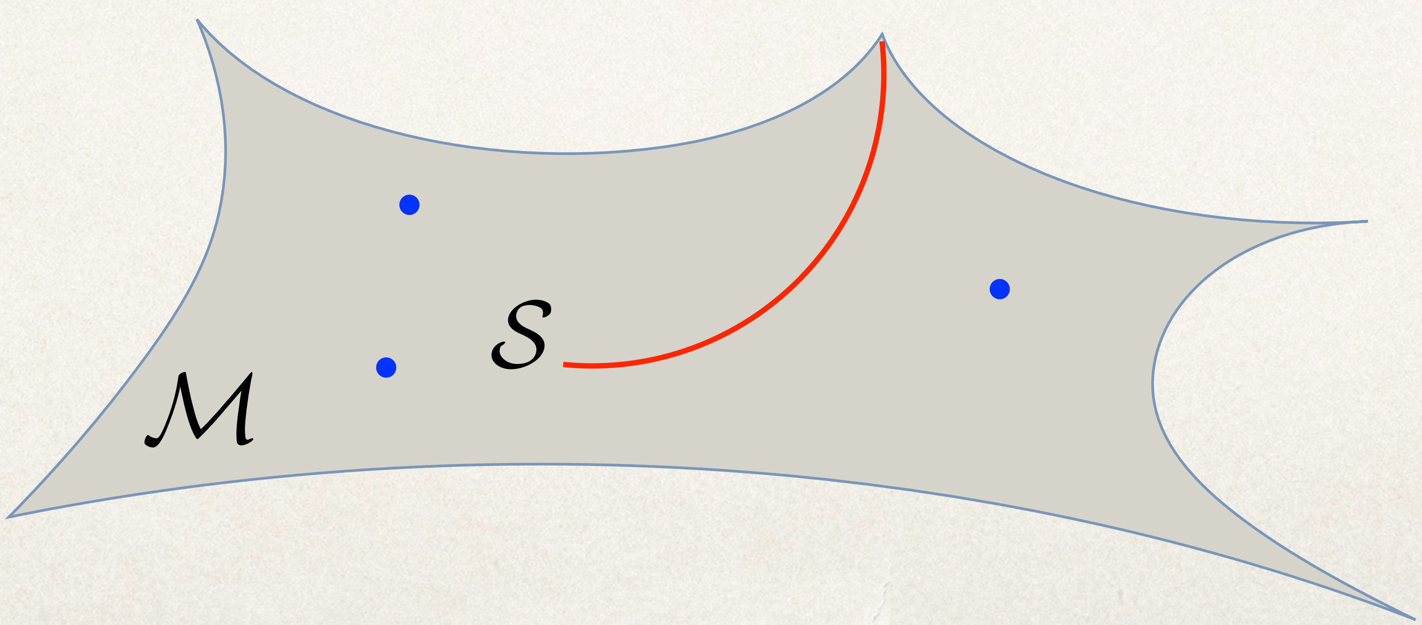

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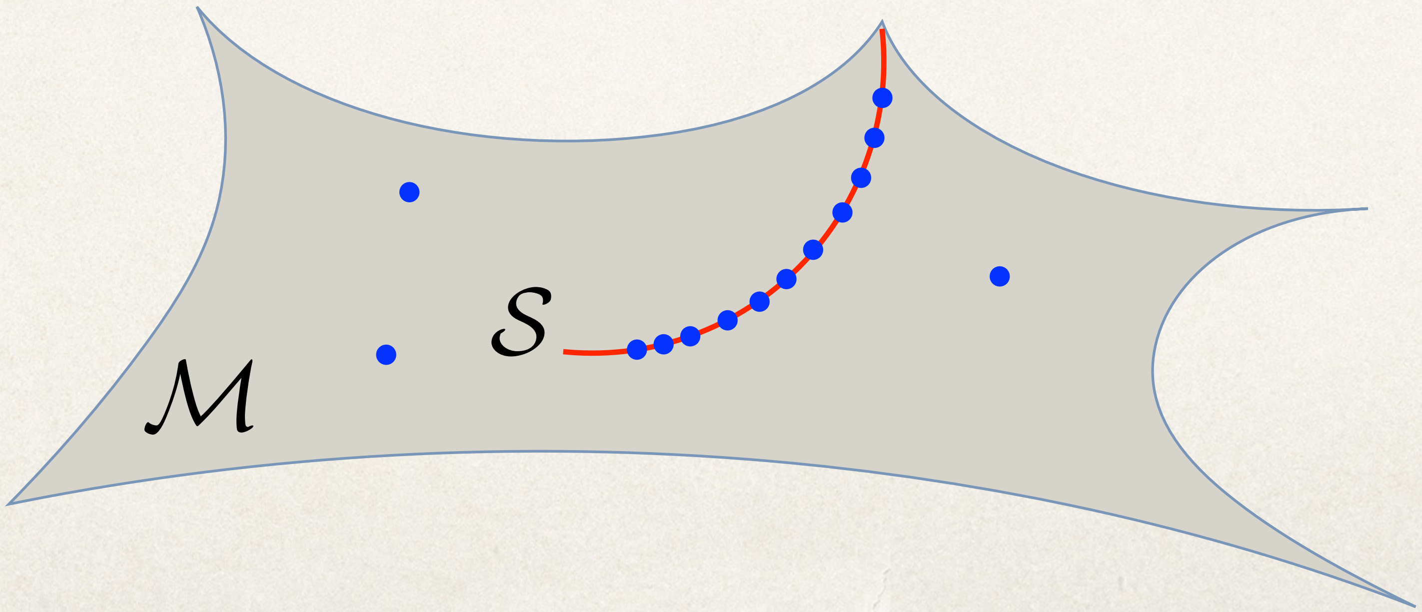

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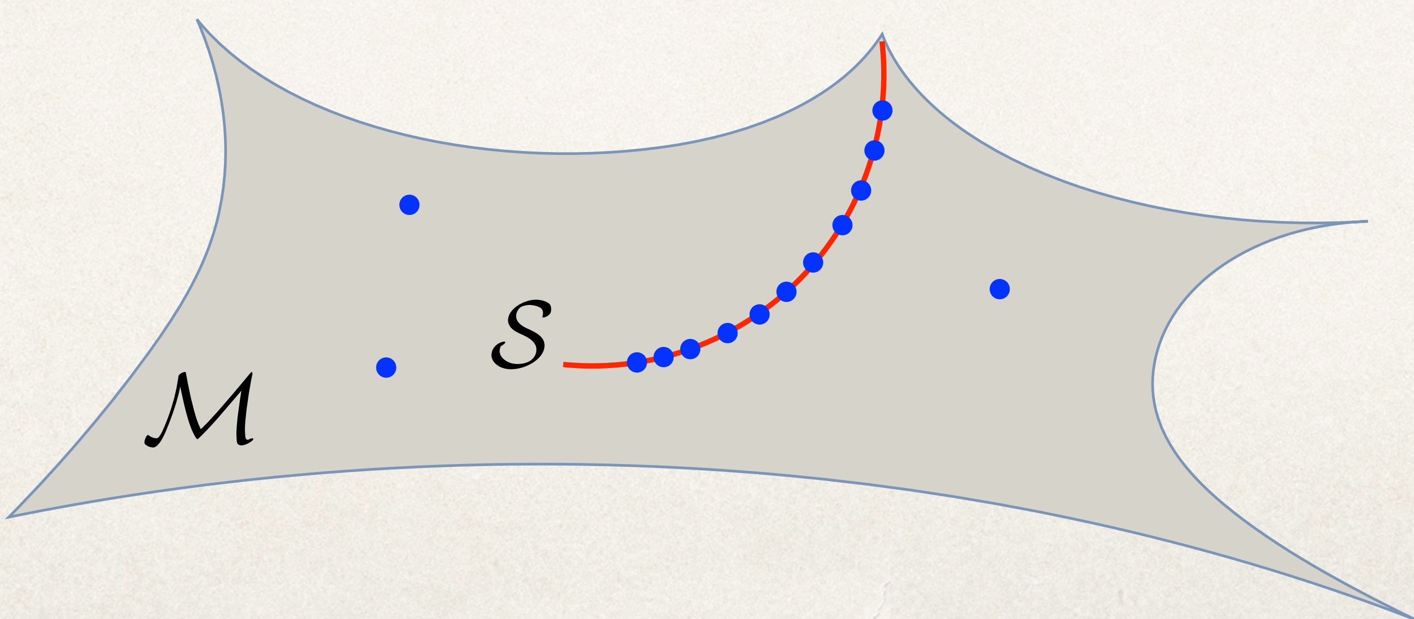

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generalizing [Gukov,Vafa '02]

Summary and comments

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- observed in examples that algebraicity of periods is related to symmetry of compactification space

$\rightarrow$ Generalized symmetry for higher Hodge tensors? 2D CFT interpretation?

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Future:

- extend beyond W=0 vacua
- sharp o-minimal structures (have notion of complexity) $\rightarrow$ precise statements about the number of vacua

  initiated in [TG, Schlechter, van Vliet '23][TG,Monnee '23]

Thanks!