Open Strings and Heterotic Instantons

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With Rafael Álvarez-García, Christian Kneißl, Nicole Righi

Why are heterotic string theories so strange?

Why are Heterotic Theories so Strange?

- Type I & Type II superstrings feature a rich non-perturbative structure of D-branes
- These seem largely absent in the heterotic theories
- But *something* should be there

$\text{HE} := (\mathrm{E}_8 \times \mathrm{E}_8) \rtimes \mathbb{Z}_2$

$\text{HO} := \mathrm{Spin}(32)/\mathbb{Z}_2$

[Polchinski,'05]

[Kaidi, Ohmori, Tachikawa, Yonekura ,'23]

Why are Heterotic Theories so Strange?

- Argument from S.Shenker – consider a ***closed*** string amplitude

[Gross & Periwal,'88]
[Shenker,'90]

$$\mathcal{A}_0 = \sum_{n=0}^{\infty} g_s^{2n-2} a_{2n}$$

$$a_{2n} \simeq C^{-2n}(2n!)$$

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- Amplitude has radius of convergence r=0
- True non-perturbative amplitude is a *trans-series*

$$\mathcal{A} = \mathcal{A}_0 + \mathcal{A}_1 + \cdots$$
$$\mathcal{A}_1 = \mathcal{O}(e^{-C/g_s})$$

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  - Explicit results from 11d SUGRA **[Green & Rudra,'16]**

$$\mathcal{S}_{10}^{(\mathrm{TI/HO})} \supset \int d^{10}x\sqrt{-G}\, t_8 t_8 R^4 E_{\frac{3}{2}}(ig_s^{-1})$$

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  - Also tied to swampland principles: species scale

[van de Heisteeg, Vafa, Wiesner, Wu,'23]

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  - Naively, this should arise from an HO instanton-like objects

$$\pi_9(\mathrm{Spin}(32)/\mathbb{Z}_2) \simeq \mathbb{Z}_2$$

    What are these, and why do they give rise to Shenker effects?

Plan for Today

- Explain the objects that give rise to $\mathcal{O}(e^{-1/g_s})$ effects in heterotic theories

- Motivate why these objects produce such effects

Quotients of Type IIB

Type IIB

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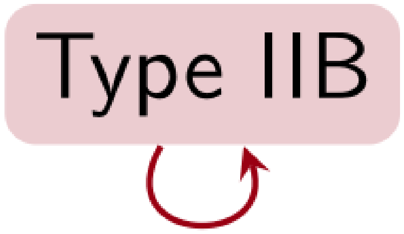

Non-perturbative: $\mathrm{SL}(2,\mathbb{Z})$

Perturbative: $\{\Omega, (-1)^{\mathrm{F}_L}\}$

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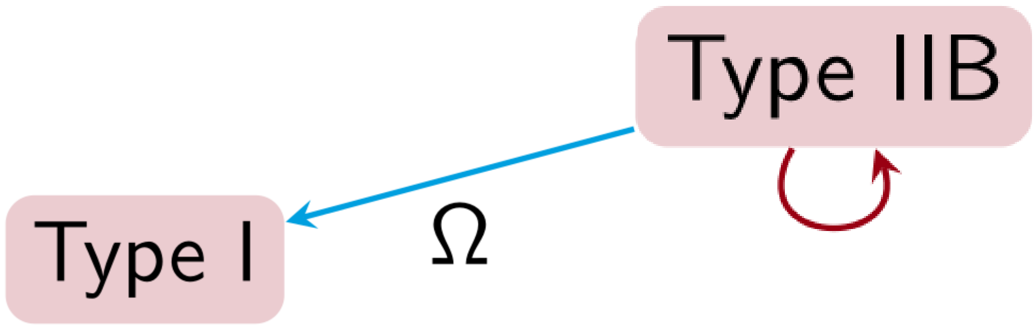

Orientifolds

$\mathrm{SL}(2,\mathbb{Z})$

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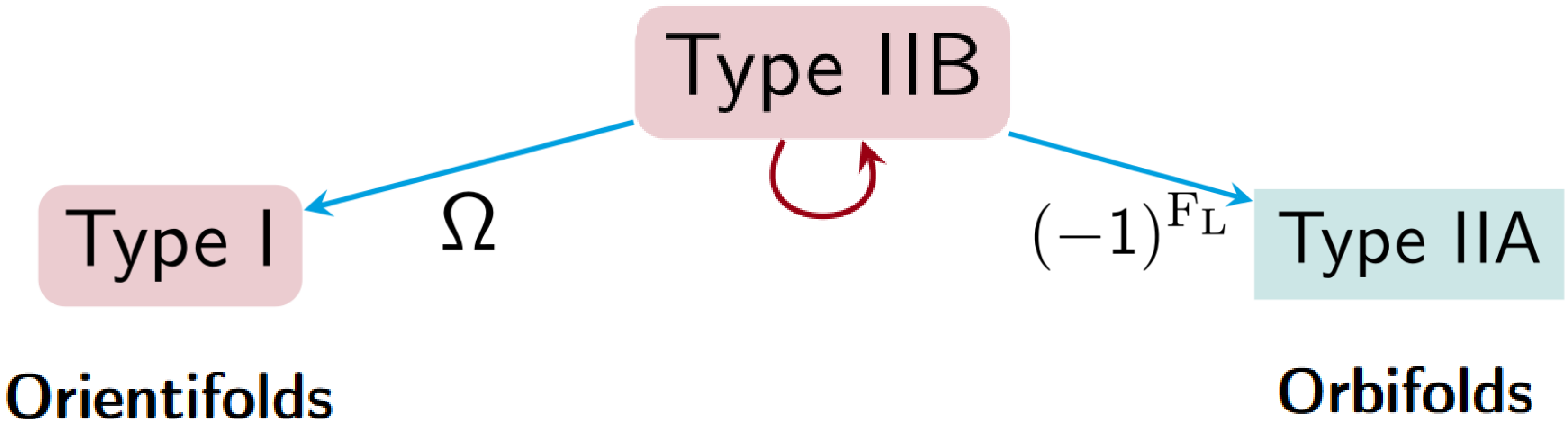

[Dabholkar,'98]

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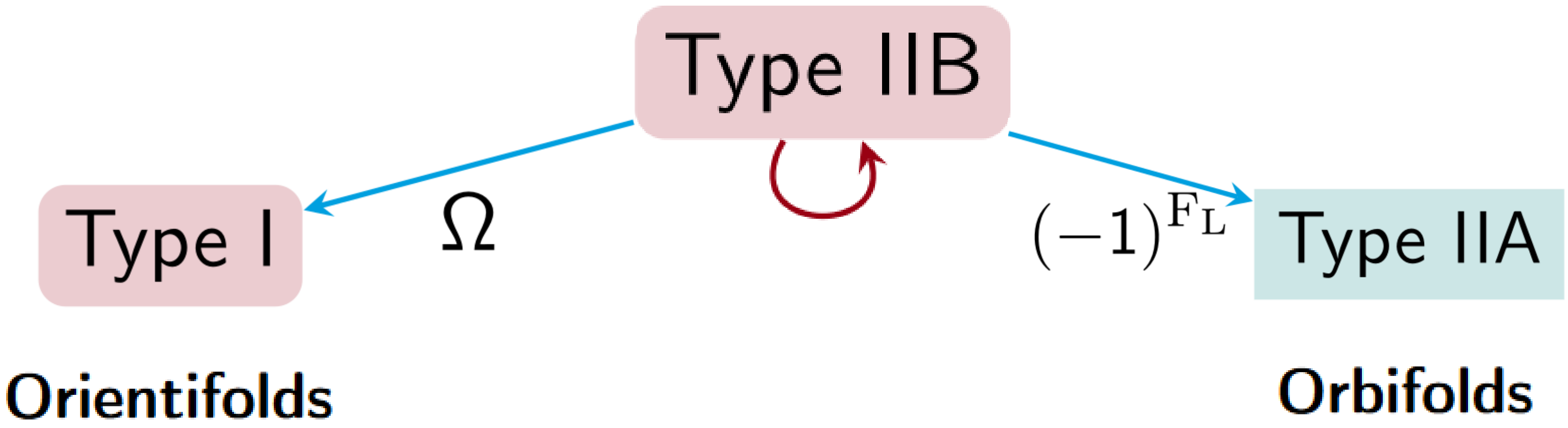

$$\tilde{\Omega} := S^{-1}\Omega S$$

[Hull,'97 & '98]

[Bergshoeff, Eyras, Halbersma, van der Schaar, Hull, '99]

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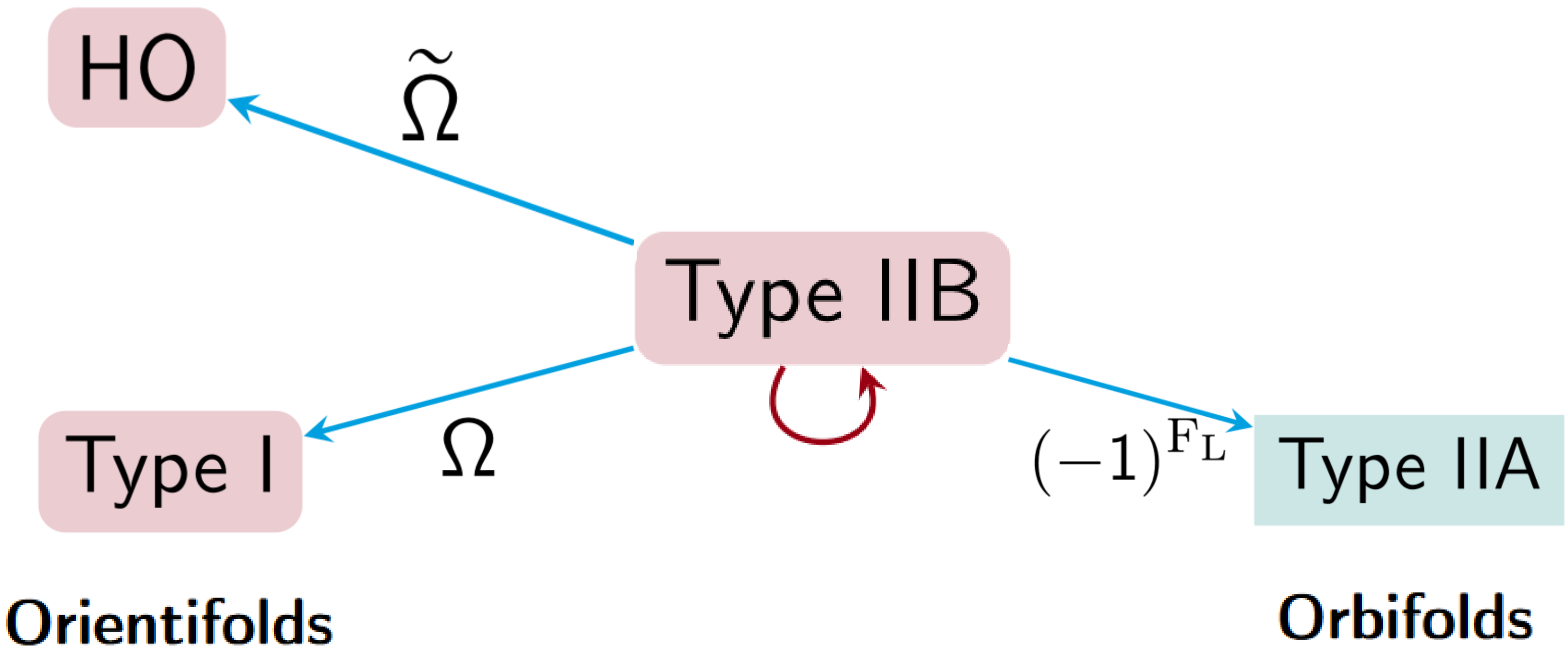

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HO as an Orientifold

- The Hull orientifold of Type IIB has many interesting properties
  - There is a non-orientable D-string
  - Signals the presence of S-dual of O9-plane, charged under B_{10}
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Heterotic gauge kinetic terms!

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Green-Schwarz anomaly cancellation terms from NS9 WZ terms

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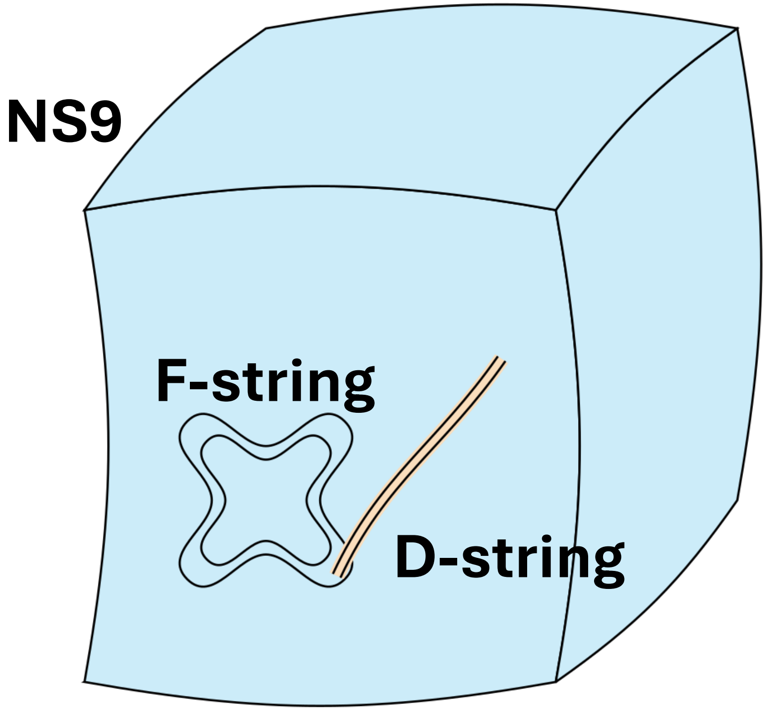

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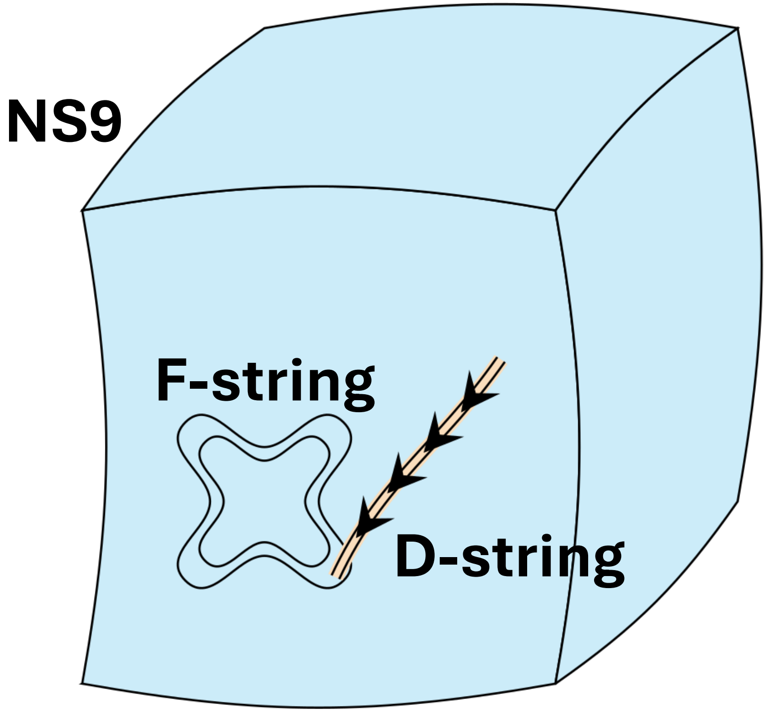

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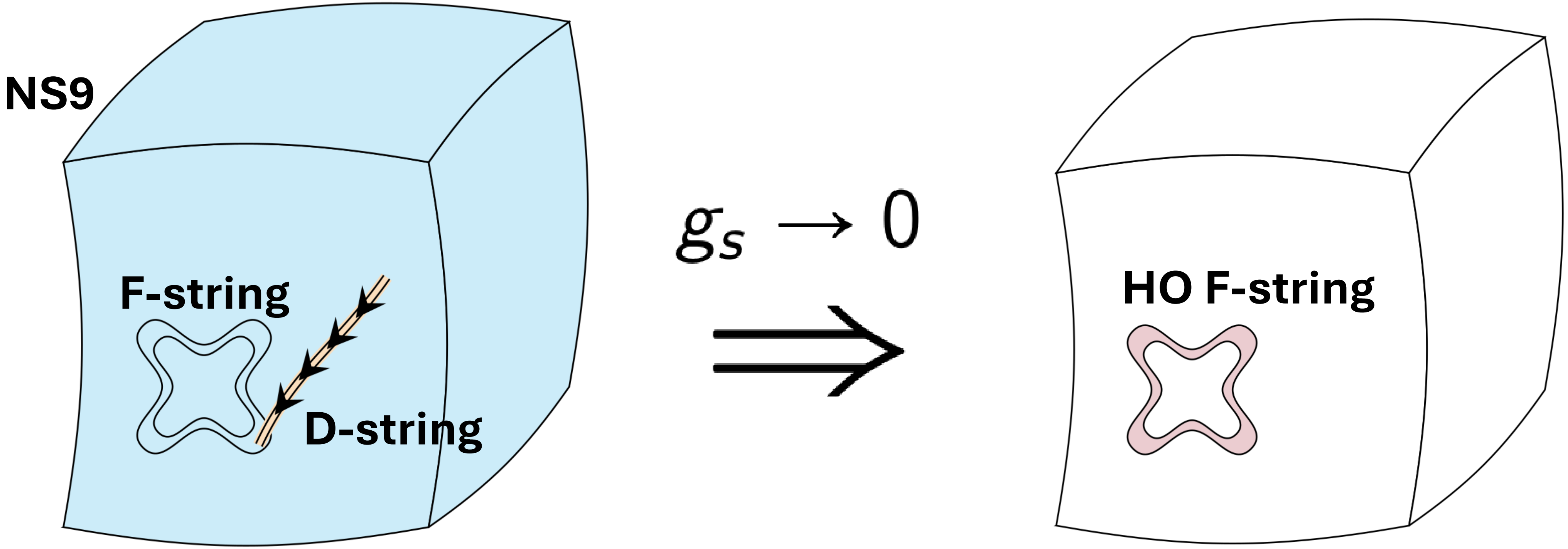

Inheriting Branes from Type IIB

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- Does HO also have this tower? Yes, but it is complicated

Inheriting Branes from Type IIB

- What have we gained from the picture?

  - HO has D-strings $\Longrightarrow$ Explains Silverstein's proposal
  - HO has inflow from the NS9-brane + D-string sector
  - There is *some set* of heterotic instantons that HO inherits from IIB via the Hull orientifold

How can we understand the instanton & its effect on the theory?

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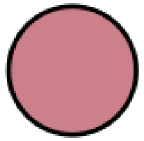
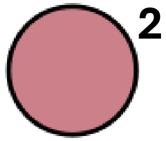
- Consider 1-instanton contribution & disconnected disks

$\mathcal{A}_1$ = (1 +  + ½ ^{2} + • • •) $\mathcal{A}_1^{\text{connected}}$

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$$\mathcal{A}_1 = \exp\Big(\bigcirc + \cdots \Big) \mathcal{A}_1^{\text{connected}}$$

- From Euler number of disk,

$$\bigcirc \simeq -\frac{C}{g_s}$$

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Nonetheless, we propose they do

Why can't there be Open Heterotic Strings?

- From the worldsheet perspective, heterotic theories are *defined* as theories of only closed strings

$$\delta S_{WS} = \frac{1}{2\pi}\int d\tau \left\{ \lambda^a \delta\lambda^a - \psi^\mu \delta\psi_\mu \right\}\Big|_{\sigma=0,\pi}$$

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Cannot be satisfied by heterotic CFTs

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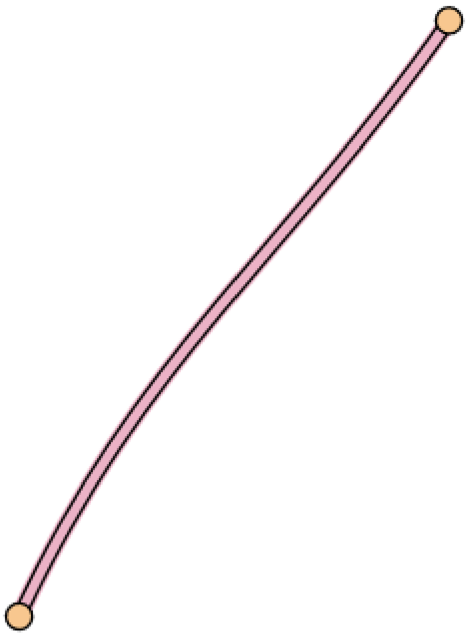
- In the context of cosmic strings: **[Polchinski,'05]**
  - Lorentzian spacetime
  - HO F-string ends on 0-branes



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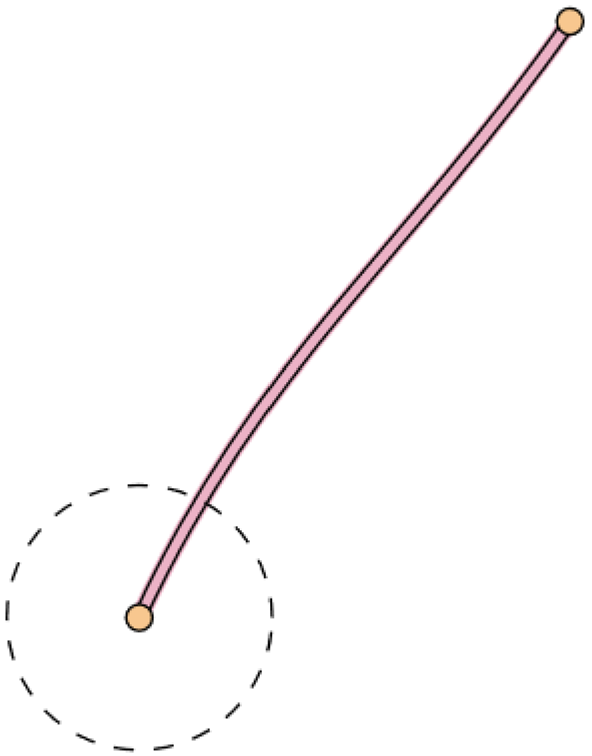

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- 0-branes have gauge profile:

$$\pm 24 = \frac{1}{4!(2\pi)^4} \int_{S^8} \mathrm{Tr}(F_2^4)$$

- Related an index theorem & zero modes
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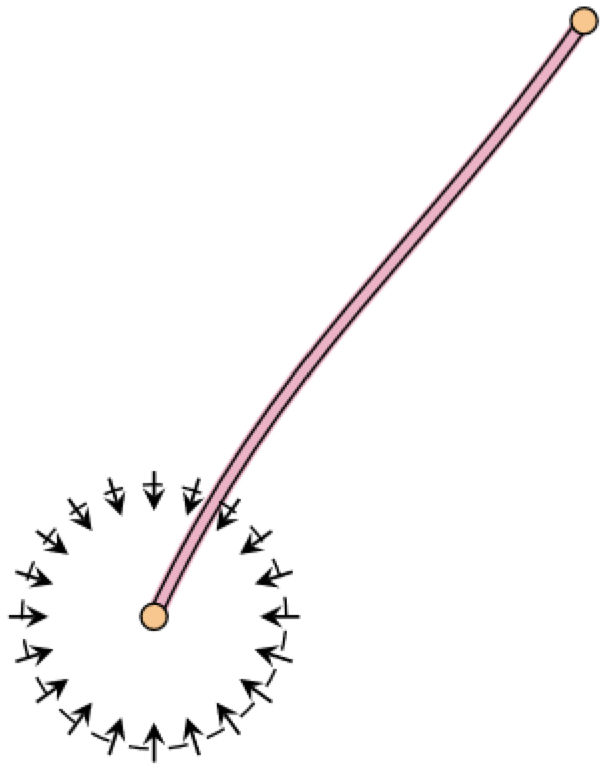

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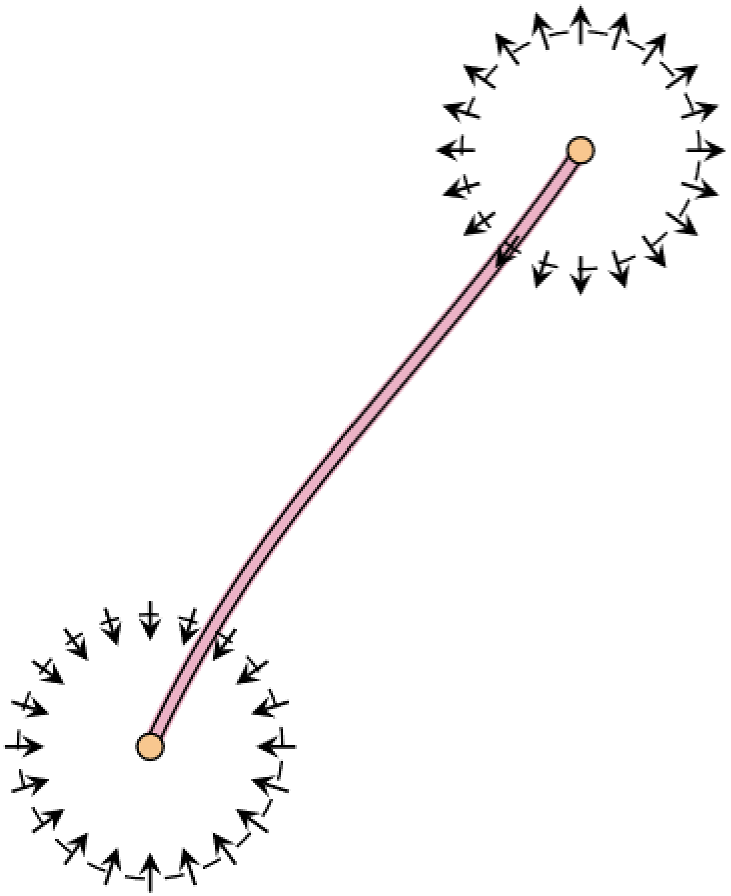

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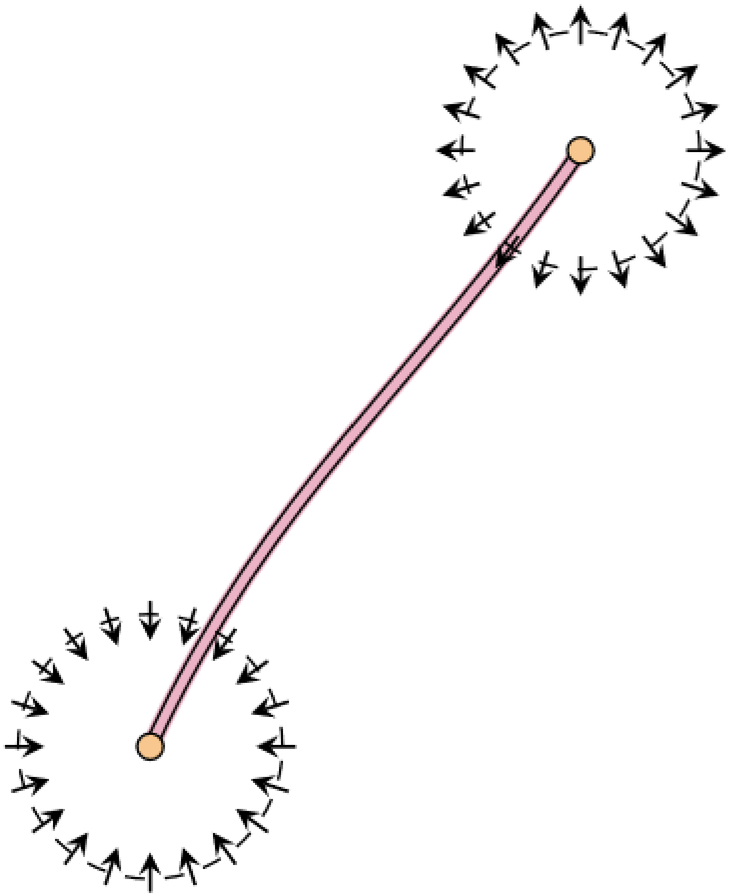

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Similar ideas applied to 2d heterotic string theories **[Seiberg,'05]**

The HO Instanton

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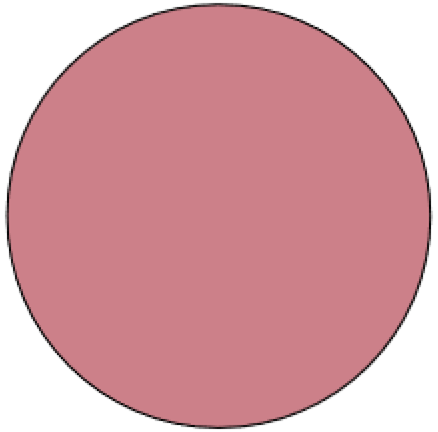

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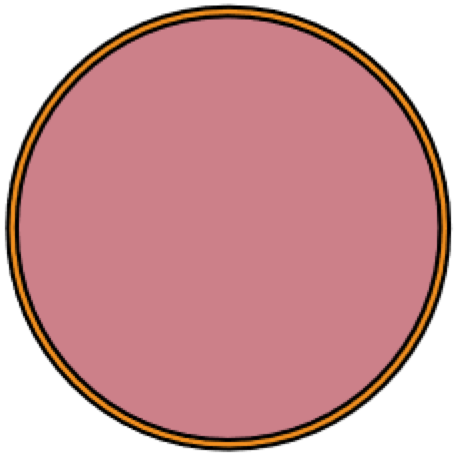

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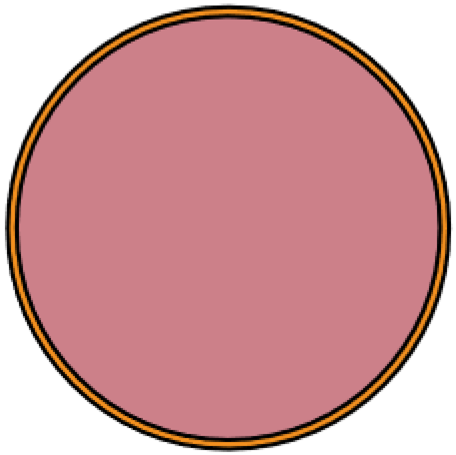

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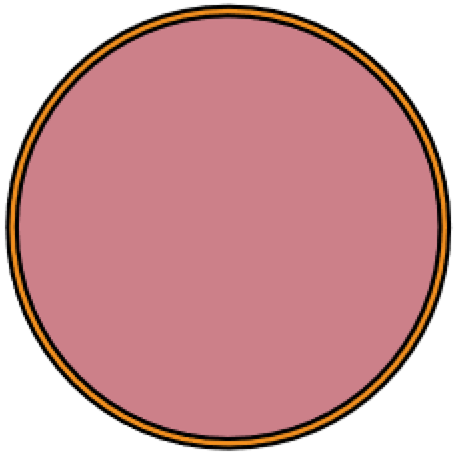

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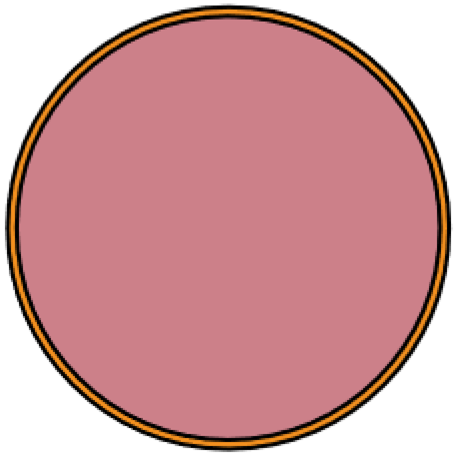

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Heterotic disks become consistent by matching spacetime and worldsheet modes

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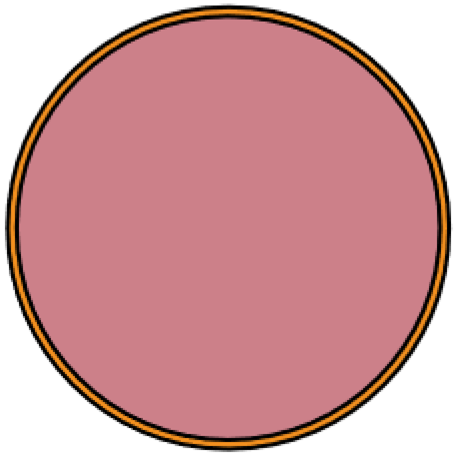

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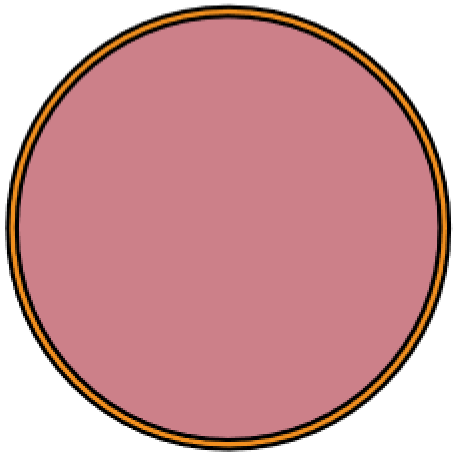

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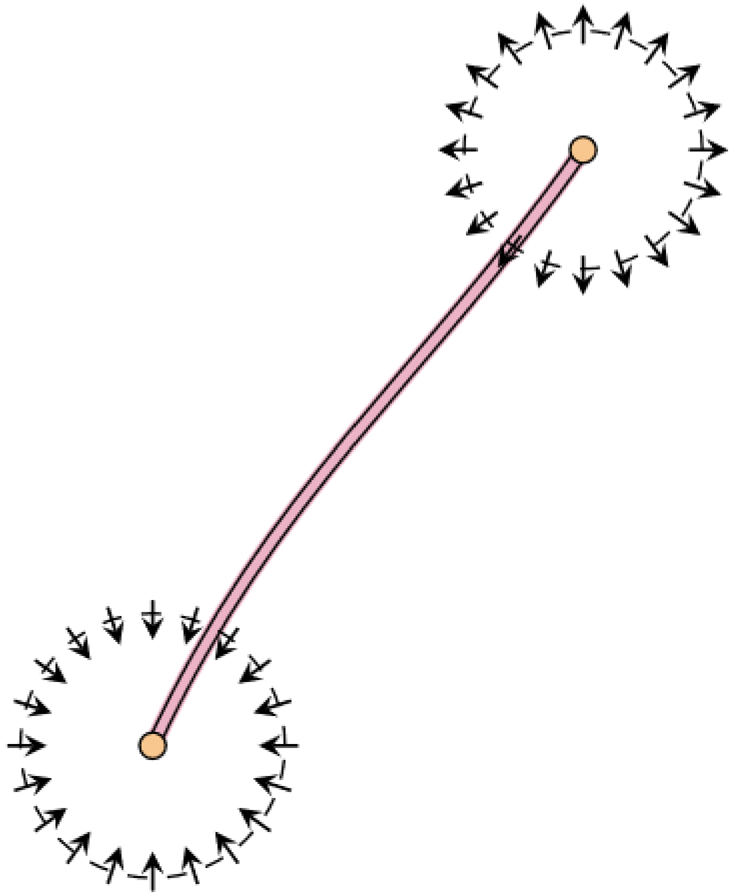
Intuitively, the D-string from the Hull orientifold is acting as "glue"

What about other heterotic theories?

Why can't there be Open HE Strings?

- HE F-string **can not** have endpoints in 10d Minkowski background:

$$\pi_7(\mathrm{E}_8) = 0$$



Why can't there be Open HE Strings?

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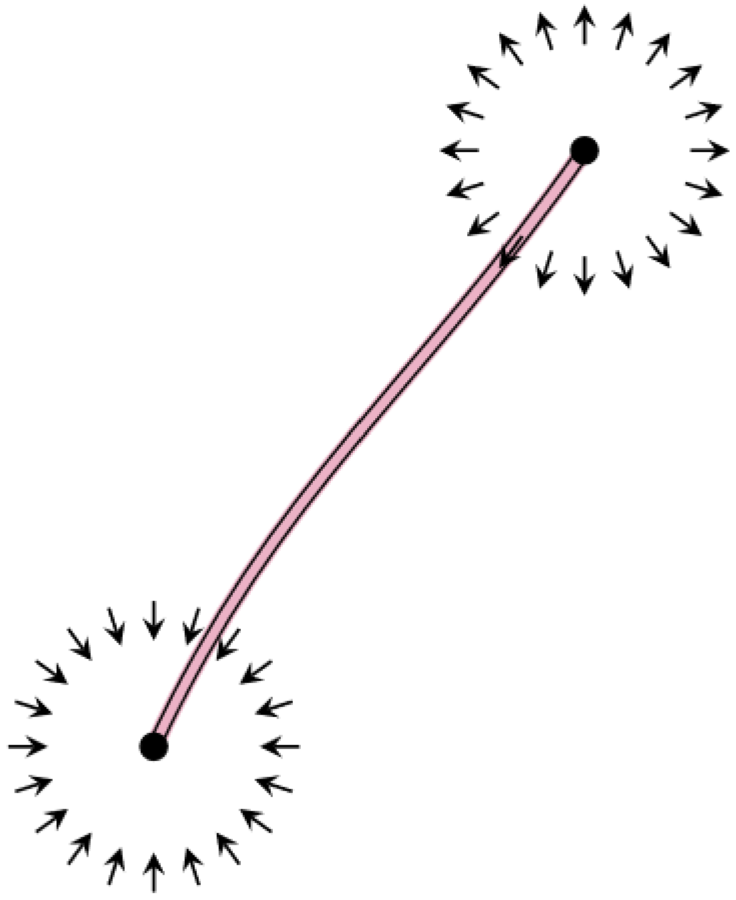

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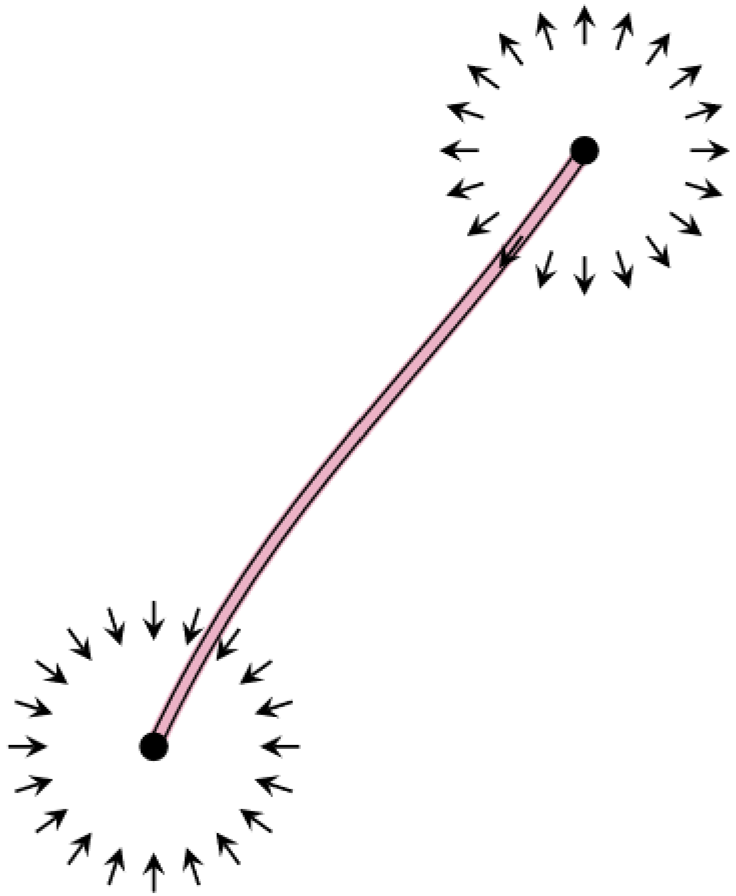

$$\pm 24 = \frac{1}{4!(2\pi)^4} \int_{S^8} \mathrm{Tr}(F_2^4)$$

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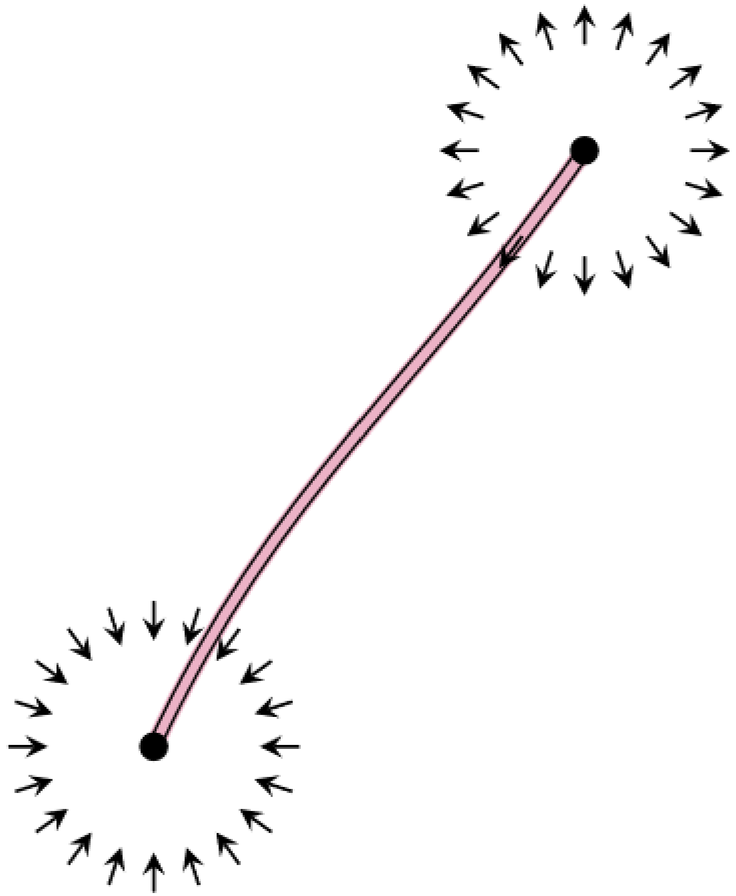

$$\pm 24 = \int_{M^8} \hat{A}(TM^8)|_{8\text{-form}}$$

Shenker effects in 10d HE are disks from open Strings on a gravitational instanton

Similar to unstable, non-BPS D(-1)-brane in IIA

Conclusion

- From Shenker, closed string theories need $\mathcal{O}(e^{-1/g_s})$ effects
- These seem mysterious from heterotic perspective, but we propose they arise from "D-branes"
- These D-branes have inflow mechanisms
    - D-string: Hull
    - 0-brane: Polchinski
    - Instanton: Today
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$$\Omega_{10}^{\mathrm{Spin}}(B\mathrm{Spin}(32)/\mathbb{Z}_2) \cong 10\mathbb{Z}_2$$

See Christian's talk tomorrow afternoon!

- All 10d supersymmetric theories have some open string sector

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$$\text{open strings} \Rightarrow \text{closed strings}$$
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