

DIETER LÜST (LMU, MPP)



Swampland and Dark Relations

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See review article [arXiv:2405.04427](https://arxiv.org/abs/2405.04427)

Introduction:

Puzzle of small (big) numbers in particle physics and cosmology:

Dark energy: $\Lambda \sim 10^{-120} M_p^4$

Dark matter: $\rho_{DM} \sim 10^{-19} M_p/m^3$

Higgs mass: $\Lambda_{Higgs} \sim 10^{-16} M_p$

Expansion rate (e-folds)
of universe during inflation: $e^N \sim e^{60}$

Are there hidden, i.e. dark relations between these numbers ?

Can we explain or predict these numbers by quantum gravity?

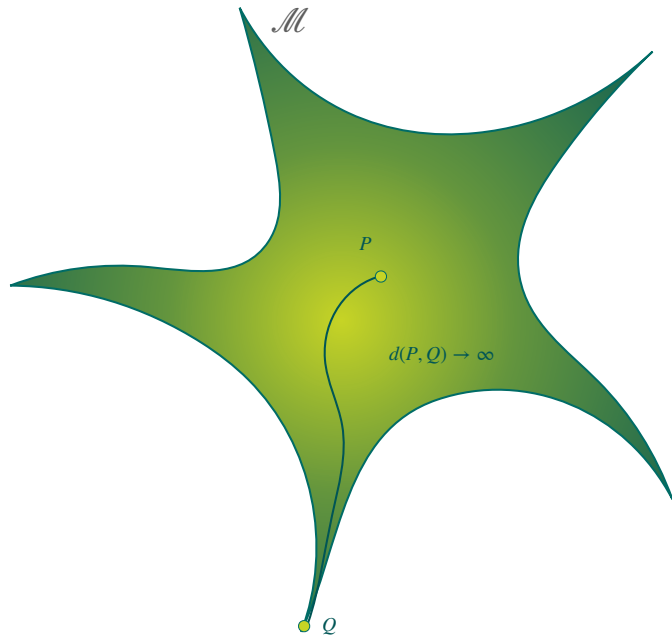
Outline

- I) Swampland Distance Relations
- II) Dark universe
- III) Dark Matter
- IV) Dark Inflation
- V) Summary

I) Swampland Distance Conjectures

Infinite distance conjecture:

[H. Ooguri, C.Vafa (2006)]



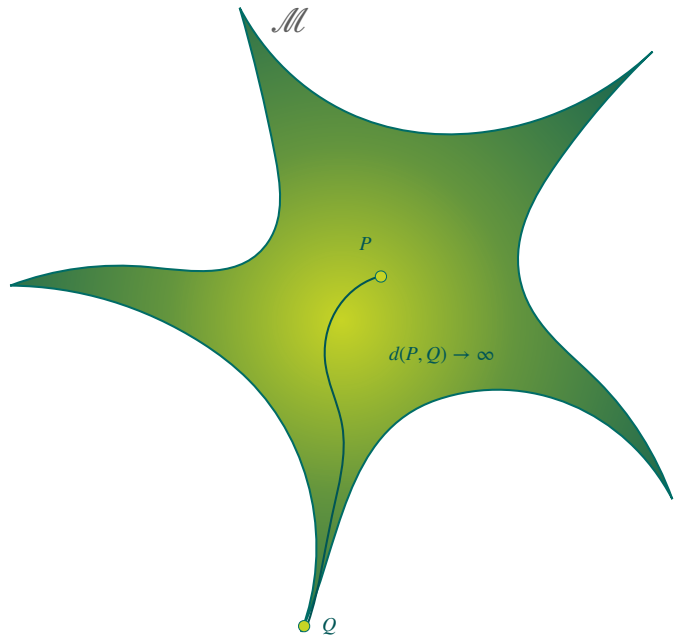
At the boundary of the moduli space,
there is tower of light states:

$$m(Q) \sim m(P) e^{-\lambda d(P, Q)}$$

I) Swampland Distance Conjectures

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At the boundary of the moduli space, there is tower of light states:

$$m(Q) \sim m(P) e^{-\lambda d(P, Q)}$$

Emergent string conjecture:

[S. Lee, W. Lerche, T. Weigand (2019)]

The light tower of states, **i.e. species**, at large distances are given by either

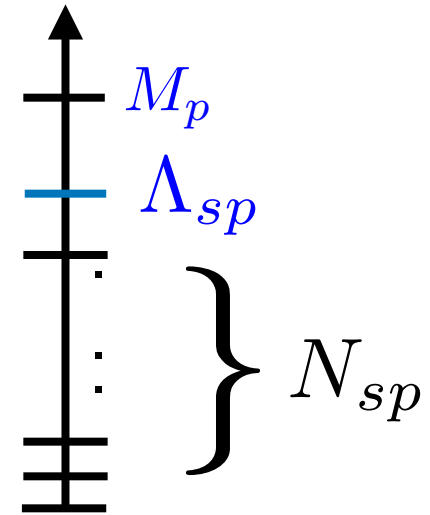
- KK particles (decompactification): $d(R_i, R_f) = |\ln(R_f/R_i)| \rightarrow \infty$
- String oscillators (tensionless string): $d(g_{s,i}, g_{s,f}) = |\ln(g_{s,f}/g_{s,i})| \rightarrow \infty$

Species scale: UV cutoff of EFT:

Discussed recently by many authors: L. Anchordoqui, I. Antoniadis, I. Basile, R. Blumenhagen, J. Calderon-Infante, A. Castellano, N. Cribiori, M. Delgado, S. Demulder, A. Gligovic, D. van de Heisteeg, A. Herraez, L. Ibanez, D.L. S. Lüster, J. Masias, M. Montero, C. Montella, A. Paraskevopoulou, T. Raml, I. Ruiz, M. Scalisi, G. Staudt, A. Uranga, C. Vafa, I. Valenzuela, M. Wiesner, D. Wu, ..

$$\Lambda_{sp} \simeq \frac{M_p}{(N_{sp})^{\frac{1}{d-2}}} \quad [\text{G. Dvali (2007)}]$$

N_{sp} : Number of particles below Λ_{sp} .



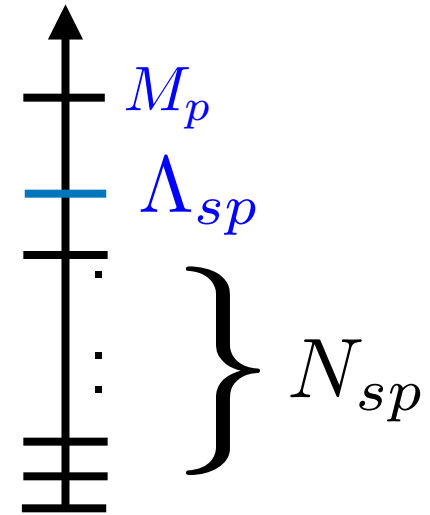
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N_{sp} : Number of particles below Λ_{sp} .

At the boundary of the moduli space: $\Lambda_{sp} \rightarrow 0$



Species scale distance:

$$d(\Lambda_{sp,i}, \Lambda_{sp,f}) = |\ln(\Lambda_{sp,f}/\Lambda_{sp,i})| \rightarrow \infty$$

In the limit of a small species scale there is light tower of states

$$m \sim (\Lambda_{sp})^\alpha \quad \text{with} \quad \alpha > 0$$

KK species: compactification on a p-dimensional compact space

$$N_{\text{sp}} = m_{\text{KK}}^{-\frac{p(d-2)}{p+d-2}} \quad \Lambda_{\text{sp}} = m_{\text{KK}}^{\frac{p}{p+d-2}} M_{\text{pl},d} = M_{\text{pl},d+p}$$

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Consider the following
scale factor:

$$e^N \sim \frac{\Lambda_{\text{sp}}}{m_{\text{KK}}} = m_{\text{KK}}^{\frac{2-d}{p+d-2}} \rightarrow \infty$$

Corresponding distance:

$$d(N_i, N_f) = N_f - N_i \rightarrow \infty$$

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String species: string excitations

$$N_{\text{sp}} = g_s^{-2}, \quad \Lambda_{\text{sp}} = M_s = g_s^{\frac{2}{d-2}} M_{\text{pl},d}$$

[G. Dvali, D.L. (2009);
G. Dvali, C. Gomez (2010)]

Consider AdS_d vacua in quantum gravity with varying **negative** cosmological constant Λ_{cc} .

AdS Distance conjecture (ADC):

[D.L., E. Palti, C.Vafa (2019)]

In the limit of small Λ there exist an infinite tower of states with mass scale m , which behaves as

ADC:

$$m \sim \Lambda^\alpha \quad \text{with} \quad \alpha > 0$$

AdS distance: $d(\Lambda_i, \Lambda_f) = |\ln(\Lambda_f/\Lambda_i)| \rightarrow \infty$

[See also: Y. Li, E. Palti, N. Petri (2023); E. Palti, N. Petri (2024)]

Gradient flow distance conjectures:

[A. Kehagias, S. Lüst, D.L. (2019)]

Consider the (Ricci) flow between manifolds of (constant) curvature \mathcal{R}

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Including the dilaton and a potential:

[A. Kehagias, S. Lüst, D.L. (2019)]

$$d(\mathcal{W}_i, \mathcal{W}_f) = |\ln(\mathcal{W}_f/\mathcal{W}_i)| \rightarrow \infty$$

$$m \sim \mathcal{W}^\alpha$$

Perelman entropy functional:

$$\mathcal{W}(g, \phi) = \int d^d \sqrt{-g} \left(\mathcal{R} + (\nabla \phi)^2 + \phi - d \right) e^{-\phi}$$

[For other discussions of distances with potential see also S. Demulder, T. Raml, D.L. (2023), arXiv:2312.07674;
A. Mohseni, M. Montero, C. Vafa, I. Valenzuela, talk at Swamplandia conference, May 2024]

Black hole entropy distance conjecture:

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019);
N. Cribiori, M. Dierigl, A. Gnechi, M. Scalisi, D.L. (2022)]

$$d(S_{\text{BH},i}, S_{\text{BH},f}) = |\ln(S_{\text{BH},f}/S_{\text{BH},i})| \rightarrow \infty$$

Tower of light states in the limit of large black hole entropy:

$$m \sim (S_{\text{BH}})^{-\alpha} \quad \text{with} \quad \alpha > 0$$

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Minimal black hole:

$$L_{\text{sp}} = \Lambda_{\text{sp}}^{-1} \equiv R_{\text{BH},\text{min}}$$

$$\Lambda_{\text{sp}} = \frac{M_p}{(S_{\text{BH},\text{min}})^{\frac{1}{d-2}}}$$

Minimal BHs have led to the notion of **species thermodynamics**. With that one can argue from the bottom-up that the associated light tower must be either a KK or a string tower.

[N. Cribiori, D.L., C. Montella (2023); I. Basile, D.L. C. Montella (2023); I. Basile, N. Cribiori, D.L. C. Montella (2024);
A. Herraez, D.L., J. Masias, M. Scalisi, to appear. See also: A. Bedroya, R. Mishra, M. Wiesner, arXiv:2405.00083]

II) The Dark Universe

Consider (meta-stable) vacua with **positive** cosmological constant and **assume that the ADC is still valid** :

Cosmological constant distance conjecture:

The limit of small **positive** cosmological constant leads to a light tower of states with mass scale m :

[E. Palti, C.Vafa, D.L. (2019);
M. Montero, C.Vafa, I.Valenzuela (2022)]

$$m \sim \lambda^{-1} \Lambda^\alpha \sim \lambda^{-1} 10^{-120\alpha}$$

with $\frac{1}{4} \leq \alpha \leq \frac{1}{2}$

Casimir energy of effective potential

Higuchi bound

[For further (controversial) discussion on this relation see C. Burgess, F. Quevdo, arXiv:2304.03902;
C. Branchina, V. Branchina, F. Contino, A. Pernace, arXiv:2308.16548, arXiv:2404.10068;
L. Anchordoqui, I. Antoniadis, S. Lüst, D.L., arXiv:2309.09330]

Collider experiments:

$$(M_s \geq \mathcal{O}(\text{TeV}))$$

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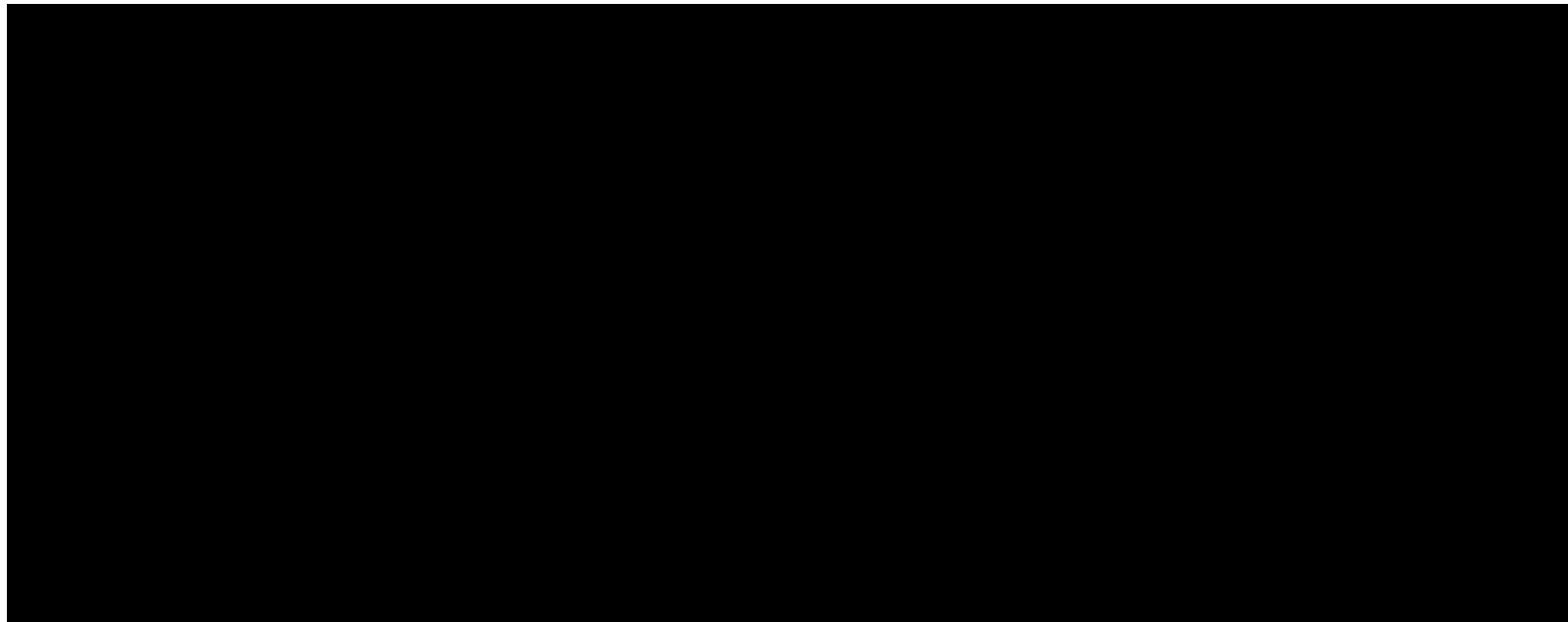
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[M. Montero, C. Vafa, I. Valenzuela (2022)]



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Dark Relations:

[M. Montero, C. Vafa, I. Valenzuela (2022)]

$$m_{\text{KK}} \simeq \Lambda^{1/4} \sim 2.31 \text{ meV}$$

$$R \simeq \Lambda^{-1/4} \sim 1 \mu\text{m}$$

$$\Lambda_{sp} \simeq \Lambda^{1/12} \sim 10^9 \text{ GeV}$$

This dark prediction can be tested by measurements of Newtonian force at micrometer scale.

[ISLE collaboration, Adelberger et al.]

Supersymmetry breaking: Gravitino mass conjecture (GMC):

[N. Cribiori, M. Scalisi, D.L. ; A. Castellano, A. Font, A. Harraez, L. Ibanez (2021)]

In the limit of small gravitino mass there exist an infinite tower of states with mass scale m , which behaves as:

$$m \sim (m_{3/2})^\beta \quad \text{with} \quad \beta > 0$$

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Assume that the towers related to the cosmological constant and the gravitino are the same:

[L. Anchordoqui, I. Antoniadis, N. Cribiori, M. Scalisi, D.L. (2023)]

$$\Lambda = (m_{3/2})^k, \quad k = \frac{\beta}{\alpha}$$

IIA and heterotic Scherk-Schwarz compactifications:

[I. Antoniadis, C. Bachas, D. Lewellen, T. Tomaras (1988); I. Antoniadis, C. Kounnas (1991)]

$$\beta = 1 \quad \text{i.e.} \quad k = 4$$

SUSY breaking scale:

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SUSY breaking scale: $M_{\text{SUSY}} \simeq (m_{3/2})^{1/2} \simeq \Lambda^{1/8} \sim 10 - 100 \text{ TeV}$

Top down string realisations of dark dimension:

- F-theory [M. Montero, C. Vafa, I. Valenzuela (2022)]
- IIB: CY with a long warped throat [R. Blumenhagen, M. Brinkmann, A. Makridou (2022)]
- **M-theory: Manifold with two boundaries:** [J. Schwarz; L. Anchordoqui, I. Antoniadis, D.L. (2024)]

$$M_6 \times S_1/Z_2 : \quad M_{11} = \kappa^{-2/9}, \quad \rho = V_6^{1/6} \quad \text{and} \quad R$$

The SM is realised at one end of the interval, with SM gauge coupling α :

$$\kappa = (2\alpha)^{3/4} \rho^{9/2}, \quad R = (\alpha/2)^{3/2} M_p^2 \rho^3$$

Requiring $R \sim 1 \mu\text{m}$ and $\alpha \equiv g^2/(4\pi) \sim 1/25$

One gets $M_{11} \sim 10^9 \text{GeV}$ $\rho^{-1} \sim 7 \times 10^8 \text{GeV}$

III) Dark matter and primordial black holes (PBHs)

[L.Anchordoqui, I.Antoniadis, D.L., arXiv:2206.07071, arXiv:2210.02475, arXiv:2401.09087, arXiv:2403.19604]

Initial mass of PBHs when produced in the early universe:

$$M_{\text{BH}} \sim \frac{c^3 t}{G} \sim 10^{15} \left(\frac{t}{10^{-23} \text{ s}} \right) \text{ g}$$

A priori there is an enormous range of possible masses:

$$t = t_{\text{Planck}} \sim 10^{-43} \text{ s} \quad \rightarrow \quad M_{\text{BH}} = M_p \sim 10^{-5} \text{ g}$$

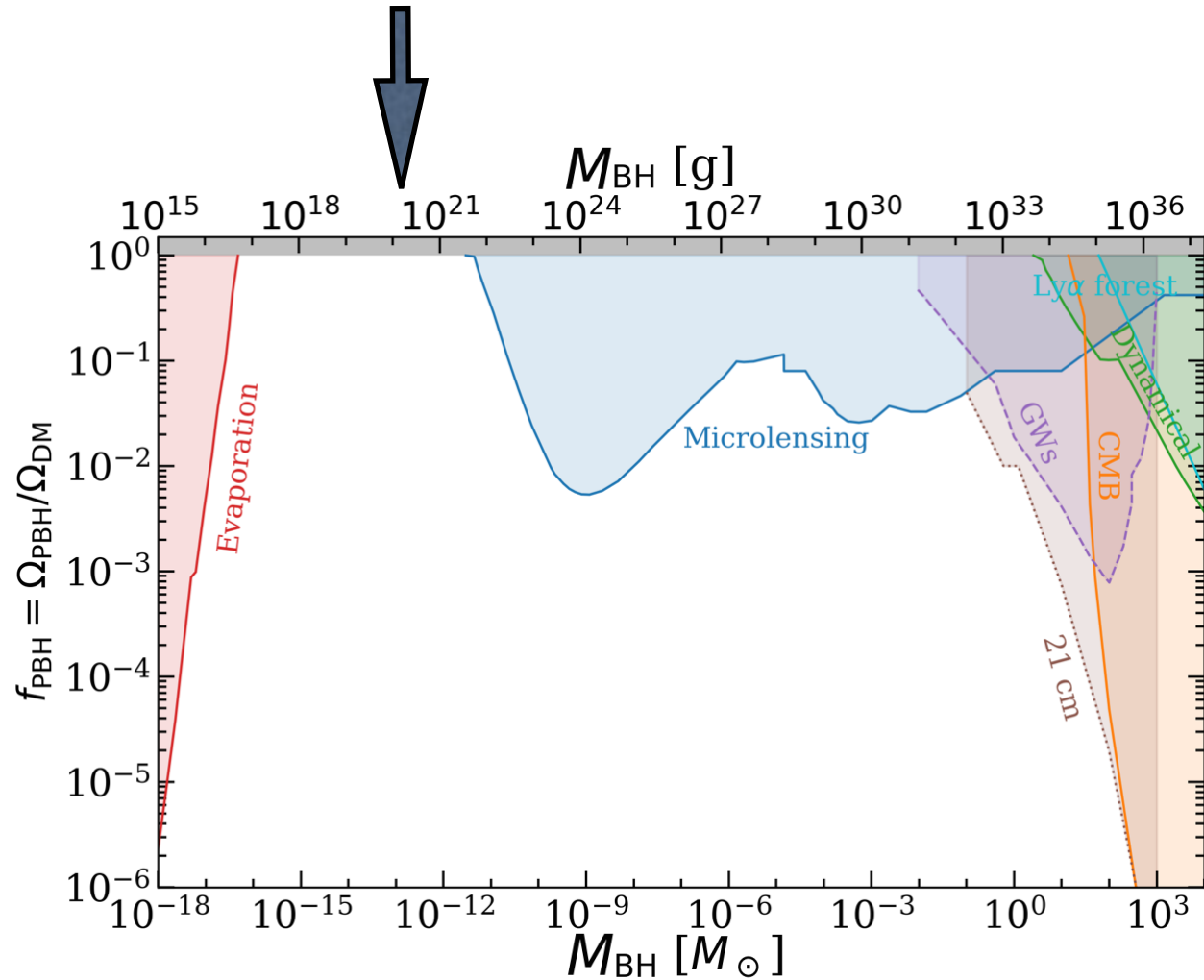
$$t = t_{\text{QCD}} \sim 10^{-5} \text{ s} \quad \rightarrow \quad M_{\text{BH}} \sim 1 M_{\odot}$$

Strong observational constraints for PBHs to be all dark matter candidates:

Upper limit on PBH mass: micro-lensing - wave length of visible light

Lower limit on PBH mass: Life time must be as long as the age of universe

Experimental status of 4D PBHs as dark matter:



P. Villanueva-Domingo, O. Mena, S. Palomares-Ruiz (2021)

However the window might become narrower or even close in the future. So we need mechanisms to slow down the decay process of PBHs.

Slow down processes:

- Near extremal black holes

[J. De Freitas Pacheco, E. Kiritsis, M. Lucca, J. Silk, arXiv:2301.13215; L. Anchordoqui, I. Antoniadis, D.L., arXiv:2401.09087]

- Memory burden due to quantum effects

[A. Alexandre, G. Dvali, E. Koutsangelas, arXiv:2402.14069; V. Thoss, A. Burkert, K. Kohri, arXiv:2402.17823; G. Dvali, M. Zantedeschi, arXiv:2405.13117; L. Anchordoqui, I. Antoniadis, D.L., arXiv:2403.19604]

- Higher dimensional PBHs in dark dimensions

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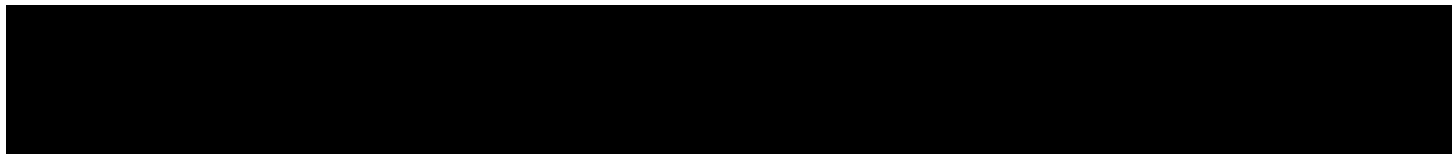
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Therefore the BPHs in the allowed window have Schwarzschild radius of the order or smaller than the size of the dark dimension.



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$$M_{PBH} \lesssim \Lambda^{-1/4} \sim 10^{21} \text{ g}$$

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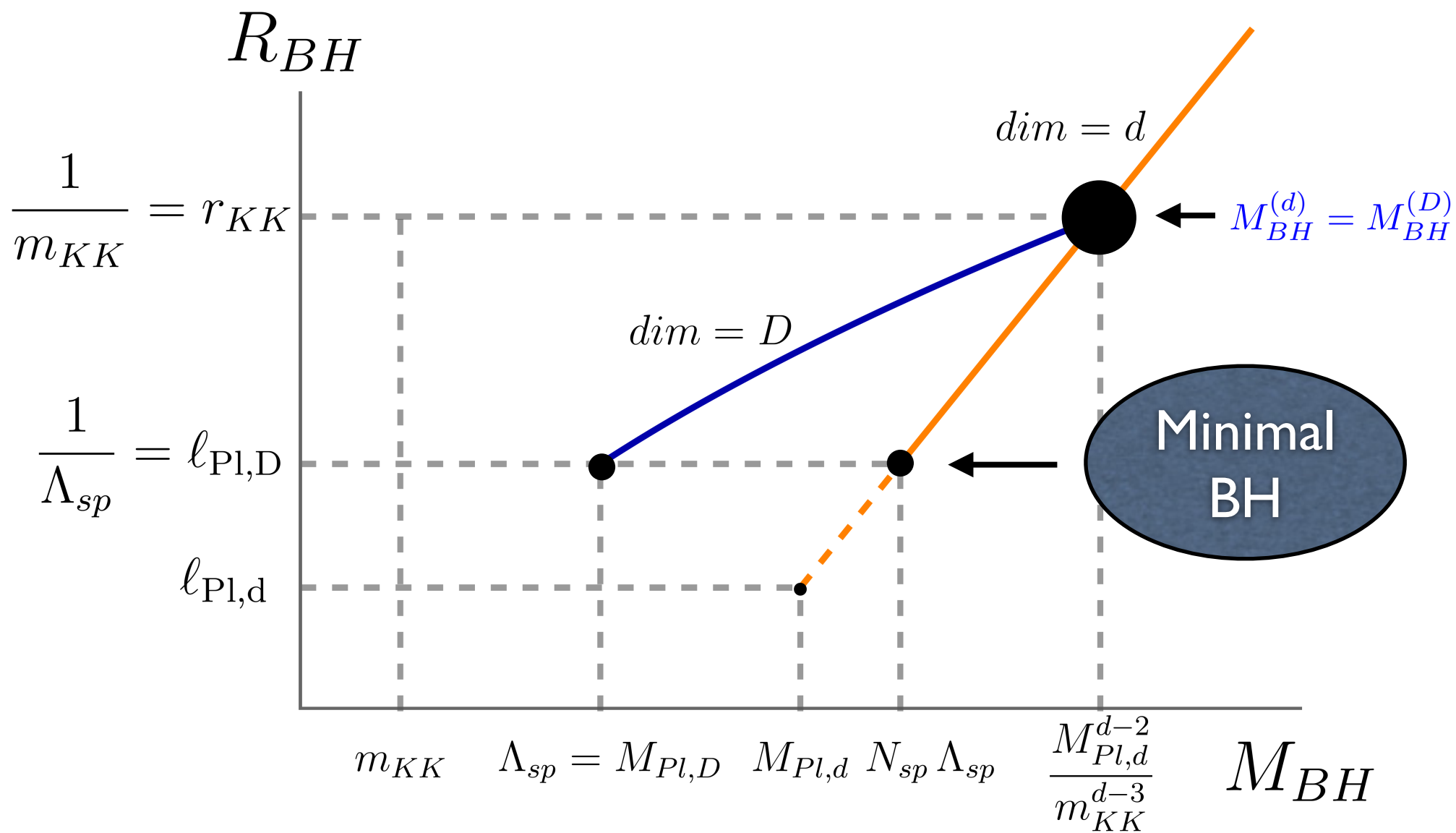
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They are described by BH solutions in 5 dimensions.

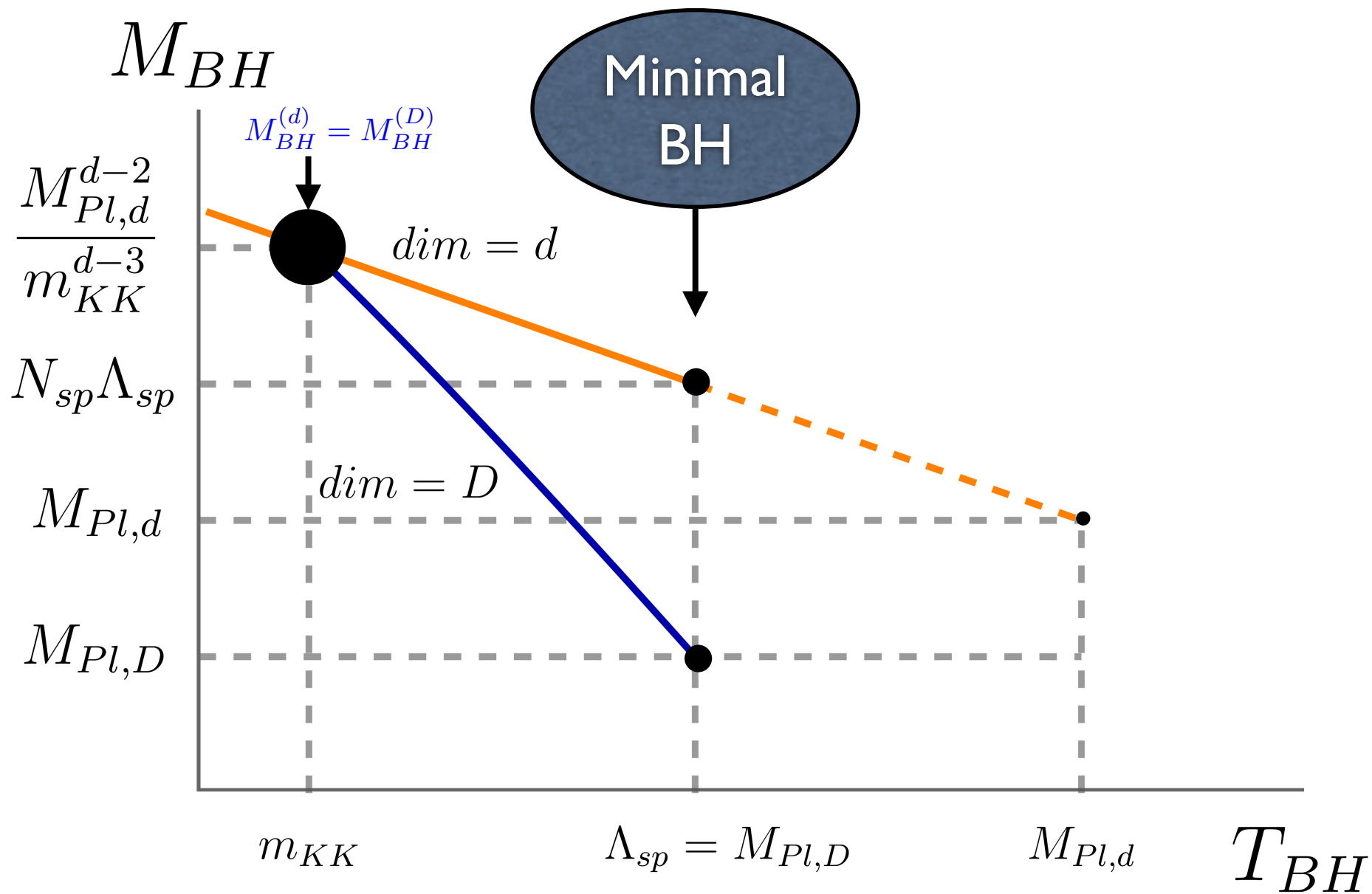
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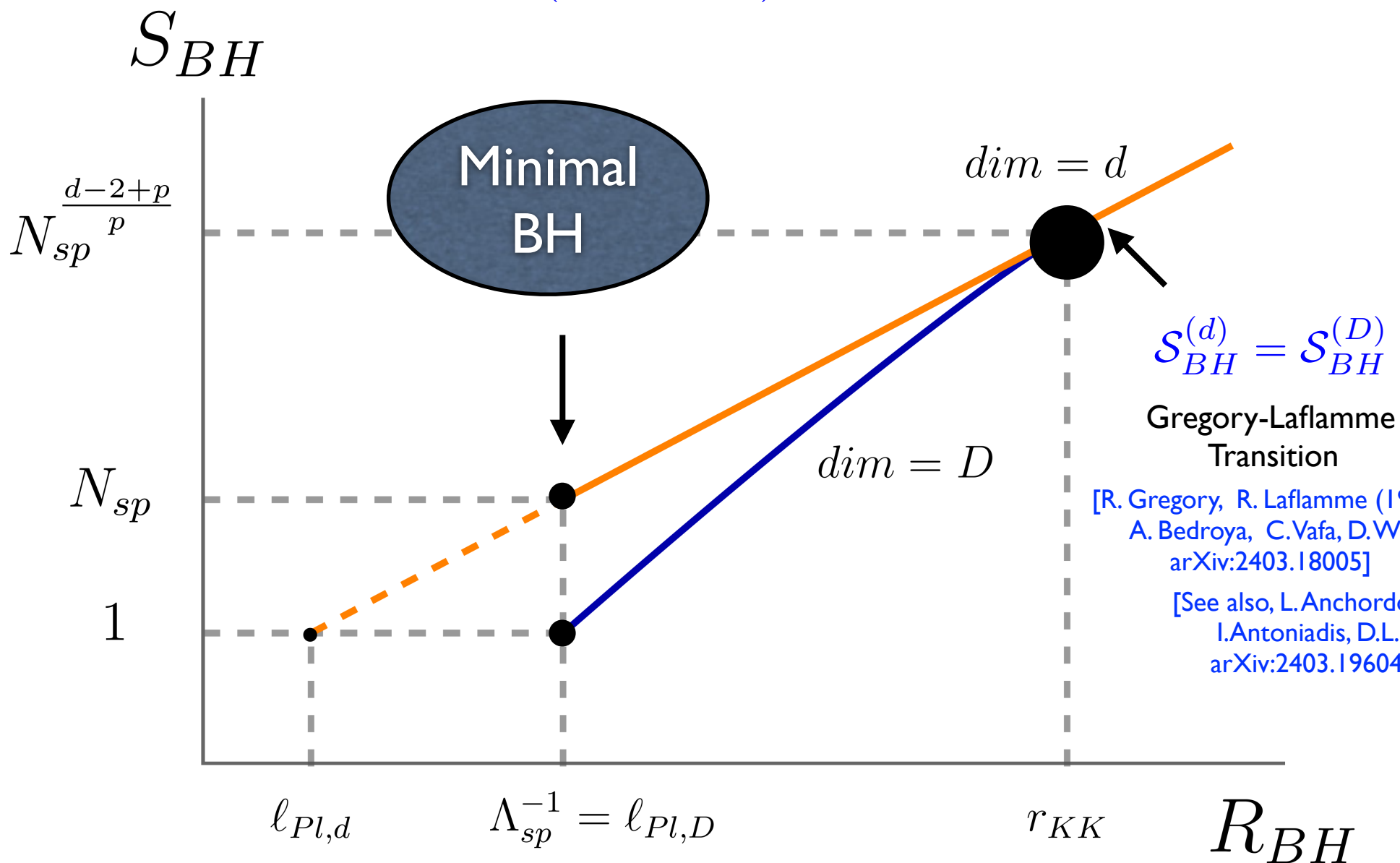
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$$(D = d + p)$$



For a given mass, the 5D PBHs have larger Schwarzschild radius and a smaller temperature than their 4D counterparts.

Hence they have a longer life time.

Some 5D dark dimension relations:

Schwarzschild radius: $r_s(M_{\text{BH}}) \sim \frac{1}{M_*} \left[\frac{2}{3\pi} \frac{M_{\text{BH}}}{M_*} \right]^{1/2}$

Hawking temperature: $T_H \sim \frac{1}{r_s} \sim \left(\frac{M_{\text{BH}}}{10^{12} \text{ g}} \right)^{-1/2} \text{ MeV}$

Life time: $\tau_s \sim 13.8 \left(\frac{M_{\text{BH}}}{10^{12} \text{ g}} \right)^2 \left(\frac{6}{\sum_i c_i(T_s) \tilde{f} \Gamma_s} \right) \text{ Gyr}$

Allowed window: $10^{15} \lesssim M_{\text{BH}}/\text{g} \lesssim 10^{21}$

Near extramal PBHs:

Entropy suppression:

$$T_{ne} \sim \frac{\beta^{1/2} T_H}{S_{\text{BH}}^{1/2}}$$

Extended window

$$10^5 \sqrt{\beta} \lesssim M_{\text{BH}}/g \lesssim 10^{21}$$

[L.Anchordoqui, I.Antoniadis, D.L., arXiv:2401.09087]

Memory burden can provide further entropy suppression.

III) Cosmology: Dark Inflation and Hubble tension

[L. Anchordoqui, I. Antoniadis, D.L., arXiv:2212.08527]

Dark dimension: Higuchi bound would imply a very low scale for inflation.

Way out: Inflation takes place in the 5 dimensions.

Radius inflates from species lengths to the size of the dark dimension.

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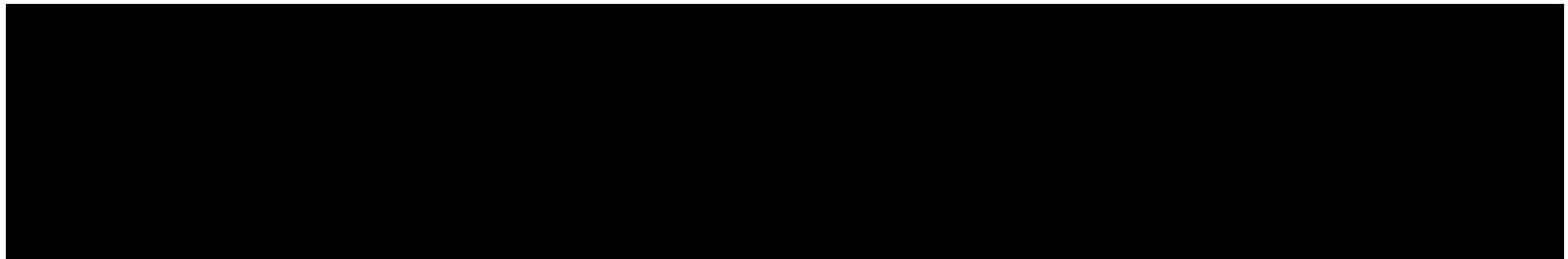
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In terms of 4d metric: $N_4 = 3/2 N_5 \sim 62$

This is just correct for solving the horizon problem.

The 5 d inflation produces a power spectrum consistent with CMB observations:

[L.Anchordoqui, I.Antoniadis, arXiv:2310.20282]

$$r = 24\epsilon_V \quad \epsilon_V < 0.0013 \quad r < 0.032$$

Cosmic discrepancies, Hubble tension:

[L.Anchordoqui, I.Antoniadis, D.L., arXiv:2312.12352; L.Anchordoqui, I.Antoniadis, N. Noble, J. Soriano, D.L., arXiv:2404.173334]

5 sigma tension between

$$H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc}$$

Planck data

$$H_0 = 73.04 \pm 1.04 \text{ km/s/Mpc}$$

SH0ES supernova

This tension can be resolved if Lambda changes sign at redshift $z=2$.

[O.Akarsu, E.Valentino, S. Kumar, R. Nunez, J.Vazquez, A.Yadav, arXiv:2307.10899]

The AdS to dS transition seems hard to be implemented, since due to ADC such a transition is at infinite distance in moduli space.

However it could happen due to finite temperatures or due to quantum tunnelling effects.

Toy model for potential
with AdS minimum:

$$V_{AdS}(R) = \frac{1}{R_0^6} (R - R_0)^2 - \frac{1}{R_0^4}$$

Satisfies scale separation with $\alpha = 1/4$.

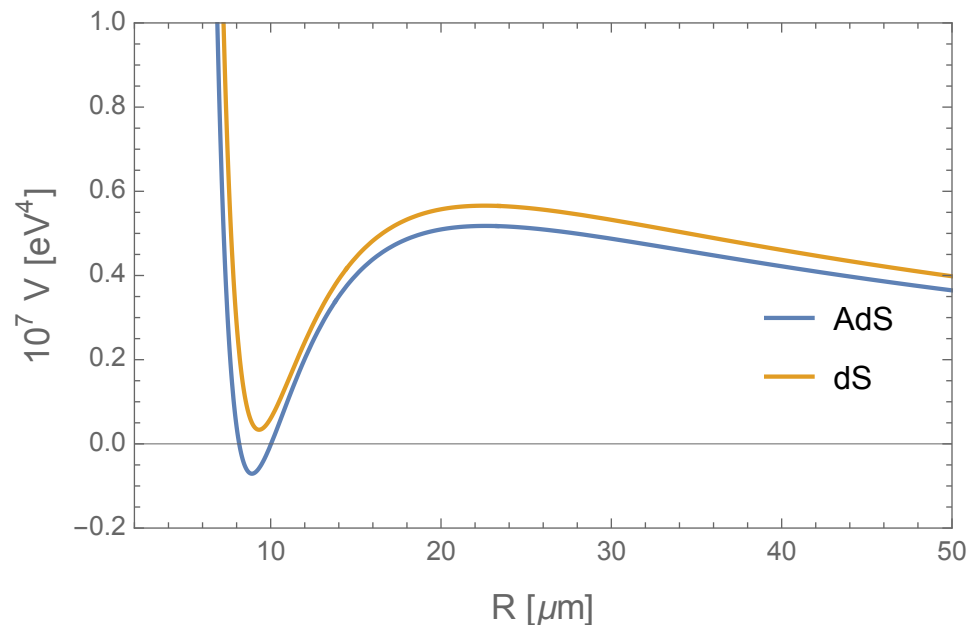
Uplift by Casimir energy:

$$V_C(R) = \frac{c(t)}{R^4}$$

$c(t)$ can change during the expansion of the universe.

Total potential:

$$V(R, a) = V_{AdS}(R) + V_C(R)$$



Return to 5D cosmological model:

$$V(R) = \frac{2\pi \Lambda_5 r^2}{R} + \left(\frac{r}{R}\right)^2 T_4 + V_C(R)$$

Casimir energy from 5d bulk particles: $(T_4 < 0)$

$$V_C(R) \simeq \sum_i \frac{2\pi r^2}{32\pi^7 R^6} (N_F - N_B)$$

AdS to dS transition due to false vacuum decay in 5 dimensions:

$$N_F - N_B = 6 \quad \text{for} \quad z > z_c \quad \Longrightarrow \quad \text{AdS}$$

$$N_F - N_B = 7 \quad \text{for} \quad z < z_c \quad \Longrightarrow \quad \text{dS}$$

There is also an alternative model depending on the decay of particles (5d vector bosons) at a certain temperature.

During the transition **the radius** of the dark dimension and the **species entropy** increase.

IV) Concluding remarks

Swampland conjectures applied to the dark dimension scenario provide new dark relations among low energy observables.

Left-out topics:

- Neutrino masses

[E. Gonzalo, L. Ibanez, I. Valenzuela (2021);
L. Anchordoqui, I. Antoniadis, J. Cunniff (2023)]

- Kaluza Klein dark matter

[E. Gonzalo, M. Montero, G. Obied, C. Vafa (2022);
G. Obied, C. Dvorkin, E. Gonzalo, C. Vafa (2023)]

- Fuzzy dark matter

[L. Anchordoqui, I. Antoniadis, D.L. (2023)]

- Axions

[N. Gendler, C. Vafa (2024)]

- Neutrino-modulino oscillations

[L. Anchordoqui, I. Antoniadis, K. Benakli, J. Cunniff, D.L. (2023)]

Thank you !